Name____

Date____

Vector Calculus Independent Study

Unit 8 Sample Test

1. [25 points] Let R be a region in the plane with area A, and let ∂R be R's boundary. Use Green's theorem to show that the center of mass $(\bar{x}, \bar{y})$ of R has coordinates

$$\bar{x} = \frac{1}{2A} \int_C x^2 \, dy$$

and

$$\bar{y} = \frac{1}{2A} \int_C y^2 \, dy$$

2. [25 points] Suppose

   (a) $\nabla \cdot \vec{F} = 0$ everywhere except at $(1, 0, 0)$ and $(3, 0, 0)$,

   (b) $\iint \vec{F} \cdot d\vec{S} = 5$ over the sphere $x^2 + y^2 + z^2 = 4$ oriented with outward pointing normal, and

   (c) $\iint \vec{F} \cdot d\vec{S} = 7$ over the sphere $x^2 + y^2 + z^2 = 16$ oriented with outward pointing normal.

   Use Gauss' Theorem to determine all the other possible values of

$$\iint \vec{F} \cdot d\vec{S}$$

   evaluated over spheres (not necessarily centered at the origin) with outward pointing normals.

3. [25 points] Calculate the surface integral

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

   where S is the hemisphere $x^2 + y^2 + z^2 = 1$, $x \leq 0$ and

$$\vec{F}(x, y, z) = (x^3, -y^3, 0).$$

4. [25 points] Prove that the work done by a particle moving along a closed path against a constant force field $\vec{F}(x, y, z) = \vec{v}$ is 0.