Precision Machine Design

Topic 6

Servo system effects on machine design

Purpose:

The machine designer must be aware of the issues that the control system designer will face, in order for concurrent engineering to be effectively practiced.

Outline:

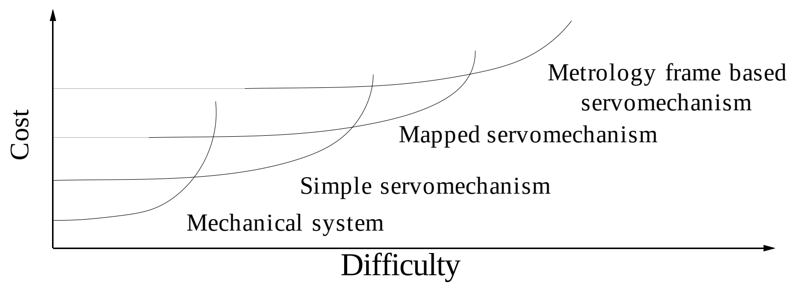
- **Introduction**
- **Dynamic characterization of systems**
- **Servo bandwidth**
- **Types of controllers**
- **Sensor mounting**

"They are able because they think they are able."

Virgil

Introduction

- **Modern precision machines could not function without servo systems:**



- **It is vital to consider the servo system design in conjunction with the mechanical system design.**

- **In the past, actuator (ballscrew) inertia so dominated the system:**
  - **Simple fixed gain controllers were adequate for almost all applications.**
  - **Mechanical friction allowed for the use of phase lag (controllers with an integrator) controllers.**

- **As faster speeds, greater contouring accuracy, longer life, and/or abrasive environment applications become more common:**
  - **Linear electric motors will become more common.**
    - **Damping will become ever more important.**
  - **The need will grow for adaptive phase lead controllers.**
  - **The need will grow for more accurate sensors with time synchronous feedback to allow for accurate numerical differentiation.**

- **Often engineers use modular controllers (e.g., a card in a PC) without developing good dynamic models of the system.**
  - **A good model is needed for proper tuning.**
- **Develop a dynamic model of the system.**
  - **Model elements as springs, masses, dampers, (resistors, inductors, capacitors).**
  - **Use controller design techniques to see what performance levels can be achieved.**
  - **Simulate system response with different controllers and conditions.**
    - **Ballscrew driven systems have a very large drive inertia (the ballscrew) and are easier to control.**
    - **Linear electric motor driven systems have a low drive inertia and are harder to control.**
    - **The quality of the position and velocity feedback can be crucial to the controller.**
  - **Use the results to guide design modifications:**
    - **Modify mechanical systems, *before they are built*, to obtain the desired performance.**

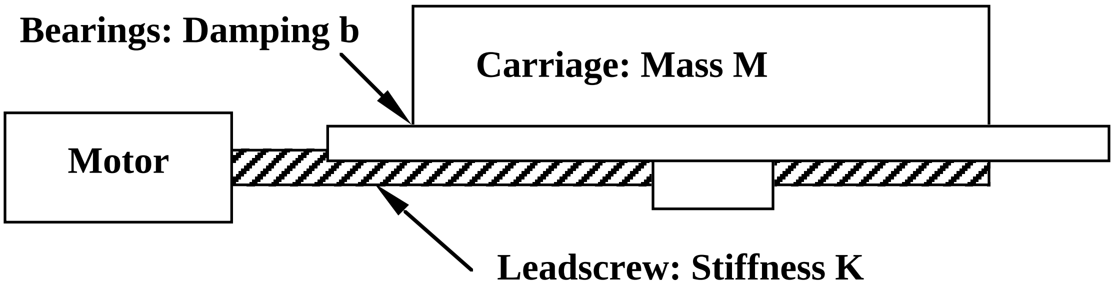
- **For precision machines, servo systems can be used to control the following:**
    - **Position of moving axes.**
    - **Velocity (and higher derivatives) of moving axes.**
    - **Temperature.**
- **The details of servo system design are beyond the scope of this course.**
- **Dynamic characterization of systems should be understood:**
    - **It can influence decisions regarding selection of components.**
- **Designers must also be able to make preliminary selection of sensor components so the design process can proceed.**
- **Controller design can proceed as a parallel effort.**

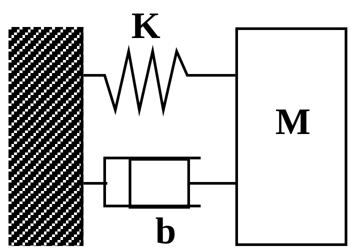
Dynamic characterization of systems:

- **In order to design a servo system for a machine:**
    - **The dynamic characteristics of the machine, sensors, and actuators must be known.**
- **System output depends on the characteristics of the system and the frequency of the input.**
- **The frequency response of a system is the system's ability to respond to changes in the input.**
- **Mass, stiffness, and damping of a system affect the relative magnitude and phase of the output with respect to the input.**
- **Generally, the faster the process, the less accurate it is.**
- **Example:**
    - **The human eye cannot see a good video terminal flicker because the screen is updated at 60 times per second**
    - **The human eye has a greater overall range of perception (e.g. color, shape, texture, size, depth of field...) than perhaps any artificial vision system.**

Frequency domain analysis of dynamic systems:

- **Consider a mass, spring, damper system:**





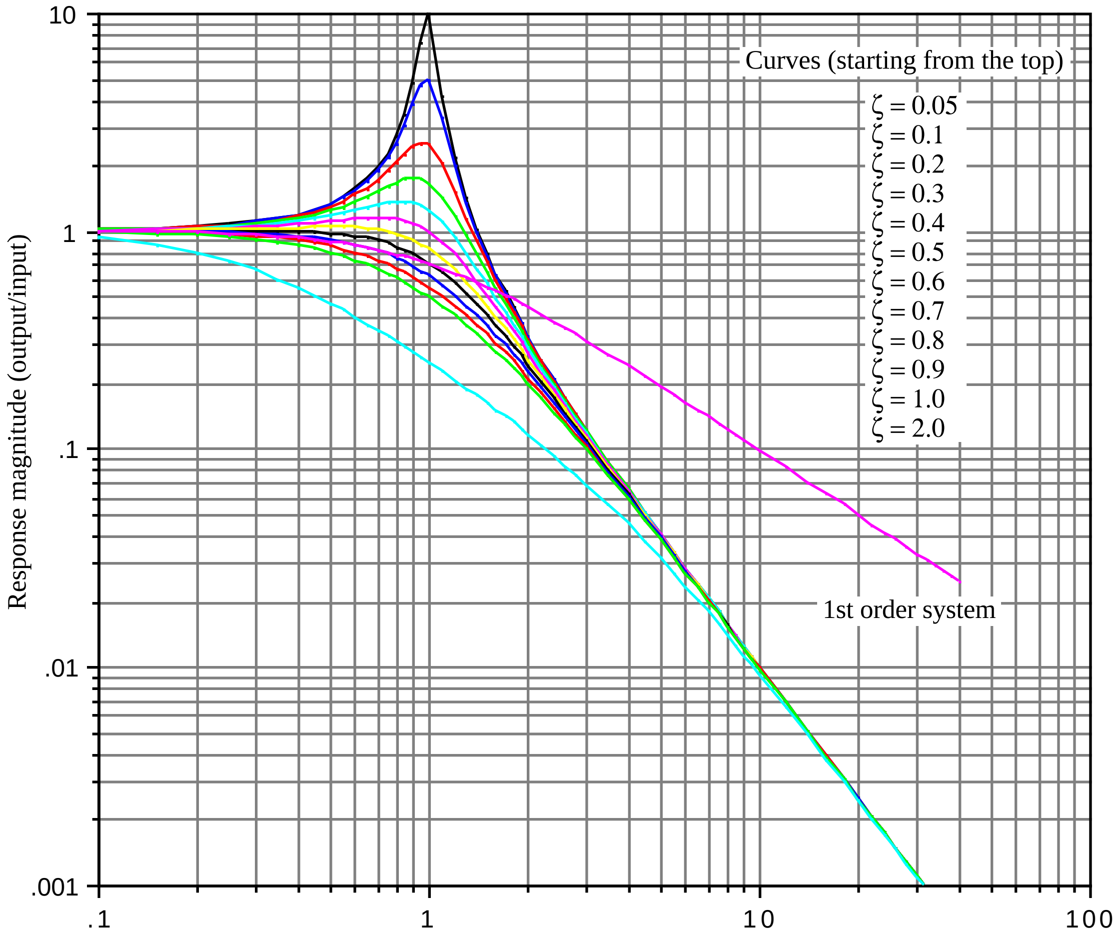
$$\frac{md^2x}{dt^2} + \frac{bdx}{dt} + kx = u(t)$$

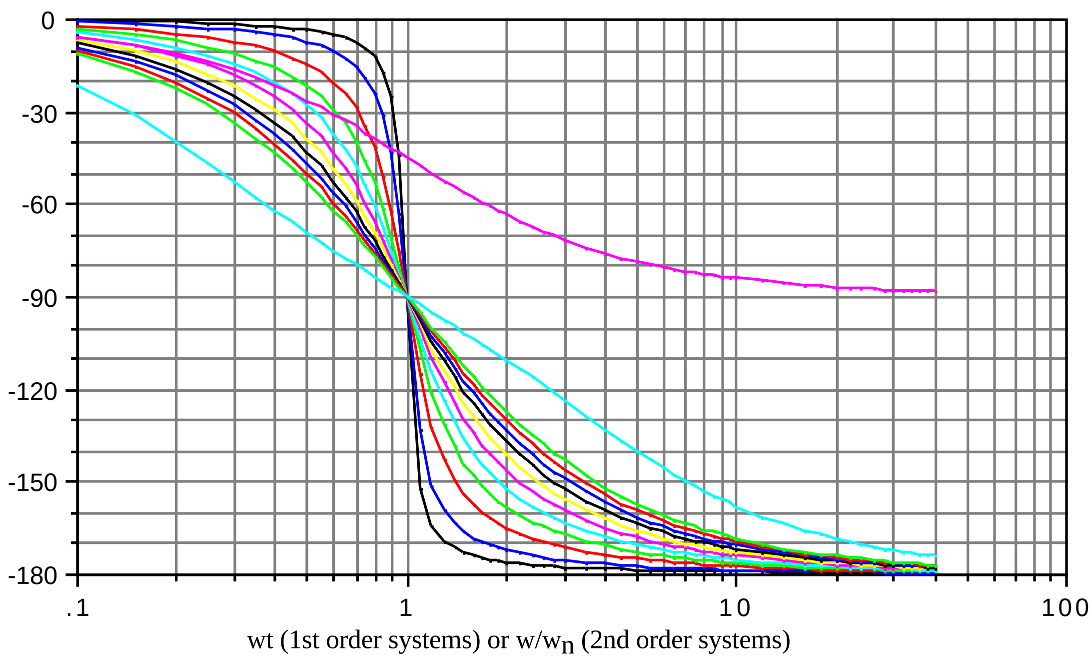
- **When substituted into 3.1.3 the equations of motion are:**

$$ms^2x + bsx + kx = U(s)$$

- **U(s) = 1 for a unit impulse and U(s) = 1/s for a unit step.**

- **Magnitude and phase of first and second order systems:**





Checking system servo bandwidth

- **Most systems null high frequency disturbance forces with their own mass.**
- **Lower frequency forces must be offset by forces from the controller/actuator.**
- **The servo bandwidth required is a function of the types of moves that the system will be required to make.**
  - **For start and stop moves (e.g., in a wafer stepper), the bandwidth should be on the order of:**

$$f(Hz) \approx \frac{10}{2\pi t_{settlingtime}}$$

  - **When contouring, the X and Y axes move according to:**

$$x = A\sin\omega t$$
$$y = B\cos\omega t$$

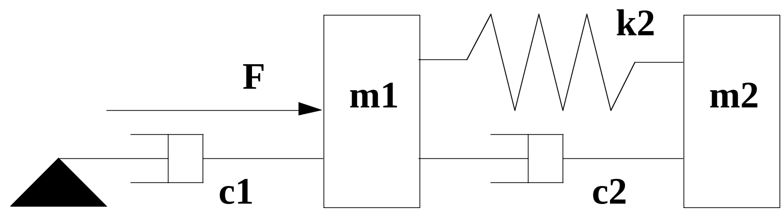
    - **The frequency *w* is a function of the linear velocity of the cutter through the material and the radius of the contour:**

$$\omega_{angular} = \frac{v_{linear}}{r_{pathradius}}$$

$$f(Hz) = \frac{v_{linear}}{2\pi r_{pathradius}}$$

    - **For a large circle (e.g., 200 mm D) being cut at modest speeds (e.g., 0.1 m/s), the bandwidth required is only 0.15 Hz.**
    - **For a sharp turn in a corner, the radius of the cutter path trajectory may only be 1 mm, so *w* > 15 Hz.**

- **A system with a motor driving a carriage can be modeled in the following manner:**



- **m_1 is the mass of a linear motor forcer or:**

- **m_1 is the reflected inertia of the motor rotor and leadscrew (or just a linear motor's moving part):**

$$M_{reflected} = \frac{4\pi^2 J}{l^2}$$

- **M_2 is the mass of the carriage.**

- **C_1 is the damping in the linear and rotary bearings.**

- **C_2 is the damping in the actuator-carriage coupling and the carriage structure.**

- **K_2 is the stiffness of the actuator and actuator-carriage-tool structural loop.**

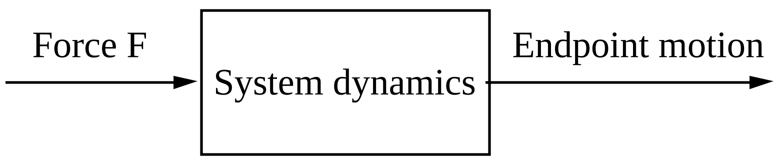
- **The equations of motion are:**

$$\begin{vmatrix} m_1 & 0 \\ 0 & m_2 \end{vmatrix} \begin{vmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{vmatrix} + \begin{vmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{vmatrix} \begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} + \begin{vmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} F(t) \\ 0 \end{vmatrix}$$

- **The transfer function x2/F (dynamic response of the carriage) is:**

$$\frac{x_2}{F} = \frac{k_2 + c_2 s}{k_1 k_2 + (c_2 k_1 + c_1 k_2)s + (c_1 c_2 + k_2 m_1 + k_1 m_2 + + k_2 m_2) s^2 + (c_2 m_1 + c_1 m_2 + c_2 m_2) s^3 + m_1 m_2 s^4}$$

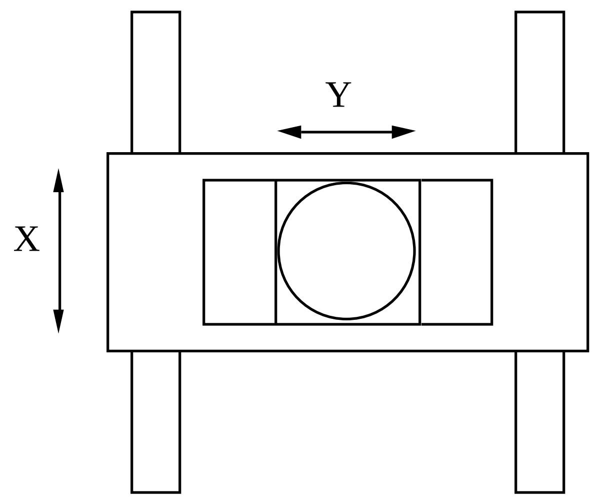
- **Note the product of the masses term which tends to dominate the system.**



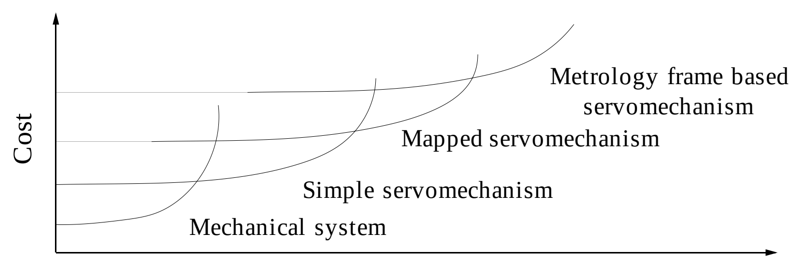
- **Calculated parameters of four possible systems are:**

Actuator	**ballscrew**	**lin. motor**	**lin. motor**	**lin. motor**
Bearings	**linear ball**	**linear ball**	**air**	**air**
Structural damping	**no**	**no**	**no**	**yes**
material damping zeta	**0.005**	**0.005**	**0.005**	**0.1**
actuator to ground zeta	**0.05**	**0.03**	**0**	**0**
m1 (actuator) (kg)	**50**	**5**	**5**	**5**
m2 (carriage), kg	**50**	**50**	**50**	**50**
c1 (N/m/s)	**355**	**187**	**0**	**0**
c2 (N/m/s)	**19**	**19**	**19**	**374**
k1 (N/m)	**1.75E+08**	**1.75E+08**	**1.75E+08**	**1.75E+08**
Bandwidth (Hz)	**25**	**100**	**30**	**100**

- **Example: the goal is to be able to move the bridge in the X direction without exciting resonances.**

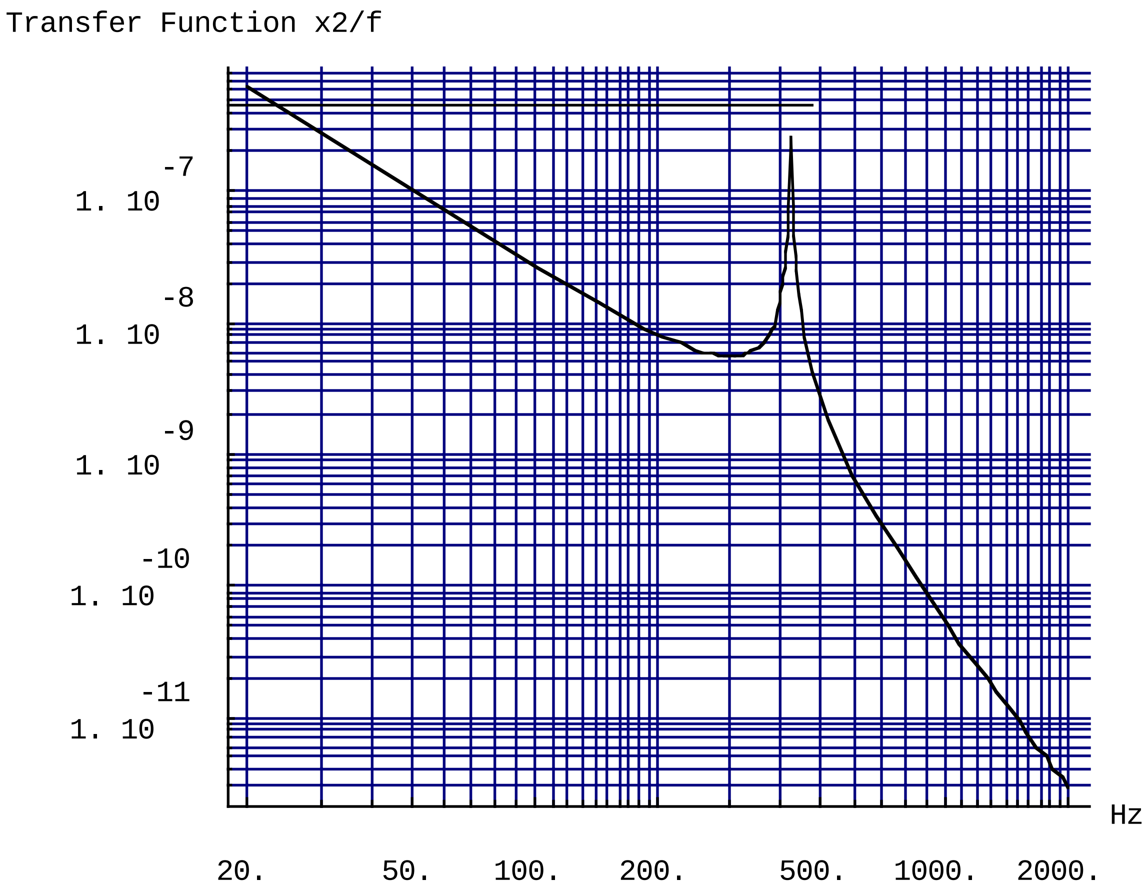


- **As a guideline, the servo bandwidth of the system is:**
  - **Generally limited by the frequency the servo can drive the system at without exciting structural modes.**
  - **Without special control techniques can be no higher than the frequency:**
    - **Found by drawing a horizontal line 3 dB above the resonant peak to intersect the response curve.**
- **This method is used only to initially size components.**
- **A detailed controls simulation must be done to verify performance, and guide further system optimization.**
- **Use sensor and software updates to create a “new machine” even when the mechanical components are not changed:**



$$\text{Difficulty} = \frac{\text{Environment} \times \text{Load} \times \text{Range} \times \text{Speed}}{\text{Accuracy}}$$

- **The response of the ballscrew driven carriage supported by rolling element linear bearings will be:**



- **The system bandwidth will be limited to about 25 Hz.**
- **Preloaded linear guides and a ballscrew provide good axial damping.**
- **The inertia of the screw will lower the system frequency considerably (note the $m_1m_2s^4$ term in the TF)**
- **The Ballscrew was inertia matched to the carriage:**

Diameter	**0.025**
Lead	**0.003**
Length	**0.5**
Axial K (N/µm)	**203**
J	**9.6E-06**
Reflected inertia = $4\pi^2 J/l^2$	**42**

Basic control algorithms

Proportional (position feedback)

$K(X_{desired}-X_{actual})$

- **Moderate stability.**
- **Poor following ability.**
- **Large steady state error.**
- **If system parameters change (M, K, b), response changes.**
- **Easy to implement with analog or digital components.**
- **Often used in conjunction with an analog tachometer-based velocity loop, thereby effectively yielding a PD controller.**

PD: Proportional + derivative (position and velocity feedback):

$$(X_{desired} - X_{actual})(K_P + sK_D)$$

- **Derivative term adds phase lead (predicts what is going to happen and compensates)**
- **Good stability.**
- **Good following ability.**
- **Moderate steady state error.**
- **If system parameters change (M, K, b), response changes.**
- **Easy to implement with analog or digital components.**
- **The derivative term is often obtained with velocity feedback to minimize errors caused by digital differentiation.**

PID: Proportional + integral + derivative (position and velocity feedback and an integrator)

$$(X_{desired}-X_{actual})(K_P + K_I/s + sK_D)$$

- **Integral term adds phase lag.**
- **Moderate stability.**
- **Moderate following ability.**
- **Zero steady state error.**
- **If system parameters change (M, K, b), response changes.**
- **Easy to implement with analog or digital components.**
- **The derivative term is often obtained with velocity feedback to avoid numerical errors.**

Full state feedback control

- **All states (voltage, position, velocity, acceleration...) are used in the control law.**
    - **Acceleration can be measured with an accelerometer, if it is mounted on a soft-mount, so vibration or shock do not send the loop unstable.**
- **When states are not directly measurable, an observer can sometimes be used to infer them.**
- **Often used on high speed, high performance systems.**

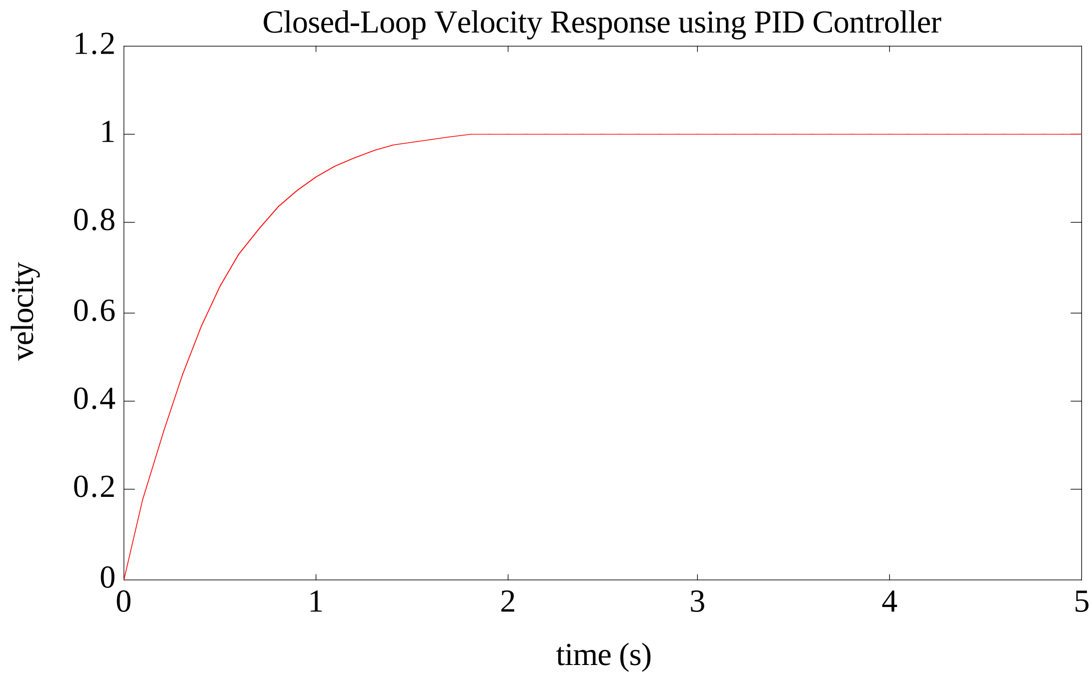
Adaptive control

- **The controller contains a model of the system.**
- **Changing dynamics (e.g., mass) are sensed and then compensated for.**

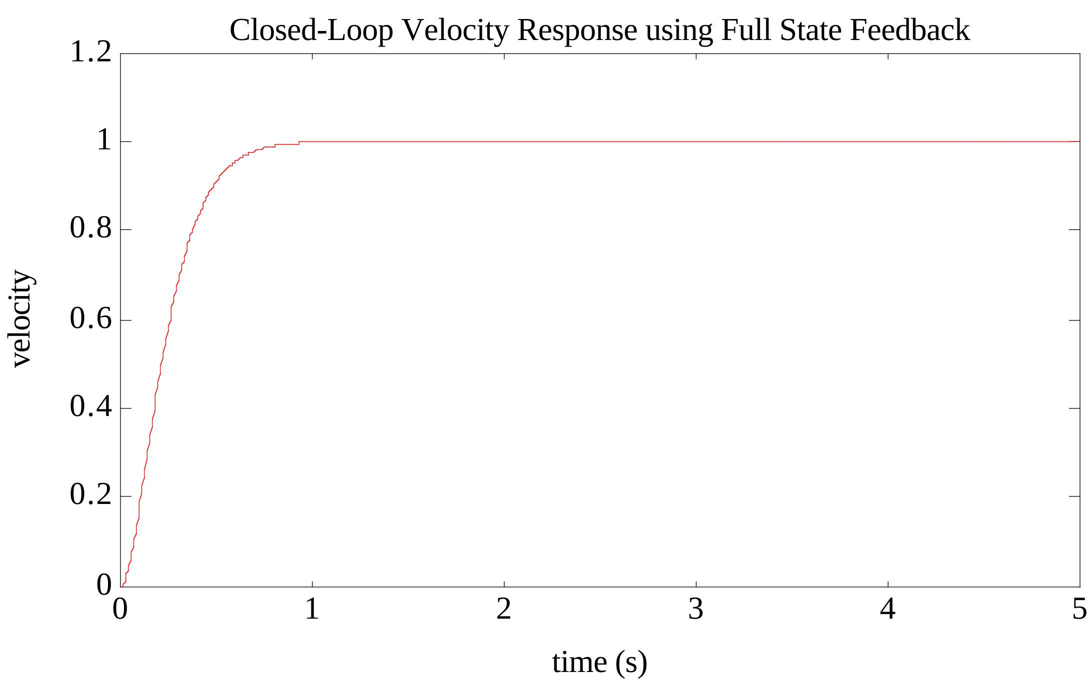
Gain Scheduling

- **The controller gains change with mapped parameters (e.g., changing inertia matrix of a robot arm).**

Closed-loop system response with a PID Controller:



Closed-loop system response with full state feedback control:



Advanced control algorithms: Input Shaping™

- **Advanced control algorithms can increase performance by an order of magnitude.**
- **Tall towers, robots, and other floppy structures are often used in automation equipment.**
  - **Floppy vibrating equipment can cause software errors.**
    - **The servo doesn't get were it was supposed to go in the allotted time because the structure was shaking.**
  - **Input Shaping™ is a very powerful tool that cancels vibrations with shaped input commands.**[1]
  - **It is better to use intelligence to overcome a problem, rather than brute force.**
  - **However, even martial arts masters have to keep fit.**
  - **Optimal performance comes about from the optimal combination of optimal components.**

1 Contact Neil Singer at Convolve Inc., 914-273-4042, neil@convolve.com

Example: Controlling robot vibration

- **The top of a tall robot tower for an XRZθ robot was vibrating excessively during high speed moves.**
- **Input Shaping™ was able to virtually eliminate any residual vibrations in the motion of the tower.**
- **Input Shaping™ was found to be very insensitive to modeling errors:**
  - **Changing the system frequency by 14% did not adversely affect the vibration reducing ability of the controller.**

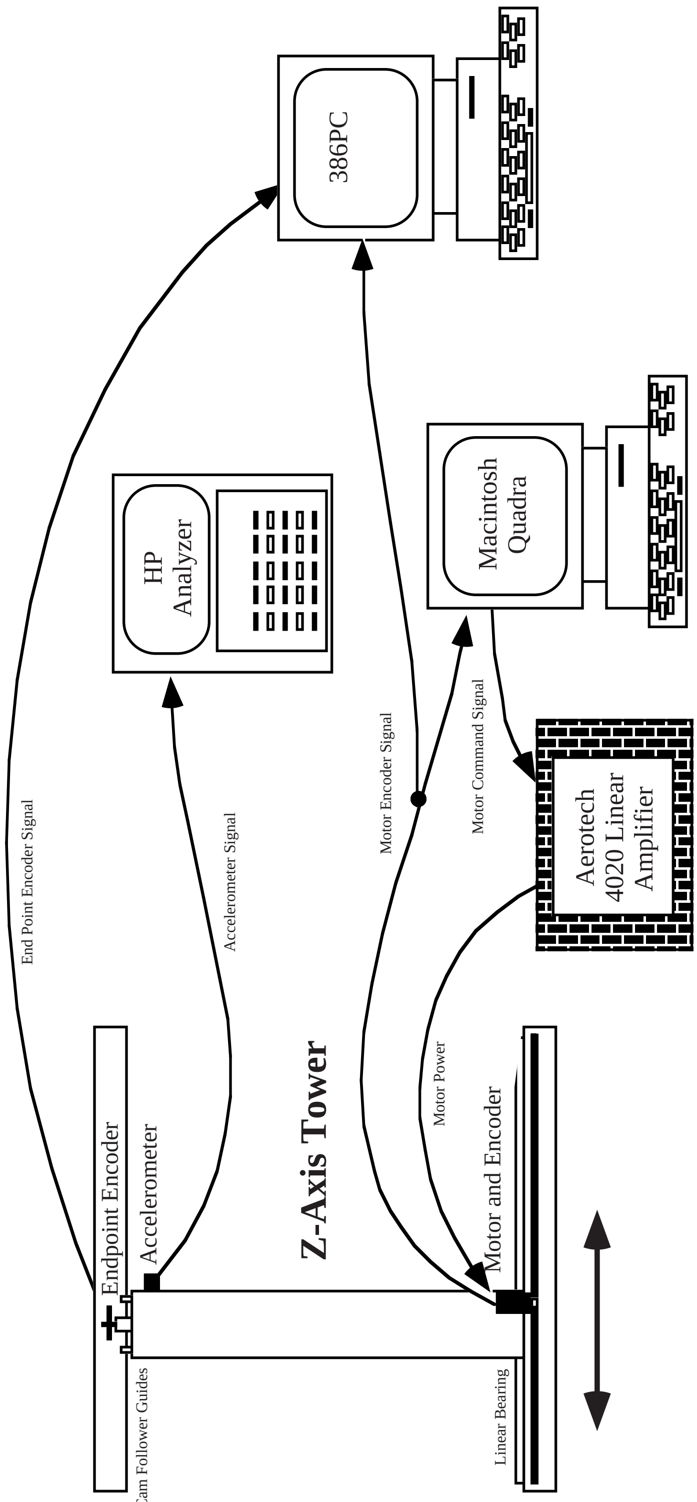
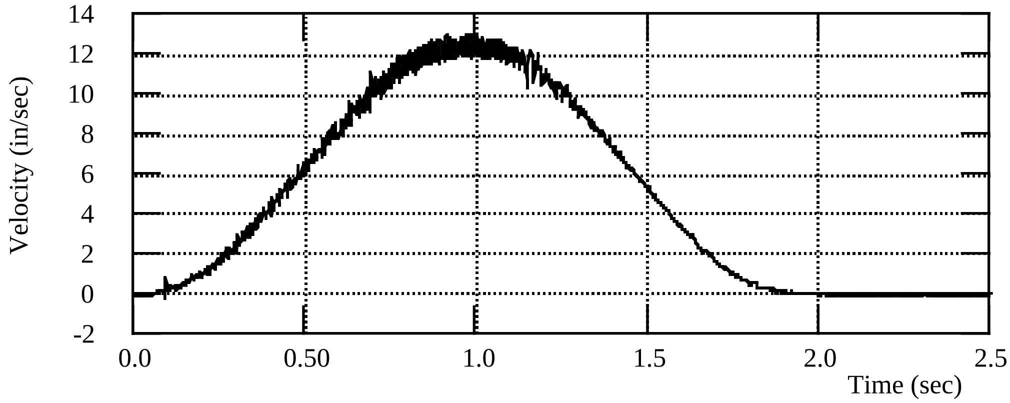
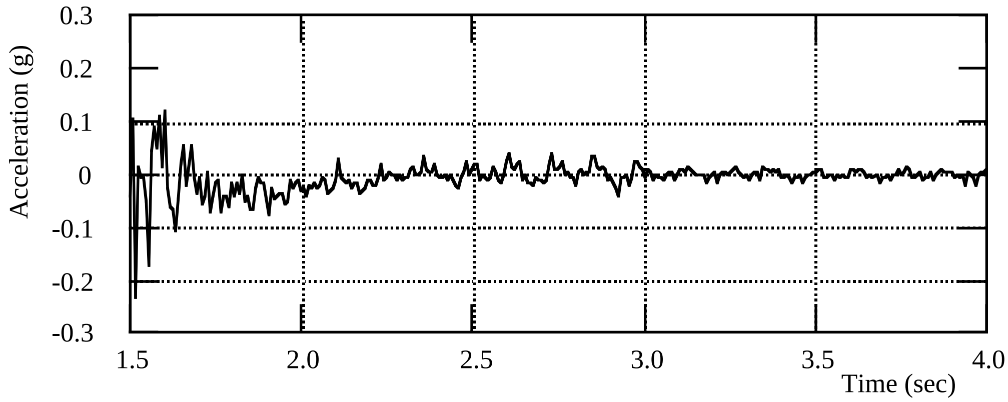


Figure 1: Experimental Setup

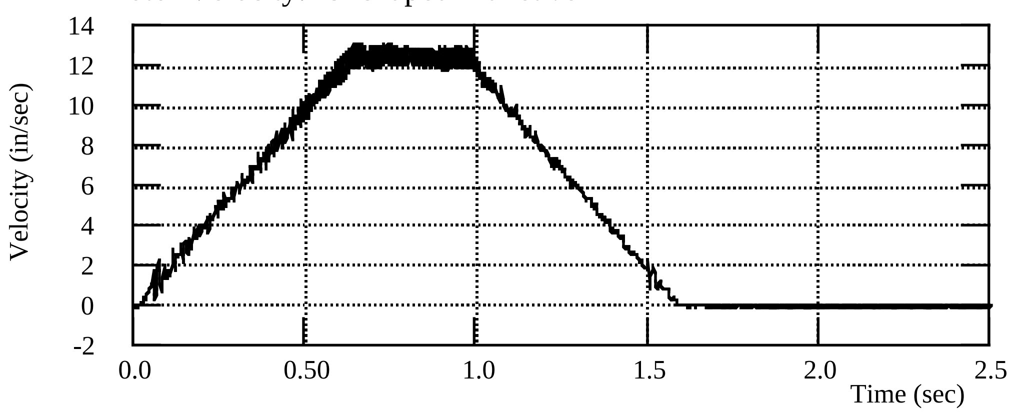
Motor Velocity: Shaped Maneuver with Tuned Sequence



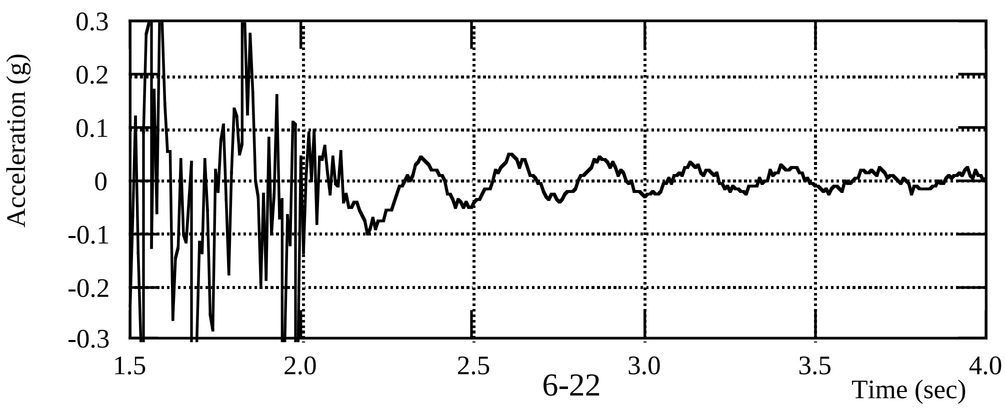
End Point Acceleration: Shaped Maneuver with Tuned Sequences



Motor Velocity: Unshaped Maneuver

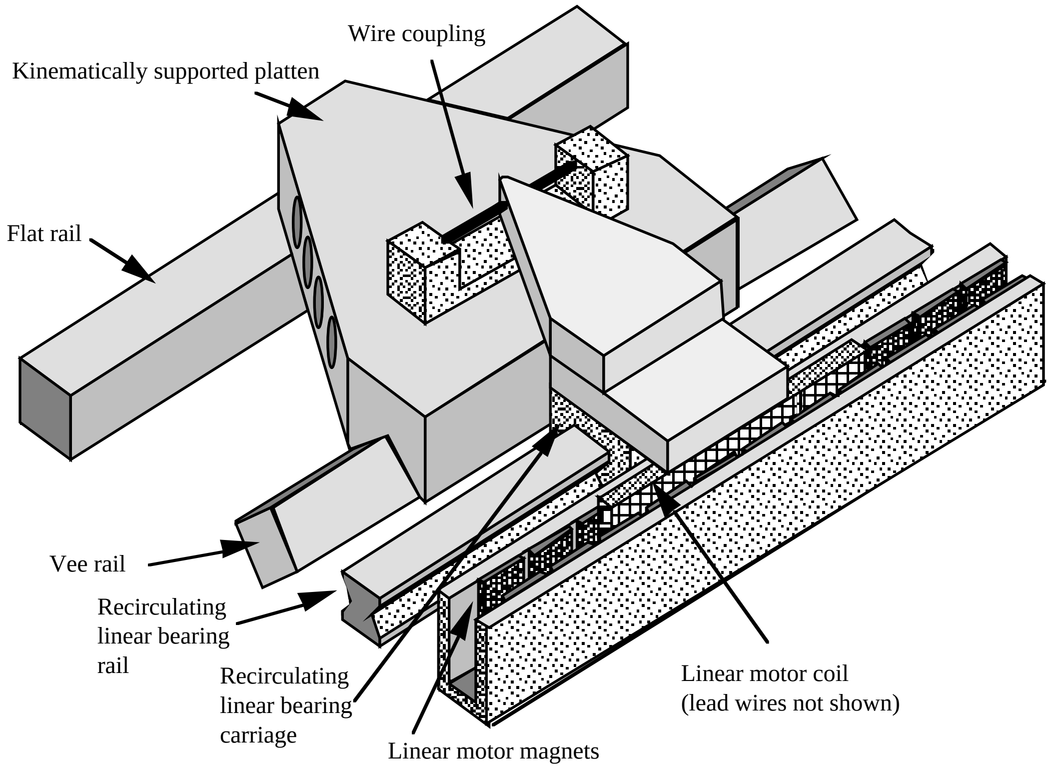


End Point Acceleration: Unshaped Maneuver*



Example: Increasing axial stiffness through dual loop control

- **Instrument platen with sliding contact bearings, linear motor, and wire-type flexural coupling:**



- **With the use of a flexible wire coupling, lateral errors and heat transfer from the motor will be greatly attenuated.**
- **However, the axial responsiveness of the platen will also be greatly reduced.**

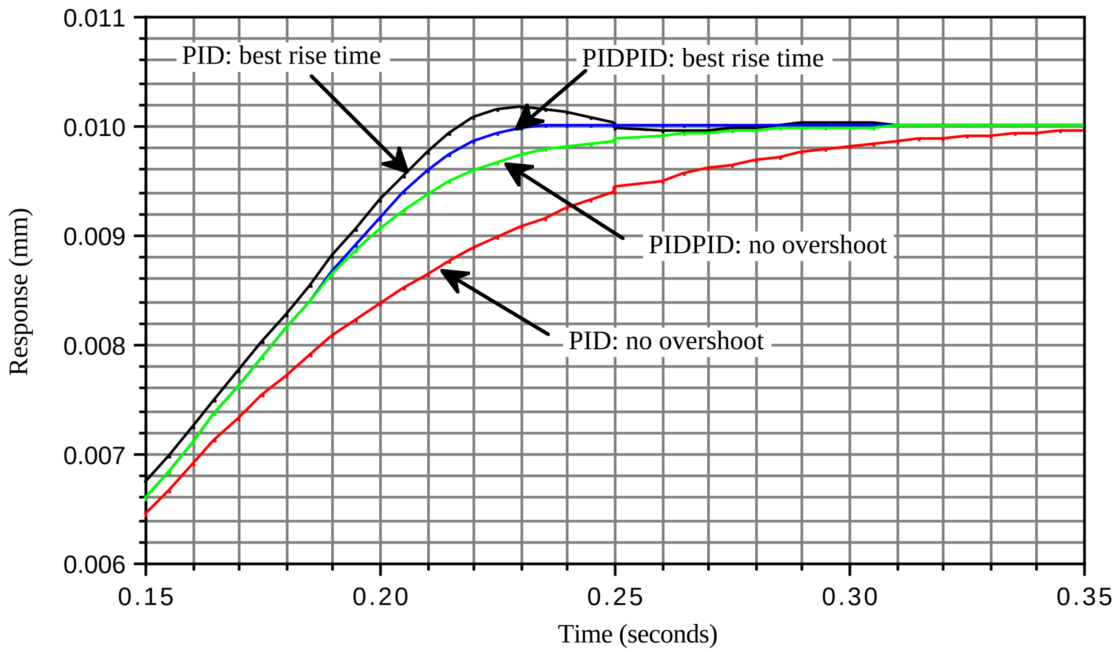
- **Implementation: close a PID loop around the position feedback from the platen.**
    - **Close a second loop (e.g. PD or PID) using the position feedback from the motor carriage.**
- **The logic used in arriving at this double algorithm is simple:**

  If the distance between the actuator and platen at rest is a stable constant, then after a move the stable distance must again be established, hence used a closed loop algorithm around the position of the platen and the motor so the difference between the two will be the equilibrium separation.

- **A section of the digital servo algorithm would look like:**

1 **Output u to DAC**
Read Lasers
$e_1 = x_1$desired - $x_1$actual (motor position error)
$e_2 = x_2$desired - $x_2$actual (platen position error)
$u_1 = a_{11}*e_1+a_{12}*e_{1old1}+a_{13}*e_{1old2}+$
$a_{14}*u_{1old1}+a_{15}*u_{1old2}$
IF ($u_1 > u_{max}/2$) THEN $u_1 = u_{max}/2$
$u_2 = a_{21}*e_2+a_{22}*e_{2old1}+a_{23}*e_{2old2}+$
$a_{24}*u_{2old1}+a_{25}*u_{2old2}$
IF ($u_2 > u_{max}/2$) THEN $u_2 = u_{max}/2$
{store old values}
$u = u_1 + u_2$
Wait for timing, then GOTO 1

- **Two degree of freedom system simulated fastest rise time with saturation and resolution non- linearities in the model:**



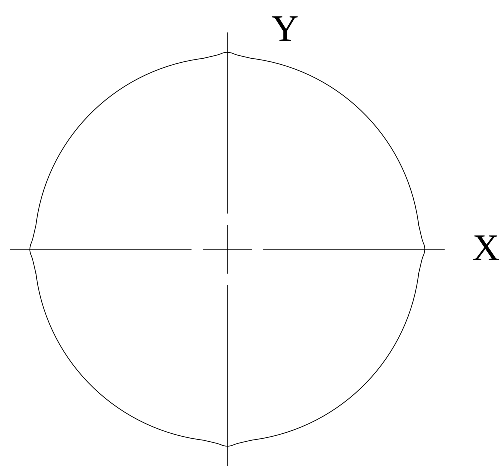
- **Many other algorithms exist that also increase performance using advanced algorithms.**
    - **For example, injecting a time delayed scaled impulse that cancels vibrations due to an initial impulse.**
- **The use of creative control techniques and multiple sensor measurements illustrates the following philosophies:**
    - **To minimize control problems, maximize stiffness and damping.**
    - **To maximize system performance:**
        - **One must often use one discipline to overcome deficiencies in another.**
- **This system, with its extra sensor, is the next step after the shaped input command to obtain greater performance.**

Velocity feedback

- **Analog velocity feedback is great, except that to be accurate at slow speeds requires a very expensive tachometer.**
- **Digital velocity obtained by differentiation is noisy at low speeds when a single loop is used:**

$$\text{Velocity} = \frac{\text{Pulses counted}}{\text{Fixed time increment } \tau}$$

  - **The encoder pulse can change between synchronized time steps.**
  - **When the pulses are divided by the fixed servo loop time:**
    - **Large errors can result if only one or two encoder pulses went by.**
    - **This occurs at zero velocity points (e.g., when a milling machine is used to cut a circle):**



- **Note where the X (Y) axis reverses direction, is a sensitive direction!**

- **Ideally, the controller has a separate loop which looks for an encoder pulse change, and then records the time.**

$$\text{Velocity} = \frac{\text{1 Pulse}}{\text{Measured time } \Delta\tau}$$

  - **This makes differentiation very accurate and not noisy.**
  - **When the synchronized loop requires a velocity value, an interpolated value is sent to it.**
  - **Ideally, the controller has an observer that predicts the velocity between updates.**
    - **This value is sent to the synchronized time control loop.**
    - **This can reduce the error by a factor of 2.**

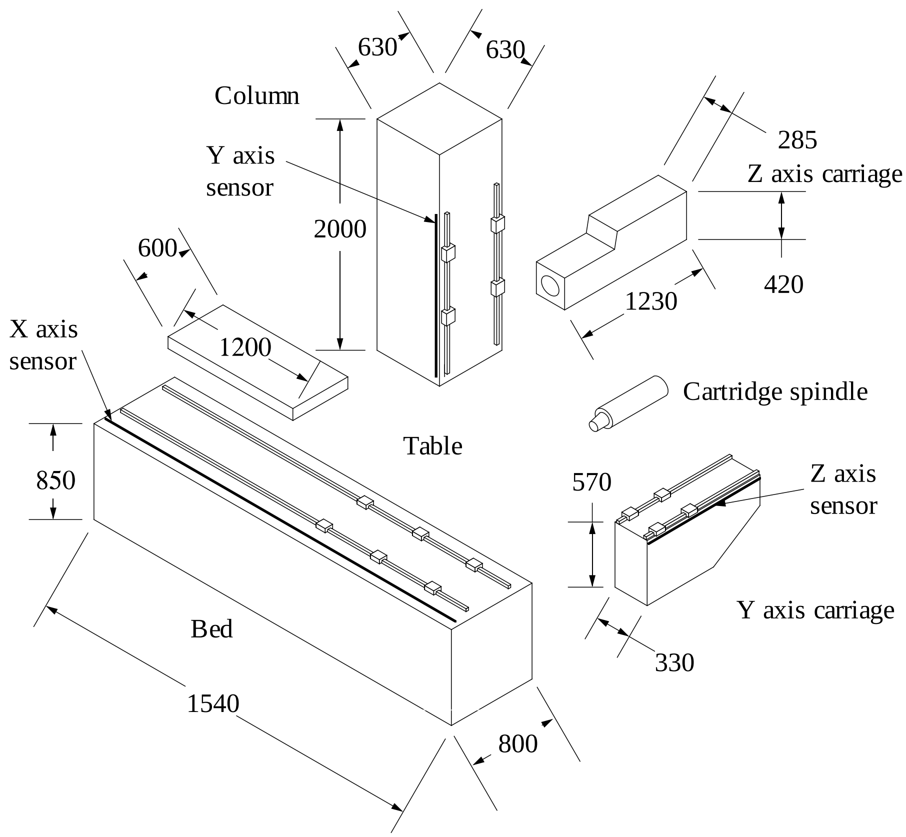
Design coupling between mechanical and control systems:

- **System mass:**
    - **The greater the system mass, the slower the dynamic response.**
    - **The greater the system mass, the more the system rejects high frequency disturbances.**
    - **In general, the lower the mass the better.**
- **System stiffness:**
    - **The greater the system stiffness, the faster the dynamic response.**
    - **As system stiffness increases, coupling between axes can increase.**
        - **For example, increasing the ballscrew stiffness axial, increases lateral stiffness.**
        - **This may increase straightness errors if the ballscrew "bumps" the linear axis.**
    - **In general, the stiffer the system the better.**

- **System damping:**
    - **The greater the damping, the less the amplification at resonance.**
    - **The greater the damping, the greater the attenuation of vibration.**
    - **The greater the damping, the better, unless it generates too much heat (in high speed systems).**
    - **Use "electronic damping" to help stabilize systems, and minimize heat generation at high velocity.**
- **Friction:**
    - **Static friction causes the system to store energy when it starts from zero velocity.**
    - **Dynamic friction is less than static friction near zero velocity and stored energy acts like an impulse to the system.**
    - **Difference in static and dynamic friction leads to the condition known as stiction.**
        - **One of the most troublesome control problems in precision machines.**
- **Hysteresis (non-linear spring) and backlash (gaps open and close):**
    - **When motion is reversed, there is a period of time where the motor is taking up the slack in the actuator (lost or greatly reduced motion).**
    - **This hesitation can be one of the largest sources of error.**
    - **Try to avoid hysteresis, and definitely avoid backlash.**

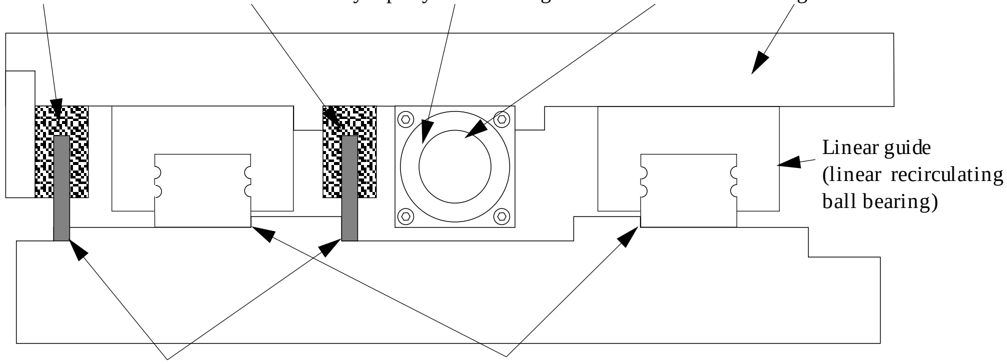
Linear position sensor location

- **The sensors must be carefully integrated into the system design.**
- **Example: Sensor locations in a precision surface grinder:**



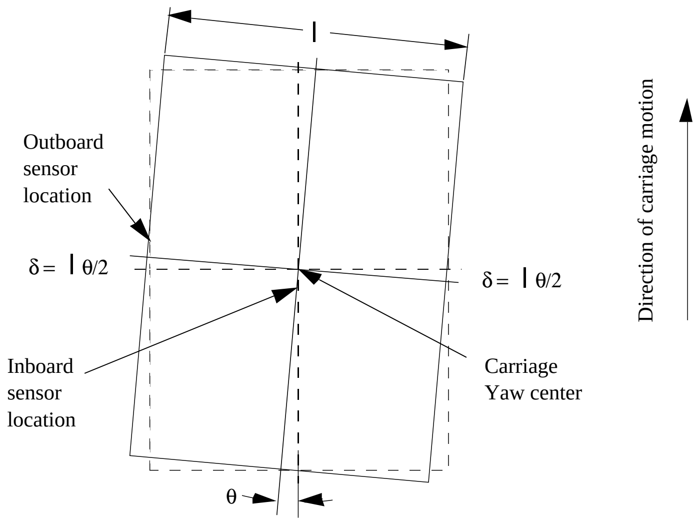
- **Y axis is between center of stiffness and tool**
- **A axis is non-sensitive so sensor placed where easy to mount and repair.**

- **Linear displacement sensor mounting locations:**



- **Inboard mounting:**
  - **Abbe error minimized.**
  - **Protection is offered by the table.**
  - **Alignment is more difficult because of access restriction. Use reference edges.**
  - **Closer to heat sources.**

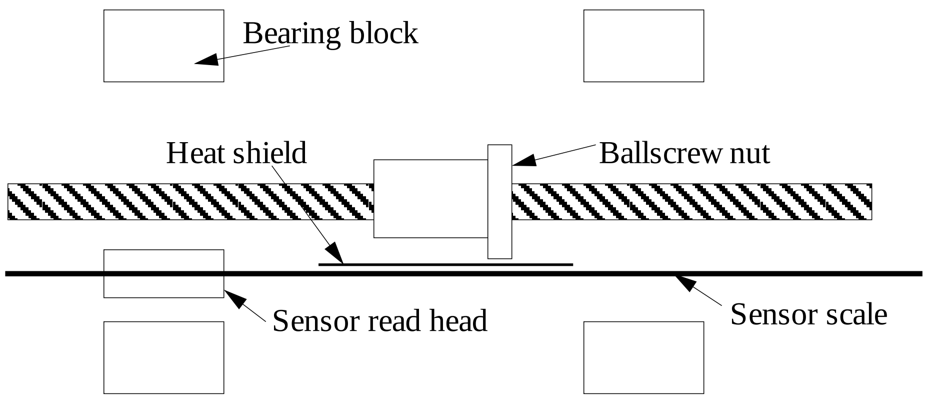
- **Outboard mounting:**
    - **Abbe error is large.**
    - **Less protection is offered by the table.**
    - **Alignment is simplified.**
    - **Far from heat sources.**
    - **Easy to install and maintain.**



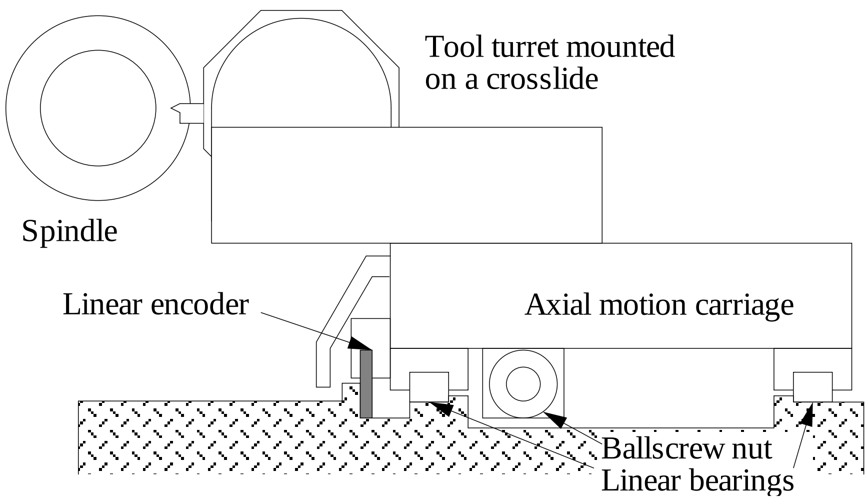
- **With error mapping, effect of yaw error, usually highly repeatable, is low.**

Examples

- **For a machining center table, place the read head away from the ballscrew nut.**
- **Place a heat shield between the ballscrew nut and the scale:**

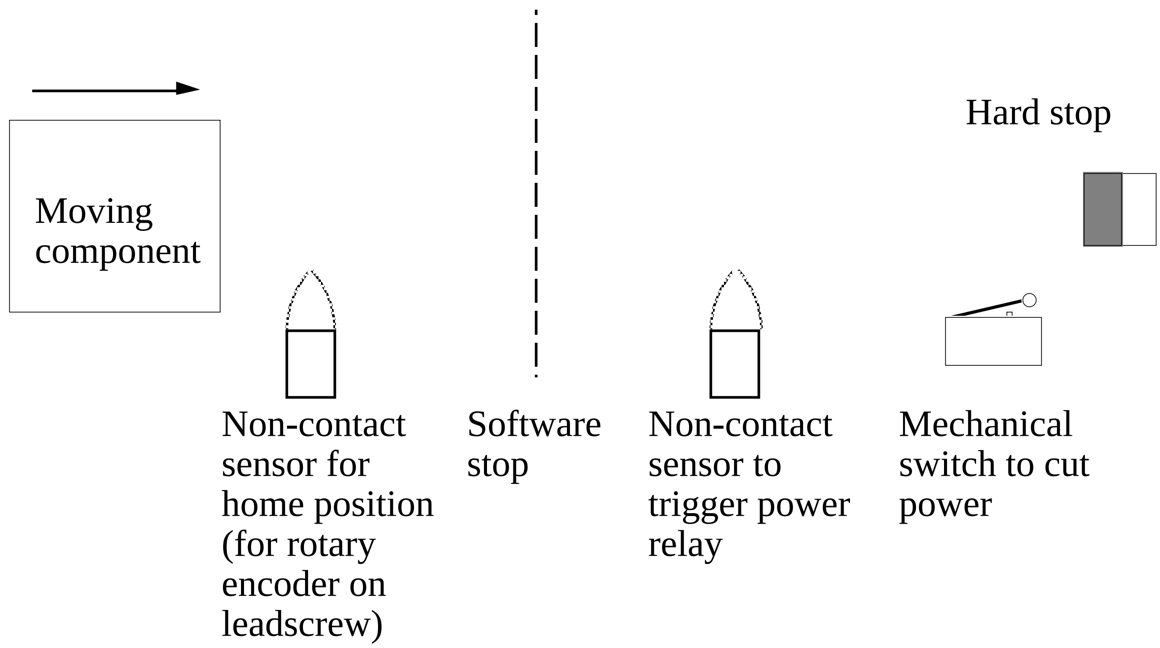


- **For a lathe carriage, one wants the scale to be close to the spindle centerline:**



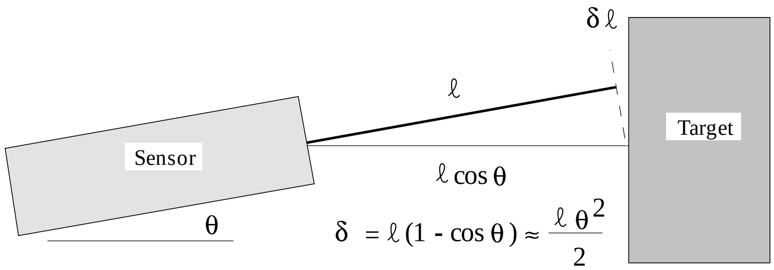
- **Crowding of the components illustrates the utility of integrated bearing/sensor units.**
  - **Example, Schneeburger's Monorail linear guide/magnetic scale system.**

Linear motion end-of-travel sensor requirements



- **Linear encoders have a home mark, so the first non-contact sensor is not needed.**
- **The second non-contact sensor is often optional.**
- **Take the space of the last sensors and replace them with a longer deceleration travel hard stop.**

- **Sensor alignment causing a *cosine error*:**

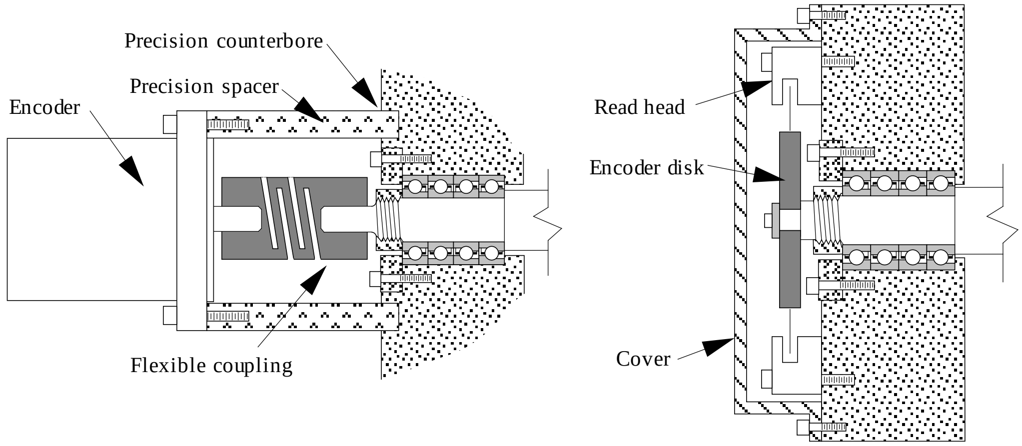


- **For precision machines (e.g., a wafer stepper), this can be substantial:**

Maximum allowable error (m)	**1.00E-09**
Distance traveled (m)	**0.5**
Maximum misalignment error (μrad)	**63**

- **63 μrad is 63 μm over 1 meter!**
- **Alignment surfaces machined in with other precision surfaces (e.g., bearings) can thus be very very useful!!**

- **Angular displacement sensor mounting methods:**



- **Note that the precision spacer's ground diameters enforces alignment.**
- **Coupling an encoder shaft to another shaft with a belt should be avoided whenever possible.**
    - **Side loads from the belt can decrease accuracy.**
    - **Slight cogging effects decrease accuracy.**

Effect of ballscrew shaft windup on rotational measurement for linear displacement control

- **The twist $\Delta\phi$ of a circular shaft is a function of:**
  - **Applied torque Γ**
  - **Length L**
  - **Polar moment of inertia I_p**
  - **Shear modulus G**

$$\Delta\phi = \frac{\Gamma L}{I_p G}$$

- **The axial displacement of a machine tool carriage driven by a leadscrew with a lead ℓ is:**

$$\Delta X = \frac{\ell\,\Delta\phi}{2\pi}$$

- **The equivalent axial displacement error caused by the sensor measuring the twist (wind-up) is:**

$$\Delta X = \frac{\ell\,L\Gamma}{I_p G 2\pi}$$

- **The relation between force and torque is::**

$$F = \frac{2\pi\Gamma}{\ell}$$

- **The equivalent axial stiffness of the torsional system is thus:**

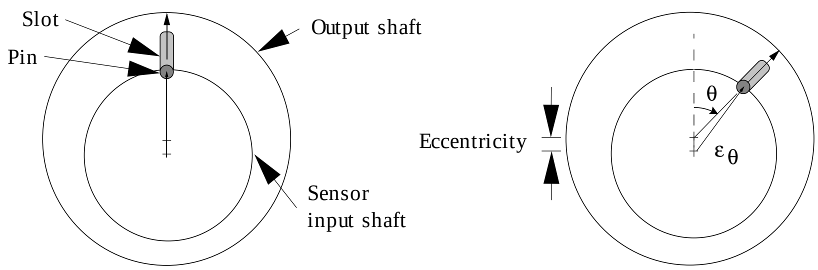
$$K_{\Gamma\text{axial eq.}} = \frac{GI_p 4\pi^2}{\ell^2 L} = \frac{\pi^3 r^4 E}{(1+\eta)\ell^2 L}$$

- **The ratio of the torsional-axial equivalent stiffness and the axial stiffness is:**

$$\frac{K_{\Gamma\text{axial eq.}}}{K_{\text{axial}}} = \frac{\pi^2 r^2}{(1+\eta)\,\ell^2}$$

- **The lead is usually less than the radius.**
  - **Torsional-axial equivalent stiffness is an order of magnitude greater than the axial stiffness of the leadscrew.**
- **Any equivalent axial displacement errors due to torque windup of the shaft are:**
  - **Often insignificant compared to the axial compression of the shaft.**
- **A linear position sensor would measure the shaft compression whereas a rotary sensor on the shaft would not.**
  - **But, in general, the servo could not respond fast enough (bandwidth issue) to compensate for ballscrew compression.**

- **Periodic rotational error caused by shaft eccentricity. Pin-in-slot is used to visualize the effect:**



- **The maximum error is approximately:**

$$\varepsilon_\theta \approx \frac{e}{r_c}$$

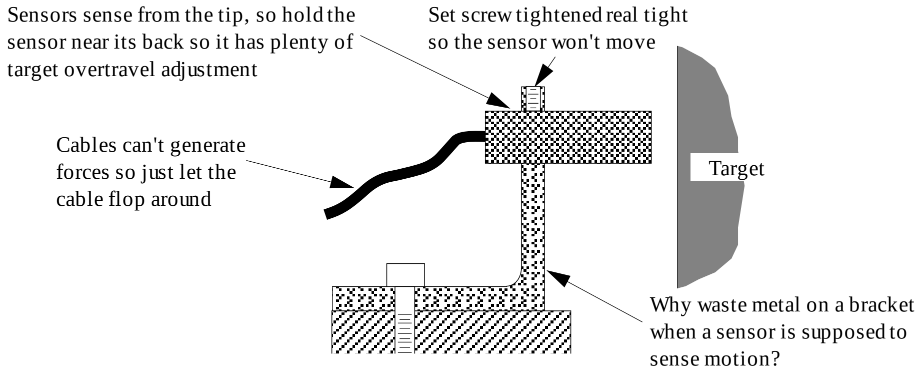
- **For flexible couplings, errors can be virtually eliminated.**
- **Coupling bore eccentricity and coupling shaft eccentricity can cause a parasitic error that can be estimated by:**

$$\varepsilon_{\text{flexible coupling}} = \left[\frac{\text{shaft eccentricity}}{\text{small shaft radius}}\right]\left[\frac{\text{coupling bore eccentricity}}{\text{small bore radius}}\right]$$

- **Example:**
  - **30 mm to 10 mm diameter shaft**
  - **Shaft eccentricity is 0.05 mm**
  - **Coupling's bore eccentricity is 0.025 mm**
  - **Coupling error estimate is (0.05/30)(0.025/10) = 17 μrad.**
  - **Axial displacement error in a ballscrew with a 10 mm lead will only be 0.03 μm.**

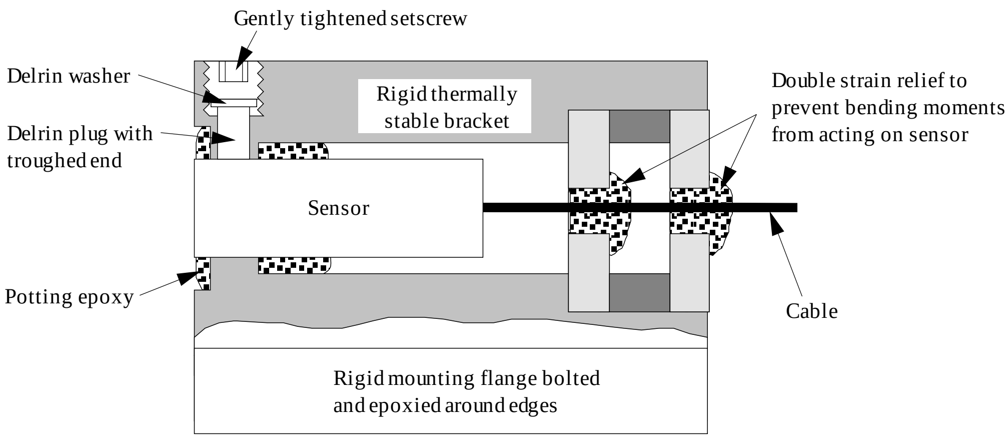
Mounting structure design:

- **How not to mount a sensor, and some false assumptions:**



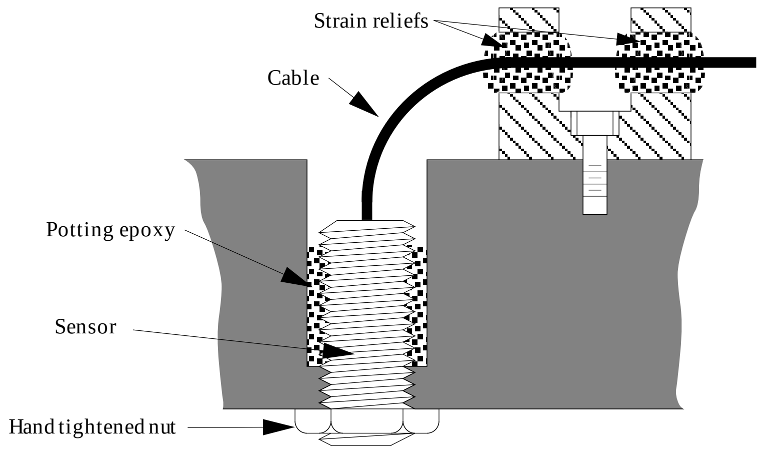
  - **Note that the wimpy bracket is subject to vibration and thermal distortion errors.**

- **An acceptable method for mounting a smooth body sensor.**



- **Alternatively, a large split housing can be used which applies uniform circumferential pressure to the sensor body.**

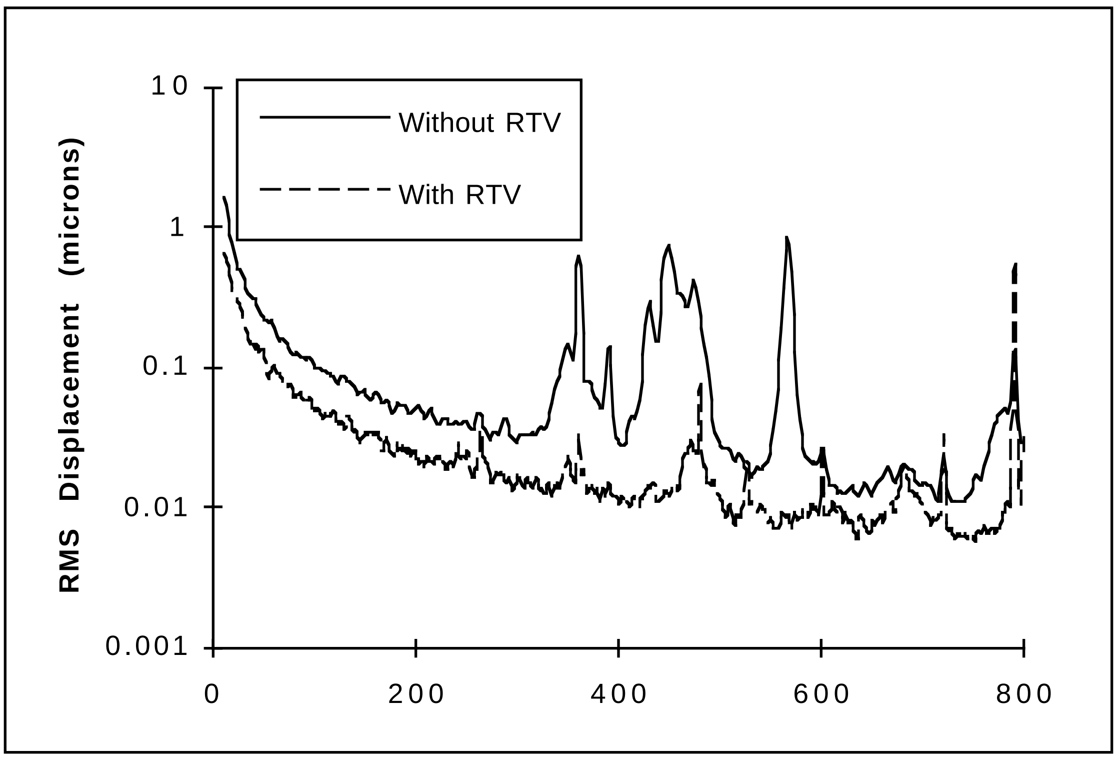
- **An acceptable method for mounting sensors with threaded bodies:**



- **Be careful to obey the same rules of accuracy in design as should be applied to the rest of the structure.**
- **Be careful to match thermal and mechanical time constants of the sensor structure to the rest of the structure.**

Example

- A plane mirror for an interferometer was mounted on a machine to measure straightness error of an axis.
- A tool holding axis' position was measured with the interferometer and was used to also correct the other axis' straightness errors.
- The tool had apparent vibration problems, which were traced using modal analysis to the mirror vibrating.
- The solution was to inject silicone rubber behind the mirror:



Mounting environment:

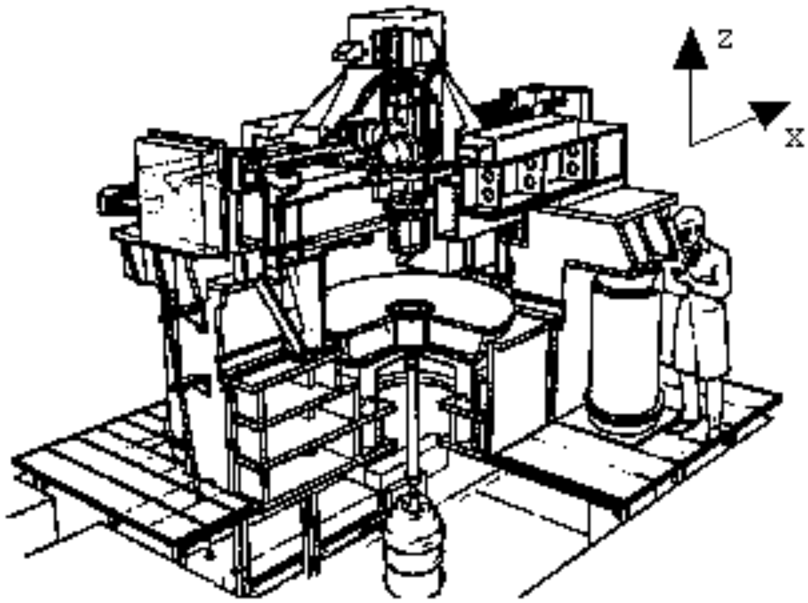
- **In order to decrease the Abbe error, the sensor should be mounted as close to the process as possible.**
- **The closer the sensor is mounted to the process, the harsher the environment.**
- **Many sensors are sealed and can be used in harsh environments.**
    - **However one should never take chances if one does not have to.**
- **Beware of disturbance forces and change capacitance caused by flexing cables.**
- **Do not put sensor cables in the same conduit as power cables.**
    - **Keep sensors well shielded and grounded to minimize electromagnetic interference.**
- **Evacuated beampaths provide the best answer for interferometers.**
- **In-process gauging structures may have a different thermal time constant than the rest of the system.**
    - **When the machine door is opened, evaporated cooling effects can cause the process gage system to change.**

Metrology frames:

- **Because any structure deforms when subjected to any type of load:**
    - **An independent structure is needed to make measurements if the highest level of accuracy is to be obtained.**
- **Metrology frames are expensive:**
    - **They must be designed with the same structural and thermal considerations as the machine itself.**
- **Often a pseudo (partial) metrology frame can be designed as a compromise.**
- **Careful error budgeting is required in order to minimize cost and maximize performance.**

Metrology frame design for the Large Optics Diamond Turning Machine (LODTM):

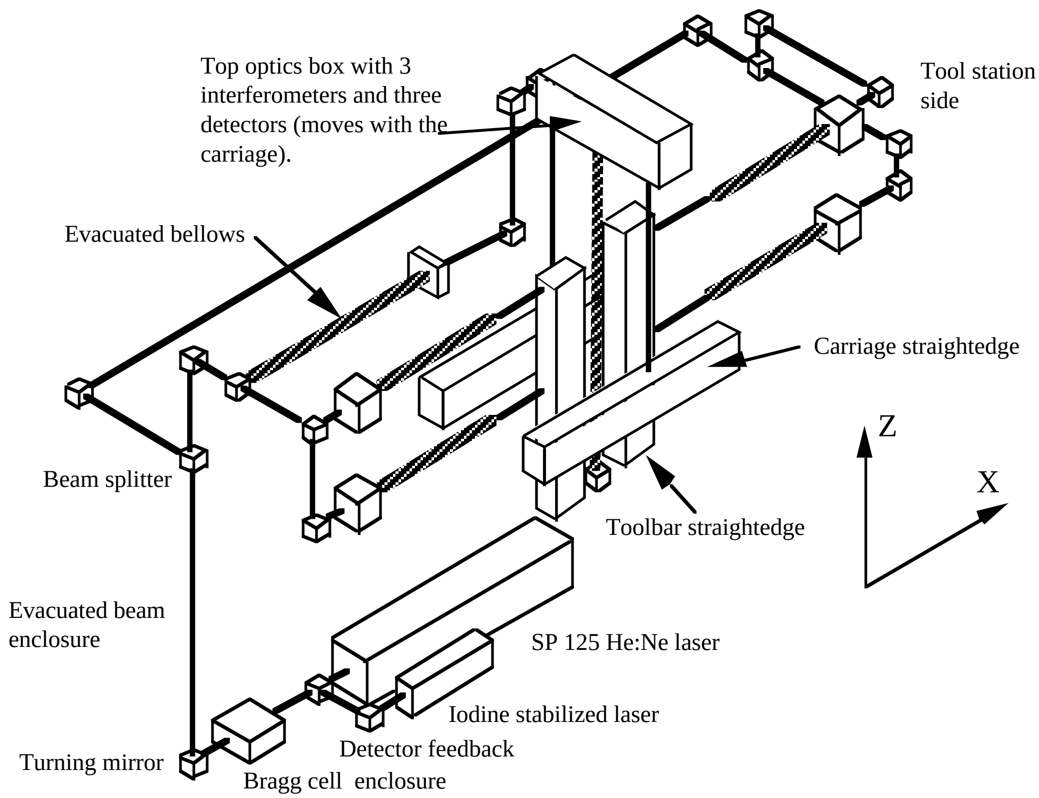
- **The Large Optics Diamond Turning Machine (LODTM), built at the Lawrence Livermore National Laboratory (LLNL), is a vertical spindle bridge-type (portal) machine:**



- **LODTM was designed to machine large optical components using a diamond tool:**
    - **Accuracy of 0.028 μm rms (1.1 μin) in the work volume (30 parts per billion).**
    - **Surface finish on the order of 42 Ångstroms R_a (0.17 μin).**
- **The machine was designed with a vertical spindle:**
    - **Minimized non-axisymmetric deformations of the part being machined.**
    - **Maximum part weight of 13.4 kN (3000 lbs) and dimensions of 1.63 m diameter by 0.51 m thick (64"x20").**

- **LODTM's structural system is made from welded mild steel plates.**
  - **The X axis, the *carriage*, moves horizontally (radially along the part) and carries the Z axis.**
  - **The Z axis, the *toolbar*, moves the tool vertically along the axial direction of the part.**
- **The metrology frame is made from Super Invar.**
  - **Virtually zero coefficient of expansion.**
    - **100 gpm of water, temperature controlled to 0.001 F^{o}, is circulated through the structure to control its temperature.**
    - **The external air temperature can be controlled to 0.01 F^{o} when the machine is operated remotely.**
  - **It is kinematically mounted to the structural frame with three flexures.**
    - **Thermal and mechanical loads cannot be transmitted between the two systems.**

- **Seven measurements are made by the interferometric measuring system.**
  - **The resolution of the interferometric system is about 6 Ångstroms (0.025 μin).**
  - **Position of the metrology frame wrt the spindle faceplate is continually measured using four capacitance probes.**



- **The metrology frame can undergo small rigid body motions in the XZ plane:**
  - **This will not change the measured relative position of the tool and workpiece.**
  - **This system allows the position of the tool point to be accurately determined with respect to the workpiece.**
    - **Even though the structural frame may deform under load by as much as 6^1/$_4$ μm (250 μin).**

- **Toolbar X direction straightness and pitch and carriage axial position and pitch are measured:**
  - **With straightedges mounted to the toolbar and four interferometers on the metrology frame.**
- **Y direction straightness and Yaw of the carriage cause motion of the tool tip in non-sensitive directions.**
- **Z displacement of the tool tip with respect to the carriage is measured.**
  - **It is combined with Z direction straightness of the carriage to yield the position of the tool tip in the Z direction.**
  - **The Z direction straightness of the carriage is measured by the two outer interferometers on the top optics box.**
- **Interferometers measure differential motion of the top optics box with respect to the carriage's two straightedges mounted to the main metrology frame.**
- **LODTM's straightedges are accurate to about 150 nanometers.**
  - **Greater accuracy is achieved after installation by measuring artifacts placed in the work zone.**

Sensor calibration:

- **All measurement depends on calibration of the sensor.**
- **Regardless of the method used to calibrate a sensor:**
  - **The certainty of the calibration can be obtained from a careful evaluation of the calibration system's error budget.**
- **In general, there are five dominant error categories to be considered:**
  - **R_I The repeatability of the sensor.**
  - **R_C The repeatability of the calibrator.**
  - **R_L The repeatability of the link between the calibration instrument and the calibrator.**
  - **S_L The systematic uncertainty of the link.**
  - **S_C The systematic uncertainty of the calibrator.**
- **The error budget must include errors caused by:**
  - **Temperature changes.**
  - **Signal digitization truncation (least significant bit errors).**
- **The systematic uncertainty of the calibrator is a function of how well it relates to the recognized National Standard.**
  - **It is usually supplied with the calibrator.**

- **Designing the calibration experiment:**
    - **The sensor should be physically mounted in the same manner in which it will be used.**
    - **The ideal goal is to make the calibration system 5-10 times more accurate than the application requires.**
    - **To eliminate Abbe errors:**
        - **The axis of measurement of the calibration standard should be coincident with the axis of measurement of the sensor.**
    - **Using a stage with a finite amount of static friction allows the servo control system to be turned off when it's time to make a measurement.**
        - **This eliminates possible limit cycling errors caused by discretization limits present in most digital servos.**
    - **One must always beware of heat generated by friction.**

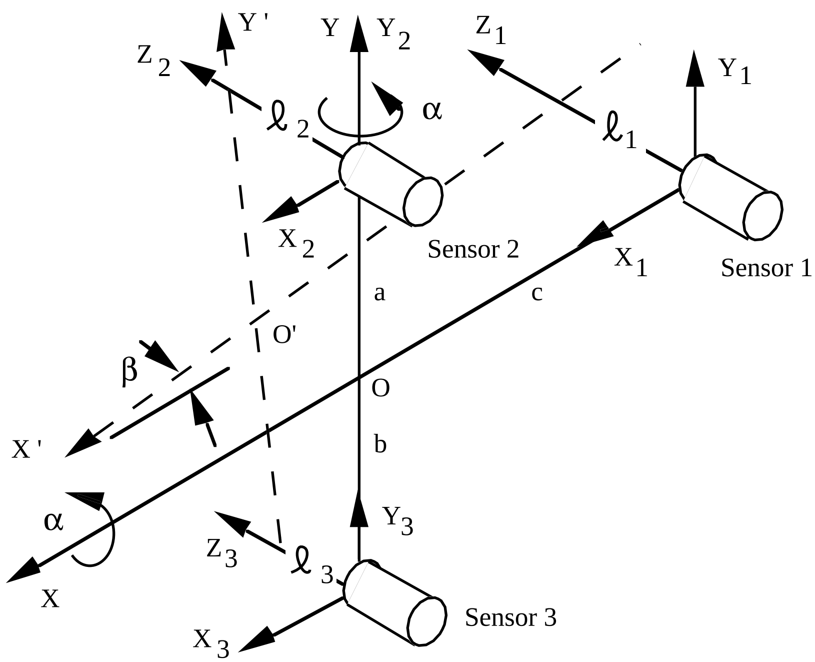
- **Stability determination:**
  - **The sensor must be mounted in a stable environment where the sensor is measuring a fixed quantity (e.g. distance).**
  - **For probe type sensors:**
    - **A cap is made from the same material as the probe body which fits over the end of the sensor.**
    - **Secured with a light squeeze type collar, or by its own weight if the sensor is oriented vertically.**
  - **The system is allowed to sit and measure the distance from the sensor to the cap for an extended period of time.**
  - **Any drift, is characteristic of the stability of the sensing system, and thus will be measured.**

- **Static response calibration:**
  - **A static calibration is one where the measurand is incremented.**
    - **The sensor's response is not measured until the system has stabilized.**
    - **Often the response includes mechanical and electrical components,**
  - **In order to increase mapping accuracy:**
    - **A set of maps of the sensor response may be made and used to find an averaged set of coefficients.**

- **Dynamic response calibration:**
    - **One method requires the use of a shaker table whose oscillation frequency and amplitude can be measured.**
    - **White noise or a swept sine wave is used as the input command to the table:**
        - **The sensor output is measured and stored.**
    - **If the transfer function of the table is known:**
        - **Digital signal processing techniques can then be used to determine the sensor's transfer function.**
        - **The transfer function for the sensor obtained by this technique:**
            - **Assumes that the response of the sensor is linear with respect to changing distance for a constant frequency.**
            - **Must be combined with the static response map generated for the sensor:**
                - **This yields a frequency-amplitude calibration for the sensor.**
    - **A second method involves input of a range of displacements and frequencies to the sensor:**
        - **This produces a map of the sensor's output as a function of these two variables.**
    - **If accuracy of this magnitude is required from a position sensor:**
        - **It is often easier to use a laser interferometer or digital encoder as the sensor in the first place.**

Effects of sensor output and location errors on system accuracy

- **Consider a triad of distance measuring sensors used to determine distance and yaw and pitch between two plates.**



- **Assume that rotations of the target plane (X'Y' plane) occur about the X and Y axes, respectively, in the sensor coordinate plane.**

- **The error associated with this non-Eulerian selection of the angles is on the order of $\alpha\sin\beta$.**

- **If α and β are < 200 μrad, the angular error in α or β due to this assumption will be about 0.04 μrad.**

- **The relations for the system are:**

$$\ell_{XY} = \ell_3 + (b + Y)\sin\alpha - X\sin\beta$$

$$\alpha = \tan^{-1}\left(\frac{\ell_2 - \ell_3}{a + b}\right)$$

$$\beta = \tan^{-1}\left(\frac{\ell_1 - (\ell_2 b + \ell_3 a) / (a + b)}{c}\right)$$

- **Using small angle approximations, the error in computing ℓ_{XY} over the expected range of α and β will be 4 ppm.**

- **Thus small angle approximations should only be used to evaluate the error caused by parameter variation.**

- **For the sensitivity of ℓ_{XY} to variations in ℓ_i, a, b, and c, it can be assumed that $\tan^{-1}(\ ) = (\ )$ and upon substituting:**

$$\ell_{XY} = \frac{-X\ell_1}{c} + \frac{\ell_2}{a + b}\left(b + Y + \frac{Xb}{c}\right) + \ell_3\left(1 - \frac{b + Y - Xa/c}{a + b}\right)$$

- **Effect of sensor output errors:**
  - **To determine the error $\delta\ell_{XY}l_i$ in the calculation of the distance between plates due to an error $\delta\ell_i$ in sensor #i's output:**
    - **The partial derivative $\partial\ell_{XY}/\partial\ell_i$ of Equation 5.9.4 is evaluated.**
  - **The angular errors $\delta\alpha\ell_i$ due to errors in the sensor reading $\delta\ell_i$ are determined in a similar manner**
- **Effect of errors in probe spacing:**
  - **Found from the partial differential of ℓ_{XY} with respect to a, b, and c:**
- **Effect of probe misalignment:**
  - **Cosine errors result in the measurements, and an Abbe error is introduced from the rotation of the plate.**