Precision Machine Design

Topic 12

Point and line contact between surfaces & Kinematic coupling design

Purpose:

This lecture describes analysis methods for estimating the stress and deformation that occur when there is point or line contact between two bodies.

Outline:

- **"Exact" solutions for point contact**
- **Line contact**
- **Failure criteria**
- **Rolling contact stress considerations**
- **Fretting corrosion**
- **Tangential stiffness**
- **Application of Hertz theory: Kinematic coupling design**

"We reason deeply, when we forcibly feel"

Mary Wollstonecraft

"Exact" solutions for point contact

- **Hertz theory can be used to accurately determine the stress and deflection in the contact region.**
- **Contact between curved surfaces results in an elliptical contact region.**
- **The equivalent modulus of elasticity of the system:**
  - **Based on the elastic modulii and Poisson's ratio η of the two materials in contact:**

$$E_e = \frac{1}{\frac{1-\eta_1^2}{E_1} + \frac{1-\eta_2^2}{E_2}}$$

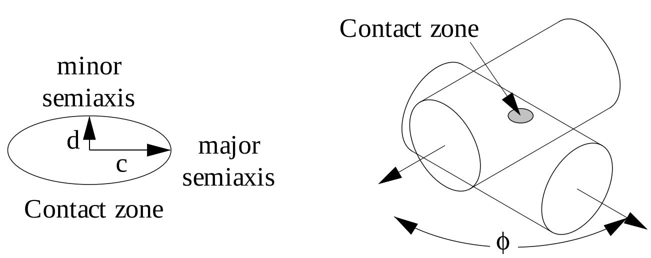
- **The equivalent radius or curvature is found:**

$$R_e = \frac{1}{\frac{1}{R_{1_{major}}} + \frac{1}{R_{1_{minor}}} + \frac{1}{R_{2_{major}}} + \frac{1}{R_{2_{minor}}}}$$

- **A function cosθ is defined as:**

$$\cos\theta = R_e \left[\left(\frac{1}{R_{1_{major}}} - \frac{1}{R_{1_{minor}}} \right)^2 + \left(\frac{1}{R_{2_{major}}} - \frac{1}{R_{2_{minor}}} \right)^2 + 2\left(\frac{1}{R_{1_{major}}} - \frac{1}{R_{1_{minor}}} \right)\left(\frac{1}{R_{2_{major}}} - \frac{1}{R_{2_{minor}}} \right)\cos 2\phi \right]^{1/2}$$

- **ϕ is the angle between the planes of principal curvature of the two bodies:**



- **For a friction drive roller on a round drive bar, ϕ = 90º.**

- **cosθ is used to find the factors α, β, and λ.**
- **Curve fits to α, β, and λ.:**

$$\alpha = 1.939e^{-5.26\theta} + 1.78e^{-1.09\theta} + 0.723/\theta + 0.221$$

$$\beta = 35.228e^{-0.98\theta} - 32.424e^{-1.04754\theta} + 1.486\theta - 2.634$$

$$\lambda = -0.214e^{-4.95\theta} - 0.179\theta^2 + 0.555\theta + 0.319$$

- **The major and minor semiaxis dimensions (one-half of the total width) of the elliptical contact area are given by:**

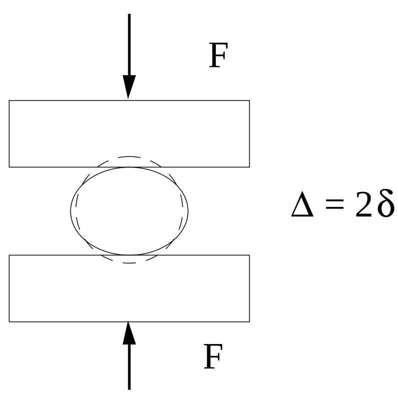
$$c = \alpha\left(\frac{3FR_e}{2E_e}\right)^{1/3} \qquad d = \beta\left(\frac{3FR_e}{2E_e}\right)^{1/3}$$

- **The contact pressure q is given by:**

$$q = \frac{3F}{2\pi cd} \approx 1.5\sigma_{\text{tensile for metals}}$$

- **For contact at a single interface (e.g. an elastic hemisphere on an elastic flat plate), the distance of approach of two far field points in the bodies is:**

$$\delta = \lambda\left(\frac{2F^2}{3R_eE_e^2}\right)^{1/3}$$

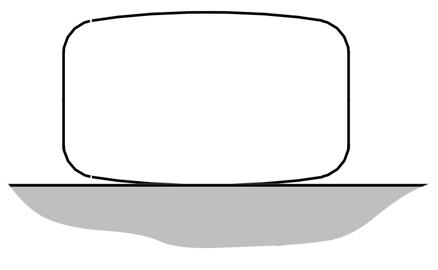


Example:

- **A 3 mm diameter (0.118") steel ball probe is used to measure the location of a steel surface.**
- **Applied force on the order of 1N (0.22 lbf).**
- **The equivalent modulus of elasticity for the system is about 110 GPa (16×10^6 psi).**
- **The equivalent radius is 0.75 mm (0.030").**
- **The deflection is about 0.3 μm (12 μin).**
- **A variation in the force by 10% causes a change in the deflection by only about 6% or 0.021 μm (0.83 μin).**
- **If the characteristic surface finish dimension of the part is much larger than the size of the contact area:**
  - **Then measurement errors will be dominated by surface finish effects.**
- **Most precision surfaces have a high surface finish:**
  - **The relatively large contact area creates an elastic averaging effect.**

Line contact:

- "Exact" solutions exist for line contact problems.
- Line contact is difficult to achieve:
    - Slight misalignments cause line contact problems to become point contact problems.
- If possible, use a design where the object that was to make line contact instead has a very large major radius:
    - On the order of 100 times the minor radius.
- This will make the design more tolerant of misalignment, and thus make the design more deterministic.



Failure criteria:

- **For metals, one can usually assume $\tau_{max\ allowable} = 1/2\sigma_{max\ tensile}$:**

$$q_{Hertz\ metals\ max} = \frac{3\sigma_{allowable\ tensile\ stress}}{2}$$

- **For brittle materials (e.g., ceramics), the flexural strength is the limiting design factor:**

$$q_{Hertz\ brittle\ materials\ max} = \frac{2\sigma_{allowable\ flexural\ stress}}{1 - 2\eta}$$

- **As the size of the specimen decreases, the likelihood of a defect initiating failure decreases.**
- **In practice, the allowable Hertz stresses may be 50% greater than the Equations allow for.**
- **For example, hot pressed silicon nitride bearing components can withstand millions of cycles at a Hertz stress level of 6.9 GPa (10^6 psi).**
  - **The single cycle failure load is often in the neighborhood of 10 GPa (1.5×10^6 psi).**

Rolling contact stress considerations:

- **The addition of a tangential stress (e.g. in a friction drive) can lead to the formation of tensile stresses in the surface.**

- **Assume the following:**

  1. **The maximum tangential load (tractive effort) that can be supported is equal to the product of the coefficient of friction μ and the normal load.**

  2. **The tangential stress is equal to the tangential load divided by the contact area.**

  3. **For metals, the maximum shear stress is equal to the sum of the shear stresses due to the normal and tangential loads.**

  4. **For brittle materials (e.g., ceramics):**

     - **The allowable flexural stress is decreased by an amount equal to the tangential stress.**

Fretting corrosion:

- **Imagine every time your fingers touched the skin grew together. Your hands would soon be scarred and damaged.**
- **The asperites on like metal surfaces can atomically bond to each other if they are pressed together.**
- **When the surfaces move, the asperites which have fused, are ripped apart.**
- **The freshly exposed surfaces can oxidize and then the oxide is pushed aside and the process repeats.**
- **This is called *fretting corrosion*.**
- **Vibration causes asperites to work their way through lubrication layers to make intimate contact.**
- **Sub-micron levels of vibration have been known to cause fretting.**
- **There are methods to prevent fretting:**
    - **Make sure the stress level is very low between elements in contact (usually not practical).**
    - **Make sure rolling elements in contact periodically roll enough to rebuild the lubricant layer between them.**
    - **Use dissimilar materials so bonding cannot occur between the asperites.**
        - **Ceramic rollers and steel races, or stainless steel elements.**

Tangential stiffness:

- **The tangential displacement of a point on either body with respect to a far-field point on the body is given by**

$$\delta_{tan} = \frac{3\mu F\,(2 - \eta)\,(1 + \eta)}{8aE}\left[1 - \left(1 - \frac{F_{tan}}{\mu F}\right)^{2/3}\right]\Phi$$

$$\delta_{tan\ unload} = \frac{3\mu F\,(2 - \eta)\,(1 + \eta)}{8aE}\left[2\left(1 - \frac{T^* - F_{tan}}{2\mu F}\right)^{2/3} - \left(1 - \frac{T^*}{\mu F}\right)^{2/3} - 1\right]\Phi$$

- **T^* is the initial tangential force, F_{tan} is the new lower force, F is the preload force, and Φ is given by**

$$\Phi = \left[\frac{4a}{\pi b\,(2 - \eta)}\right]\left[\left(1 - \frac{\eta}{k^2}\right)\mathbf{K} + \frac{\eta \mathbf{E}}{k^2}\right] \qquad a < b$$

$$\Phi = 1 \qquad \text{(spherical contact)} \qquad a = b$$

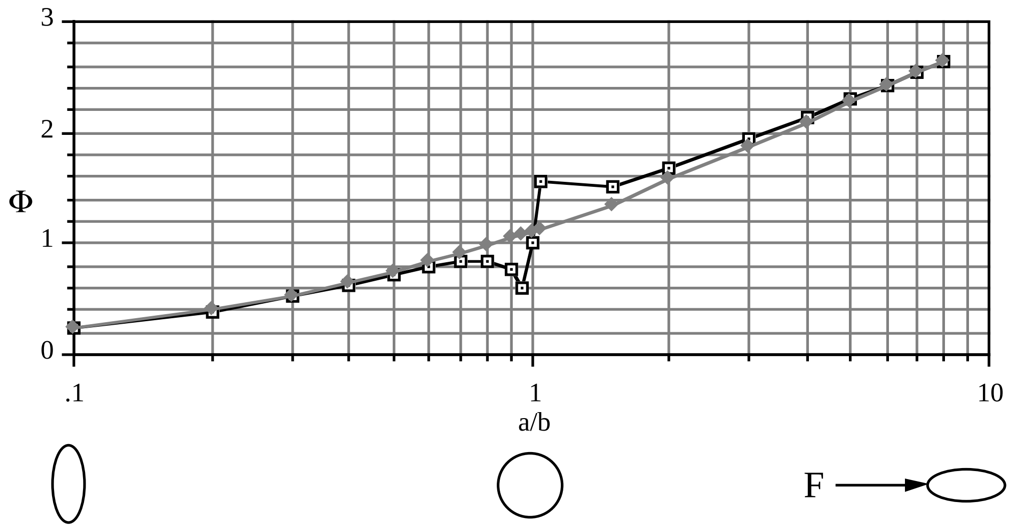
$$\Phi = \left[\frac{4}{\pi\,(2 - \eta)}\right]\left[\left(1 - \eta + \frac{\eta}{k_1^2}\right)\mathbf{K}_1 + \frac{\eta \mathbf{E}_1}{k_1^2}\right] \qquad a > b$$

- **Take care to note that *a* is the dimension of the semiaxis *parallel* to the direction of the applied tangential force.**

- **Also note, that the material constants η and E are for the body (e.g. roller or drive bar in a friction drive) of interest.**

- **The constants k and k_1 are the arguments of the complete elliptic integrals K, E, and K_1, E_1, respectively.**

- **Φ is a function of the ratio of semiaxis dimensions:**



- **Polynomials of reasonable accuracy can be found for Φ to aid in spreadsheet analysis:**

$$\Phi = 0.13263 + 1.4325\,(a/b) - 0.54754\,(a/b)^2 + 0.12303\,(a/b)^3 - 0.013591\,(a/b)^4 + 0.0005729\,(a/b)^5 \quad \text{for } 0.1 \le a/b \le 8$$

$$\Phi = 1.9237 + 0.11029\,(a/b) - 2.8323 \times 10^{-3}\,(a/b)^2 + 4.3109 \times 10^{-5}\,(a/b)^3 - 3.3497 \times 10^{-7}\,(a/b)^4 + 1.0257 \times 10^{-9}\,(a/b)^5 \quad \text{for } 8 < a/b < 90$$

- **The stiffness of each of the two contacting bodies is found as the inverse of the compliance:**

$$K_{tan} = \frac{1}{\frac{\partial \delta_{tan}}{\partial F_{tan}}} = \frac{4Ea}{(2 - \eta)\,(1 + \eta)\,\Phi}\left(1 - \frac{F_{tan}}{\mu F}\right)^{1/3}$$

- **This equation reflects the condition where the slip zone size increases with the tangential force.**

- **Example: Spreadsheet used for the design of a friction drive with silicon nitride roller and steel drive bar:**

Portion of max load at which stiffness is evaluated	50%	Roller modulus of elasticity (GPa)	310
Usable force portion of maximum load	75%	Drive bar modulus of elasticity (GPa)	204
Coefficient of friction	0.1	Roller Poisson ratio	0.27
Maximum desired contact stress (GPa)	2.07	Drive bar Poisson ratio	0.29

Roller D_{minor} (mm)	25	30	35	40	45	50	55	60	65	70	75
Roller D_{major} (mm)	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000	7500
Preload force (N)	5963	8587	11687	15265	19320	23852	28861	34347	40309	46749	53666
Usable (75%) F (N)	447	644	877	1145	1449	1789	2165	2576	3023	3506	4025
Semiaxis c (mm)	3.95	4.74	5.53	6.32	7.10	7.89	8.68	9.47	10.3	11.1	11.8
Semiaxis d (mm)	0.348	0.418	0.488	0.558	0.627	0.697	0.767	0.836	0.906	0.976	1.05
K along c (N/m)	243	291	340	388	437	486	534	583	631	680	728
K along d (N/m)	245	293	342	391	440	489	538	587	636	685	734

- **A stainless, carbide, or ceramic roller will not cause fretting should one axis be locked for an extended period.**

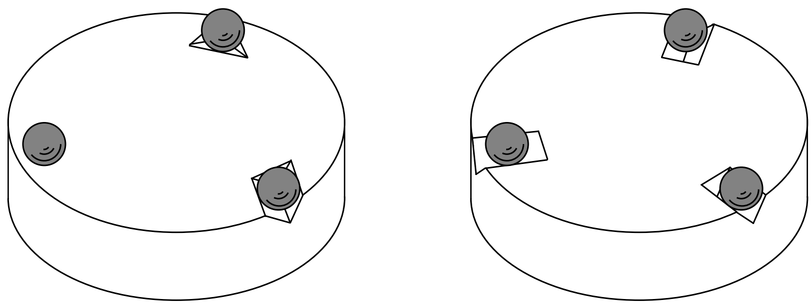
- **A stainless steel roller would actually allow for the use of a higher preload.**
  - **It has a lower modulus and thus would not indent the steel drive bar so deeply.**

Hertz theory application: Kinematic coupling design

- **Industrial significance**
- **Static equilibrium analysis**
- **Deformation at the contact points**
- **Analysis of error motions**
- **Practical design considerations**
- **Design spreadsheet**
- **Three tooth couplings**
- **Tetrahedron-groove-flat couplings**

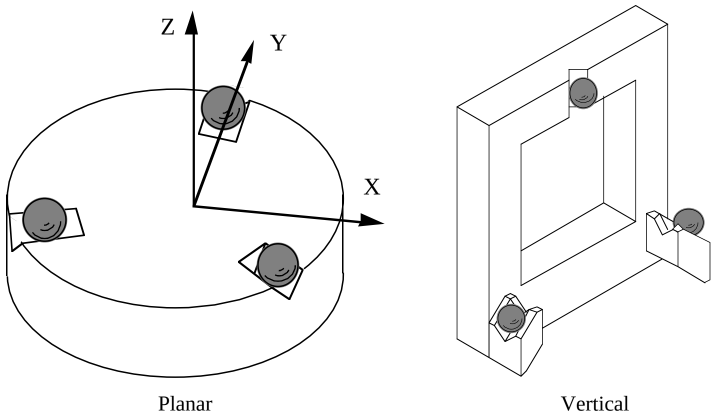
Industrial significance

- Kinematic couplings provide the most economical and dependable method for attaining high repeatability.
- To be a kinematic coupling, six degrees of freedom must be restrained by the coupling:



- Tetrahedron-groove-flat couplings can be made stable and rigid with virtually any aspect ratio.
- For equilateral couplings, three-groove couplings are easier to produce, and have equivalent performance.
- Merely restraining six degrees of freedom is not enough:
    - The popular 3-2-1 concept of fixturing is not as repeatable as a three groove or Tetrahedron-groove-flat coupling.
    - The first three points indent the part and resist motion of the part as it is pushed against two points.
    - Five points are then indented which resist motion into the final point.
    - With a three groove or Tetrahedron-groove-flat coupling:
        - The six contact points engage virtually simultaneously.

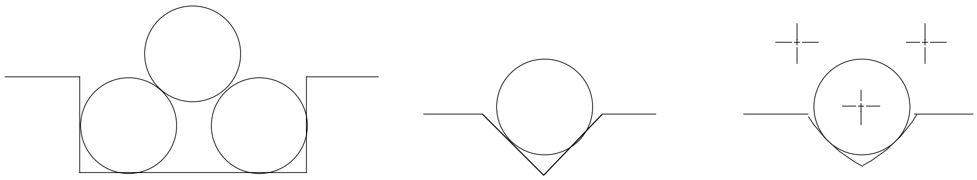
- **Examples of three groove kinematic couplings for horizontal and vertical fixturing applications:**



- **Vertical couplings can be used to position machine modules on larger assemblies.**
- **Properly designed kinematic couplings are deterministic.**
  - **Repeatability is related to surface finish and preload repeatability.**
- **Being deterministic makes performance predictable and also helps to reduce design and manufacturing costs.**

However:

- **The contact stresses can be very high and no elasto-hydrodynamic lubrication layer exists.**



Mounting (Rc 58 on Rc 58)	Maximum load
20 mm ball on 20 mm rollers	173 N
20 mm ball in a Vee groove	359 N
20 mm ball in a 22 mm Gothic arch groove	4,015 N
500 mm "ball" (contact points) in a vee groove[1]	224,419 N

[1] US Patent pending on this "canoe" shape, Aesop, Inc. Concord, NH fax 603-224-5369.

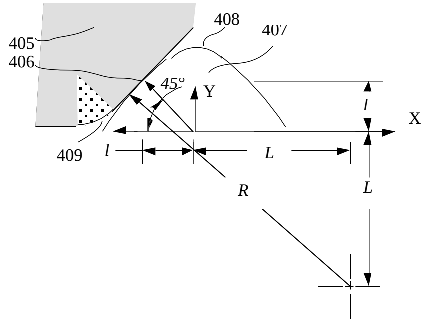


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Fig. 14

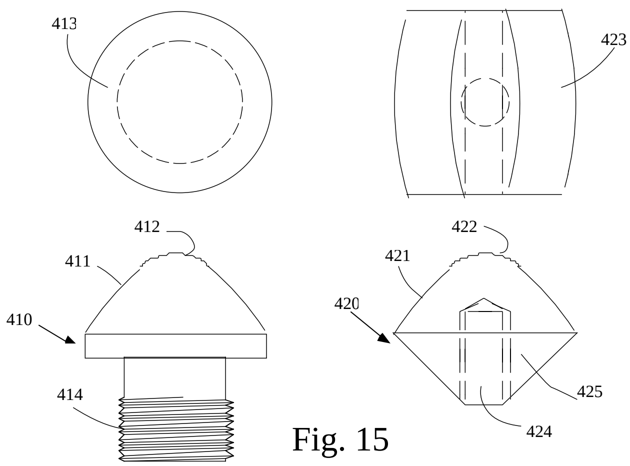

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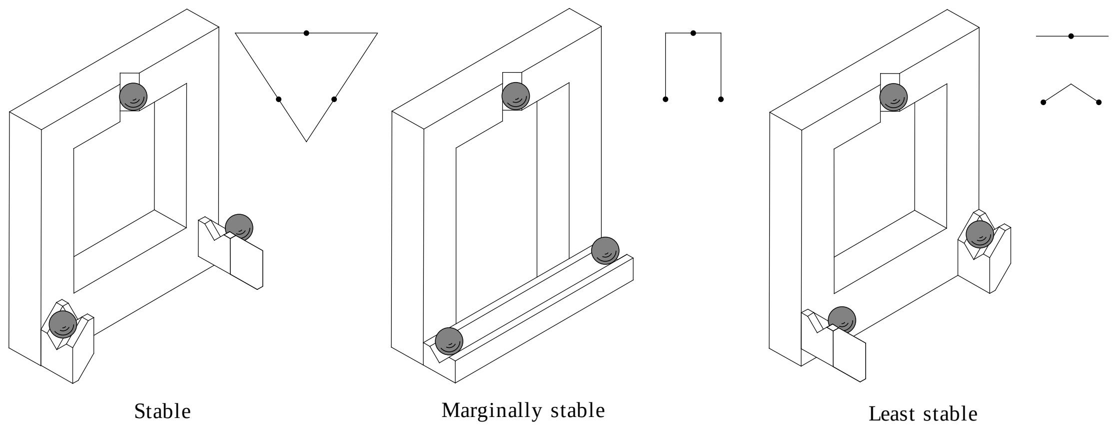
Fig. 15

- **For high cycle applications, the contact surfaces should be made from corrosion resistant materials (e.g., stainless, carbide or ceramic materials).**
    - **When steel components are used, one must be wary of fretting at the interface.**
        - **Steel couplings should only be used for low cycle applications.**
- **In general, the use of grooves minimizes the overall stress state in the coupling.**
- **Application examples:**
    - **A three groove coupling can be used to hold a grinding fixture on a profile grinder.**
        - **A matching coupling on a CMM can be used to hold the fixture.**
        - **A part can be measured and then returned to the grinder.**
    - **A vertically oriented coupling with gravity preload can be used to hold photolithographic masks in a large wafer stepper.**

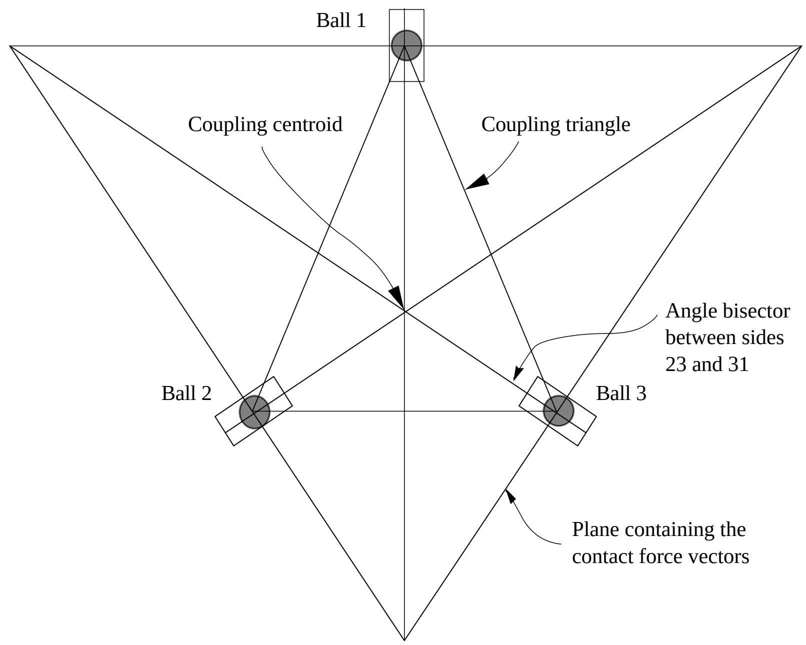
Static equilibrium analysis

- **Any three groove kinematic coupling's stability can be quickly assessed:**
    - **Examine the intersections of the planes that contain the contact force vectors.**
    - **For stability, the planes must form a triangle:**

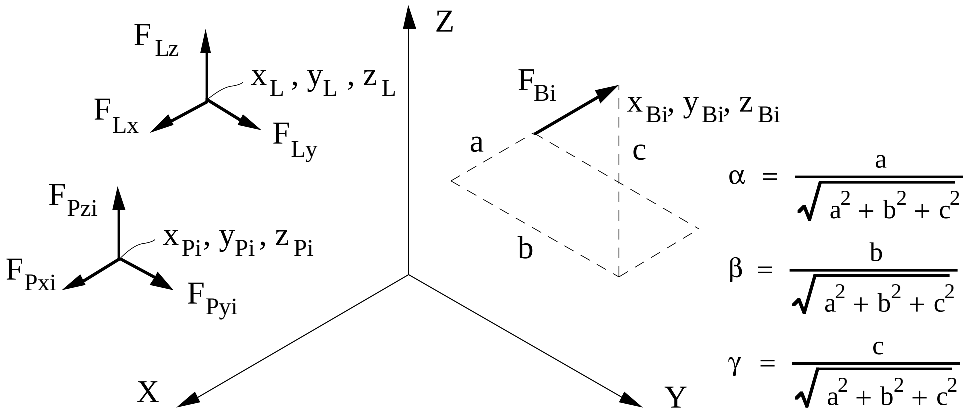


- **Degenerate forms of kinematic couplings are sometimes employed.**
    - **They may be easier to manufacture, but extreme care must be taken in their application.**

- **Ideally, the normals to the contact planes should bisect the coupling triangle's angles:**



- **The force and moment equilibrium equations are:**



$$\sum_{i=1}^{6} F_{Bi}\,\alpha_{Bi} + \sum_{i=1}^{3} F_{Pxi} + F_{Lx} = 0$$

$$\sum_{i=1}^{6} F_{Bi}\,\beta_{Bi} + \sum_{i=1}^{3} F_{Pyi} + F_{Ly} = 0$$

$$\sum_{i=1}^{6} F_{Bi}\,\gamma_{Bi} + \sum_{i=1}^{3} F_{Pzi} + F_{Lz} = 0$$

$$\sum_{i=1}^{6} F_{Bi}\,(-\beta_{Bi} z_{Bi} + \gamma_{Bi} y_{Bi}) + \sum_{i=1}^{3} - F_{Pyi} z_{Pi} + F_{Pzi} y_{Pi} - F_{Ly} z_L + F_{Lz} y_L = 0$$

$$\sum_{i=1}^{6} F_{Bi}\,(\alpha_{Bi} z_{Bi} - \gamma_{Bi} x_{Bi}) + \sum_{i=1}^{3} F_{Pxi} z_{Pi} - F_{Pzi} x_{Pi} + F_{Lx} z_L - F_{Lz} z_L = 0$$

$$\sum_{i=1}^{6} F_{Bi}\,(-\alpha_{Bi} y_{Bi} + \beta_{Bi} x_{Bi}) + \sum_{i=1}^{3} - F_{Pxi} y_{Pi} + F_{Pyi} x_{Pi} - F_{Lx} y_L + F_{Ly} x_L = 0$$

- **The magnitudes of the six contact point forces are easily calculated using a spreadsheet.**

- **The contact forces can be used to determine the stress and deflection at the contact points using Hertz theory.**

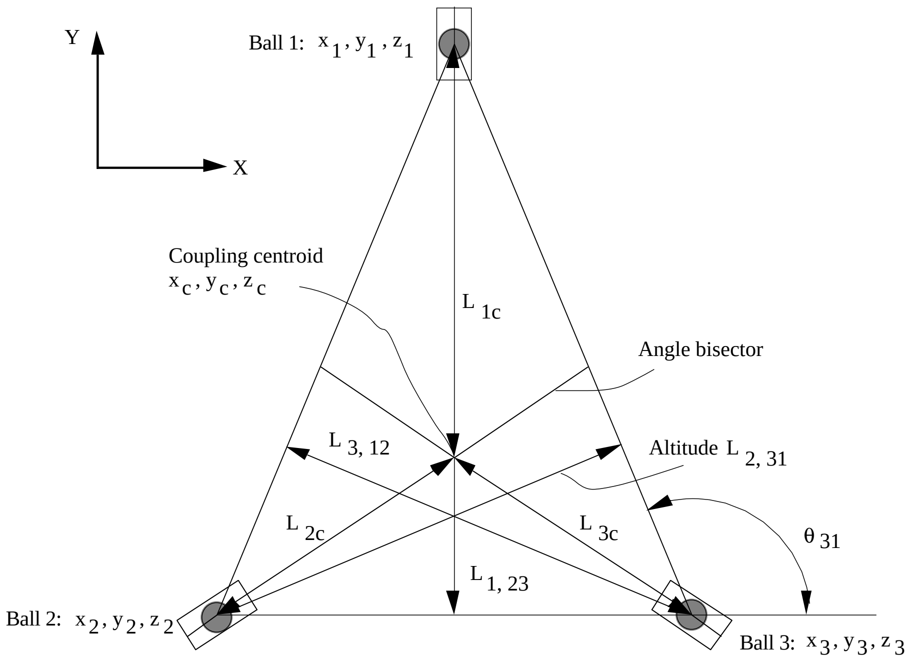
Analysis of error motions

- **Use Hertz theory and the force analysis results to predict the deflection at each ball-groove interface.**
- **Because of friction and elastic indentation:**
    - **It is reasonable to assume that there is no relative tangential motion at the contact interface.**
- **If the new position of the balls' centers is found using the contact displacements and contact forces' direction cosines:**
    - **There will not be a unique homogeneous transformation matrix that relates the old and new ball positions.**
    - **Energy methods can be used to develop approximate solutions also.**
- **In general, these factors make the calculation of a kinematic coupling's error motions a non-deterministic problem.**
- **However, reasonably accurate estimations can be made of the error motions**

- **The product of the deflection of the balls with the contact forces' direction cosines are used to calculate the ball's deflections.**
- **The displacements of the coupling triangle's centroid:**
  - $\delta\xi_c$ **(ξ = x, y, z),**
  - **Assumed to be the equal to the weighted average (by the distances between the balls and the coupling centroid) of the ball's deflections:**

$$\delta_{\xi c} = \left(\frac{\delta_{1\xi}}{L_{1c}} + \frac{\delta_{2\xi}}{L_{2c}} + \frac{\delta_{3\xi}}{L_{3c}}\right)\frac{L_{1c} + L_{2c} + L_{3c}}{3}$$

- **If the distances between the balls remained the same, then the theory should be exact.**
- **The change in distance between the balls is typically five to ten times less than the deflection at the contact points.**
- **The ratio of the change in the distance between the balls to the distance between the balls,**
  - **Is typically an order of magnitude less than the ratio of the deflection of the balls to the ball diameters.**



- **To determine the rotations about the x and y axes:**
  - **The distances between the balls, altitude lengths, and orientation angles must be determined:**

$$\varepsilon_x = \frac{\delta_{z1}}{L_{1,23}} \cos\theta_{23} + \frac{\delta_{z2}}{L_{2,31}} \cos\theta_{31} + \frac{\delta_{z3}}{L_{3,12}} \cos\theta_{12}$$

$$\varepsilon_y = \frac{\delta_{z1}}{L_{1,23}} \sin\theta_{23} + \frac{\delta_{z2}}{L_{2,31}} \sin\theta_{31} + \frac{\delta_{z3}}{L_{3,12}} \sin\theta_{12}$$

- **For ball 1, the rotation about the z axis is**

$$\varepsilon_{z1} = \frac{\sqrt{(\alpha_{B1}\,\delta_1 + \alpha_{B2}\,\delta_2)^2 + (\beta_{B1}\,\delta_1 + \beta_{B2}\,\delta_2)^2}}{\sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2}}\ \mathrm{SIGN}(\alpha_{B1}\,\delta_1 + \alpha_{B2}\,\delta_2)$$

- **Rotation about the z axis is assumed to be the average of the rotations about the z axis calculated for each ball:**

$$\varepsilon_z = \frac{\varepsilon_{z1} + \varepsilon_{z2} + \varepsilon_{z3}}{3}$$

- **The errors can then be assembled into a homogeneous transformation matrix (HTM) for the coupling:**

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_c \\ y - y_c \\ z - z_c \\ 1 \end{bmatrix} - \begin{bmatrix} x - x_c \\ y - y_c \\ z - z_c \\ 0 \end{bmatrix}$$

- **Note that in this HTM it has been assumed that the rotations are small so.**

- **With a spreadsheet, the design engineer can easily play "what-if" design games.**

Practical Design Considerations:[2]

- **The balls must be permanently affixed to one side of the coupling:**
  - **A hole with a tapered bottom can be jig ground very accurately.**
  - **A surface can be ground flat and then annular grooves ground around ball locations.**
    - **Sleeves can then be pressed into the grooves.**
    - **The balls can then be pressed into the sleeves until they contact the flat surfaces.**
  - **Tetrahedrons can be EDM's into the coupling and then the balls burnished or lapped into place.**
  - **A hemispherical seat can be cut with a ball-end mill and then the ball can be epoxied or brazed into the seat.**
- **In all cases, once the balls are affixed, they should be epoxied into place.**
- **In some cases the balls should be affixed to a rough machined fixture, and then the fixture finished in situ.**

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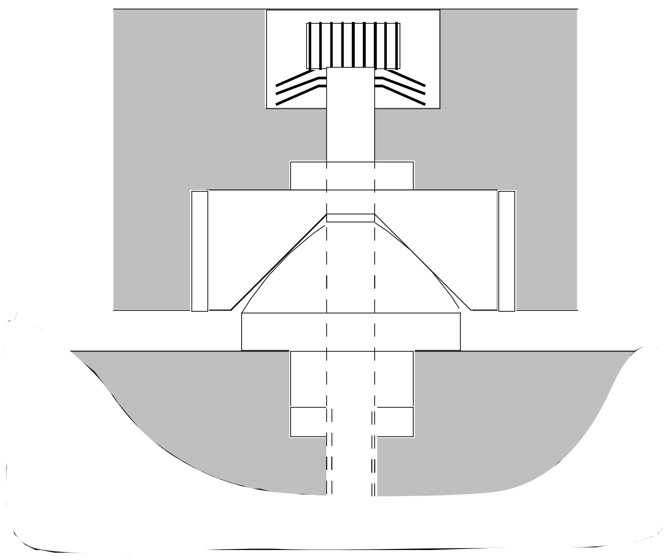
2 Modular components can be purchased from Eugene Gleason, Bal-Tec, 1550 E. Slauson Avenue, Los Angeles, CA 90011, (800) 322-5832.

There are many forms of modular components:

- **A hardened groove can be press-fit into a hole**
- **A contoured pin can be pressed into a hole in the other part**
- **The assemble can be preloaded with a bolt and spring washers.**



- **The coefficient of friction between the ball and the groove should be minimized.**
- **The ideal material for the balls and grooves would be a hard ceramic-type material because it would not corrode or fret.**
  - **Tungsten carbide (carbide), silicon carbide, silicon nitride, and zirconia work well.**
- **Hardened stainless steel can also be used.**
- **The grooves can be profile ground in a monolithic plate using a profile grinder and an index table.**
- **The grooves can be made in the form of modular inserts.**
- **The grooves preferably should have a greater modulus than the balls.**
  - **This minimizes the depth of elastic indenting.**
  - **The less indenting, the greater the kinematicity.**

Example: Kinematic coupling used to hold probe arm to an electronic indicator (US patent 4,574,625, assigned to Federal Products Corp.).

- **The coupling is preloaded via magnets (70 and 71).**

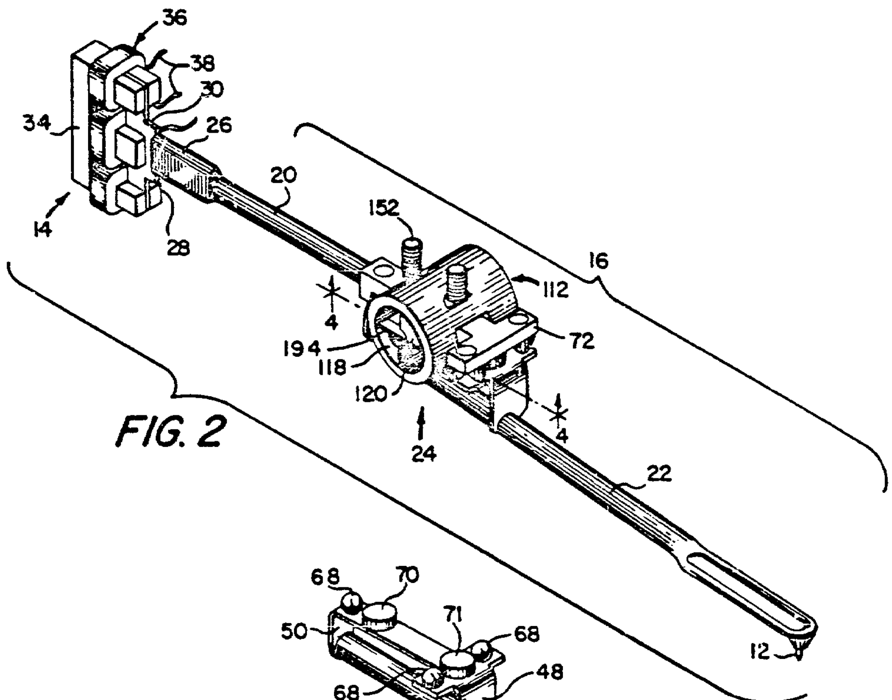


FIG. 2

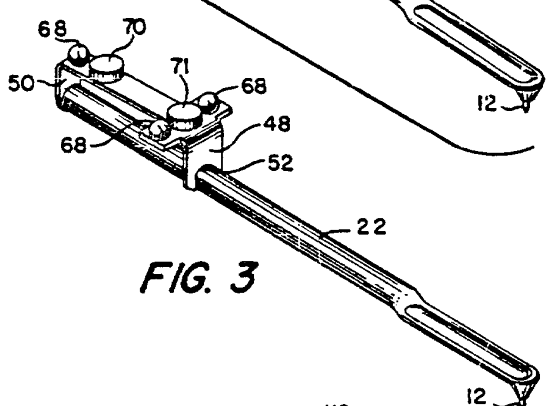


FIG. 3

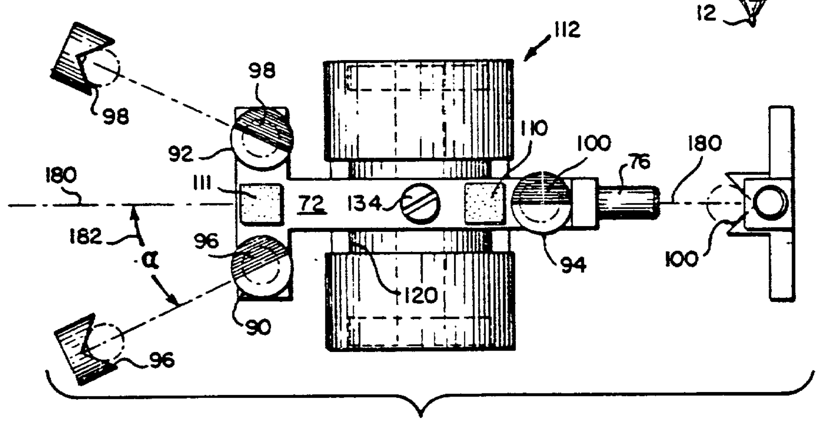


FIG. 4

Kinematic Coupling Design Spreadsheet

KINCOUP.XLS
To design three groove kinematic couplings
Written by Alex Slocum. Last modified 6/16/95 by Alex Slocum
Only change cells with boldface numbers
XY plane is assumed to contain the ball centers
For standard coupling designs, contact forces are inclined at 45 to the XY plane
Standard 120 degree equal size groove coupling? **TRUE**
For non standard designs, enter geometry after results section

Dbeq=	**0.05**	Equivelent diameter ball that would contact the groove at the same points			
Rbminor =	**0.025**	Minor radius	0.02439024		
Rbmajor =	**1**	Major radius	0.56	Material properties	
Rgroove =	**1.00E+06**	Groove radius (negative for a trough)		*Hertz stress*	
Costheta =	**FALSE**	Is ball major radius along groove axis?		SiN	**6.75E+09**
Dcoupling=	**0.250**	Coupling diameter		plastic	**1.04E+08**
Fpreload =	**-5000**	Preload force over each ball (N)		RC 62 Steel	**1.81E+09**
Xerr =	**0.000**	X location of error reporting		*Elastic modulus*	
Yerr =	**0.000**	Y location of error reporting		SiN	**3.11E+11**
Zerr =	**0.400**	Z location of error reporting		plastic	**1.04E+10**
Auto select material values assume that metric units are used (mks)				RC 62 Steel	**2.04E+11**
Matlab =	**4**	Enter 1 for plastic ball, plastic groove		*Poisson ratio*	
		Enter 2 for Steel ball, plastic groove		SiN	**0.27**
		Enter 3 for SiN ball, RC 62 Fe groove		plastic	**0.2**
		Enter 4 for RC 62 Fe ball, RC 62 Fe groove		RC 62 Steel	**0.29**
		Enter 5 for other values and enter them for each ball and groove			
Min. yield strength (Pa, psi)	######	262500			

Applied forces' Z,Y,Z values and coordinates		**Coupling centroid**		
FLx =	**2000.00**	XL =	**0.000**	xc	0.000
FLy =	**0.00**	YL =	**0.000**	yc	0.000
FLz =	**0.00**	ZL =	**0.400**	zc	0.000
					146323.536

Results: Hertz stresses and deformations
Error displacements at the point of interest (micron)

DeltaX	-1.89E-05	DeltaY	-1.17E-06	DeltaZ	-6.49E-07	-6.49E-07

							Contact ellipse	
Groove normal forces		*Contact stress*		*Stress/Allow*	*Deflection (+into ball)*		*Major radius*	*Minor radius*
Fbnone	***4.48E+03***	sigone	***1.01E+09***	***0.56***	delone	***2.22E-06***	***4.73E-03***	***4.47E-04***
Fbntwo	***2.59E+03***	sigtwo	***8.41E+08***	***0.46***	deltwo	***-2.43E-06***	***3.94E-03***	***3.73E-04***
Ball-Groove 2							*Contact ellipse*	
Groove normal forces		*Contact stress*			*Deflection*		*Major radius*	*Minor radius*
Fbnthree	***4.51E+02***	sigthree	***4.70E+08***	***0.26***	delthree	***-9.69E-06***	***2.20E-03***	***2.08E-04***
Fbnfour	***1.39E+03***	sigfour	***6.84E+08***	***0.38***	delfour	***-6.00E-06***	***3.21E-03***	***3.03E-04***
Ball-Groove 3							*Contact ellipse*	
Groove normal forces		*Contact stress*			*Deflection*		*Major radius*	*Minor radius*
Fbnfive	***5.68E+03***	sigfive	***1.09E+09***	***0.60***	delfive	***4.82E-06***	***5.12E-03***	***4.84E-04***
Fbnsix	***6.62E+03***	sigsix	***1.15E+09***	***0.63***	delsix	***6.74E-06***	***5.39E-03***	***5.10E-04***

Results: Error motions

Error motions are at X,Y,Z coordinates:		***0.000***	***0.000***	***0.400***	
deltaX	**-1.89E-05**				
deltaY	**-1.17E-06**	*Homogenous Transformation Matrix:*			
deltaZ	**-6.49E-07**	**1.00E+00**	**9.13E-07**	**-4.45E-05**	**-1.11E-06**
EpsX	**3.50E-06**	**-9.13E-07**	**1.00E+00**	**-3.50E-06**	**2.29E-07**
EpsY	**-4.45E-05**	**4.45E-05**	**3.50E-06**	**1.00E+00**	**-6.49E-07**
EpsZ	**-9.13E-07**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**1.00E+00**

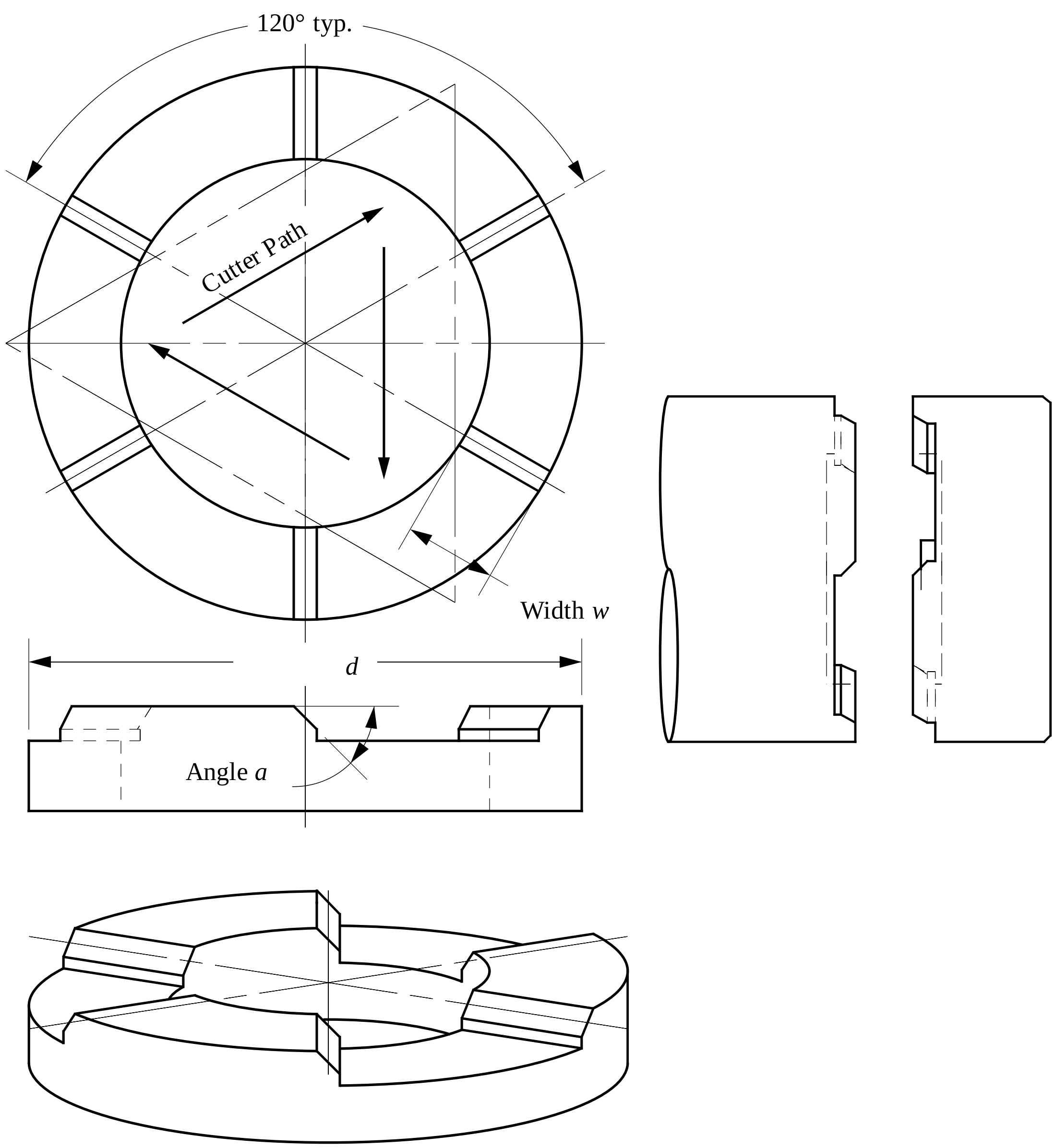
Summary

- **The analysis methods described above were successfully implemented on a spreadsheet[3].**
- **In order to test the spreadsheet:**
  - **Forces were applied along axes of symmetry.**
  - **It was checked to ensure that the expected displacements were obtained.**
  - **For example, a Z direction force should yield equal forces at all the contact points and only a z displacement should occur.**
  - **For all test cases, the results were as expected.**
  - **The magnitude of the errors corresponded well with those found in earlier experiments.**
- **The theory implemented on the spreadsheet provides the designer of precision fixtures with a very useful tool.**

3 You should be using a version of the spreadsheet that has a date of 4/5/92 or later. An updated version can be obtained from the ASPE, P.O. Box 10826, Raleigh NC 27605-0826 (phone 919 839 8444).

Three tooth couplings: The Hale Coupling[4]

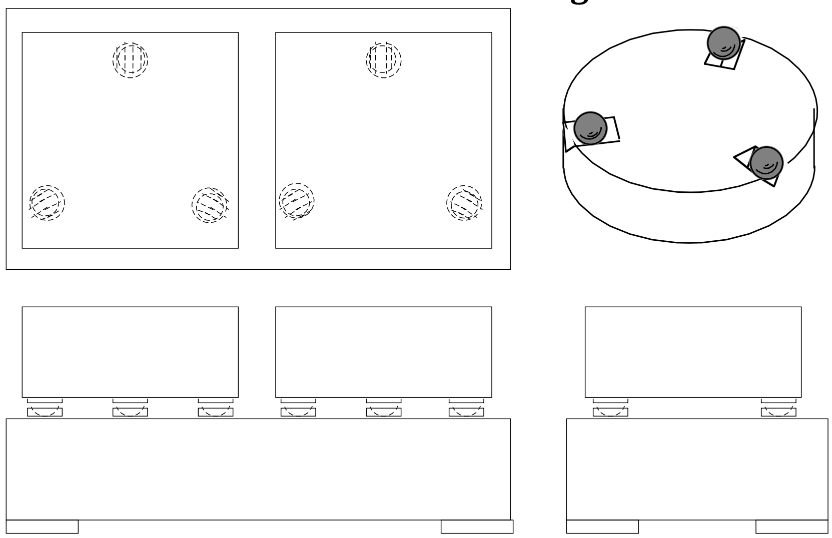
- **Virtual balls and grooves can be used:**
    - **Only the portion of the ball and the groove that are at the 45 degree latitude are used.**
- **When the load capacity cannot be attained, a quasi-kinematic coupling can be used with line/surface contact:**



4 Invented by Layton Hale of Lawrence Livermore National Laboratory.

Gallary of Kineamtic Coupling Usage

- **Conventional: to aid in machines being modular**



Kinematic Fluid Coupling: Utilizes Hertz Contact area as a tight seal-less seal![5]

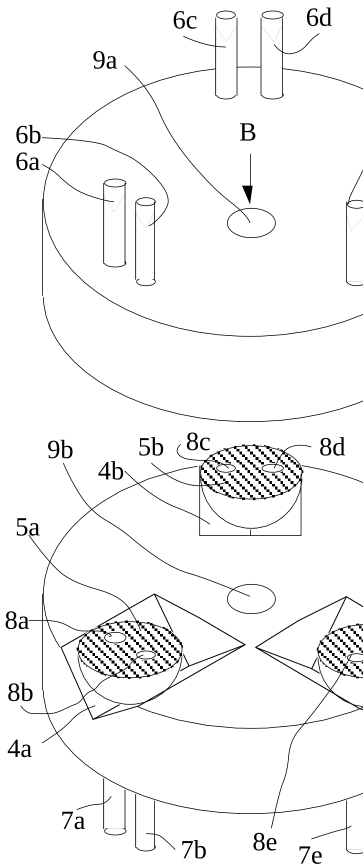


Fig. 1

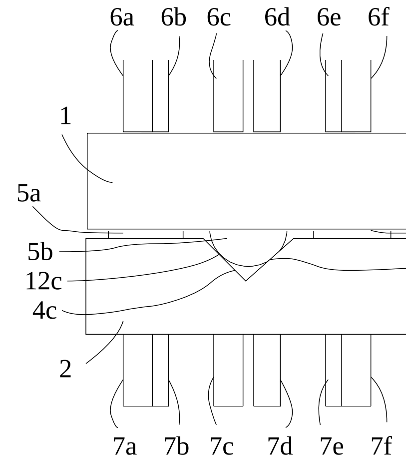


Fig. 2

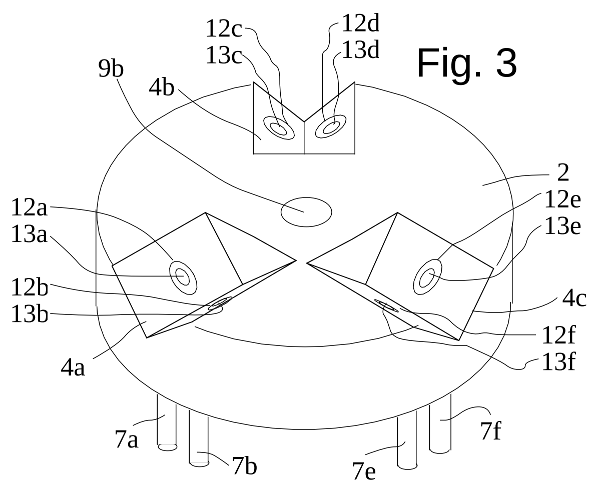


Fig. 3

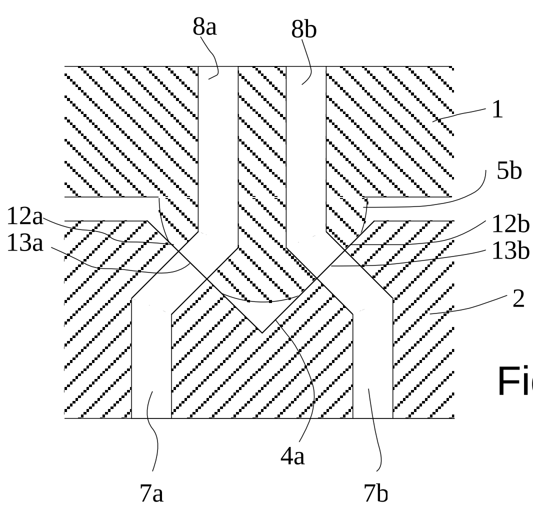


Fig. 4

[5] US Patent pending, Aesop, Inc. Concord, NH fax 603-224-5369

Kinematic alignment of two soft-bodies using a nard ball that indents[6]. Used for aligning sand cores, OR parts that align and then mate when bolted together.

6 US Patent pending, Aesop, Inc. Concord, NH fax 603-224-5369

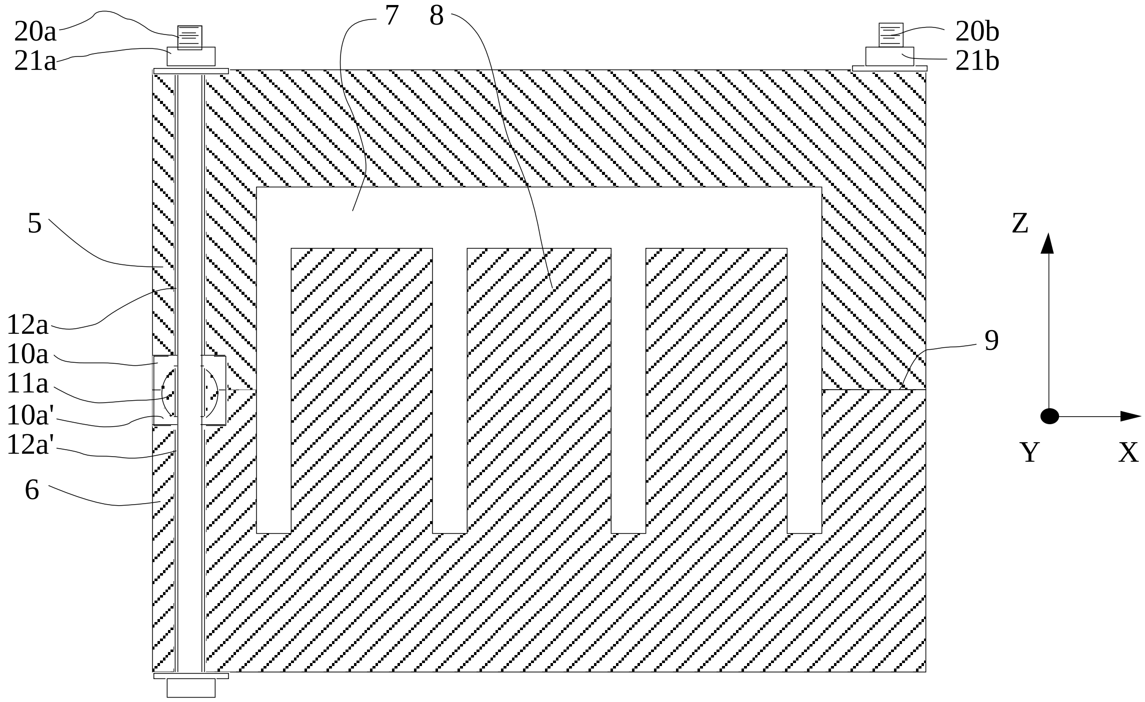


Fig. 1

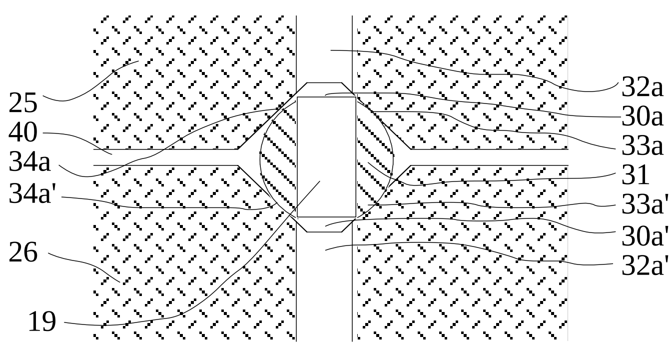


Fig. 2a

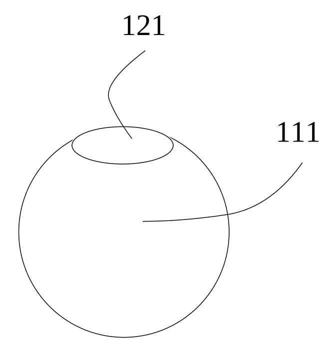


Fig. 3

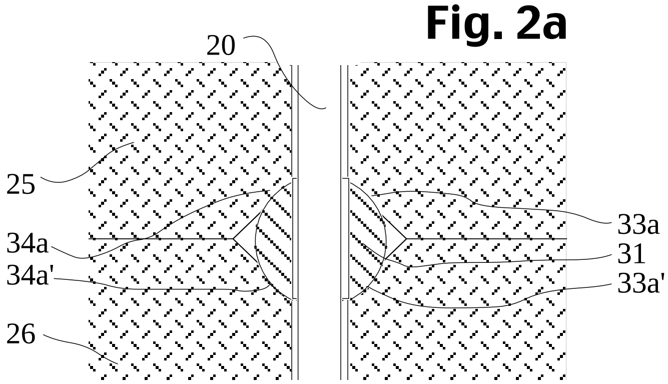


Fig. 2b

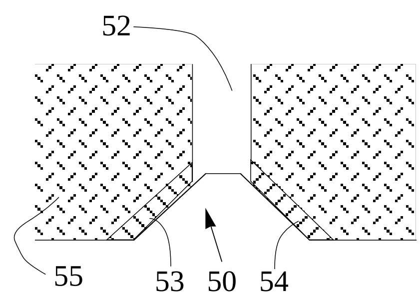
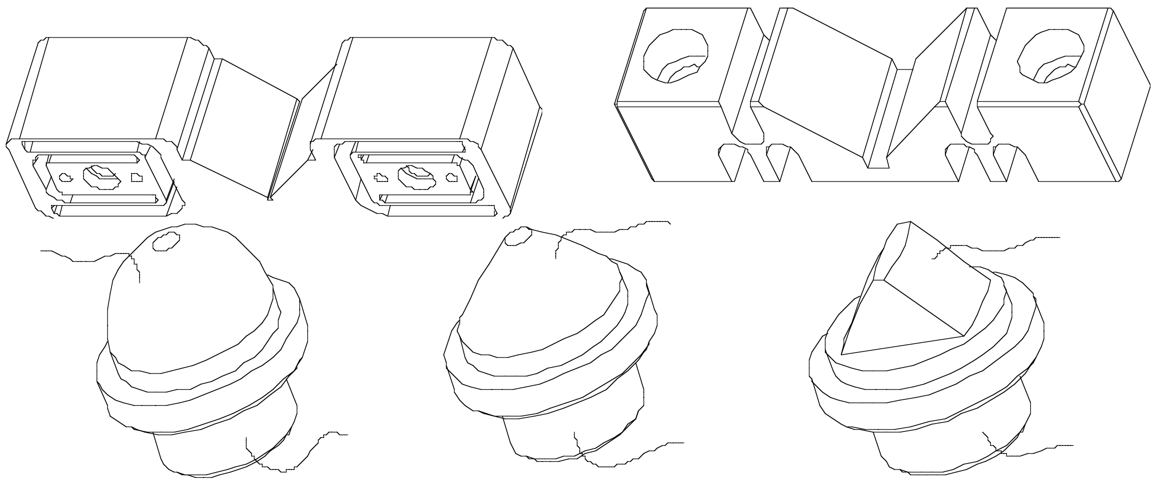
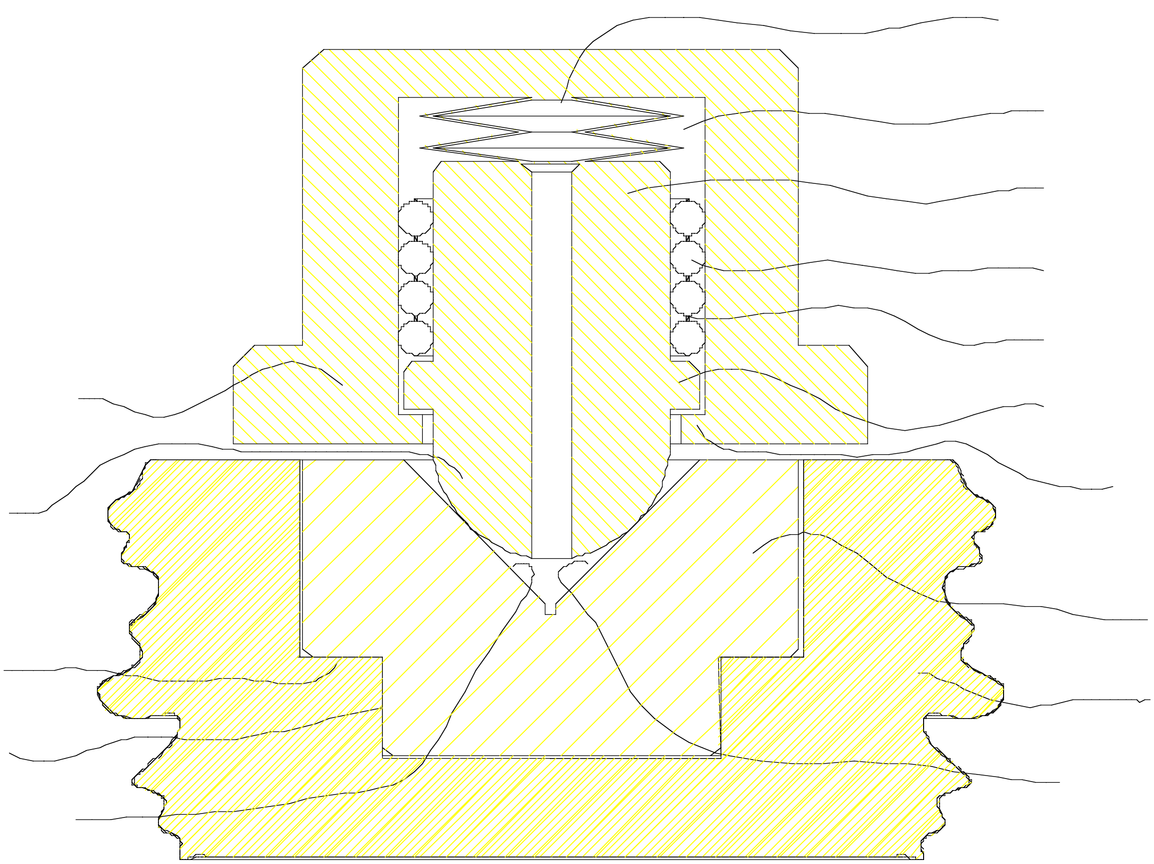


Fig. 4

Translational Kinematic Coupling[7]

To allow for precision alighnment in a plane prior to clamping





[7] US Patent 5,678,944 Aesop, Inc. Concord, NH fax 603-224-5369

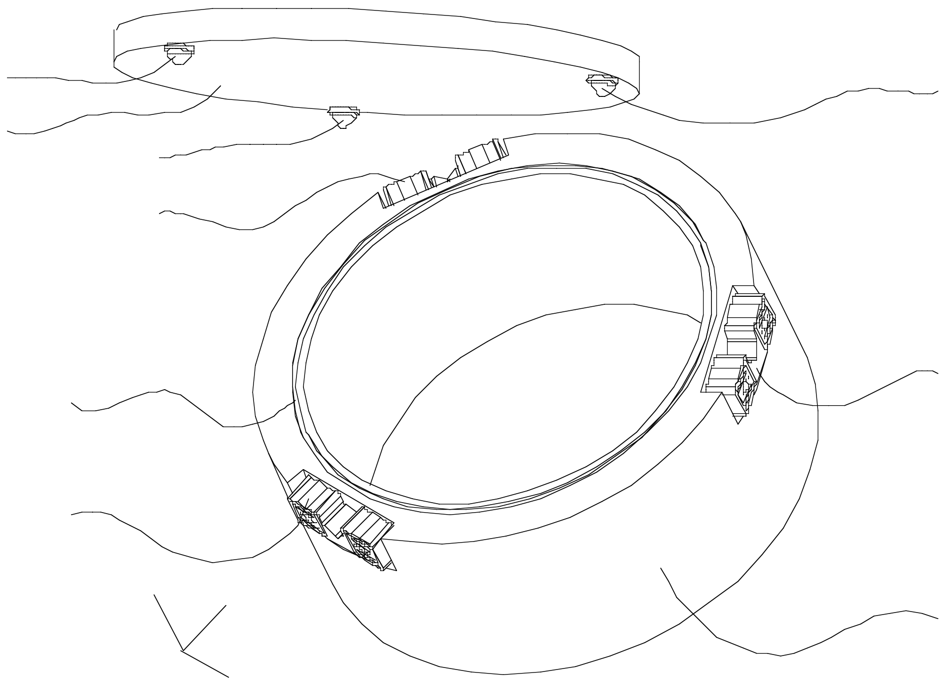


FIG. 5

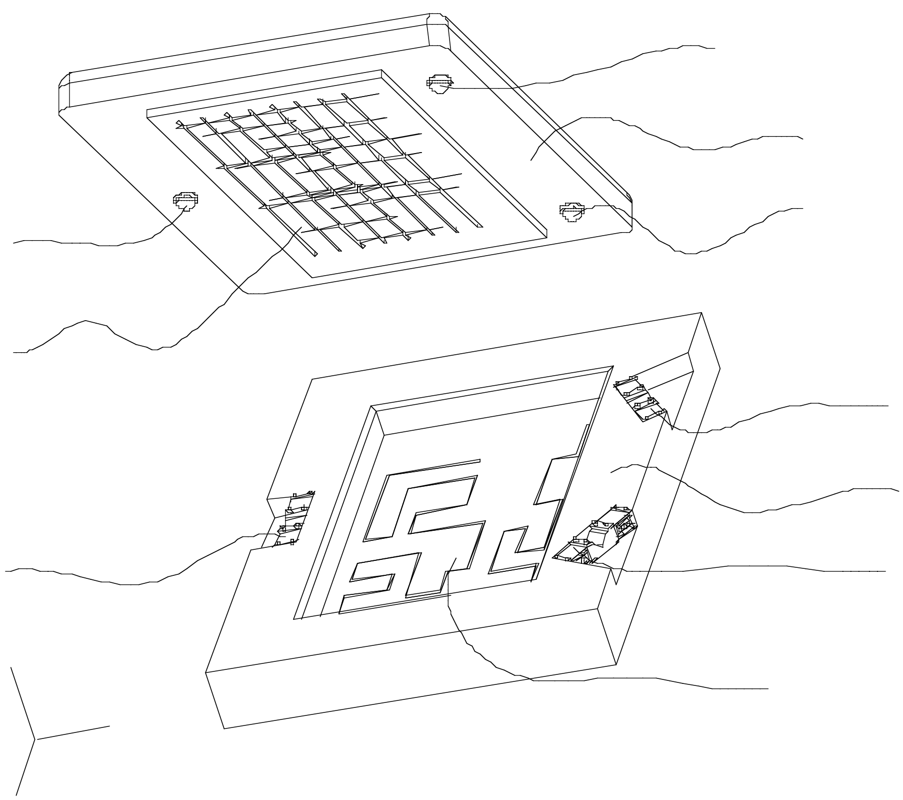


FIG. 6

Kinematic thin sheet alignment (for making printed circuit boards):[8]

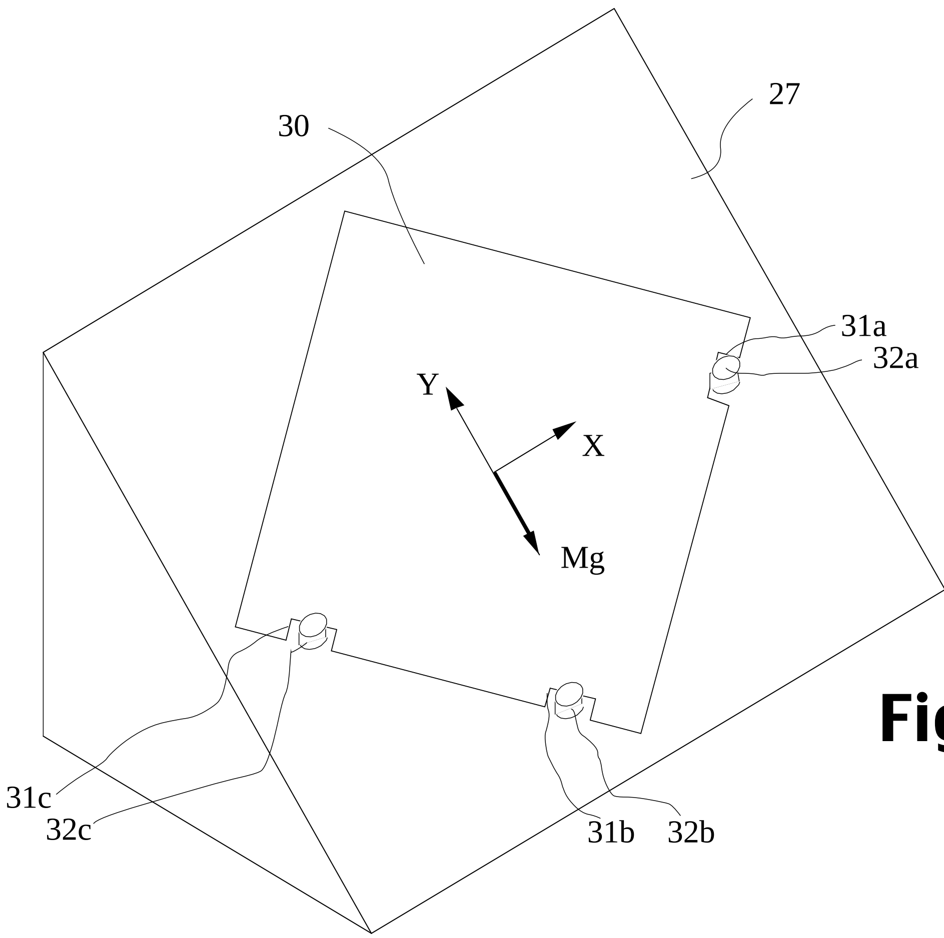


Fig. 2

[8] US Patent pending, Aesop, Inc. Concord, NH fax 603-224-5369