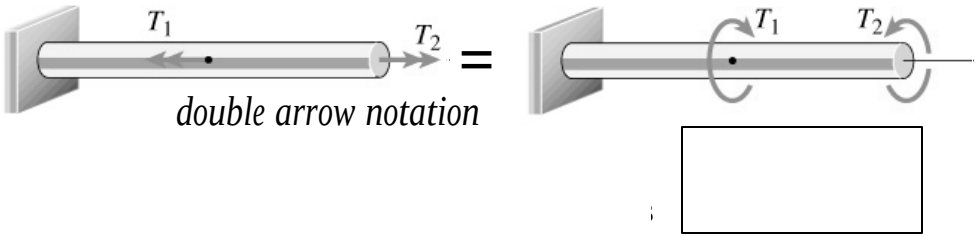
LECTURE #13 :
3.11 MECHANICS OF MATERIALS F03

INSTRUCTOR : Professor Christine Ortiz
OFFICE : 13-4022 **PHONE** : 452-3084
WWW : http://web.mit.edu/cortiz/www

- Review : Torsion and Beam Bending 1&2
- Beam Bending 3 : Stresses in Beams

Review Lecture 10 : Torsion

1. Torsion : loading of a bar by an external torqu
the longitudinal (z) axis of the bar



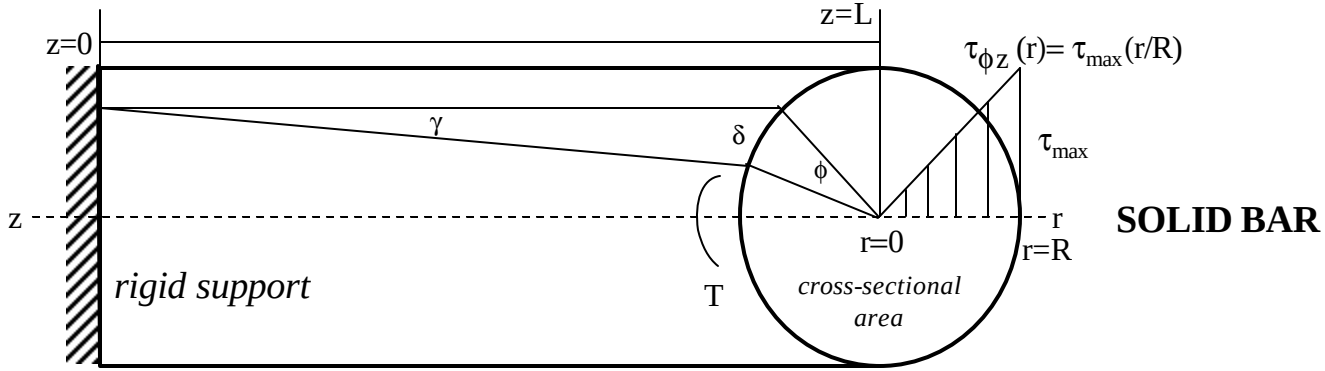
2. Pure Torsion :

ASSUMPTIONS :

- every cross-section of the bar is identical and subjected to sa
- all cross-sections remain plane, the same shape, and the radii
- if the angle of rotation is small, the change in length and radi
- only twisting and pure shear stresses, no bending or warping
- linear elastic, isotropic

Parameters :

L=bar length
ϕ=angle of twist at end of bar
$\gamma_{\phi z}$=shear strain
δ =shear deformation
r=radial coordinate
z=axial coordinate
R=cross-sectional area radius
T=torque
$\tau_{\phi z}$=shear stress



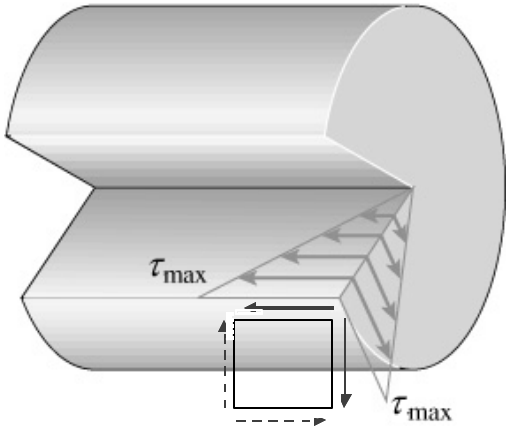
Review Lecture 10 : Torsion

DERIVATION:

1. Geometrical Statement : $d = rdf$

2. Shear Strain Displacment Relation : $g_{fz} = r\dfrac{df}{dz}$

3. Constitutive Law : $t_{fz} = Gg_{fz} = Gr\dfrac{df}{dz}$



4. Equations of Static Equilibrium (Rotational) :

constant T : $f = \dfrac{TL}{JG}$

[p radians=180^o, 1 rad=57.3^o]

J(m^4) = polar moment of inertia,

solid bar, circular cross-sectional area : $J = \dfrac{p r^4}{2}$

hollow bar : $J = \dfrac{p (r_o^4 - r_i^4)}{2}$

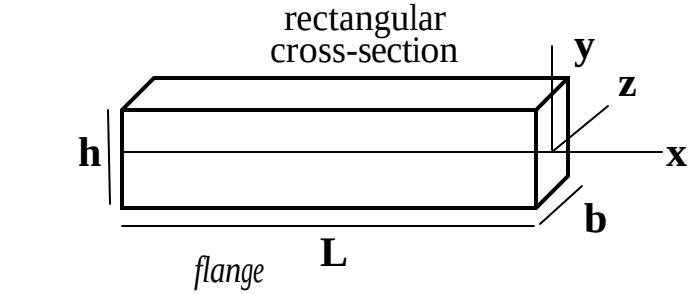
JG(Nm^2)="torsional rigidity", k(Nm)$= \dfrac{JG}{L}$="torsional stiffness"

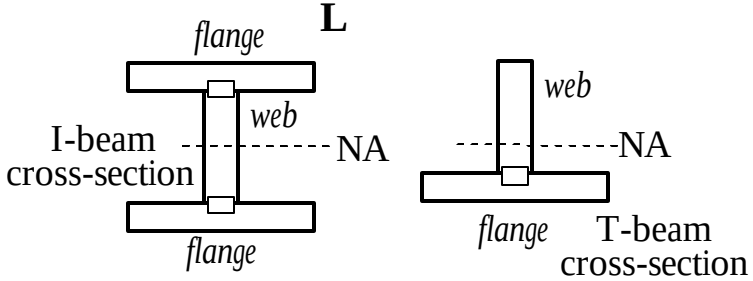
5. Shear Stresses : $t_{fz} = \dfrac{Tr}{J}$

$\left(t_{fz}\right)_f = \dfrac{T_f r}{J}$ =modulus of rupture in torsion

Review : Beam Theory

I. Basics : BEAM : structural member subjected to lateral loads, i.e. forces or moments having their vectors perpendicular to the axis of the bar





nomenclature :

L=length or span, b=width, h=height

NA=neutral axis passes through centroid ($\bar{x},\bar{y}=0$) of cross-section

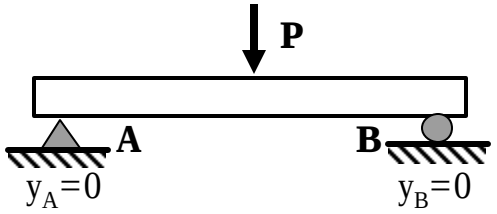
I=moment of inertia of cross section about NA

$$I_{rectangular}=\frac{bh^3}{12},\ I_{circular}=\frac{\pi r^4}{4}$$

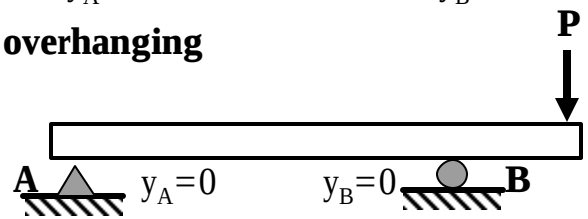
EI=flexural modulus

II. Types of Supports and Boundary Conditions

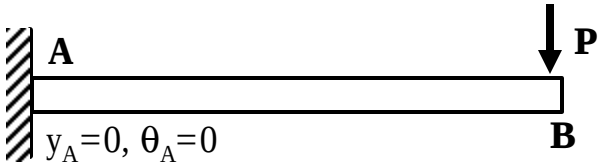
1. simply supported



2. overhanging

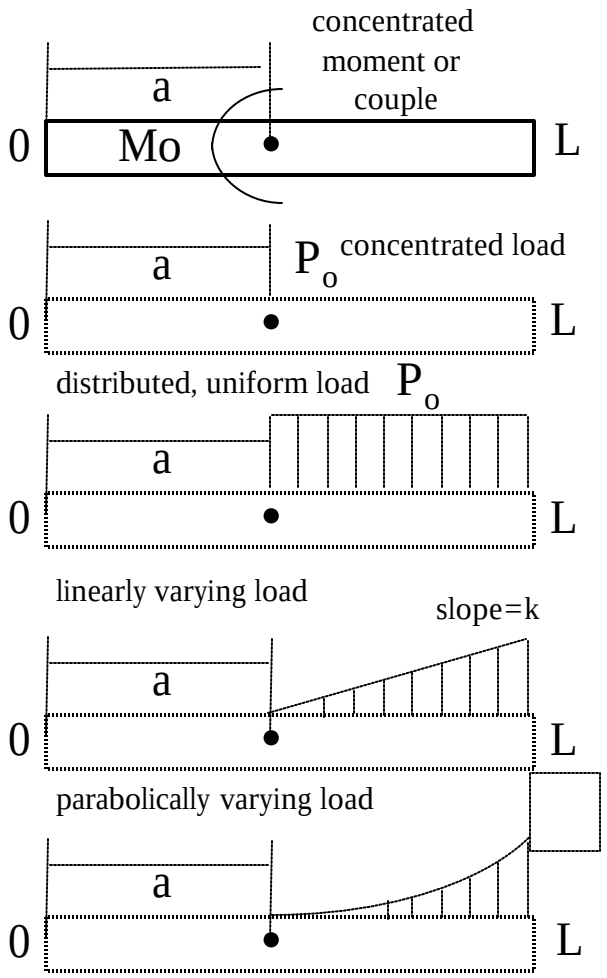


3. cantilever



Review : Beam Theory (Cont'd)

III. Types of Loading



Review : Beam Theory (Cont'd)

IV. Successive Integration Method / Shear and Bending Moment Diagrams :

$q(x)$ $\left(q=-\frac{dV}{dx}\right)$ q= loading function

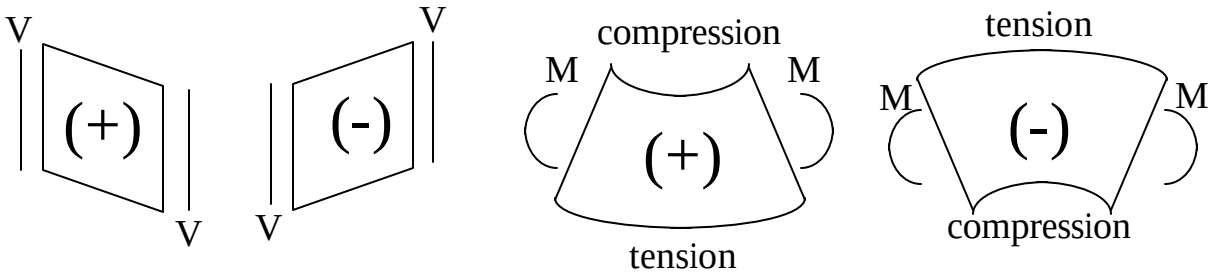
$V(x) = -\int q(x)dx + C_1$ $\left(V=\frac{dM}{dx}\right)$ V=shear force

$M(x) = \int V(x)dx + C_1 x + C_2$ $\left(M=\frac{dq}{dx}\right)$ M=bending moment

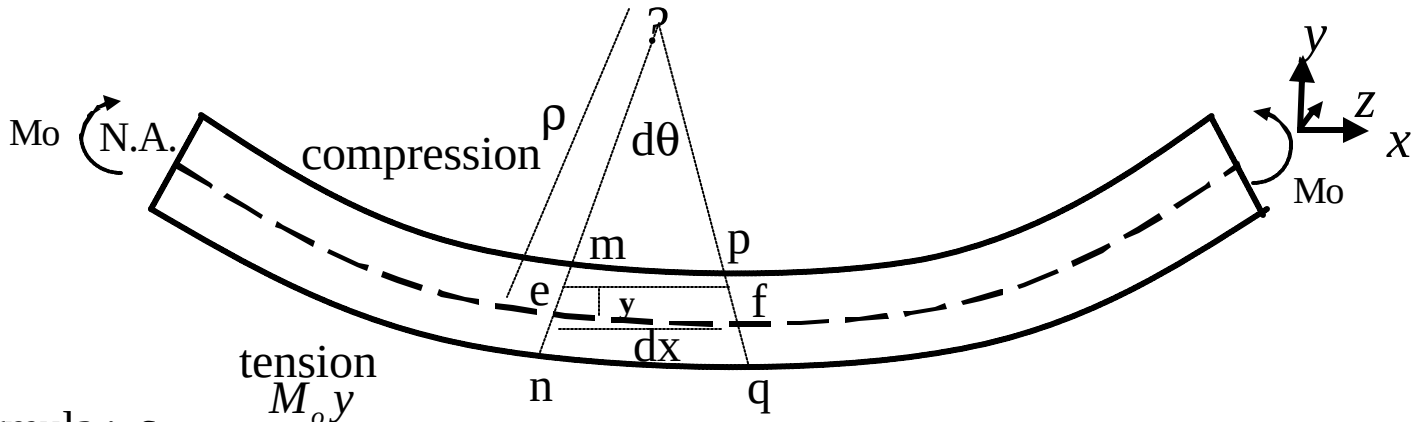
$q(x) = \frac{1}{EI}\int M(x)dx + \frac{C_1 x^2}{5} + C_2 x + C_3$ $\left(q=EI\frac{dV}{dx}\right)$ q =curvature=slope of y-displacement curve

$y(x) = \int q(x)dx + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{5} + C_3 x + C_4$ y=vertical displacement

V. Sign Conventions :



Beam Theory 3 : Normal Stresses and Strains



Flexure formula : $s_x = -\frac{M_o y}{I}$

where : s_x = normal stress in x-direction

M_o = internal bending moment

y = vertical distance from NA axis

(see Gere Chapter 12, Appendix D, p 321)

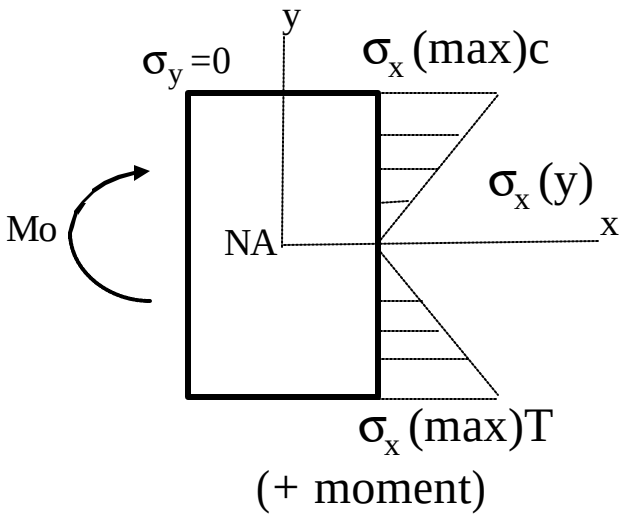
I = moment of inertia of cross-sectional area

$$(s_x)_{\max} = -\frac{[M_o(x)]_{\max} y_{\max}}{I}$$

for rectangular beams : $y_{\max} = \frac{h}{2}$

$$e_x = -\frac{M_o y}{EI}, (e_x)_{\max} = -\frac{[M_o(x)]_{\max} y_{\max}}{EI}$$

EI = "flexural modulus"



Beam Theory (Cont'd) : Shear Stresses and Strains

Derivation in Gere Section 5.8 :

$$t_{xy}(\text{rectangular cross section}) = -\frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$$

where : t_{xy} = shear stress

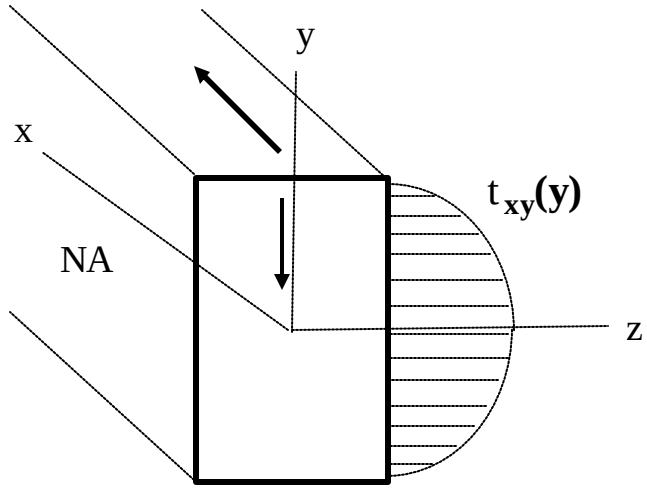
V = shear force

h = height of cross sectional area

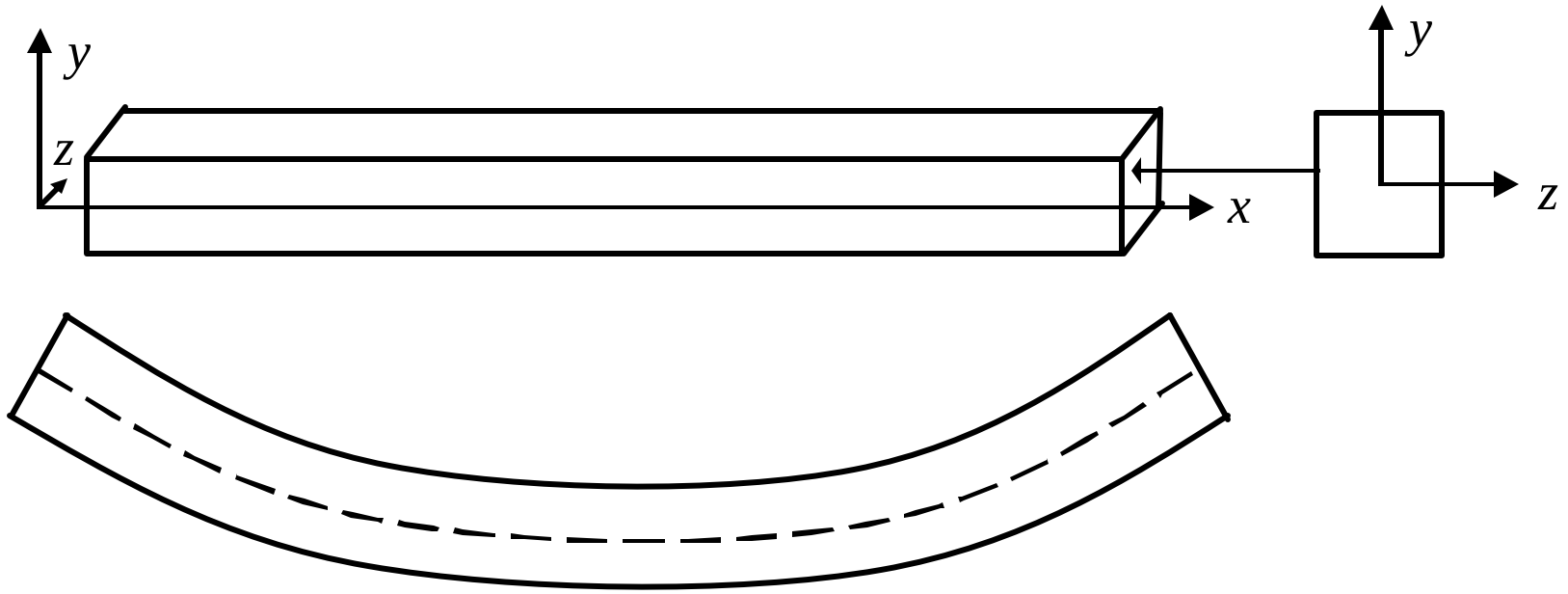
y= distance from NA

$$t_{xy}(\text{rectangular cross section})_{\max} = -\frac{3V}{2A}$$

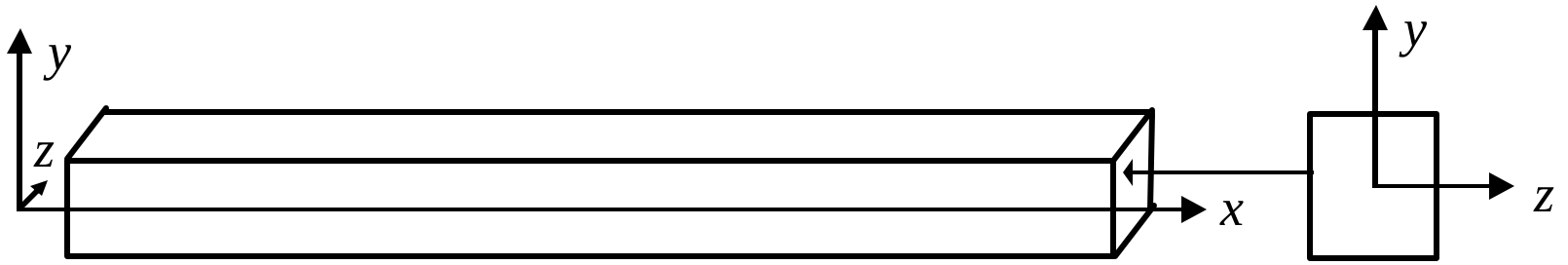
A = cross sectional area

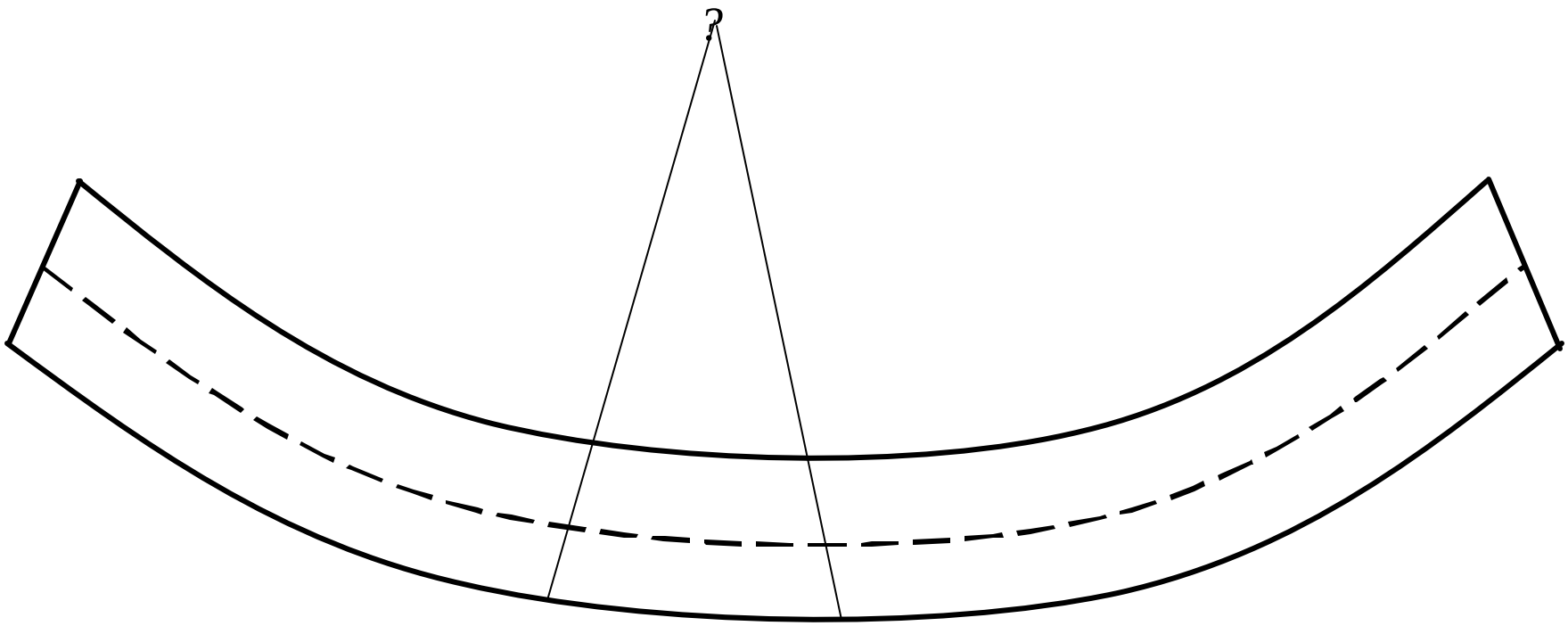


Beam Theory 3 : Stresses and Strains

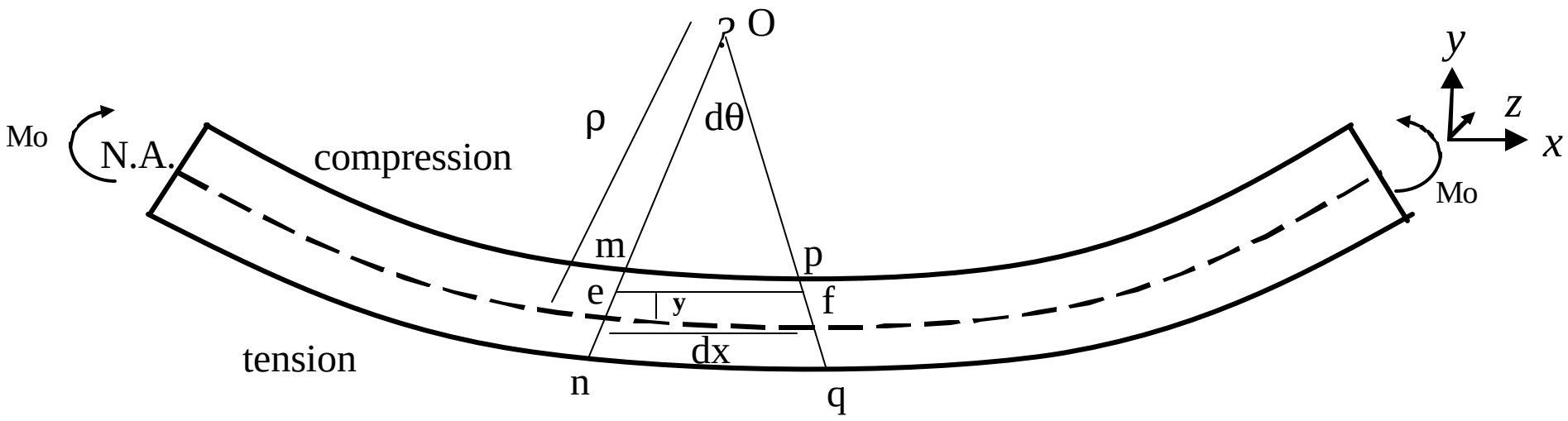


Beam Theory 3 : Stresses and Strains





Beam Theory 3 : Stresses and Strains



Beam Theory 3 : Relate Stress to Moment

