Vector Calculus Independent Study

Unit 1: Vectors and Three Dimensional Geometry

In this unit you should/will learn:

1. Points and Vectors in 3 space.
    - How to associate cartesian coordinates to a point in $\mathbf{R}^3$.
    - How to find the distance between two points (Pythagorean theorem).
    - The definition of a vector, and the meaning of $\vec{0}$, $\vec{i}$, $\vec{j}$, and $\vec{k}$.
    - How to find the length of a vector, $|\vec{v}|$.
    - How to find the vector between two points, $\vec{v}_{a,b} = \vec{b} - \vec{a}$.
    - How to add and subtract vectors. Geometric meaning.
    - How to multiply/divide a vector by a constant. Geometric meaning.
2. Lines in 3 space.
    - Parametric definition: $l(t) = \vec{p} + t\vec{v}$
    - How to find the line between 2 points.
    - How to find the vector lying along the line ($\vec{v}$).
    - How to find the line going through a point along a given vector.
    - How to convert between parametric and implicit definitions of a line. (Implicit is $A_1x+B_1y+C_1z+D_1 = 0, A_2x+B_2y+C_2z+D_2 = 0$).
3. Planes in 3 space.
    - Implicit definition: $Ax + By + Cx + D = 0$.
    - How to find the plane containing 3 points.
    - How to find the plane containing a line and a point.

- How to find the plane containing 2 intersecting lines.
- How to find the normal to a plane. (A, B, C).
- How to find the plane through a point with a given normal vector: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.
- How to convert between implicit and parametric $P(r, s) = \vec{p} + r\vec{v_1} + s\vec{v_2}$. ($\vec{p}$ is a point on the plane, $\vec{v_1}$ and $\vec{v_2}$ are vectors lying on the plane.)

4. Dot Products.

   - Algebraic definition: $\vec{a} \cdot \vec{b} = a_1b_2 + a_2b_2 + a_3b_3$.
   - Geometric definition: $\vec{a} \cdot \vec{b} = |a||b| \cos\theta$.
   - How to find the angle between 2 vectors using the dot product.
   - How to project a vector onto another vector.
   - How to project a point onto a line.
   - How to find the distance between a point and a line.
   - How to test if two vectors are perpindicular ($\vec{a} \cdot \vec{b} = 0$?)

5. Cross Products.

   - Geometric definition: $|\vec{a} \times \vec{b}| = |a||b| \sin\theta$, with direction given by right hand rule.
   - Algebraic definition: $\vec{a} \times \vec{b} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = (a_2b_3 - b_2c_3)\vec{i} - (a_1b_3 - a_3b_1)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$
   - How to find a vector perpendicular to two other vectors.
   - How to find the area of a parallelogram spanned by two vectors.
   - How to find the area of a triangle spanned by two vectors.
   - How to find the plane going through two given vectors.

6. Other Coordinate Systems

   - The definition of the cylindrical coordinate system (r, θ, z).

- The definition of the spherical coordinate system (ρ, θ, ϕ).
- How to convert a point in cartesian coordinates to cylindrical or spherical coordinates, or vice-versa.
- How to convert an equation in cartesian coordinates to cylindrical or spherical coordinates, or vice-versa.

Suggested Procedure:

1. Read and do some problems from
   - Rogers Chapters 1, 2, 3, 4, and sections 7.1, 7.2, and 17.1.
   - Marsden and Tromba, chapter 1. [Section 1.5 is optional.]
   - Thomas & Finney chapter 13, or
   - Simmons section 17.3, and chapter 18.
2. Take the sample test.
3. Take a unit test.