16.61 Spring 2006
Lecture Presentation Thu 23-Mar-06 ver 2.0

Charles P. Coleman

March 23, 2006

TODAY

- Single Link Robotic Manipulator
- Block Sliding on a Wedge
- Atwood's machine

Single-Link Manipulator with Friction

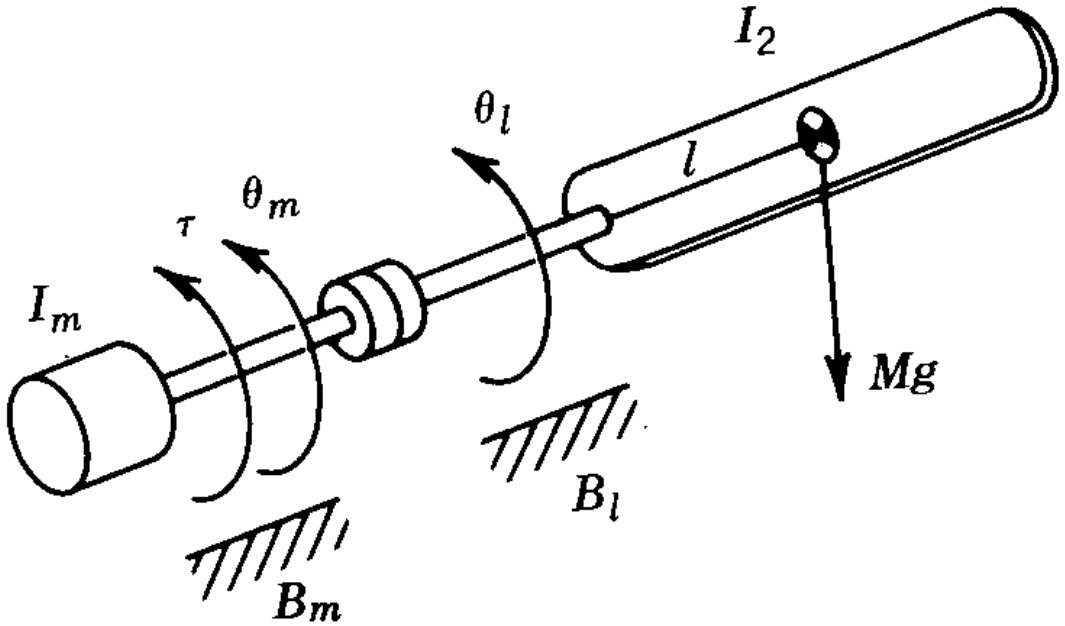
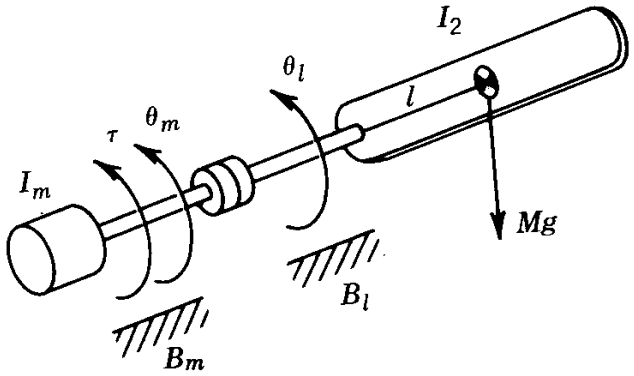


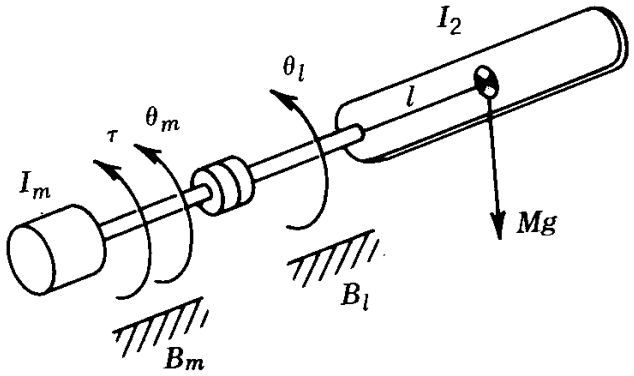
Figure 1: Single-Link Manipulator

Single-Link Manipulator



θ_l: angle of link

Single-Link Manipulator



θ_l: angle of link

θ_m: angle of motor shaft

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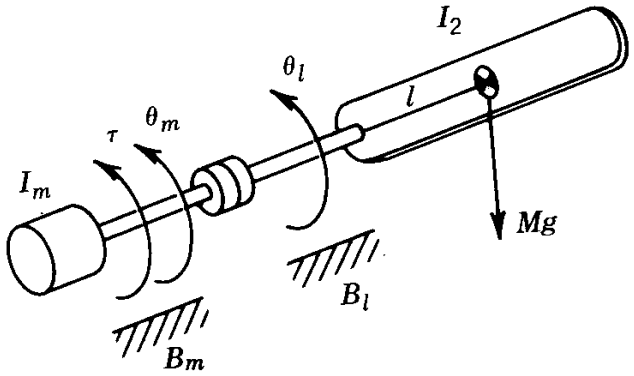

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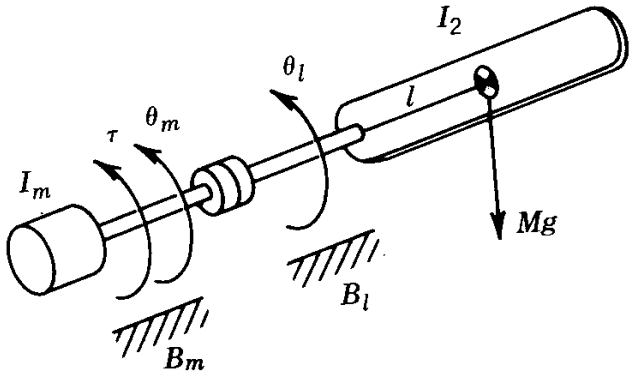
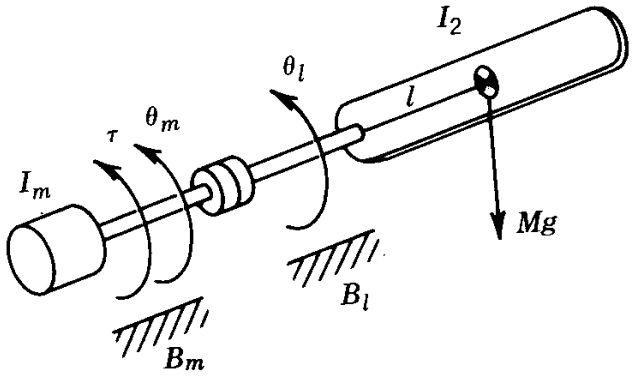


θ_l: angle of link
θ_m: angle of motor shaft
$n:1$: gear ratio
$\theta_l = 1/n\,\theta_m$

Single-Link Manipulator



I_m and I_2 are the rotational intertias of the motor and link.

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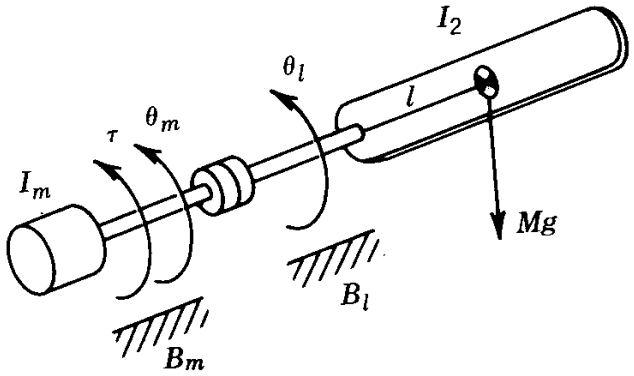

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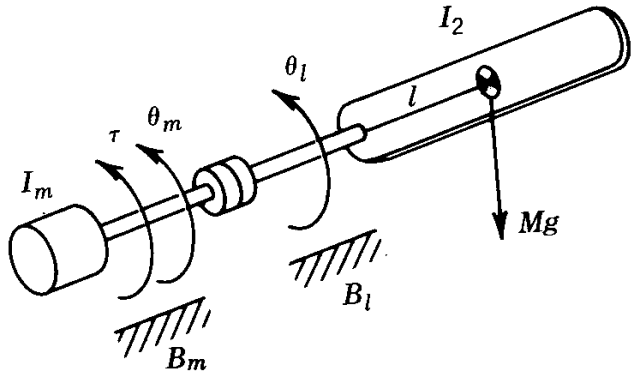
Kinetic Energy

Single-Link Manipulator



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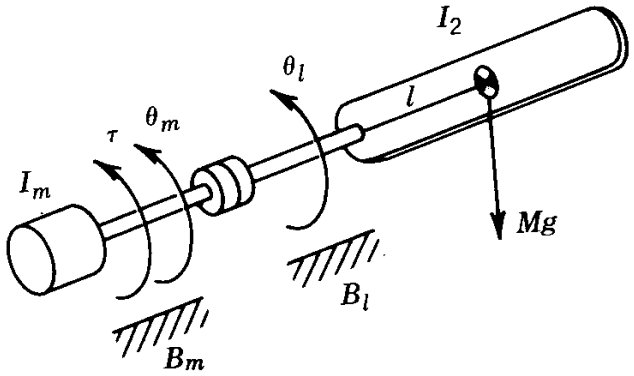
Kinetic Energy

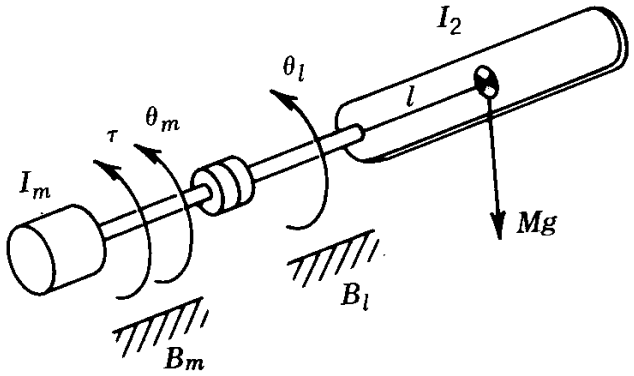
$$T = 1/2 I_m \dot{\theta_m}^2 + 1/2 I_2 \dot{\theta_l}^2$$
$$= 1/2 (I_m + I_2/n^2) \dot{\theta_m}^2$$

Single-Link Manipulator



M is the total mass of the link and l is the distance from the joint axis to the link center of mass.

Single-Link Manipulator



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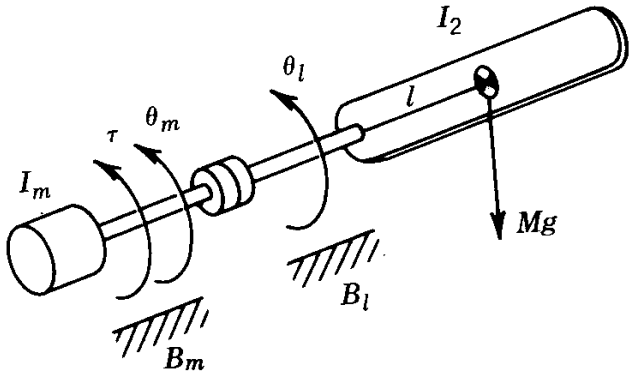
Potential Energy

Single-Link Manipulator



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Potential Energy

$$\begin{aligned} V &= Mgl(1 - \cos\theta_l) \\ &= Mgl(1 - \cos\theta_m/n) \end{aligned}$$

Single-Link Manipulator

The Langrangian is then

$$\begin{aligned} L &= T - V \\ &= 1/2(I_m + I_2/n^2)\dot{\theta_m}^2 - Mgl(1 - \cos\theta_m/n) \end{aligned}$$

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Langrange's Equation is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = \tau_j$$

Single-Link Manipulator

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$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = \tau_j$$

Which leads to

$$(I_m + I_2/n^2)\ddot{\theta_m} + \frac{Mgl}{n}\sin\theta_m/n = \tau_j$$

Single-Link Manipulator

Generalized Forces:

Single-Link Manipulator

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The generalized force τ consists of the motor torque u, and the (nonconservative) link-motor shaft damping torques: $B_m\dot{\theta}_m$ and $B_l\dot{\theta}_l$.

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Complete Dynamics:

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- In robotics and multibody applications the resulting system of equations will have the following form:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

Single-Link Manipulator

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D will be called the mass or inertial matrix, C will contain centrifugal and Coriolis terms, and the g terms arise from the conservative forces.

Block Sliding Down a Ramp

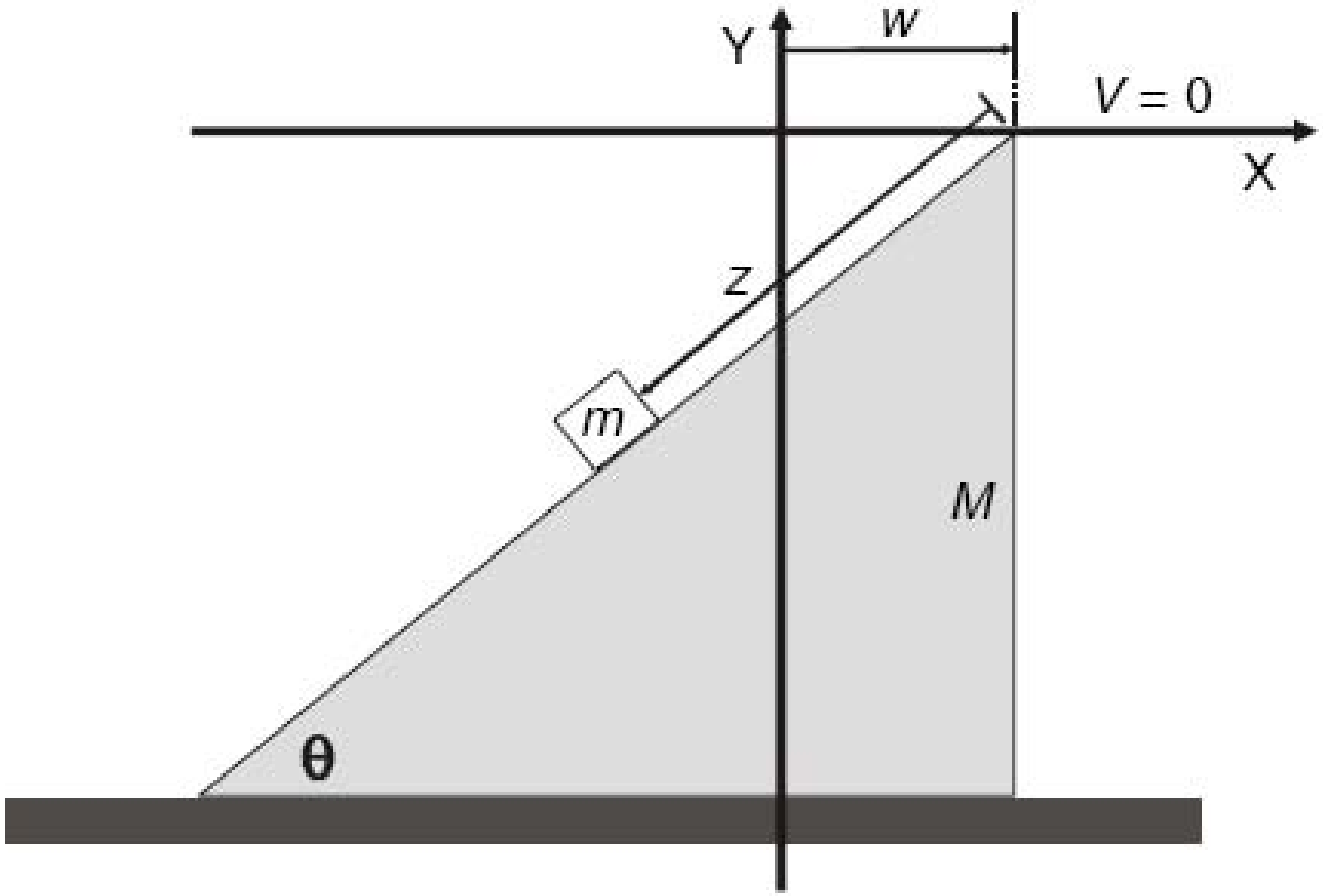

Figure 2: Sliding Block

Atwood's Machine

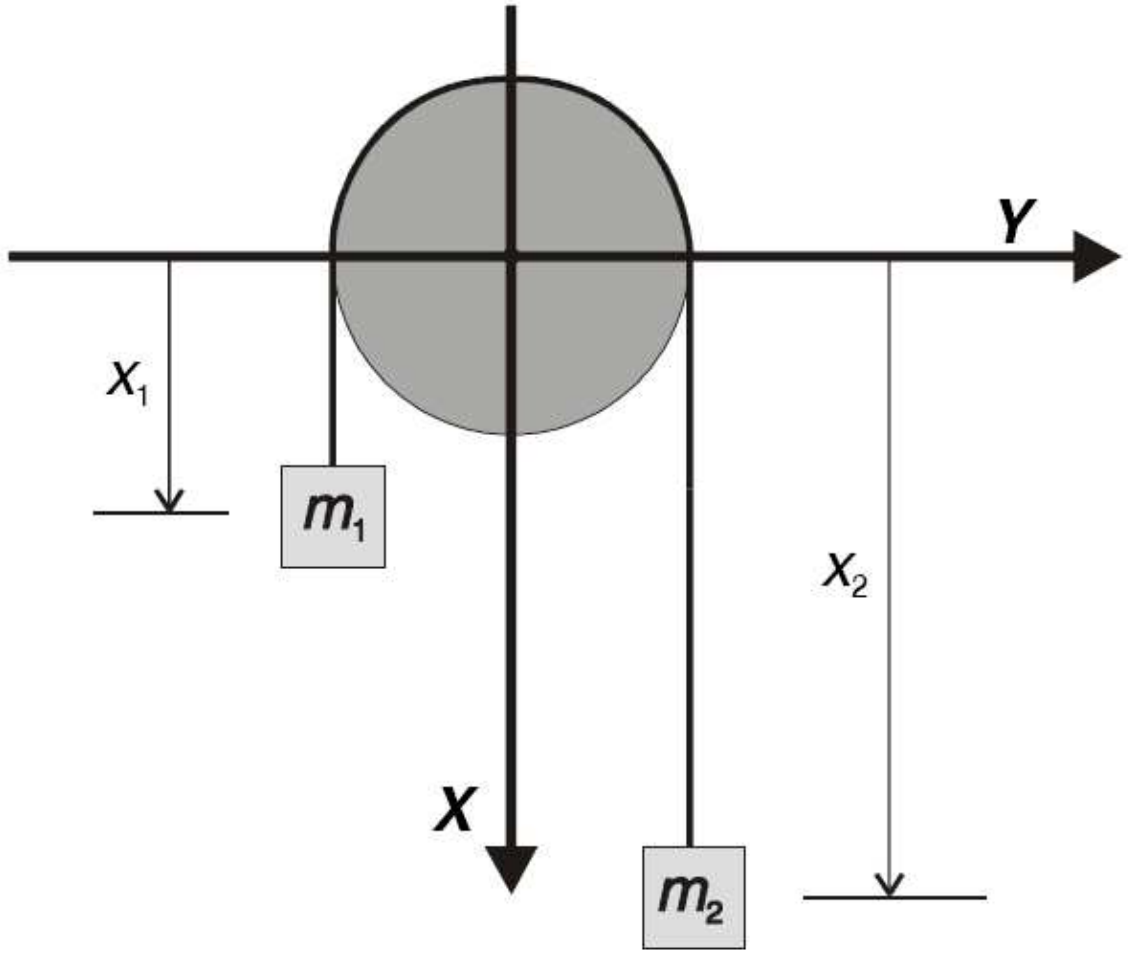

Figure 3: Atwood's Machine