Topics in Reinforcement Learning: AlphaZero, ChatGPT, Neuro-Dynamic Programming, Model Predictive Control, Discrete Optimization
Arizona State University
Course CSE 691, Spring 2024

Links to Class Notes, Videolectures, and Slides at
http://web.mit.edu/dimitrib/www/RLbook.html

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Lecture 4

POMDP, Systems with Changing Parameters, Adaptive Control
Model Predictive Control

Outline

Review: How do we Formulate DP Problems?

An informal recipe: First define the controls, then the stages (and info available at each stage), and then the states

- Define as state x_k something that "summarizes" the past for purposes of future optimization, i.e., as long as we know x_k, all past information is irrelevant.
- Rationale: The controller applies action that depends on the state. So the state must subsume all info that is useful for decision/control.

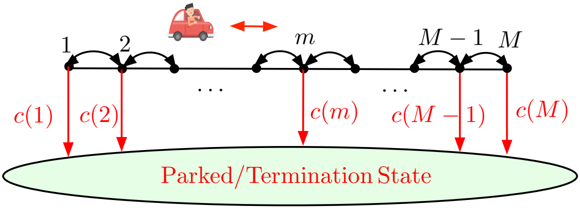
There may be multiple useful ways to define a valid state

- An important example is POMDP (Partial Information Markovian Decision Problems).
- At time k, instead of observing the state x_k, we obtain a measurement z_k that is "related" to x_k.
- Thus at time k all we have is the information vector

$$I_k = (z_0, u_0, z_1, u_1, \ldots, z_k, u_{k-1})$$

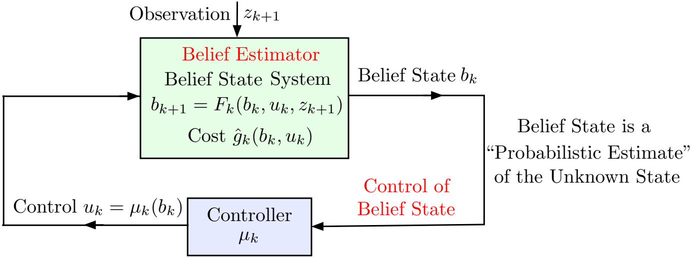
- It can serve as state, but there are also other possibilities.

Parking with a Deadline: An Example of Partial State Observation



- At each time step, move one spot in either direction. Decide to park or not at spot m (if free) at cost $c(m)$. If we have not parked by time N there is a large cost C
- We observe the free/taken status of only the spot we are in. Parking spots may change status at the next time step with some probability.
- The free/taken status of the spots is “estimated" in a “probabilistic sense" based on the observations (the free/taken status of the spots visited ... when visited)
- What should the “state" be? It should summarize all the info needed for the purpose of future optimization
- First candidate for state: The entire information vector up to the present time.
- Another candidate: The “belief state", i.e., the conditional probabilities of the free/taken status of all the spots: $p(1), p(2), \ldots, p(M)$, conditioned on all the observations so far

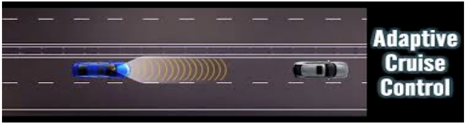
Partial State Observation Problems: Reformulation via Belief State



The reformulated DP algorithm has the form

$$J_k^*(b_k) = \min_{u_k \in U_k} \left[\hat{g}_k(b_k, u_k) + E_{z_{k+1}} \left\{ J_{k+1}^*\big(F_k(b_k, u_k, z_{k+1})\big) \mid b_k, u_k \right\} \right]$$

- $J_k^*(b_k)$ denotes the optimal cost-to-go starting from belief state b_k at stage k
- U_k is the control constraint set at time k
- $\hat{g}_k(b_k, u_k)$ denotes expected cost of stage k: expected stage cost $g_k(x_k, u_k, w_k)$, with distribution of (x_k, w_k) determined by b_k and the distribution of w_k
- Belief estimator: $F_k(b_k, u_k, z_{k+1})$ is the next belief state, given current belief state b_k, u_k is applied, and observation z_{k+1} is obtained



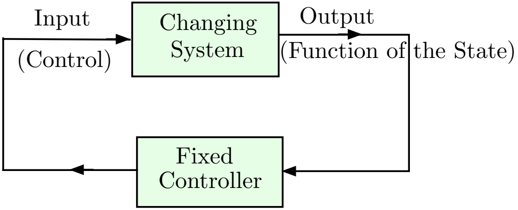
Example: A cruise control-type problem (keep velocity close to a target level)

- Control car velocity evolution: $x_{k+1} = ax_k + bu_k$ ($a < 1$ models friction, wind drag, etc, $b > 0$ depends on road, number of passengers, etc)
- Cost over N stages: $(x_N - \bar{x})^2 + \sum_{k=0}^{N-1} \big((x_k - \bar{x})^2 + ru_k^2\big)$, where $r > 0$ is given
- ... but a, b, and $\bar{x}$ are changing all the time; they may be measured with error (?)

Adaptive control deals with such situations. Some possibilities:

- Ignore the changes in parameters; design a controller that is robust ("works" for a broad range of parameters).
- Try to estimate the parameters, and use the estimates to modify the controller
  - On-line replanning by optimization; modify the controller to make it optimal for the current set of estimates.
  - On-line replanning by rollout with a base policy whose cost values are computed using the current parameter estimates. This is a simpler (approximate) reoptimization.
- View the adaptive control problem as a POMDP and try to deal approximately.

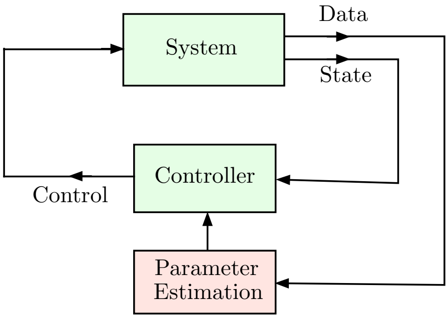
Robust Control



- We ignore the changes in the system
- Make no attempt to estimate/learn them
- Hope that a fixed controller will yield “acceptable" performance throughout the range of system changes
- A simple approach ... if it works

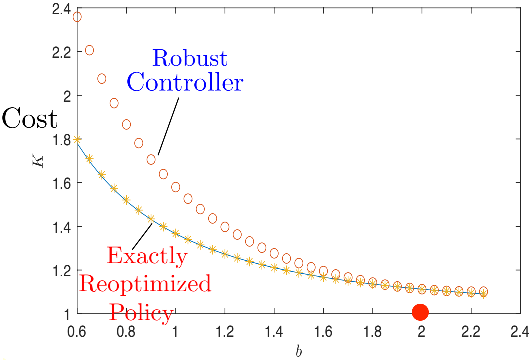
Unsophisticated ... but there is an important time-honored successful example

- PID control (Proportional-Integral-Derivative)
- Involves three parameters, which are tuned experimentally/heuristically
- Applies to single input-single output case (output: the error from some “set point")
- No math model of the system is needed



Two-phase alternation: System identification <–> Controller reoptimization

- Introduce on-line estimation/identification of changing parameters
- Recompute the controller so it is optimal for the current set of parameters
- This can be time-consuming
- There are some serious issues regarding reliable parameter identification (simultaneously with control); see the class notes

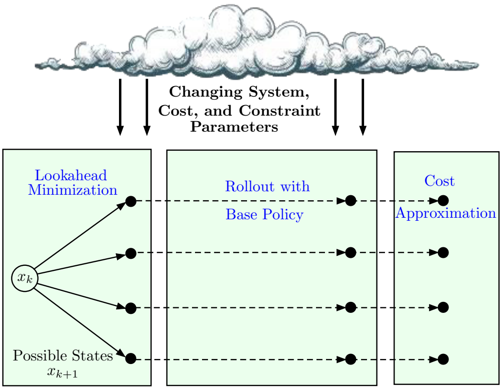


One-dimensional linear-quadratic example:

$$x_{k+1} = x_k + bu_k, \qquad \text{Cost} = \lim_{N\to\infty} \sum_{k=0}^{N-1} (x_k^2 + ru_k^2)$$

Quadratic cost coefficient as b changes. Robust controller is optimal for $b = 2$

On-Line Replanning by Rollout/Approximation in Value Space

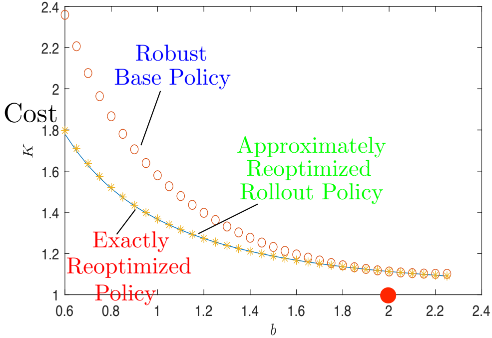


Use a robust policy as a base policy for rollout

No controller reoptimization; this is faster

- Introduce new parameter estimates in the lookahead minimization and the rollout
- Continue to use the same base/robust policy
- Possibly recalculate the base/robust policy in the background

Comparison of On-Line Replanning by Reoptimization and by Rollout



Performance comparison of on-line replanning by rollout and by optimization

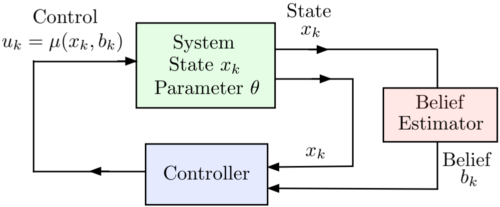
Note the effect of Newton's method

One-dimensional linear-quadratic example:

$$x_{k+1} = x_k + bu_k, \qquad \text{Cost} = \lim_{N\to\infty} \sum_{k=0}^{N-1} (x_k^2 + ru_k^2)$$

Quadratic cost coefficient as b changes. Base policy is optimal for $b = 2$

A (More Principled) POMDP Approach to Adaptive Control



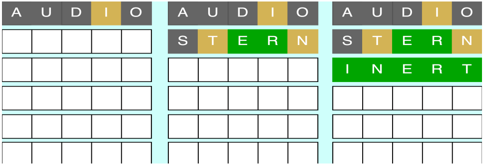
Deterministic system $x_{k+1} = f(x_k, \theta, u_k)$, $\theta \in \{\theta^1, \ldots, \theta^m\}$: unknown parameter

- View θ as part of an augmented state (x_k, θ) that is partially observed
- Belief state: $b_{k,i} = P\{\theta = \theta^i \mid I_k\}$ (estimated on-line), where $I_k = (x_0, \ldots, x_k, u_0, \ldots, u_{k-1})$ is the information vector
- Bellman equation for optimal cost function J_k^*:

$$J_k^*(I_k) = \min_{u_k} \sum_{i=1}^{m} b_{k,i} \Big(g(x_k, \theta^i, u_k) + J_{k+1}^*\big(I_k, f(x_k, \theta^i, u_k), u_k\big) \Big)$$

- Approximation in value space: Use approximation $\tilde{J}^i\big(f(x_k, \theta^i, u_k)\big)$ in place of $J_{k+1}^*\big(I_k, f(x_k, \theta^i, u_k), u_k\big)$. Minimize over u_k to obtain one-step lookahead policy
- Example 1: $\tilde{J}^i$ is the cost function of the optimal policy corresponding to θ^i
- Example 2: $\tilde{J}^i$ is the cost function of a known policy assuming $\theta = \theta^i$ (this is rollout)

An Example: The Wordle Puzzle



θ: the unknown mystery word

x_k: list of possible mystery words (given the guesses so far)

u_k: guess word selected at time k

- Find a mystery word θ using a minimal number of successive 5-letter guess words
- View θ as part of an augmented state (x_k, θ) that is partially observed
- Apply rollout with one of several base heuristics
- Joint work with S. Bhambri and A. Bhattacharjee (started as term paper for the 2022 offering of this class); see ArXiv paper on-line
- To be discussed in more detail at a later lecture

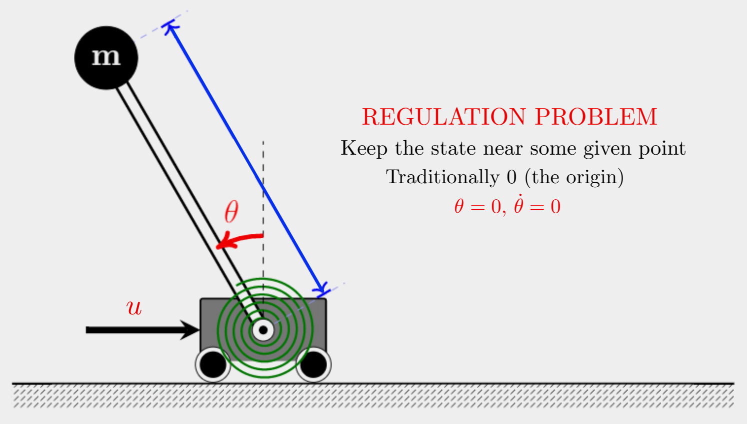
Rollout Performance = 3.5231 vs the Optimal = 3.5084 average # of guesses

Within $<$ 0.5% more guesses from the optimal policy - On-line answer within 1-3 secs

IT SCALES WITH PROBEM SIZE

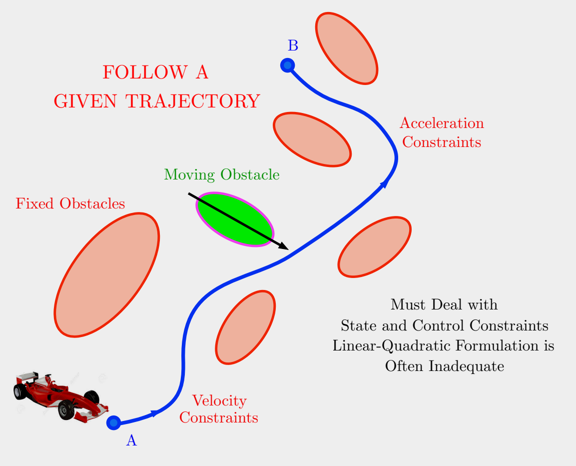
A Fifteen-Minute Break

Catch our breath and think about issues relating to the first half of the lecture.
Ask questions when you return.



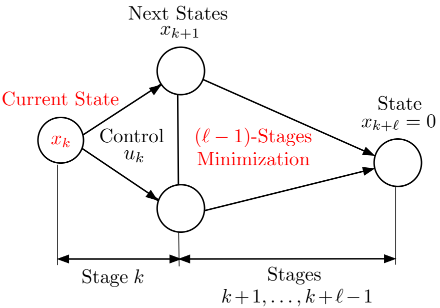
Keep the state of the system close to a reference point (usually 0)

Classical Control Problem II: Path Planning



Keep the state of the system close to a planned trajectory
State constraints can be very important
On-line path replanning may be needed

Model Predictive Control - A Form of Approximation in Value Space



System: $x_{k+1} = f(x_k, u_k)$

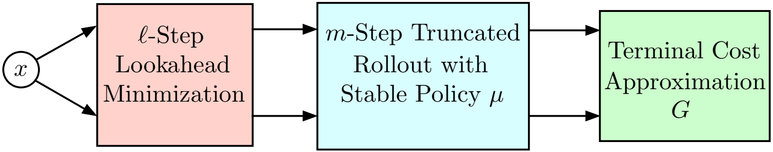
Cost: $g(x_k, u_k) \geq 0$, for all (x_k, u_k)

The system can be kept at the origin at zero cost by some control

Consider undiscounted infinite horizon; we want to keep the system near 0

- Original form of MPC: We minimize the cost function over the next ℓ stages while requiring $x_{k+\ell} = 0$
- At the current state x_k, we apply the first control of the minimizing sequence, discard the other controls
- This is rollout w/ base heuristic the min that drives $x_{k+\ell}$ to 0 in $(\ell - 1)$ steps
- Well-suited for on-line replanning
- We neglect for the moment (the often very important) state constraints

Model Predictive Control with Terminal Cost Approximation



- In a common variant of MPC, we use a nonnegative terminal cost $G(x_{k+\ell})$ in the ℓ-stage MPC problem, instead of driving the state to 0 in ℓ steps:

$$\min_{u_t,\ t=k,\ldots,k+\ell-1} \left[G(x_{k+\ell}) + \sum_{t=k}^{k+\ell-1} g(x_t, u_t) \right]$$

- This can be viewed as approximation in value space with multistep lookahead
- Truncated rollout with some base policy may also be introduced
- The problem may also include state constraints, in which case we obtain one of the most general forms of MPC
- When is the MPC policy stable? G must be in the region of stability; see the class notes. Truncated rollout helps in this respect.

Model Predictive Control with State Constraints

- It is often important to deal with additional state constraints of the form $x_k \in X$, where X is some subset of the state space
- Then the MPC problem to be solved at the kth stage must be modified
- Assuming that the current state x_k belongs to X, the MPC problem is

$$\min_{u_t,\, t=k,\ldots,k+\ell-1} \left[G(x_{k+\ell}) + \sum_{t=k}^{k+\ell-1} g(x_t, u_t) \right],$$

subject to the control constraints

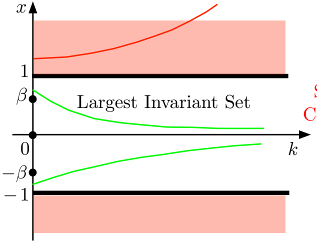
$$u_t \in U(x_t), \qquad t = k, \ldots, k+\ell-1,$$

and the state constraints

$$x_t \in X, \qquad t = k+1, \ldots, k+\ell$$

- The control $\tilde{u}_k$ thus obtained is required to generate a state $x_{k+1} \in X$
- Important difficulty: There is no guarantee that this problem has a feasible solution for all initial states $x_k \in X$
- This is particularly true for unstable systems

A One-Dimensional Example for MPC with State Constraints



- Consider state constraints of the form $x_k \in X$, for all k, where

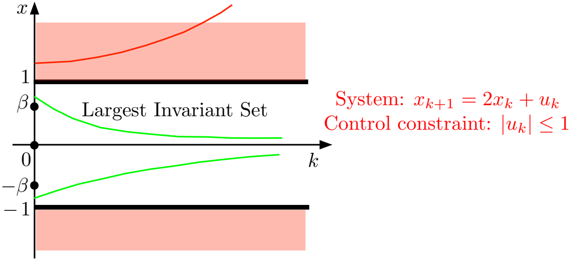
$$X = \{x \mid |x| \leq \beta\}$$

- If $\beta > 1$, the state constraint cannot be satisfied for all initial states $x_0 \in X$.
- Reason: If we take $x_0 = \beta > 1$, then $2x_0 > 2$ and $x_1 = 2x_0 + u_0$ will satisfy $x_1 > x_0 = \beta$ for any value of u_0 with $|u_0| \leq 1$.
- Arguing similarly, the entire sequence of generated states $\{x_k\}$ will satisfy

$$x_{k+1} > x_k \quad \text{for all } k, \qquad x_k \uparrow \infty.$$

- The state constraint can be satisfied only for x_0 in the set $\hat{X} = \{x \mid |x| \leq 1\}$
- Sets like $\hat{X}$ are called invariant (see the next slide)

Invariance Requirement for the State Constraints



- To guarantee feasibility of the MPC problem with state constraints $x_k \in X$ for all k, an invariance condition must be satisfied by X

$$\text{for every } x \in X, \text{ there exists } u \in U(x) \text{ such that } f(x, u) \in X$$

- How do we compute an invariant subset of a given constraint set?
- This is necessarily an off-line calculation. Cannot be easily performed during on-line play
- It turns out that given X, there exists a largest possible invariant subset of X and it can be computed in the limit with an algorithm that resembles value iteration.
- There also other/simpler possibilities for computing invariant subsets of X

Overview of What we Have Done

We aimed for an overview of the approximate DP/RL landscape; a foundation for deeper development of other RL topics (and getting you started on your term paper).

We have described in varying levels of depth the following:

- The algorithmic foundation of exact DP in all its major forms: deterministic and stochastic, discrete and continuous, finite and infinite horizon.
- Approximation in value space with one-step and multistep lookahead, the workhorse of RL, which underlies its major success stories, including AlphaZero.
- The fundamental division between off-line training and on-line play in the context of approximation in value space. Their synergy through Newton's method.
- The fundamental methods of policy iteration and rollout, the former being primarily an off-line method, and the latter being primarily a less ambitious on-line method. Connections with Newton's method.
- Some major models with a broad range of applications, such as discrete optimization, POMDP, multiagent problems, adaptive control, and model predictive control.
- The use of function approximation, which has been a recurring theme in our presentation. We have hinted at several points some of the principal schemes for approximation, based on neural networks and feature-based architectures.

Overview of What Lies Ahead; What we Will and Will Not Cover

The first chapter of the class notes provides a foundational platform for exploring at a deeper level various algorithmic methodologies, such as:

- Rollout, its variants, and its applications; e.g., discrete/deterministic, multiagent, etc; Chapter 2.
- Sequential decision making in special contexts involving changing system parameters, sequential estimation, Bayesian optimization, and simultaneous system identification and control; Chapter 2.
- Off-line training for approximation in value and policy space using neural networks and other approximation architectures; Chapter 3.
- Stochastic algorithms, such as temporal difference methods and Q-learning, used for off-line policy evaluation, in the context of approximate policy iteration.
- Sampling methods to collect data for off-line training, in the context of cost and policy approximations.
- Statistical estimates and efficiency enhancements of various sampling methods used in simulation-based schemes.
- A deeper exploration of control system design methodologies such as model predictive control, and its applications in robotics and automated transportation.

About the Next Lecture

We will cover:

- Deterministic rollout and variations
- Rollout for stochastic problems

Homework to be announced next week

Watch videolecture 5 from 2023 ASU course offering