Name

# 5.73

## Quiz 22

## October 28, 2002

$$\left[\mathbf{L}_i,\mathbf{p}_j\right] = i\hbar\sum_k \varepsilon_{ijk}\mathbf{p}_k$$

$$\left[\mathbf{L}_i,\mathbf{q}_j\right] = i\hbar\sum \varepsilon_{ijk}\mathbf{q}_k$$

$\varepsilon_{ijk} =$ +1 if ijk are in cyclic order (i.e. xyz, yzx, or zxy)
–1 if ijk are in anti-cyclic order
0 if any index is repeated.

$$\mathbf{L} = (\mathbf{q}\times\mathbf{p}) = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z \end{pmatrix}$$

A. What are $\mathbf{L}_y$ and $\mathbf{L}_z$ in terms of $(\mathbf{x},\mathbf{y},\mathbf{z})$ and $(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)$?

B. Use $\varepsilon_{ijk}$ notation to evaluate $[\mathbf{L}_x, \mathbf{x}]$, $[\mathbf{L}_x, \mathbf{z}]$, $[\mathbf{L}_x, \mathbf{p}_x]$, and $[\mathbf{L}_x, \mathbf{p}_z]$.

$[\mathbf{L}_x, \mathbf{x}] =$

$[\mathbf{L}_x, \mathbf{z}] =$

$[\mathbf{L}_x, \mathbf{p}_x] =$

$[\mathbf{L}_z, \mathbf{p}_z] =$

C. Use the results of part B to show that $[\mathbf{L}_x, \mathbf{L}_y] = i\hbar\mathbf{L}_z$. Recall that $[\mathbf{A},\mathbf{BC}] = \mathbf{B}[\mathbf{A},\mathbf{C}] + [\mathbf{A},\mathbf{B}]\mathbf{C}$.