Name

5.73

Quiz 13

October 4, 2002

1.

$$\mathbf{a} = \begin{pmatrix} 0 & 1^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2^{1/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3^{1/2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \ddots & \ddots \\ 0 & 0 & 0 & 0 & 0 & \ddots \end{pmatrix} \quad \mathbf{a}^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 2^{1/2} & 0 & 0 & 0 \\ 0 & 0 & 3^{1/2} & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \ddots \end{pmatrix}$$

A. Write the values of the following quantities:

$\mathbf{a}^{\leq}_{2,3}$

$\mathbf{a}^{\leq}_{5,4}$

$(\mathbf{a}^{\dagger}\mathbf{a})_{6,7}$

$(\mathbf{a}^{\dagger}\mathbf{a})_{7,7}$

$[\mathbf{a},\mathbf{a}^{\dagger}]_{2,2}$

$[\mathbf{a}^{\dagger},\mathbf{a}]_{3,3}$

B. $\mathbf{a}^\dagger\mathbf{a}$ is called the "number operator", $\mathbf{N}$. Why?

C. Selection rules are specified as the value of the quantum number on the left minus the value of the quantum number on the right. For the following products of six $\mathbf{a}$ or $\mathbf{a}^\dagger$ operators, what are the selection rules for nonzero matrix elements of:

$\mathbf{aaaa}^\dagger\mathbf{aa}$ $\quad \Delta n =$

$\mathbf{a}^\dagger\mathbf{a}^\dagger\mathbf{aaaa}^\dagger$ $\quad \Delta n =$

$\mathbf{aaa}^\dagger\mathbf{aa}^\dagger\mathbf{a}$? $\quad \Delta n =$

C. Which of the following matrices are Hermitian?

$\mathbf{a}$

$\mathbf{a}^\dagger$

$\mathbf{a} + \mathbf{a}^\dagger$

$\mathbf{aa}^\dagger$

$\mathbb{1}$

D. Remember to start by first applying the operator on the far right: Evaluate

$(\mathbf{a}^\dagger)^7|n\rangle$

$\langle n|\mathbf{a}^{\leq}\mathbf{aaa}^{\leq}\mathbf{a}^{\leq}|n+1\rangle$.