Sample Calculus Final

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This is a sample, three hour calculus final. As originally given, it was closed book (no notes), but calculators were allowed. Have fun.

1. Perform the following calculations:

(a)
$$\frac{d}{dx}\left(\frac{1}{x^2-1}\right)$$

(b)
$$\frac{d}{dx}((x+3)^2(x-4)^2)$$

(c)
$$\int \sec\theta\,(\sec\theta-\tan\theta)\,d\theta$$

(d)
$$\int \frac{dx}{\sqrt{2+2x-3x^2}}$$

(e)
$$\int \frac{2}{\sqrt{e^x}}\,dx$$

(f)
$$\int \sin 4\theta\,d\theta$$

(g)
$$\int \frac{dx}{x\sqrt{9+4x^2}}$$

(h)
$$\int x^2\tan^{-1}x\,dx$$

(i)
$$\lim_{x\to 0}\frac{3x}{\tan x}$$

(j)
$$\lim_{x\to\infty}(x+e^x)^{\frac{2}{x}}$$

(k)
$$\lim_{x\to 0^+}x^x$$

(l)
$$\int_0^\infty e^{-x}\cos x\,dx$$

2. Use the definition of the derivative to calculate
$$\frac{d}{dx}(4x^2+1).$$

3. Graph the following function. Include all the usual important information.

$$y = \frac{x(x-2)}{(x+1)^2}$$

4. Find the area of the region bounded by $y = 4x - x^2$ and $y = x$.

5. Find the Taylor series expansion (about zero) of hyperbolic cosine. In what region does this converge?

6. Determine (by any method) whether the following series converge or diverge:

   (a)

   $$\Sigma_{n=o}^{\infty} \left(-\frac{4}{5}\right)^n$$

   (b)

   $$\Sigma_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$$

   (c)

   $$\Sigma_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$$

   (d)

   $$\Sigma_{n=0}^{\infty} \frac{(2n+2)}{3^n (n)^2}$$

7. A tank has the shape of an inverted cone with height 10m and radius 4m. Water is being pumped into it at the rate of 2π m^3/min.

   (a) How fast is the depth increasing when the water is 1m deep?

   (b) If we stop adding water when the water is 1m deep, how much work would be done by then pumping all the water over the edge of the tank? (Use w_0 for the weight density of water.)

8. Give the limit definition of e.