Topics in Reinforcement Learning: Rollout and Approximate Policy Iteration

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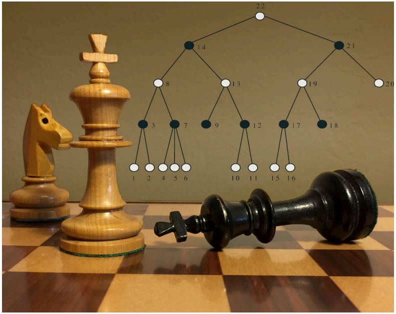
Lecture 1

Outline

1. Introduction, History, General Concepts
2. About this Course
3. Exact Dynamic Programming - Deterministic Problems
4. Organizational Issues



AlphaZero

Plays much better than all chess programs

Plays different!

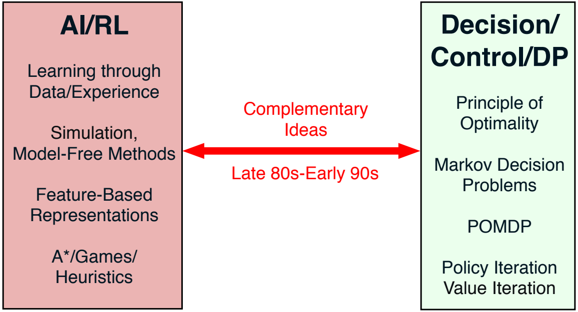
Learned from scratch ... with 4 hours of training!

Same algorithm learned multiple games (Go, Shogi)

AlphaZero is not just playing better, it has discovered a new way to play!

With a methodology closely related to the special RL topics of this course

Evolution of Approximate DP/RL



Historical highlights

- Exact DP, optimal control (Bellman, Shannon, and others 1950s ...)
- AI/RL and Decision/Control/DP ideas meet (late 80s-early 90s)
- First major successes: Backgammon programs (Tesauro, 1992, 1996)
- Algorithmic progress, analysis, applications, first books (mid 90s ...)
- Machine Learning, BIG Data, Robotics, Deep Neural Networks (mid 2000s ...)
- AlphaGo and Alphazero (DeepMind, 2016, 2017)

Approximate DP/RL Methodology is now Ambitious and Universal

Exact DP applies (in principle) to a very broad range of optimization problems

- Deterministic <—-> Stochastic
- Combinatorial optimization <—-> Optimal control w/ infinite state/control spaces
- One decision maker <—-> Two player games
- ... BUT is plagued by the curse of dimensionality and need for a math model

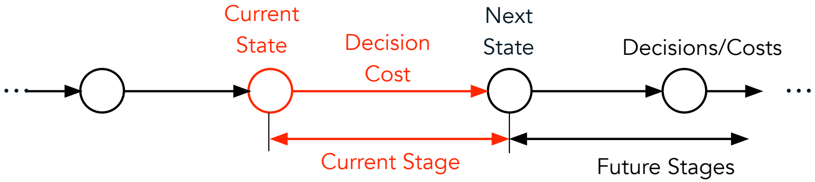
Approximate DP/RL overcomes the difficulties of exact DP by:

- Approximation (use neural nets and other architectures to reduce dimension)
- Simulation (use a computer model in place of a math model)

State of the art:

- Broadly applicable methodology: Can address a very broad range of challenging problems. Deterministic-stochastic-dynamic, discrete-continuous, games, etc
- There are no methods that are guaranteed to work for all or even most problems
- There are enough methods to try with a reasonable chance of success for most types of optimization problems
- Role of the theory: Guide the art, delineate the sound ideas

A Key Idea: Sequential Decisions w/ Approximation in Value Space



Exact DP: Making optimal decisions in stages (deterministic state transitions)

- At current state, apply decision that minimizes

$$\text{Current Stage Cost} + J^*(\text{Next State})$$

  where $J^*(\text{Next State})$ is the optimal future cost, starting from the next state.
- This defines an optimal policy (an optimal control to apply at each state and stage)

Approximate DP: Use approximate cost $\tilde{J}$ instead of J^*

- At current state, apply decision that minimizes (perhaps approximately)

$$\text{Current Stage Cost} + \tilde{J}(\text{Next State})$$

- This defines a suboptimal policy

Major Approaches/Ideas to Compute the Approximate Cost Function $\tilde{J}$

Problem approximation

Use as $\tilde{J}$ the optimal cost function of a related problem (computed by exact DP)

Rollout and model predictive control

Use as $\tilde{J}$ the cost function of some policy (computed somehow, perhaps according to some simplified optimization process)

Use of neural networks and other feature-based architectures

They serve as function approximators

Use of simulation to generate data to “train" the architectures

Approximation architectures involve parameters that are “optimized" using data

Policy iteration/self-learning, repeated policy changes

Multiple policies are sequentially generated; each is used to provide the data to train the next

Aims and References of this Course

Purpose of this course

- To explore the state of the art of approximate DP/RL at a graduate level
- To explore in some depth some special research topics (rollout, approximate policy iteration)
- To provide the opportunity for you to explore research in the area

Main references:

- Bertsekas, Reinforcement Learning and Optimal Control, Athena Scientific, 2019
- Bertsekas: Class notes based on the above, and focused on our special RL topics. Slides and videolectures from the 2019 ASU offering, and "Ten Key Ideas ..." overview lecture; check my web site
- Selected papers on AlphaGo, AlphaZero, and others

Supplementary references

- Exact DP: Bertsekas, Dynamic Programming and Optimal Control, Vol. I (2017), Vol. II (2012) (also contains approximate DP material)
- Bertsekas and Tsitsiklis, Neuro-Dynamic Programming, 1996
- Sutton and Barto, 1998, Reinforcement Learning (new edition 2018, on-line)

Terminology in RL/AI and DP/Control

RL uses Max/Value, DP uses Min/Cost

- Reward of a stage = (Opposite of) Cost of a stage.
- State value = (Opposite of) State cost.
- Value (or state-value) function = (Opposite of) Cost function.

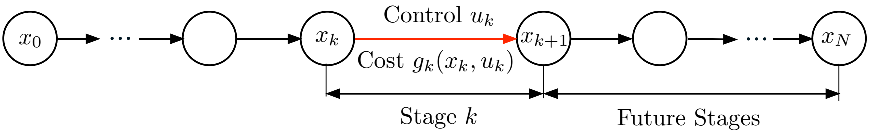
Controlled system terminology

- Agent = Decision maker or controller.
- Action = Decision or control.
- Environment = Dynamic system.

Methods terminology

- Learning = Solving a DP-related problem using simulation.
- Self-learning (or self-play in the context of games) = Solving a DP problem using simulation-based policy iteration.
- Planning vs Learning distinction = Solving a DP problem with model-based vs model-free simulation.

Finite Horizon Deterministic Problem



- System

$$x_{k+1} = f_k(x_k, u_k), \qquad k = 0, 1, \ldots, N-1$$

where x_k: State, u_k: Control chosen from some set $U_k(x_k)$

- Cost function:

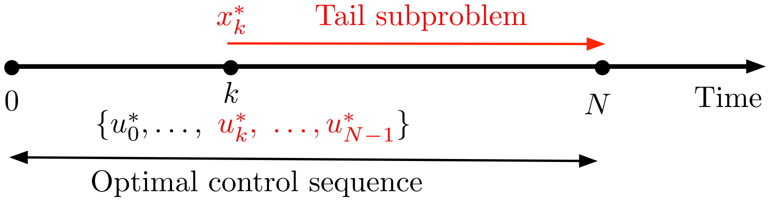
$$g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

- For given initial state x_0, minimize over control sequences $\{u_0, \ldots, u_{N-1}\}$

$$J(x_0; u_0, \ldots, u_{N-1}) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

- Optimal cost function $J^*(x_0) = \min_{\substack{u_k \in U_k(x_k) \\ k=0,\ldots,N-1}} J(x_0; u_0, \ldots, u_{N-1})$

Principle of Optimality: A Very Simple Idea



Principle of Optimality

Let $\{u_0^*, \ldots, u_{N-1}^*\}$ be an optimal control sequence with corresponding state sequence $\{x_1^*, \ldots, x_N^*\}$. Consider the tail subproblem that starts at x_k^* at time k and minimizes over $\{u_k, \ldots, u_{N-1}\}$ the "cost-to-go" from k to N,

$$g_k(x_k^*, u_k) + \sum_{m=k+1}^{N-1} g_m(x_m, u_m) + g_N(x_N).$$

Then the tail optimal control sequence $\{u_k^*, \ldots, u_{N-1}^*\}$ is optimal for the tail subproblem.

THE TAIL OF AN OPTIMAL SEQUENCE IS OPTIMAL FOR THE TAIL SUBPROBLEM

DP Algorithm: Solves All Tail Subproblems Using the Principle of Optimality

Idea of the DP algorithm

Solve all the tail subproblems of a given time length using the solution of all the tail subproblems of shorter time length

By the principle of optimality: To solve the tail subproblem that starts at x_k

- Consider every possible u_k and solve the tail subproblem that starts at next state $x_{k+1} = f_k(x_k, u_k)$. This gives the "cost of u_k"
- Optimize over all possible u_k

DP Algorithm: Produces the optimal costs $J_k^*(x_k)$ of the x_k-tail subproblems

Start with

$$J_N^*(x_N) = g_N(x_N), \qquad \text{for all } x_N,$$

and for $k = 0, \dots, N-1$, let

$$J_k^*(x_k) = \min_{u_k \in U_k(x_k)} \Big[g_k(x_k, u_k) + J_{k+1}^*\big(f_k(x_k, u_k)\big) \Big], \qquad \text{for all } x_k.$$

The optimal cost $J^*(x_0)$ is obtained at the last step: $J_0(x_0) = J^*(x_0)$.

Construction of Optimal Control Sequence $\{u_0^*, \ldots, u_{N-1}^*\}$

Start with

$$u_0^* \in \arg \min_{u_0 \in U_0(x_0)} \Big[g_0(x_0, u_0) + J_1^*\big(f_0(x_0, u_0)\big) \Big],$$

and

$$x_1^* = f_0(x_0, u_0^*).$$

Sequentially, going forward, for $k = 1, 2, \ldots, N-1$, set

$$u_k^* \in \arg \min_{u_k \in U_k(x_k^*)} \Big[g_k(x_k^*, u_k) + J_{k+1}^*\big(f_k(x_k^*, u_k)\big) \Big], \qquad x_{k+1}^* = f_k(x_k^*, u_k^*).$$

Approximation in Value Space - Use Some $\tilde{J}_k$ in Place of J_k^*

Start with

$$\tilde{u}_0 \in \arg \min_{u_0 \in U_0(x_0)} \Big[g_0(x_0, u_0) + \tilde{J}_1\big(f_0(x_0, u_0)\big) \Big],$$

and set

$$\tilde{x}_1 = f_0(x_0, \tilde{u}_0).$$

Sequentially, going forward, for $k = 1, 2, \ldots, N-1$, set

$$\tilde{u}_k \in \arg \min_{u_k \in U_k(\tilde{x}_k)} \Big[g_k(\tilde{x}_k, u_k) + \tilde{J}_{k+1}\big(f_k(\tilde{x}_k, u_k)\big) \Big], \qquad \tilde{x}_{k+1} = f_k(\tilde{x}_k, \tilde{u}_k).$$

Extensions

Stochastic finite horizon problems

The next state x_{k+1} is also affected by a random parameter (in addition to x_k and u_k)

Infinite horizon problems

The exact DP theory is mathematically more complex

Stochastic partial state information problems

Very hard to solve even approximately ... but offer great promise for applications

Minimax/game problems

The exact DP theory is substantially more complex ... but the most spectacular successes of RL involve games

Course Aims and Requirements

Our principal aim:

To get you to think about research in RL, and about how RL may apply to your current research interests

Requirements:

- Pass-Fail
- Homework (50%): A total of 2-3
- Research-oriented term paper (50%). A choice of:
  - A mini-research project. You may work in teams of 1-3 persons. You are strongly encouraged to at least try. Selected projects will be presented to the class at the end of the term. I am available to help.
  - A read-and-report term paper based on 2-3 research publications (chosen by you in consultation with me)

Our TA: Shushmita Bhattacharya, sbhatt55@asu.edu

Office hours: Tuesdays or Thursdays 4-5pm, or by appointment

Syllabus (Approximate)

- Lecture 1 (this lecture): Introduction, finite horizon deterministic exact DP
- Lecture 2: Stochastic exact DP, examples of problem formulation
- Lecture 3: Approximation in value space, introduction to rollout (start from a policy, get a better policy)
- Lecture 4: Rollout, Monte Carlo tree search, model predictive control
- Lecture 5: Rollout with an expert, multiagent rollout, constrained rollout
- Lecture 6: Applications of rollout in large-scale discrete optimization and other areas
- Lecture 7: Parametric approximation architectures, feature-based architectures, (deep) neural nets, training with incremental/stochastic gradient methods
- Lecture 8: Value and policy networks; use in approximate DP; perpetual rollout
- Lecture 9: AlphaGo and AlphaZero
- Lecture 10: Infinite horizon and policy iteration
- Lecture 11: Distributed asynchronous policy iteration
- Lecture 12: Partitioned architectures and distributed asynchronous policy iteration
- Lecture 13: Project presentations

About Machine Learning and Math

Math requirements for this course are modest

Calculus, elementary probability, minimal use of vector-matrix algebra. Our objective is to use math to the extent needed to develop insight into the mechanism of various methods, and to be able to start research.

However a math framework is critically important

Human insight can only develop within some structure of human thought ... math reasoning is most suitable for this purpose

On machine learning (from NY Times Article, Dec. 2018)

“What is frustrating about machine learning is that the algorithms can’t articulate what they’re thinking. We don’t know why they work, so we don’t know if they can be trusted ... As human beings, we want more than answers. We want insight. This is going to be a source of tension in our interactions with computers from now on."

About the Next Lecture

We will cover:

- Stochastic DP algorithm
- DP algorithm for Q-factors
- Examples of discrete deterministic DP problems
- Partial information problems

PLEASE READ AS MUCH OF CHAPTER 1 OF CLASS NOTES AS YOU CAN

MAKE SURE YOUR NAME/EMAIL IS LISTED IN THE APPROPRIATE SIGNUP SHEET