

5.73

Quiz 17

October 16, 2002

1.

$$\Psi_{v_1 v_2}^{(0)} = \phi_{v_1}(x_1) \phi_{v_2}(x_2)$$

$$E_{v_1 v_2}^{(0)} = \hbar[\omega_1(v_1 + 1/2) + \omega_2(v_2 + 1/2)]$$

$$\mathbf{H}^{(1)} = k_{122} x_1 x_2^2$$

$$\mathbf{H}_{n_1 k_1 n_2 k_2}^{(1)} = k_{122} \langle n_1 | x_1 | k_1 \rangle \langle n_2 | x_2^2 | k_2 \rangle$$

$$\mathbf{x}_1 = \left(\frac{m_1 \omega_1}{2\hbar} \right)^{1/2} (\mathbf{a}_1 + \mathbf{a}_1^\dagger)$$

$$\mathbf{x}_2 = \left(\frac{m_2 \omega_2}{2\hbar} \right)^{1/2} (\mathbf{a}_2 + \mathbf{a}_2^\dagger)$$

- A. Matrix elements of $\mathbf{H}^{(1)}$ have four indices, n_1 and k_1 specify the final and initial state quantum numbers for oscillator #1, n_2 and k_2 the final and initial state quantum numbers for oscillator #2. What are the selection rules for nonzero matrix elements of $\mathbf{H}^{(1)}$?

$$n_1 - k_1 =$$

$$n_2 - k_2 =$$

- B. What are the zero-order energy differences that correspond to each of the nonzero matrix elements of $\mathbf{H}_{n_1 k_1 n_2 k_2}^{(1)}$?

$$E_{n_1 n_2}^{(0)} - E_{k_1 k_2}^{(0)} = \hbar[\omega_1(\quad) + \omega_2(\quad)]$$

C. Evaluate at least two of the six nonzero values of the off-diagonal elements of $\mathbf{H}^{(1)}$.

(i) e.g. $\mathbf{H}_{n_1 n_1 + 1 n_2 n_2 + 2} = \gamma [(n_1 + 1)(n_2 + 2)(n_2 + 1)]^{1/2}$

(ii) $\mathbf{H}_{n_1 n_1 + 1 n_2 n_2} = \gamma [\quad]$

DO THIS ONE! (iii) $\mathbf{H}_{n_1 n_1 + 1 n_2 n_2 - 2} = \gamma [\quad]$

(iv) $\mathbf{H}_{n_1 n_1 - 1 n_2 n_2 + 2} = \gamma [\quad]$

(v) $\mathbf{H}_{n_1 n_1 - 1 n_2 n_2} = \gamma [\quad]$

(vi) $\mathbf{H}_{n_1 n_1 - 1 n_2 n_2 - 2} = \gamma [\quad]$

where $\gamma \equiv \left(\frac{m_1 \omega_1}{2\hbar} \right)^{1/2} \left(\frac{m_2 \omega_2}{2\hbar} \right)^1$.

D. The term $\frac{|\mathbf{H}_{n_1 n_1 + 1 n_2 n_2 - 2}^{(1)}|^2}{E_{n_1 n_2}^{(0)} - E_{n_1 + 1 n_2 - 2}^{(0)}}$ appears in the second-order perturbation summation for $E_{n_1 n_2}^{(2)}$. Evaluate this term (based on your answers to parts B and C(iii)).

E. What happens to the term in part D if $\omega_1 = 2\omega_2$?