

# 5.73

## Quiz 12

October 2, 2002

1.

The matrices for  $x$  and  $p$ , evaluated in the harmonic oscillator eigenbasis, have the general nonzero matrix elements:

$$\langle n | x | n+1 \rangle = x_{n,n+1} = \left[ \frac{\hbar}{2\omega\mu} \right]^{1/2} (n+1)^{1/2}$$

$$p_{n,n+1} = -i \left[ \frac{\hbar\omega\mu}{2} \right]^{1/2} (n+1)^{1/2}$$

A.  $x$  and  $p$  are Hermitian. Write the “other” nonzero matrix element (be careful about  $p$ !)

$$x_{n+1,n} =$$

$$p_{n+1,n} =$$

B. What are the general “selection rules” for nonzero matrix elements of  $x$ ,  $p$ , and  $x^2$ ?

$$\text{For } x, \Delta n = ?$$

$$\text{For } p, \Delta n = ?$$

$$\text{For } x^2, \Delta n = ? \text{ and } ?$$

C. If you multiply  $x$  times  $x$ , the matrix multiplication cartoon helps:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & \ddots \\ 0 & 0 & \ddots & \ddots \end{pmatrix}$$

Based on the cartoon, what is the general selection rule for nonzero matrix elements of  $x^k$ ?