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Quiz 16

October 11, 2002

1.

Non-degenerate Perturbation Theory

$$E_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} = \mathbf{H}_{nn}^{(0)} + \mathbf{H}_{nn}^{(1)} + \sum'_m \frac{\mathbf{H}_{nm}^{(1)}\mathbf{H}_{mn}^{(1)}}{E_n^{(0)} - E_m^{(0)}}$$

$$\Psi_n = \Psi_n^{(0)} + \Psi_n^{(1)} = \sum'_m \frac{\mathbf{H}_{nm}^{(1)}}{E_n^{(0)} - E_m^{(0)}} \Psi_n^{(0)}$$

Quasi-degenerate Perturbation Theory (Van Vleck transformation)

$$\mathbf{H}_P^{(2)} = \sum_{s \in P'} \frac{\mathbf{H}_{ns}^{(1)}\mathbf{H}_{sm}^{(1)}}{\frac{E_n^{(0)} + E_m^{(0)}}{2} - E_s^{(0)}} \text{ where } n, m \text{ belong to } P \text{ and } s \text{ belongs to } P'.$$

- A. Use non-degenerate perturbation theory to find the eigenvalues and eigenvectors of $\begin{pmatrix} 0 & 5 \\ 5 & 20 \end{pmatrix}$.
- B. Find the eigenvalues of the 2×2 matrix in part A by diagonalization.

C. Perform the Van Vleck transformation on

$$\mathbf{H} = \left(\begin{array}{cc|c} 0 & 0 & 10.02 \\ 0 & 1 & 14.18 \\ \hline 10.02 & 14.18 & 100.5 \end{array} \right)$$

D. If you were successful on part **C**, you would have obtained a matrix of the form

$$\mathbf{H}^{\text{eff}} = \begin{pmatrix} A & B & 0 \\ B & A & 0 \\ 0 & 0 & C \end{pmatrix} \quad \text{where } |C| \gg |A| + |B|.$$

What are the two lowest energy eigenvalues and eigenvectors of \mathbf{H}^{eff} ?
[**HINT:** You should be able to solve this problem by inspection!]