

Name _____

5.73

Quiz 14

October 6, 2003

1.

Quartic Oscillator

$$\mathbf{H} = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}k\mathbf{x}^2 + b\mathbf{x}^4$$

$$\mathbf{H}^{\circ} = \frac{1}{2}(\mathbf{a}^{\dagger}\mathbf{a} + \mathbf{a}\mathbf{a}^{\dagger})\hbar\omega$$

$$\mathbf{x} = \left(\frac{\hbar}{m\omega}\right)^{1/2} \mathbf{x} = \left(\frac{\hbar}{2m\omega}\right)^{1/2} (\mathbf{a} + \mathbf{a}^{\dagger})$$

$$[\mathbf{a}, \mathbf{a}^{\dagger}] = \mathbb{1}$$

$$\mathbf{x}^4 = \left(\frac{\hbar}{2m\omega}\right)^2 \left\{ \mathbf{a}^4 + 2\mathbf{a}^2(2\mathbf{a}^{\dagger}\mathbf{a} - 1) + [6\mathbf{a}^{\dagger}\mathbf{a}(\mathbf{a}^{\dagger}\mathbf{a} + 1) + 3] + 2\mathbf{a}^{\dagger 2}(2\mathbf{a}^{\dagger}\mathbf{a} + 3) + \mathbf{a}^{\dagger 4} \right\}$$

A. Give the general formula for $E_n^{(0)} = H_{nn}^{(0)}$
 $E_n^{(0)} = [\text{some function of } n]\hbar\omega.$

B. $\mathbf{H}^{(1)} = b\mathbf{x}^4$. Give the general formula for $E_n^{(1)} = H_{nn}^{(1)}$
 $E_n^{(1)} = [\text{some function of } n]b\left(\frac{\hbar}{2m\omega}\right)^2.$

C. The expression for the second-order correction to the energy is

$$E_n^{(2)} = \sum_k \frac{|H_{nk}^{(1)}|^2}{E_n^{(0)} - E_k^{(0)}}$$

(i) For which 5 values of k will $H_{nk}^{(1)}$ be nonzero?
 $k = n + 4, k = ?, \text{ etc.}$

(ii) For each of those values of k , what is $E_n^{(0)} - E_k^{(0)}$?
 $E_n^{(0)} - E_{k=n+4}^{(0)} = \hbar\omega[\text{some integer}], \text{ etc.}$

(iii) For $k = n + 4$, what is $H_{nk}^{(1)}$?
 $H_{nn+4}^{(1)} = b \left[\frac{\hbar}{2m\omega} \right]^2 [\text{some function of } n]$