

Name \_\_\_\_\_

Date \_\_\_\_\_

## Vector Calculus Independent Study

### Unit 9 Sample Test

1. [20 points] Expand  $\ln x$  in a Taylor series about  $x = 5$ , find its radius of convergence, and determine if it converges or diverges on the endpoints of its interval of convergence.

2. [20 points] Calculate

(a)

$$\lim_{n \rightarrow \infty} (-1)^n \frac{n}{n+1}$$

(b)

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} + (0.5)^n$$

(c)

$$\lim_{n \rightarrow \infty} \frac{n}{\ln n}$$

(d)

$$\lim_{n \rightarrow \infty} \frac{2^n}{n^2}$$

3. [20 points] Consider the following series:

$$a = 3 + 3(0.5) + 3(0.5)^2 + \dots + 3(0.5)^n + \dots$$

$$b = \sum \frac{n}{2n+1}$$

$$c = \sum ne^{-n^2}$$

$$d = 5 + \frac{5}{\sqrt{2}} + \frac{5}{\sqrt{3}} + \frac{5}{\sqrt{4}} + \dots$$

$$e = \sum \frac{1}{3+4^n}$$

$$f = \sum (-1)^n \frac{2n}{3n^2 - 5}$$

$$g = \sum \frac{n!}{e^n}$$

$$h = \sum (-1)^n 3^{1/n}$$

For each series, determine whether it is convergent or divergent, and explain what test you used.

4. [20 points] Let

$$f(x) = \sum \frac{n^n}{n!} x^n$$

$$g(x) = \sum_{n=1}^{\infty} \frac{(n+1)!}{10^n} (x-5)^n$$

For both  $f$  and  $g$ , find

- The radius of convergence, including whether the series converges at the endpoints of the interval of convergence.
- The power series representing the improper integral.
- The power series representing the derivative.