Precision Machine Design

Topic 19

Power transmission system requirements

Purpose:

In order to specify actuators for a system, one has to have a first-order method for determining the required system stiffness and transmission ratio. This lecture presents methods for making estimates of these required parameters. Once the system is designed, a more detailed dynamic analysis can then be done.

Outline:

- **Overall design approach**
- **Minimum required stiffness and servoloop time**
- **Optimal transmission ratio selection for lightly loaded systems**
- **Optimal transmission ratio selection for heavily loaded systems**
- **Checking system servo bandwidth**

"The management of a balance of power is a permanent undertaking, not an exertion that has a foreseeable end."

Henry Kissinger

Overall design approach

- **Method needed to provide 1st order estimate of system stiffness requirements.**
- **Allows for initial sizing of transmission system elements (e.g., ballscrew diameter):**
  - **This enables the designer to select the optimal transmission ratio.**
- **After the transmission ratio is selected, an estimate of servo bandwidth can be made.**
  - **This allows system damping to be selected, and iterations on mass, stiffness, and transmission ratio to be made.**
- **After the system is designed using these preliminary methods, a detailed controls simulation can be made.**
  - **Errors can then be accurately predicted as a result of:**
  - **Ideal trajectory moves.**
  - **Disturbance forces.**

Minimum required static stiffness and servoloop time

- **First an estimate of the time constant (in seconds/cycle) of the system is made:**

$$\tau_{mech} = 2\pi \sqrt{\frac{M}{K}}$$

- **The control system loop time τ_{loop} must be at least twice as fast to avoid aliasing (a factor of L = 10-20 is used in practice)**

- **Faster servo times create an averaging effect by the factor $(\tau_{mechanical}/2\tau_{loop})^{1/2}$.**

- **For a controller with N bits of digital to analog resolution, the incremental force input is:**

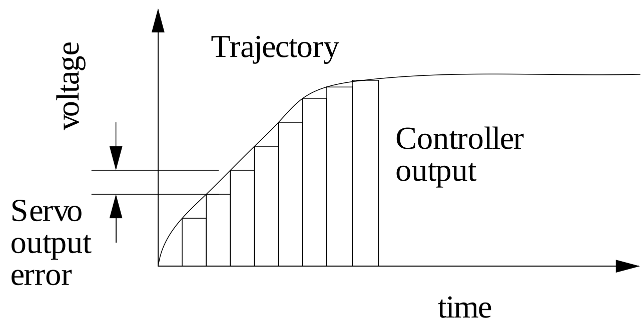
$$\Delta F = \frac{F_{max}}{2^N \sqrt{\frac{\tau_{mech}}{2\tau_{servo}}}}$$

- **The minimum axial stiffness is thus:**

$$K \geq \left\{ \frac{F_{max}\, \tau_{servo}^{1/2}}{2^N\, \pi^{1/2}\, M^{1/4}\, \delta_K} \right\}$$

- **Conservatively, assume no damping and a simple proportional control system.**
- **While the digital controller is calculating the next value to send to the DAC:**
  - **The power signal remains equal to the last value in the DAC.**
- **The error δ_M due to the mass being accelerated by the force resolution of the system for a time increment τ_{servo} is:**

$$\delta_M = \frac{1}{2}\left(\frac{\Delta F}{M}\right)\tau_{servo}^2$$



- **The maximum allowable servoloop time is:**

$$\tau_{servo} \le \sqrt{\frac{2\delta_M M}{\Delta F}}$$

- **The stiffness required to control the axis (not accounting for external force or nonlinear effects) is thus:**

$$K \ge \frac{F_{max}\,\delta_M^{1/4}}{2^{N-1/4}\,\pi^{1/2}\,\delta_K^{5/4}}$$

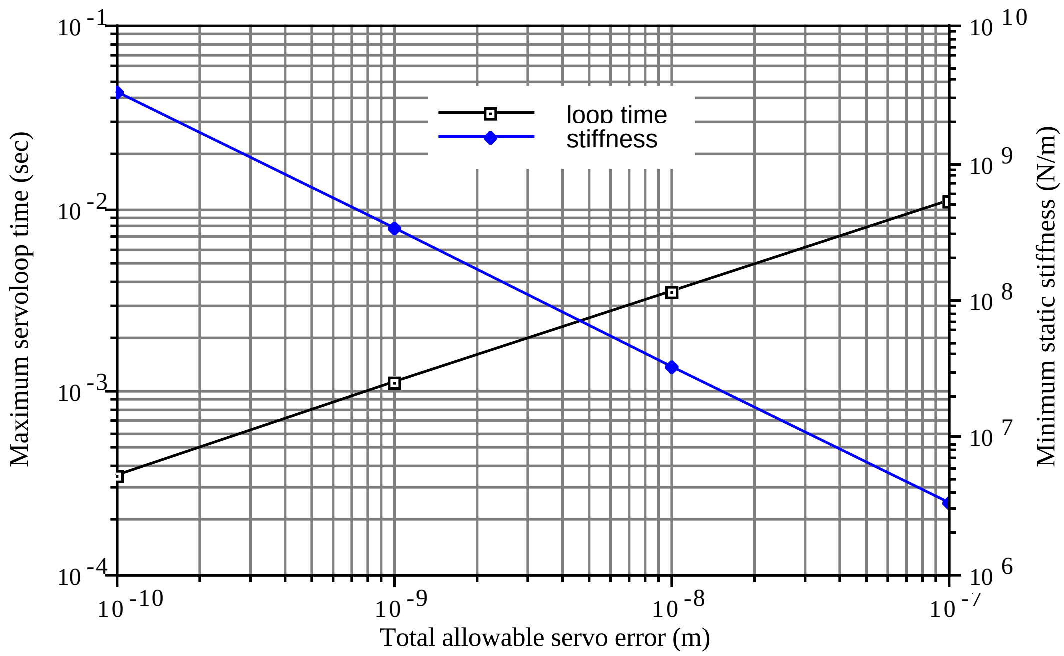
  - **This includes all the elements in the structural loop.**
- **Also determine the stiffness required so the error from applied forces will be below a desired maximum value.**
  - **Typically $\delta_K = \delta_M = 1/2\delta_{servo}$.**

- **The maximum allowable servo loop time is also found:**

$$\tau_{servo} \leq \sqrt{\frac{\pi^{1/2}\, 2^{(4N+3)/4}\, \delta_M^{3/4}\, M\, \delta_K^{1/4}}{F_{max}}}$$

- **Actual servoloop time should be at most τ_{servo}/L where L is the number of past values used by a recursive algorithm.**

- **Example: Required K_{static} and τ_{servo} for a machine with 1000 N F_{max}, 200 kg system mass, and 12 bit DAC:**



- **Assume a no-load resolution on the order of 10 nanometers (10^{-8} m) is desired**

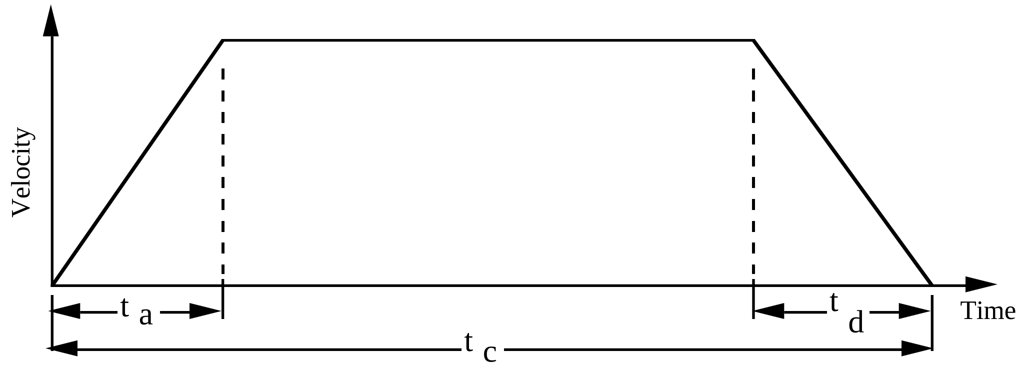
- **The actuator stiffness should be >20 N/μm (110,000 lbf/in) and the servoloop time should be <1.5 millisceconds (L = 2).**

- **These estimates are in agreement with practice.**

Optimal transmission ratio selection for lightly loaded systems

- **The mass (kg) or inertia (kg-m^2) of the load and drive train components must be known.**
    - **Ballscrew inertia comes from a knowledge of the stiffness (diameter and length).**
- **One must have an estimate for the approximate velocity profile as a function of time.**
- **It will be assumed that it is desired to move a specified load a specified distance in a given time.**
- **Parabolic profile is assumed to represent 100% efficiency**
- **Trapezoidal profile with equal acceleration, slew (constant speed), and deceleration times is 89% efficient.**
- **Triangular profile is 75% efficient.**
- **The triangular profile, although it requires more power-per-move, is the fastest way of moving a load a given distance.**
    - **To minimize impulse vibration effects, the trapazoid should actually be rounded at the corners.**

- **Trapezoidal velocity profile:**



- **Given the distance D that the load is required to move:**

$$D = \frac{a_{max}}{2}\left(\frac{t_c}{3}\right)^2 + \left(\frac{a_m t_c}{3}\right)\left(\frac{2t_c}{3}\right) + \frac{a_{max}}{2}\left(\frac{t_c}{3}\right)^2$$

- **The maximum acceleration and velocity during the move are:**

$$a_{max} = \frac{9D}{2t_c^2}$$, maximum acceleration (m/s^2 or rad/s^2)

$$v_{max} = \frac{3D}{2t_c}$$, maximum velocity (m/s or rad/s)

Power Rate

- **The next step is to determine the *power rates* for the motor and load.**
- **The unit of power rate is Watt/sec**
- **Power rate is a measure of the electrical to mechanical power conversion efficiency of a given actuator.**
- **The power rate combines both the power of the actuator and the mechanical time constant into a single figure-of-merit:**

$$P_{R\ motor} = \frac{\Gamma^2_{motor}}{J} = \Gamma_{motor}\,\alpha = \frac{I^2R}{t_a}$$

- **Γ^2_{motor}/J is a measure of how fast the motor can accelerate its own inertia and I^2R is the power into the motor.**
- **The maximum motor power is the power rate multiplied by the acceleration time:**

$$P_{motor} = PR\,\frac{t_a}{3}$$

- **The required power rate PR and motor power P_{motor} should be matched to the rest of the system.**
  - **These values can then be used to evaluate candidate motors.**
- **Since the load inertia and required acceleration are quantities specified by the machine configuration:**
  - **The load's power rate can be calculated by:**

$$P_{R\ load} = (M_{load}\, a + F_f)\, a$$

$$P_{R\ load} = (J_{load}\, \alpha + \Gamma_f)\, \alpha$$

- **The factors F_f and Γ_f are the constant friction force or friction torque opposing the motion of the load.**
- **The max. static and dynamic coefficients of friction under the max. load and speed conditions should be used.**
- **The friction effects are often of considerable, and surprising, magnitude so they must not be ignored.**
- **The power rate required can be scaled by the efficiency.**
  - **For example, if the efficiency of a ballscrew is 90%:**
    - **The power rate for the carriage driven by the ball screw should be multiplied by 1/0.9 = 1.11.**

- **The load power is thus simply:**

$$P_{load} = P_{R\ load}\ t_a$$

- **For systems dominated by inertial loads:**
  - **Maximum power is transferred from the motor to the load under what is called the *matched inertia doctrine*.**
  - **Proof is given in text's references.**
- **The power expended in causing motion of the motor's core and directly attached drive train components:**
  - **Should be the same as that expended in causing motion of the load.**
- **The motor should have a total power rating of at least twice the load power requirement to move itself and the load:**

$$P_{motor} \geq 2P_{load}$$

- **When the motor and load inertias are matched, the required motor power rate must be at least four times the load power rate:**

$$P_{R\ motor} \geq 4P_{R\ load}$$

- **With estimate of the machines mass from a preliminary layout (based on minimum required stiffness:**
  - **You can now do a preliminary motor sizing.**

- **One can search for a motor which has the correct PR_{motor} and P_{motor} characteristics.**

- **For low frictional and cutting forces, the matched inertia doctrine can be used to find the "optimal" transmission ratio.**

- **Assume the power leaving the motor arrives at the load, and power equals the product of torque and angular speed:**

$$\Gamma_{motor}\,\omega_{motor} = \Gamma_{load}\,\omega_{load}$$

- **Torque also equals the product of inertia and angular acceleration, thus:**

$$J_{motor}\,\alpha_{motor}\,\omega_{motor} = J_{load}\,\alpha_{load}\,\omega_{load}$$

- **Substituting for the angular velocity in terms of constant acceleration and time:**

$$J_{motor}\,\alpha_{motor}^{2} = J_{load}\,\alpha_{load}^{2}$$

- **The transmission ratio n relates the motor and load velocities by $\omega_{motor} = n\omega_{load}$.**

- **Differentiating this relation once with respect to time, and substituting above:**

$$J_{motor}\,n^{2}\,\alpha_{Load}^{2} = J_{Load}\,\alpha_{Load}^{2}$$

- **The "optimal" transmission ratio for a pure rotational motion system dominated by inertial loads is thus:**

$$n_{opt} = \sqrt{\frac{J_{load}}{J_{motor}}}$$

- **For a friction (capstan) or belt drive system the optimal drive wheel radius r in meters is:**

$$r_{roller} = \sqrt{\frac{J_{motor}}{M_{load}}}$$

- **For a leadscrew driven carriage, the lead in mm/rev is:**

$$\ell = 2\pi \times 10^3 \sqrt{\frac{J_{motor}}{M_{load}}}$$

- **Next, estimate the efficiency of the drive train and see if the initial estimate used to find the load power rate was correct.**

- **The iterative nature of motor size and transmission ratio calculations should now be apparent.**

- **One must also check the required velocity and torque of the motor. For rotational systems the motor speed in rpm is:**

$$\omega_{motor} = n_{opt}\,\omega_{load}$$

- **For linear friction (capstan) or belt drives with a linear carriage velocity in m/s, the motor speed in rpm is:**

$$\omega_{motor} = \frac{30V_{load}}{\pi r_{roller}}$$

- **For leadscrew drives with lead ℓ (mm/rev) and a linear carriage velocity in m/s, the motor speed in rpm is**

$$\omega_{motor} = \frac{6 \times 10^4\, V_{load}}{\ell}$$

- **When the load inertia is very large, the transmission ratio is often very large.**
- **This can require that the motor have a very high speed.**
- **Thus inertia matching methods are often impractical for choosing motors for massive or heavily loaded systems.**

Optimal transmission ratio selection for heavily loaded systems

- **For constant velocity motion with large frictional loads and cutting forces, inertia matching doesn't work.**
- **Thermal power generated by the motor during a trapezoidal move, and with a constant applied external load is:**

$$W_{thermal} = \frac{cR_{motor}\,\omega_{load}^2\,J_{load}^2}{K_t^2\,t_c}\left[n^2\left(\frac{J_{motor}}{J_{load}} + \frac{1}{n^2}\right)^2 + \frac{r}{n^2}\right]$$

$$c = \frac{t_c}{t_a} + \frac{t_c}{t_d}$$

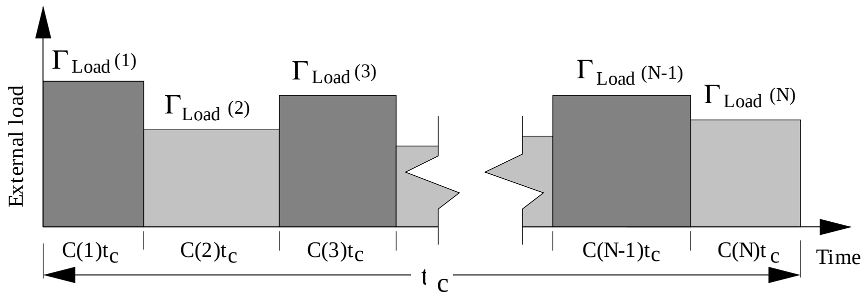
$$r = \frac{\Gamma_{load}^2\,t_c^2}{c\omega_{load}^2\,J_{load}^2}$$

- **The thermal power can be minimized with respect to the transmission ratio n by $\partial W_{thermal}/\partial(N^2) = 0$:**

$$n_{opt} = \sqrt{\frac{J_{load}\sqrt{1 + r}}{J_{motor}}} = \sqrt{\frac{J_{load}}{J_{motor}}} \;@\; r = 0$$

- **If the load is moving linearly:**
  - **Then J_{load} is the mass (kg), Γ_{load} is the force (Newtons),**
  - **ω_{load} is the maximum load speed (m/s) at the maximum force level.**

- **In many situations the load application is divided up into N segments each of length $C(i)t_c$:**



- **An overall equivalent load can be defined:**

$$\Gamma_{Lequ} = \sqrt{\sum_{i=1}^{N} r_{load}^2 (i)\, C(i)}$$

- **The sum of all C(i) must equal 1.0.**
- **The optimum transmission ratio is then determined using $\Gamma_{load} = \Gamma_{Lequ}$ in Equation 10.2.28b.**
- **If the external loads are zero, then the optimal ratio is equal to that given by the inertia matching criteria.**
- **During a normal operating period of several hours or days:**
  - **All the loads encountered during all of the N moves should be considered.**

- **In this case, the equivalent external loads $\Gamma_{Lequ}(i)$ are found for each trapezoidal move $t_c(i)$.**

- **Using Equation 10.2.30, and the total thermal power dissipated by the motor is:**

$$W_{thermal} = \sum_{i=1}^{M} k(i) \left[n^2 \left(\frac{J_{motor}}{J_{load}} + \frac{1}{n^2} \right)^2 + \frac{r(i)}{n^2} \right]$$

$$k(i) = \frac{c(i)\, R_{motor}\, \omega_{load}^2 (i)\, J_{load}^2}{K_t^2\, t_c (i)}$$

$$r (i) = \frac{\Gamma_{Lequ}^2 (i)\, t_c^2 (i)}{c(i)\omega_{load}^2 (i)\, J_{load}^2}$$

$$c (i) = \frac{t_c(i)}{t_a(i)} + \frac{t_c(i)}{t_d(i)}$$

- **The optimal reduction ratio for the system is again obtained from $\partial W_{thermal}/\partial(N^2) = 0$:**

$$n_{opt} = \sqrt{\frac{J_{load}\sqrt{1 + R}}{J_{motor}}}$$

$$R = \frac{\sum_{i=1}^{M} s (i)\, r (i)}{\sum_{i=1}^{M} s (i)}$$

$$s (i) = \frac{c (i)\, \omega_{loadmax}^2 (i)}{t_c (i)}$$

- **Equation 10.2.34b represents a portion of the i-th trapezoidal segment's contribution to the total performance requirement.**
    - **It is non-dimensionalized with respect to that of the first segment.**
- **The energy expended is weighted against the overall system characteristics.**
- **To minimize cooling requirements for a machine, minimize the thermal output during any one move.**
- **The "optimal" reduction ratio should be found for each particular move, and then the heat dissipation calculated.**
- **Note that all of the above equations are quite amenable to spreadsheet implementation.**

Checking system servo bandwidth

- **Once a minimum stiffness and optimal transmission ratio are selected:**
    - **An estimate of the system's servo bandwidth can be made.**
- **Most systems null high frequency disturbance forces with their own mass.**
- **Lower frequency forces must be offset by forces form the controller/actuator.**
- **The servo bandwidth required is a function of the types of moves that the system will be required to make.**
    - **For start and stop moves (e.g., in a wafer stepper), the bandwidth should be on the order of:**

$$f(Hz) \approx \frac{10}{2\pi t_{settlingtime}}$$

- **When contouring, the X and Y axes move according to:**

$$x = A\sin\omega t$$

$$y = B\cos\omega t$$

  - **The frequency *w* is a function of the linear velocity of the cutter through the material and the radius of the contour:**

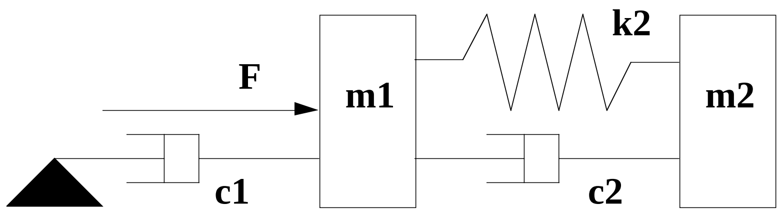
$$\omega_{angular} = \frac{v_{linear}}{r_{pathradius}}$$

$$f(Hz) = \frac{v_{linear}}{2\pi r_{pathradius}}$$

  - **For a large circle (e.g., 200 mm D) being cut at modest speeds (e.g., 0.1 m/s), the bandwidth required is only 0.15 Hz.**

  - **For a sharp turn in a corner, the radius of the cutter path trajectory may only be 1 mm, so *w* > 15 Hz.**

- **A system with a motor driving a carriage can be modeled in the following manner:**



- **m_1 is the mass of a linear motor forcer or:**
- **m_1 is the reflected inertia of the motor rotor and leadscrew (or just a linear motor's moving part):**

$$M_{reflected} = \frac{4\pi^2 J}{l^2}$$

- **M_2 is the mass of the carriage.**
- **C_1 is the damping in the linear and rotary bearings.**
- **C_2 is the damping in the actuator-carriage coupling and the carriage structure.**
- **K_2 is the stiffness of the actuator and actuator-carriage-tool structural loop.**

- **The equations of motion are:**

$$\left|\begin{matrix} m_1 & 0 \\ 0 & m_2 \end{matrix}\right| \left|\begin{matrix} \ddot{x}_1 \\ \ddot{x}_2 \end{matrix}\right| + \left|\begin{matrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{matrix}\right| \left|\begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix}\right| + \left|\begin{matrix} k_2 & -k_2 \\ -k_2 & k_2 \end{matrix}\right| \left|\begin{matrix} x_1 \\ x_2 \end{matrix}\right| = \left|\begin{matrix} F(t) \\ 0 \end{matrix}\right|$$

- **The transfer function x_2/F (dynamic response of the carriage) is:**

$$\frac{x_2}{F} = \frac{k_2 + c_2 s}{c_1 s\left(k_2 + c_2 s + m_2 s^2\right) + (m_1 + m_2) s^2 (k_2 + c_2 s) + m_1 m_2 s^4}$$

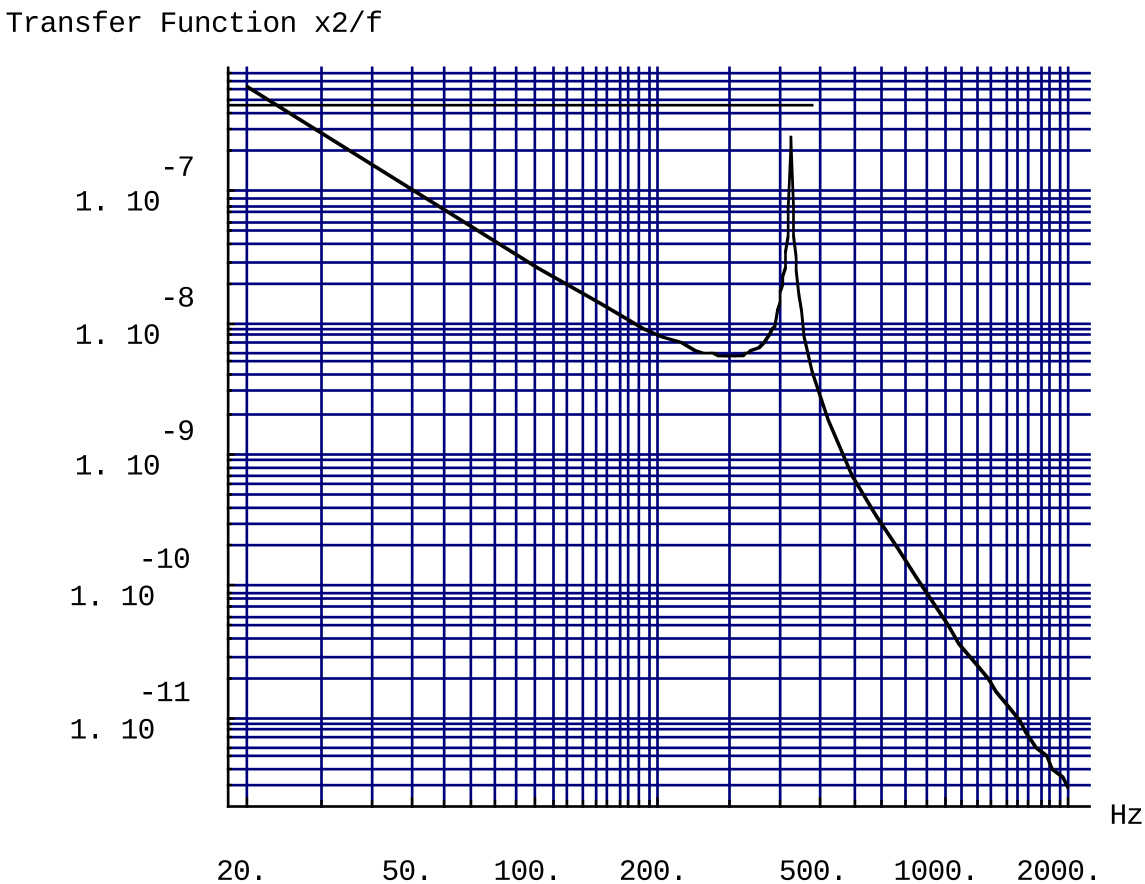
- **Note the product of the masses term which tends to dominate the system.**

- **Calculated parameters of four possible systems are:**

Actuator	**ballscrew**	**lin. motor**	**lin. motor**	**lin. motor**
Bearings	**linear ball**	**linear ball**	**air**	**air**
Structural damping	**no**	**no**	**no**	**yes**
material damping zeta	**0.005**	**0.005**	**0.005**	**0.1**
actuator to ground zeta	**0.05**	**0.03**	**0**	**0**
m1 (actuator) (kg)	**50**	**5**	**5**	**5**
m2 (carriage), kg	**50**	**50**	**50**	**50**
c1 (N/m/s)	**355**	**187**	**0**	**0**
c2 (N/m/s)	**19**	**19**	**19**	**374**
k1 (N/m)	**1.75E+08**	**1.75E+08**	**1.75E+08**	**1.75E+08**
Bandwidth (Hz)	**25**	**100**	**30**	**100**

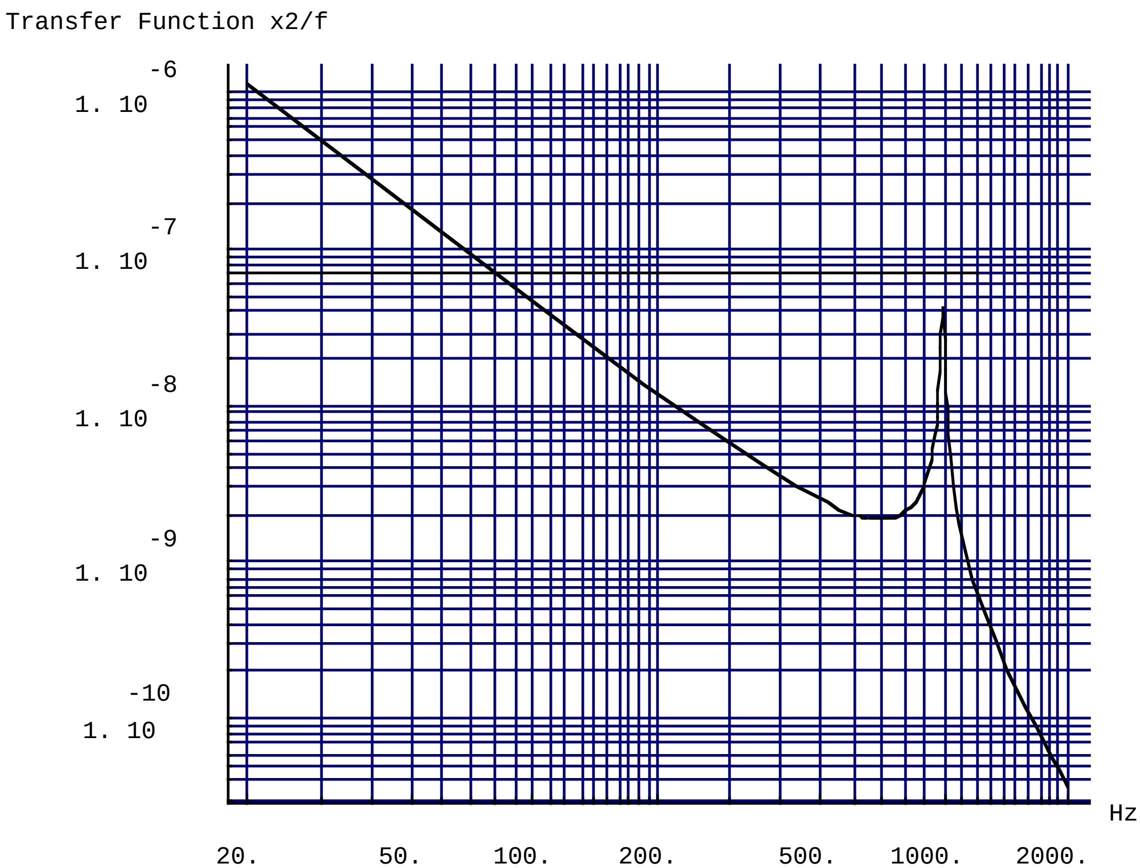
- **As a guideline, the servo bandwidth of the system is:**
  - **Generally limited by the frequency the servo can drive the system at without exciting structural modes.**
  - **Without special control techniques can be no higher than the frequency:**
    - **Found by drawing a horizontal line 3 dB above the resonant peak to intersect the response curve.**
- **This method is used only to initially size components.**
- **A detailed controls simulation must be done to verify performance, and guide further system optimization.**

- **The response of the ballscrew driven carriage supported by rolling element linear bearings will be:**



- **In this case, since preloaded linear guides and a ballnut are used, damping to ground will be high.**
- **The inertia of the screw will lower the system frequency considerably (note the $m_1 m_2 s^4$ term in the TF)**
- **The system bandwidth will be limited to about 25 Hz.**

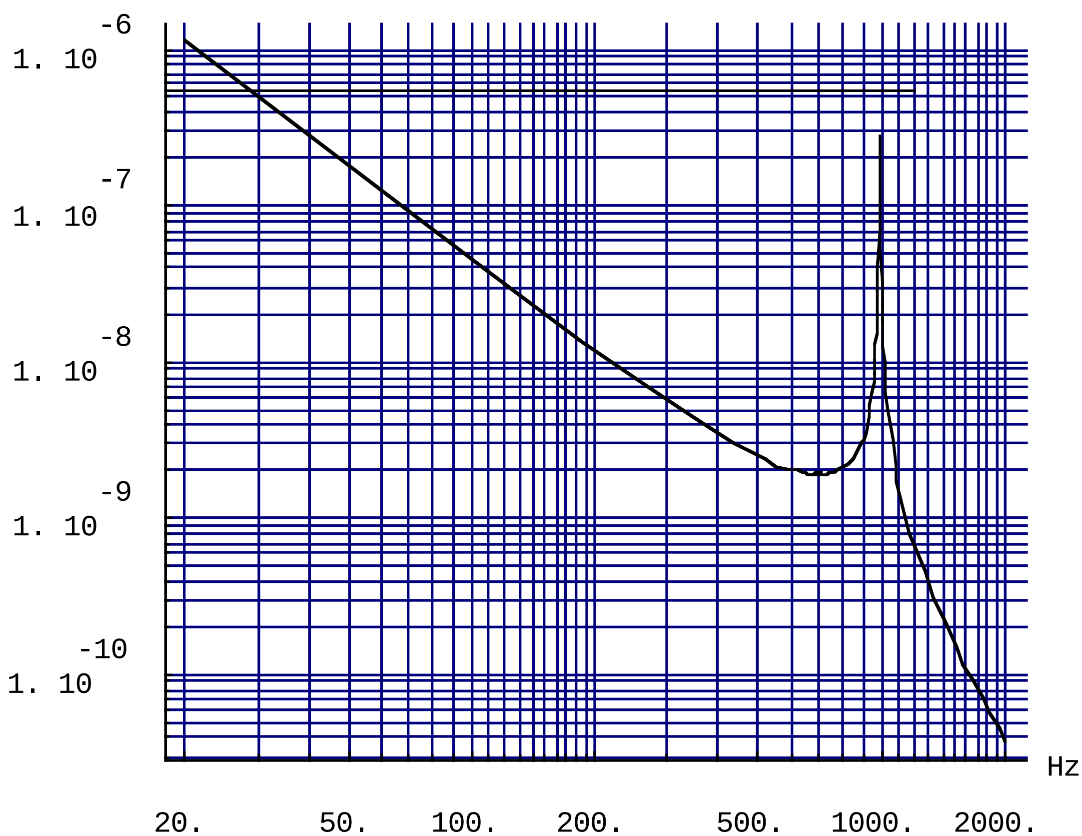
- **The response of the linear motor driven carriage supported by rolling element linear bearings will be:**



- **In this case, since preloaded linear guides are used, damping to ground will be high, to help damp modes of the carriage.**
- **The inertia of the motor is low, so the system frequency is high.**
- **The system bandwidth will be limited to about 100 Hz.**

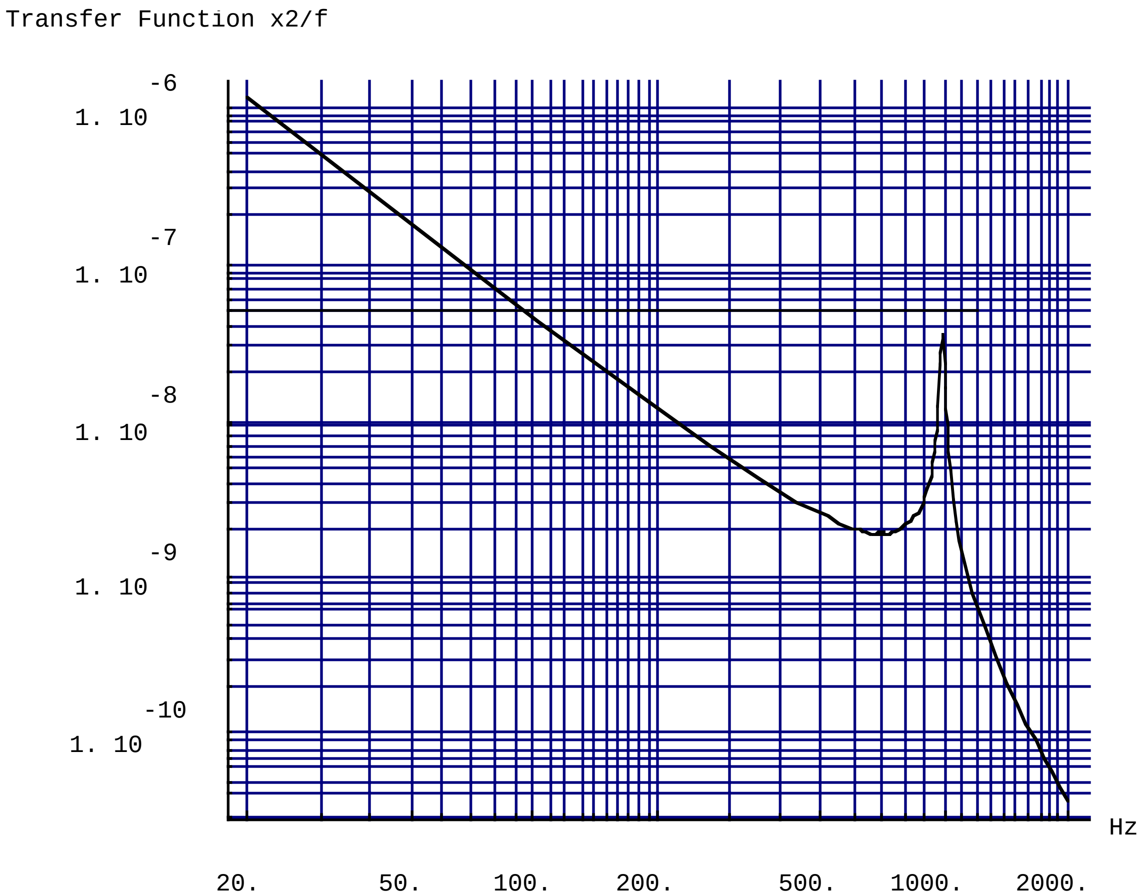
- **The response of the linear motor driven carriage supported by air bearings will be:**

Transfer Function x2/f



- **In this case, since air bearings are used, damping to ground will be zero.**
- **Since structural damping is low, only the motor can damp modes of the carriage.**
- **The inertia of the motor is low, so the system frequency is high, but lightly damped.**
- **The system bandwidth will be limited to about 30 Hz.**

- **With a high degree of structural damping:**
    - **The linear motor driven system with air bearing support will see a large increase in bandwidth:**



- **In this case, since air bearings are used, damping to ground will be zero.**
- **Since structural damping is high, the motor is not greatly needed to damp modes of the carriage.**
- **The inertia of the motor is low, so the system frequency is high, and well damped.**
- **The system bandwidth will be limited to about 100 Hz.**