

Name_____

Date_____

Vector Calculus Independent Study

Unit 6 Sample Test

1. [25 points] Re-write

$$\int_0^1 \int_x^1 \int_y^x f(x, y, z) dz dy dx$$

in all five of the other orders of integration.

2. [25 points] Suppose a region in \mathbf{R}^3 is bounded by a surface described by the equation $\rho = f(\theta, \phi)$ in spherical coordinates. Show that the volume enclosed by the surface is

$$V = \frac{1}{3} \int_0^{2\pi} \int_0^\pi f^3(\theta, \phi) \sin \phi d\phi d\theta$$

3. [50 points] The temperature inside a rectangular box is described by $0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3$ is kd^2 , where d is the distance to the origin and k is constant. What is the average temperature in the rectangular box?
4. [25 points] Find the volume of the tetrahedron bounded by the coordinate planes and the plane $z = 4 - 3x - 2y$.