Name

5.73

Quiz 25

November 4, 2002

1.

$$\mathbf{L}\bullet\mathbf{S} = \tfrac{1}{2}\left[\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2\right]$$

$$\mathbf{L}\bullet\mathbf{S} = \mathbf{L}_z\mathbf{S}_z + \tfrac{1}{2}\left[\mathbf{L}_+\mathbf{S}_- + \mathbf{L}_-\mathbf{S}_+\right]$$

$$\mathbf{H}^{\mathrm{SO}} = \zeta\mathbf{L}\bullet\mathbf{S}$$

$$\mathbf{H}^{\mathrm{Zeeman}} = -\gamma\beta_z\left(\mathbf{L}_z + 2\mathbf{S}_z\right)$$

A. For a $^2F_{5/2}$ state:

(i) What is **S**?

(ii) What is **L**?

(iii) What is **J**?

(iv) Evaluate $\left\langle {}^2F_{5/2}, M_J \middle| \mathbf{H}^{\mathrm{SO}} \middle|^2 F_{5/2}, M_J \right\rangle$.

(v) Evaluate $\left\langle {}^2F_{7/2}, M_J = 7/2 \middle| \mathbf{H}^{\mathrm{Zeeman}} \middle|^2 F_{7/2}, M_J = 7/2 \right\rangle$.

B. Apply $\mathbf{J}^- = \mathbf{L}^- + \mathbf{S}^-$ to both sides of $\left|{}^2\mathrm{F}_{7/2}, \mathrm{M_J} = 7/2\right\rangle = \left|{}^2\mathrm{F}, \mathrm{M_L} = 3, \mathrm{M_S} = 1/2\right\rangle$ where the two basis states are the "extreme" states, respectively in the coupled and uncoupled basis sets:

$$\mathbf{J}^-\left|{}^2\mathrm{F}_{7/2}, \mathrm{M_J} = 7/2\right\rangle = \left(\mathbf{L}^- + \mathbf{S}^-\right)\left|{}^2\mathrm{F}, \mathrm{M_L} = 3, \mathrm{M_S} = 1/2\right\rangle.$$

Find the normalized combination of $\left|{}^2\mathrm{F}, \mathrm{M_L} = 2, \mathrm{M_S} = 1/2\right\rangle$ and $\left|{}^2\mathrm{F}, \mathrm{M_L} = 3, \mathrm{M_S} = -1/2\right\rangle$ that corresponds to $\left|{}^2\mathrm{F}_{7/2}, \mathrm{M_J} = 5/2\right\rangle$.

C. Verify that
$\left|{}^2\mathrm{F}_{5/2}, \mathrm{M_J} = 5/2\right\rangle = -7^{-1/2}\left|{}^2\mathrm{F}, \mathrm{M_L} = 2, \mathrm{M_S} = 1/2\right\rangle + (6/7)^{1/2}\left|{}^2\mathrm{F}, \mathrm{M_L} = 3, \mathrm{M_S} = -1/2\right\rangle$
is orthogonal to $\left|{}^2\mathrm{F}_{7/2}, \mathrm{M_J} = 5/2\right\rangle$ which you obtained in part B.

D. Evaluate $\left\langle {}^2\mathrm{F}_{5/2}, \mathrm{M_J} = 5/2\right|\mathbf{H}^{\mathrm{Zeeman}}\left|{}^2\mathrm{F}_{5/2}, \mathrm{M_J} = 5/2\right\rangle$.