Vector Calculus Independent Study

Unit 3: Scalar Valued Functions

Scalar valued functions of several variables are fundamental to the study of vector calculus. For one thing, it is possible to break any vector valued function up into component functions, each of which is scalar valued. For another, they are the most direct generalization of the single variable functions you studied in calculus, and have all of the wonderful applications that you've come to love and expecof the wonderful applications that you've come to love and expect (for example, max/min problems).

In this unit you should/will learn:

1. Graphing
   - How to graph $z = f(x, y)$.
   - How to find cross sections ($x = c$ or $y = c$).
   - How to find levels sets $z = c$.
   - How to use symmetry to graph functions.
   - How to find the domain of definition of a function.
   - How to find the level sets of $w = f(x, y, z)$.
2. Partial derivatives
   - The limit definition: $\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y)-f(x,y)}{h}$
   - How to compute one.
   - Geometric meaning.
   - The gradient $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$.
   - How to compute iterated derivatives.
   - $$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$
3. Tangent planes

- The tangent plane to $z = f(x, y)$ at (x_0, y_0) is $z = f(x_0, y_0) + \frac{\partial f}{\partial x}|_{(x_0,y_0)}(x - x_0) + \frac{\partial f}{\partial y}|_{(x_0,y_0)}(y - y_0)$.
- How to approximate $f(x, y)$ at (x_0, y_0):

$$f(x, y) \approx f(x_0, y_0) + \frac{\partial f}{\partial x}|_{(x_0,y_0)}(x - x_0) + \frac{\partial f}{\partial y}|_{(x_0,y_0)}(y - y_0)$$

4. Chain rule

   - If $z = f(g(x, y), h(x, y))$, then

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial x} + \frac{\partial f}{\partial h}\frac{\partial h}{\partial x}$$

5. Directional derivatives.

   - The limit definition.
   - The gradient short-cut: $\frac{\partial f}{\partial \vec{v}} = \nabla f \cdot \vec{v}$
   - The gradient as direction of maximal directional derivative (rate of increase).
   - The gradient is perpindicular to level sets.

Suggested Procedure:

1. Read and do some problems from

   - Rogers Chapters 8, 9, 10, and 11,
   - Marsden and Tromba chapter 2 (if you have the 4th edition, read section 3.1 as well), or
   - Simmons, sections 19.1 through 19.6.

2. Take the sample test.

3. Take a unit test.