TESTS FOR THE SUSPECTED TREND IN ANNUAL RAINFALL SERIES IN CENTRAL AND WESTERN SUDAN

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ABSTRACT

It is widely noticed that the annual rainfall amounts in the entire region of central and western Africa is showing a decline for the last decade. The significance of the decline in the annual rainfall series in the region of central and western Sudan is investigated to check whether it is the random variability or there are indications of a trend in annual rainfall amounts.

INTRODUCTION

Central and western Sudan is defined in the present study as the region which extends between 10° and 15°N and from the White Nile in the east to the borders with Chad in the west. The region is an extension of the African belt severely affected by the last continental drought, known as the Sahel and Soudano-Sahel regions. The region is also known as Kordofan and Darfur. It is shown in figure (1).

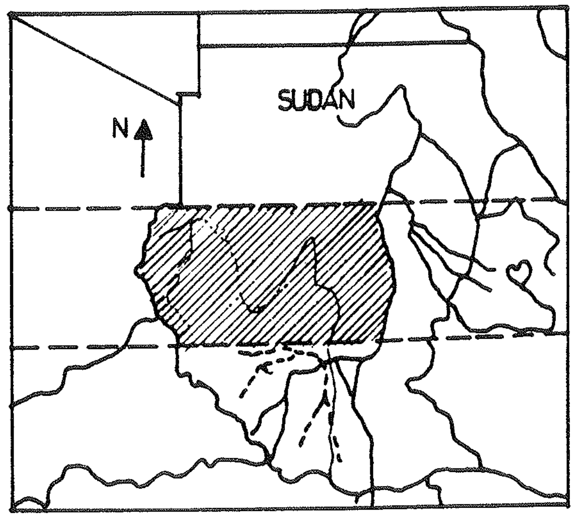
Eleven station's records from the region are investigated. A grouping of the stations was needed to decide which stations' results to consider while deriving conclusions about the stationarity of annual rainfall in the different sub-regions.

In grouping the stations four parameters were considered, the mean of the series, the coefficient of variation, the length of the rainy season and the autocorrelation coefficient at lag one. According to the values of these parameters the region was divided into five sub-regions Khartoum, Northern Kordofan, Southern Kordofan, Northern Darfur and Southern Darfur. Table (1) shows the grouping of the stations and describes the annual rainfall series for each station. Appendix A shows graphical display of the eleven annual rainfall series.

RESULTS OF THE TESTS OF STATIONARITY AGAINST A TREND

Two tests, Kendall test and Student's t test, W.M.O technical note (5), were applied to check the suspected downward trend in annual rainfall series in the region. Appendix B explains the theory of the two tests. They were applied as one-sided tests with the null hypothesis: 'the series is a sample from a population which is stationary in the mean.', against the alternative hypothesis: 'the series is showing a downward trend.'. The results of the tests are shown in table (2).

From the results of these tests it was concluded that the annual rainfall series from the sub-regions of Khartoum, Northern Kordofan and Southern Kordofan proved stationary in the mean while the annual rainfall series from the sub-regions of Northern Darfur and Southern Darfur proved to be showing a downward trend. The only exception was the result of the Kendall test for the annual rainfall series in Southern Kordofan. but results from a weaker test proved the stationarity of



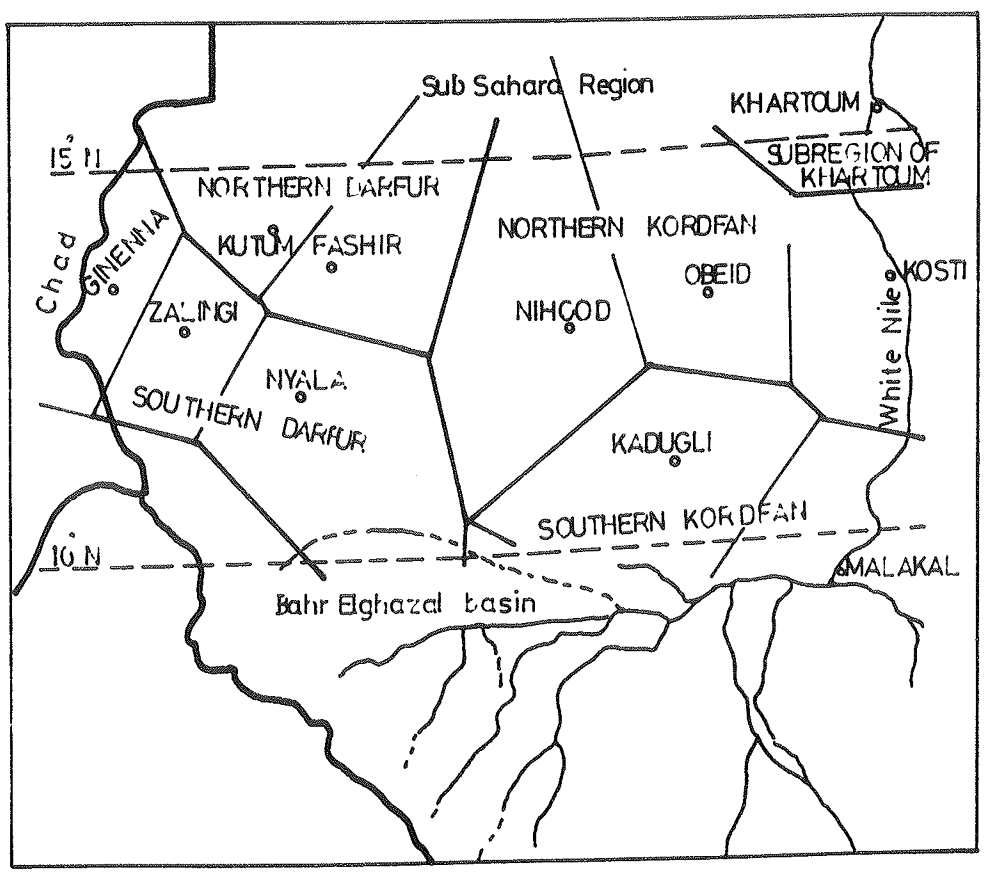


FIGURE (1) REGION OF CENTRAL AND WESTERN SUDAN

Table (1) subregions and Properties of Annual Rainfall Series.

Subregions and Stations	Period of Record	Mean (mm)	Coefficient of Variation	Autocorrelation Coefficient at lag one	Length of Rainy Season (month)
(1) Khartoum:					
Khartoum	(1899-1986)	155.7	0.48	0.21	5
(2) Northern Kordfan:					
Kosti	(1909-1986)	387.6	0.29	0.34	6
El Obeid	(1902-1986)	363.9	0.30	0.29	6
El Nihood	(1912-1986)	393.7	0.29	0.19	6
(3) Southern Kordfan:					
Kadugli	(1910-1986)	728.2	0.19	0.04	7
Malkal	(1909-1986)	777.0	0.20	0.18	8
(4) Northern Darfur:					
Fashir	(1918-1986)	267.7	0.43	0.32	5
Kutum	(1929-1986)	297.3	0.36	0.27	5
(5) Southern Darfur:					
Nyala	(1920-1986)	457.4	0.26	0.42	6
El Ginena	(1929-1986)	505.3	0.29	0.51	6
Zalingi	(1929-1986)	603.6	0.23	0.57	6

Table (2) Results of the Tests of Stationarity against a Trend.

Subregion	Station	Kendall Test	Student's t Test
(1) Khartoum	Khartoum	A*	A
(2) Northern Kordfan	Kosti	A	A
	El Obeid	A	A
	El Nihood	A	A
(3) Southern Kordfan	Kadugli	R	A
	Malkal	R	A
(4) Northern Darfur	El Fashir	R	R
	Kutum	R	R
(5) Southern Darfur	Nyala	R	R
	El Ginena	R	R
	Zalingi	R	R

*A : The null hypothesis was accepted at the 5% level of significance.
R : The null hypothesis was rejected at the 5% level of significance, the alternative hypothesis was accepted.

the annual rainfall series in Southern Kordofan at the 1% level of significance.

It was noticed that stations from the western sub-regions, which were Northern and Southern Darfur, have shorter records than the other stations. Table (1) gives a clear picture of this fact. For three of the stations, namely Kutum, Ginena and Zalingi the record starts at the year 1929, while for Elfashir and Nyala it starts at years 1918 and 1920 respectively. It was suspected that the non-stationarity shown by the annual rainfall series from the sub-regions of Darfur was due to the short records available there rather than to any other factor.

Farmer and Wigley (1). defined climate as the synthesis of weather over a period long enough to establish its statistical properties (mean values, variances, probabilities of extreme events, etc.) and is largely independent of any instantaneous (Weather) state. They concluded that any discussion of climate requires a specification of the applicable time scale (inter-annual, decadal and so on) and any statement regarding prevailing climatic conditions relative to some normal should state what reference period has been used to define the normal.

In the light of those conclusions, the same tests for checking stationarity against a trend were repeated for all the stations with the same null hypothesis: 'the series is a sample from a population which is stationary in the mean. 'against the alternative hypothesis 'the series is showing a downward trend.', considering only the records with the common period, namely (1929-1986). The results for these tests are shown in table (3). From these results it was concluded that annual rainfall amounts in the entire region are showing a downward trend with respect to the normal climatic conditions prevailing since 1929.

The Kendall rank statistic was used as a measure of the correlation between the historically observed sequence of the series and the series sorted in an ascending order. A value of the statistic close to a positive one indicates a rising trend while a value close to a negative one indicates a falling trend. The Kendall rank statistic for a stationary independent series of a sufficiently large size will asymptotically approach zero. Table (4) shows the Kendall rank statistic calculated for all the stations for the full record and also for the common period of records.

DISCUSSION OF THE RESULTS

The difference between the normal climatic conditions prevailing in the region since the year 1899 and the normal climatic conditions prevailing since 1929 can be seen very clearly in the change of the value of the Kendall rank statistic as calculated from the annual rainfall series in Khartoum for the two periods, a change from a value of -0.03 which is not significantly different from zero to a value of -0.24 which was not tolerated by a stationary process even at the 1% level of significance.

Nicholson (3) deriving anomaly types for the region of western and central Africa, including the sub-regions considered in the present study, classified the years from 1902 to 1972 into five annual rainfall anomaly types well above normal, above normal, normal, below normal and well below normal annual rainfall.

In that study it was concluded that the years 1902 to 1908 were highly correlated within each other and were having well below

Table (3) Results of the Tests of Stationarity against a Trend. (1929-1986)

Subregion	Station	Kendall Test	Student's Test
(1) Khartoum	Khartoum	R*	A
(2) Northern Kordfan	Kosti	R	A
	El Obeid	R	R
	El Nihood	R	R
(3) Southern Kordfan	Kadugli	R	R
	Malkal	A	A
4) Norhern Darfur	El Fashir	R	R
	Kutum	R	R
(5) Southern Darfur	Nyala	R	R
	El Ginena	R	R
	Zalingi	R	R

* A : The null hypothesis was accepted at the 5% level of significance.
R : The null hypothesis was rejected at the 5% level of significance, the alternative hypothesis was accepted

Table (4) Kendall Rank Statistic for Annual Rainfall Series, Central and Wewtern Sudan.

Subregion	Station	Period of Record	Kendall Rank Statistic (full record)	Kendall Rank Statistic (1929-1986)
(1) Khartoum	Khartoum	1899-1986	-0.03	-0.24
(2) Northern Kordfan	Kosti	1909-1986	-0.10	-0.19
	El Obeid	1902-1986	-0.09	-0.29
	El Nihood	1912-1986	-0.13	-0.25
(3) Southern Darfur	Kadugli	1910-1986	-0.16	-0.24
	Malkal	1909-1986	-0.16	-0.12
(4) Northern Darfur	El Fashir	1918-1986	-0.32	-0.33
	Kutum	1929-1986	-0.35	-0.35
(5) Southern Darfur	Nyala	1920-1986	-0.37	-0.38
	El Ginena	1929-1986	-0.29	-0.29
	Zalingi	1929-1986	-0.28	-0.28

normal level of rainfall. The years 1918, 1924, 1927 and 1928 were classified as years with below normal rainfall. She also described the years 1911, 1912, 1913 and 1917 as negatively correlated to the anomaly type of well above normal rainfall. None of the years from the period 1902 to 1928 were classified with the anomaly types of well above normal, above normal or normal annual rainfall.

From this study it was concluded that on average the period from 1902 to 1928 was a period of low annual rainfall amounts in the entire region of western and central Africa. Hence including this period within the records of any of the stations considered by the present study affected the statistical properties of the series.

Nicholson (3) calculated the correlation coefficients between the areally averaged normalized annual rainfall departures for different sub-regions from western and central Africa. There is a correspondence between the sub-regions of that study and the sub-regions of the present study. She found significant correlations between the regional averages of the different sub-regions. The correlation coefficient between the sub-regions 16 (which included Northern Darfur) and 17 (which included Northern Kordofan) was calculated as 0.59, while the correlation between sub-regions 21 (which included Southern Darfur) and 22 (which included Southern Kordofan) was calculated as 0.38.

Since annual rainfall series in the eastern sub-regions were proved stationary relative to the normal climatic conditions prevailing since the beginning of the present century, and the results of the correlations between the eastern sub-regions and the western sub-regions were found significant, it can be postulated that annual rainfall series in the entire region may be stationary relative to the normal climatic conditions prevailing in the region since the early years of the present century.

CONCLUSIONS

A trend in annual rainfall is a sign of a climatic change, but climate is a function of the reference period considered in defining the normal climatic conditions. Hence a conclusion about a trend in annual rainfall is also a function of the period of record investigated.

For the region of central and western Sudan the common period for the records of all stations considered extends from 1929 to 1986. For this period all the stationarity tests indicate that the present decline in annual rainfall is significant and can not be considered as a random variability in the process.

For some of the stations the records available start in early years of the present century. These are mainly the stations in Kordofan and Khartoum. The results of the tests run on these records indicate that the observed decline is insignificant. This may suggest that similar levels of rainfall were experienced in early years of the present century.

A continuation of the present low levels of annual rainfall for the next few years will favour the significance of the decline and the unfortunate climatic change.

ACKNOWLEDGMENT

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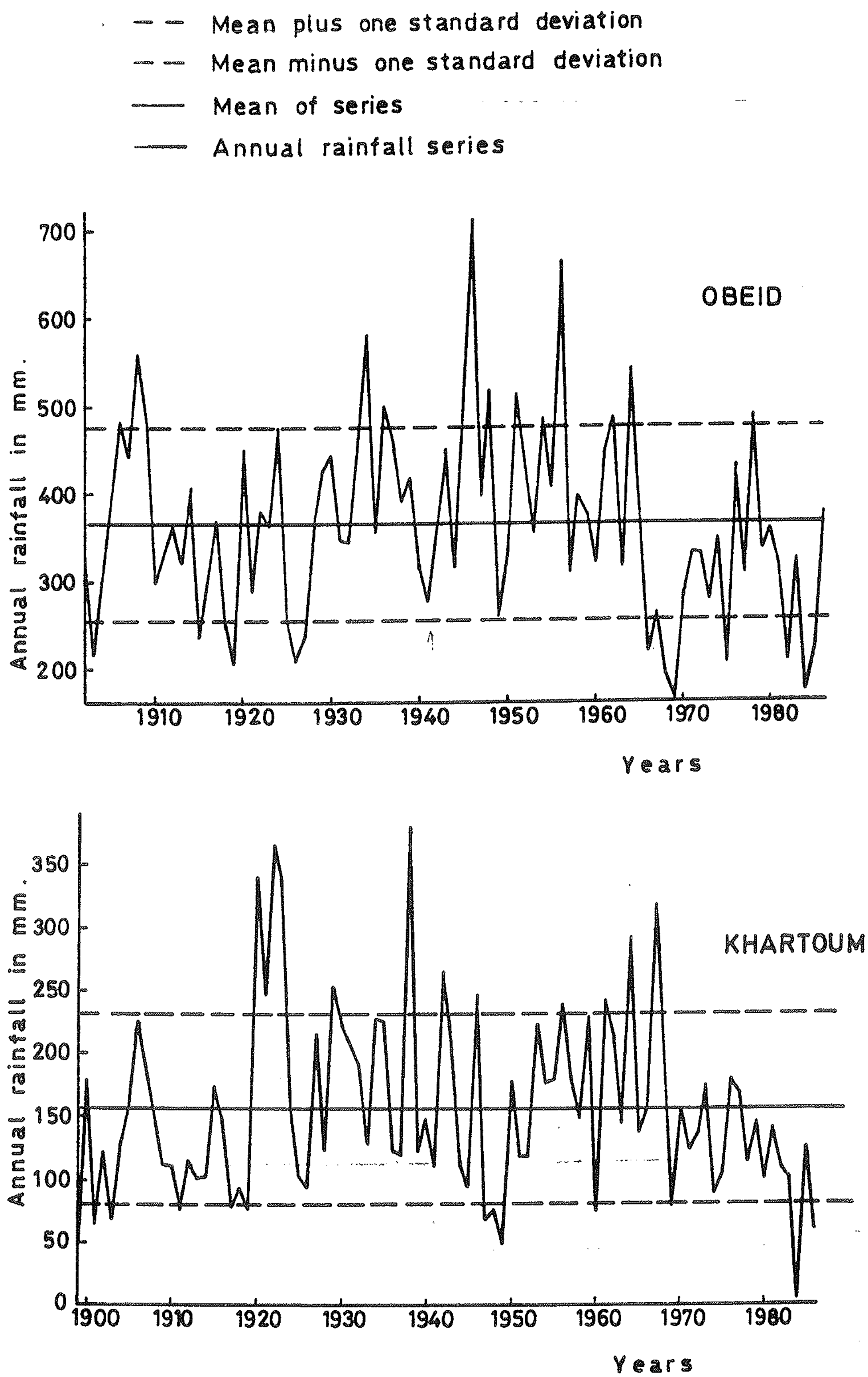
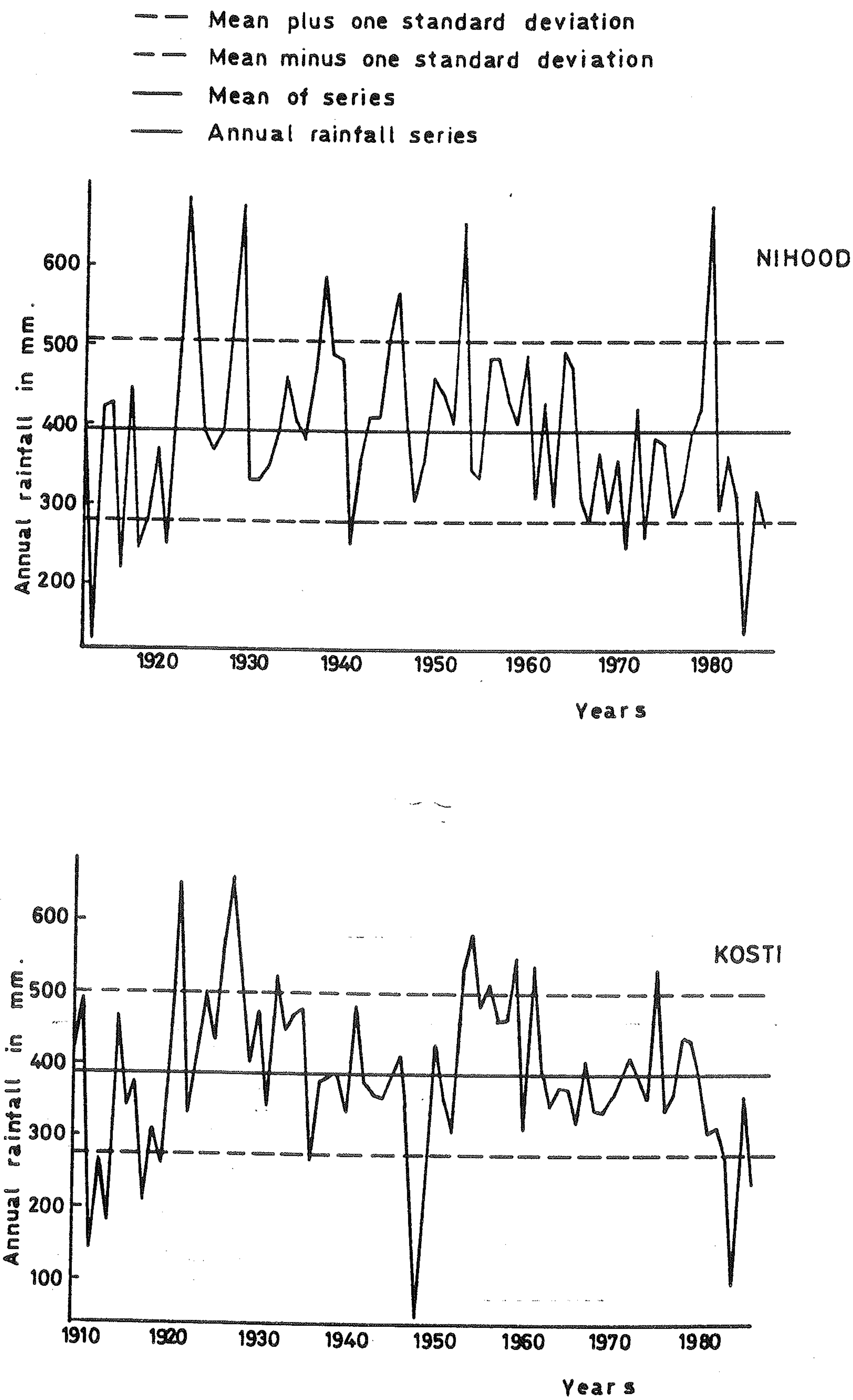
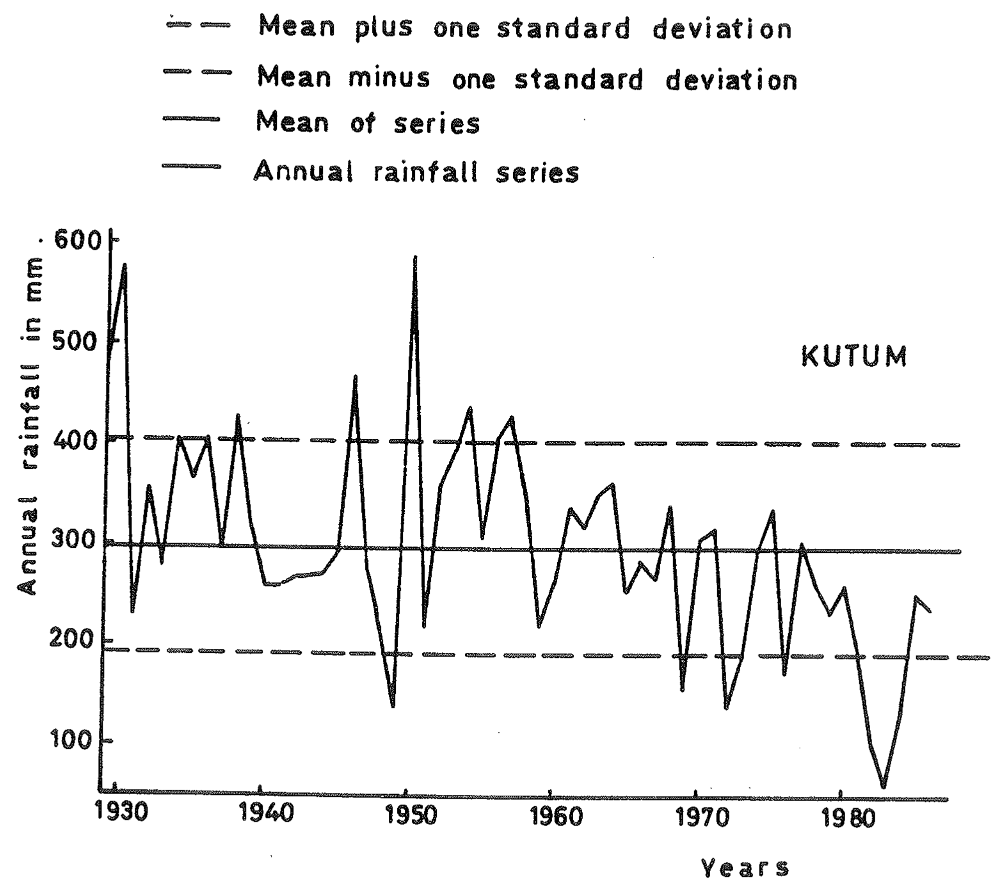
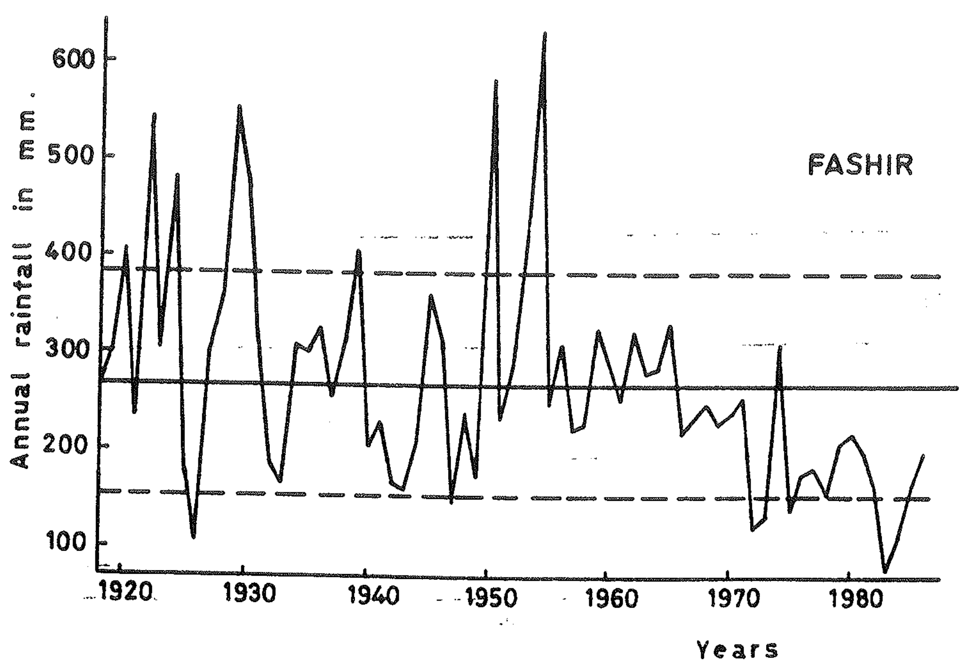


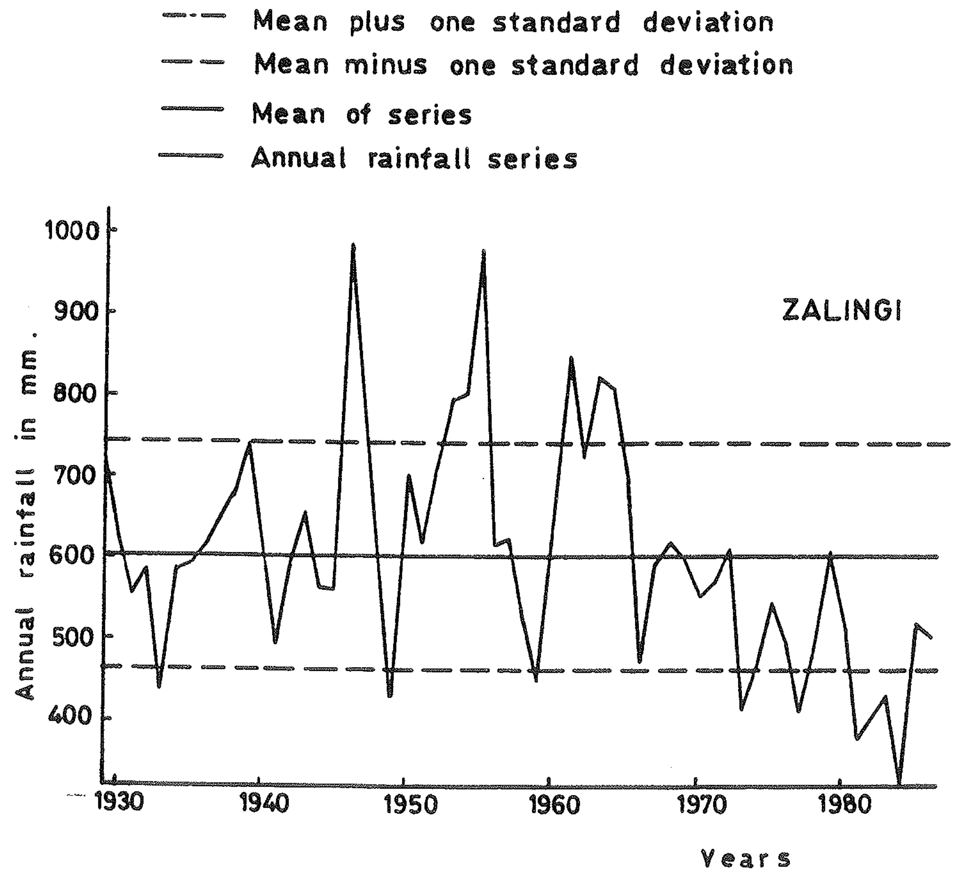
APPENDIX A

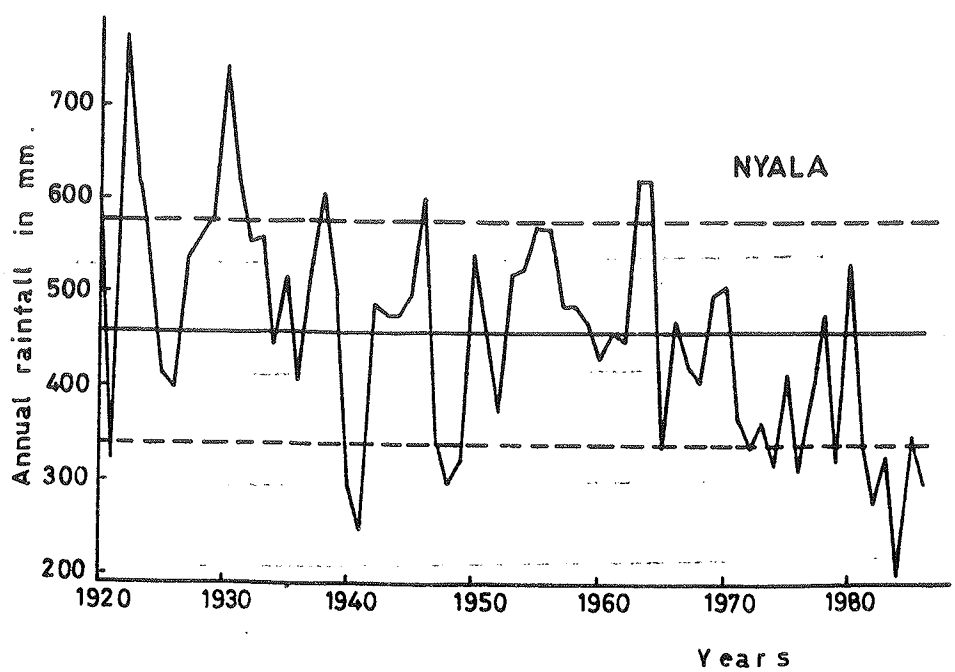
ANNUAL RAINFALL SERIES, CENTRAL AND WESTERN SUDAN

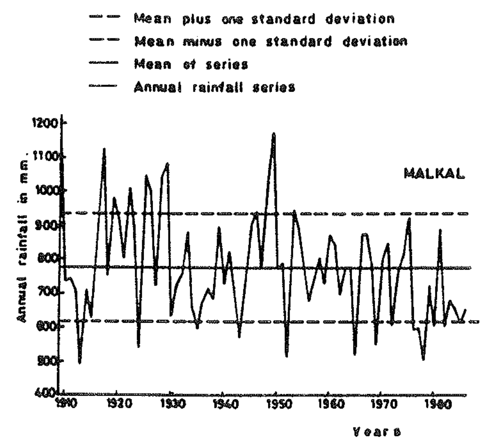

Mean plus one standard deviation
Mean minus one standard deviation
Mean of series
Annual rainfall series
NIHOOD
Annual rainfall in mm.
600
500
400
300
200
1920
1930
1940
1950
1960
1970
1980
Years
KOSTI
Annual rainfall in mm.
600
500
400
300
200
100
1910
1920
1930
1940
1950
1960
1970
1980
Years

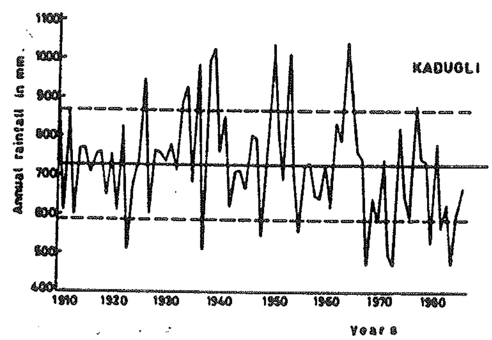

Mean plus one standard deviation
Mean minus one standard deviation
Mean of series
Annual rainfall series
Annual rainfall in mm.
600
500
400
300
200
100
KUTUM
1930
1940
1950
1960
1970
1980
Years

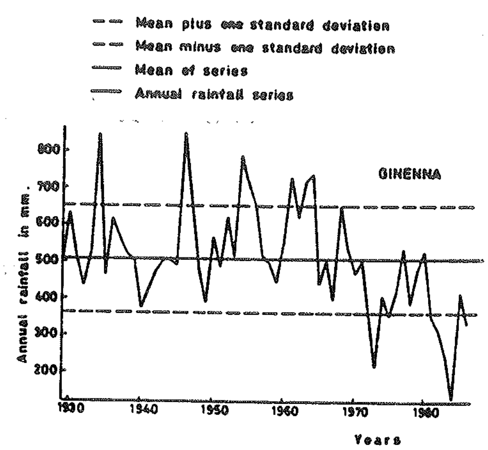

Annual rainfall in mm.
600
500
400
300
200
100
FASHIR
1920
1930
1940
1950
1960
1970
1980
Years


Mean plus one standard deviation
Mean minus one standard deviation
Mean of series
Annual rainfall series
ZALINGI
Annual rainfall in mm.
1000
900
800
700
600
500
400
1930
1940
1950
1960
1970
1980
Years


NYALA
Annual rainfall in mm.
700
600
500
400
300
200
1920
1930
1940
1950
1960
1970
1980
Years


Mean plus one standard deviation
Mean minus one standard deviation
Mean of series
Annual rainfall series
MALKAL
Annual rainfall in mm.
1200
1100
1000
900
800
700
600
500
400
1910
1920
1930
1940
1950
1960
1970
1980
Years


KADUGLI
Annual rainfall in mm.
1100
1000
900
800
700
600
500
400
1910
1920
1930
1940
1950
1960
1970
1980
Years


Mean plus one standard deviation
Mean minus one standard deviation
Mean of series
Annual rainfall series
GINENNA
Annual rainfall in mm.
800
700
600
500
400
300
200
1930
1940
1950
1960
1970
1980
Years

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APPENDIX B

Kendall Test:

It is also called Mann-Kendall test. It is a non-parametric test. Kendall test is applicable to data with a ranking scale, in other words it is needed to consider only the relative values of the variable X under consideration.

The Kendall rank statistic, τ, is a measure of the correspondance or the correlation between any two different rankings X(I) and X(I') of N terms of any variable X.

It can easily be calculated and interpreted by the following simple procedure:

Consider each possible pair of terms measured for the variable studied. Count the number of times in which the order of the two terms as appeared in the first ranking agrees with the order of the same two terms in the second ranking. Denote it by NA. The number of times in which the order of any pair in the first ranking disagrees with the order of the same pair at the second ranking ND, can be obtained by subtracting NA from the total possible number of pairs, NT.

Kendall rank statistic is defined by the following equations:

$\tau = (NA-ND)/NT$ or

$\tau = [NA-(NT-NA)]/NT$ or

$\tau = (2NA-NT)/NT = 2NA/NT-1.0$

The value for NT is given by the following equation:

$$NT = \tfrac{1}{2}.N.(N+1)$$

For the correspondance between any two rankings, there are two extremes:

(1) When both rankings are identical then,

$$NA = NT, ND = 0 \Rightarrow \tau = NT/NT = 1.0$$

(2) When one ranking is the reverse of the other then,

$$NA = 0, ND = NT \Rightarrow \tau = -NT/NT = -1.0$$

This shows the similarity between Kendall rank statistic and the other measures of correlation, e.g. Pearson correlation coefficient.

In applying Kendall test for testing first order stationarity, one of the two rankings should be the historically observed sequence, the other ranking can be the series sorted in an ascending or descending order;

usually it is taken as the rising series. A value of the statistic close to positive one indicates a rising trend while a value close to negative one indicates a falling trend.

For the above application the calculation of NA can be simplified as follows. Compare the value of the first term of the series X(1) with the values of all later terms in the series from the second to the Nth. Count up the number of terms whose values exceed X(1). Denote this number by n(1). Compare the value of the second term X(2) with the values of all the later terms. Count the number of terms which exceed X(2) and denote it by n(2). Repeat the same procedure until X(N-1) and its corresponding number n(N-1). NA is given by the following equation,

$$NA = \sum_{i=1}^{N-1} n(i)$$

Kendall rank statistic, τ, calculated for a sample from an independent population has a sampling distribution which is very nearly normal for N larger than ten. It has expected value zero and variance S^2 where,

$$S^2 = \frac{(4N+10)}{[9.N.(N-1)]}$$

This knowledge about the sampling distribution of τ provided the basis for the test.

In a two-sided test of size a, the condition for the acceptance of the null hypothesis, that the series tested is a sample from a population which is stationary in the mean, is given by the following,

$$-S \;.T_{a/2} \leq t \leq S \;.T_{a/2}$$

Where

t :Kendall rank statistic calculated from the series

$T_{a/2}$:value of the standard normal distribution corresponding to an exceedence probability a/2.

The calculation of Kendall rank statistic for a series with tied observations, i.e. with two or more terms sharing the same value, is slightly different from the described above. Siegel (4) pointed out that for series with few number of tied observations, the difference is negligible. This fact was checked during the present study, in annual rainfall, and proved true.

Student's t Test:

It is a very powerful test for detecting changes in the mean and trends.

Consider any two random samples. The first sample consists of N1 values. It is obtained from an independent population, with variable X1, mean U1 and variance $SS1^2$. The mean of the sample denoted by XM1, is defined as,

$$XM1 = \frac{1}{N1}.[(\sum_{I=1}^{N1} X1(I)]$$

The variance of the sample, denoted by $S1^2$, is defined as

$$S1^2 = \frac{1}{N1} .[\sum_{I=1}^{N1} (X1(1)-XM1)^2]$$

The second sample consists of N2 values. It is obtained from an independent population, with variable X2, mean U2 and variance $SS2^2$.

where,

$$SS2^2 = SS1^2 = SS^2$$

The mean of the sample, denoted by XM2, is defined by,

$$XM2 = \frac{1}{N2} . [\sum_{I=1}^{N2} X\ 2(1)]$$

The variance of the sample, denoted by $S2^2$, is defined by,

$$S2^2 = \frac{1}{N2} . \sum_{I=1}^{N2} [X(I)-XM2]^2$$

For applying the Student's t test for the difference between the means of these two samples, the test statistic TD is computed by,

$$TD = \frac{[(XM1-XM2)-(U1-U2)]}{[(1/N1 + 1/N2).SP^2]^{\frac{1}{2}}}$$

SP^2 is an estimate of the populations' common variance. It is obtained by pooling the variances of the two samples,

$$SP^2 = \frac{(N1.S1^2 + N2.S2^2)}{(N1 + N2-2)}$$

The test is based on assuming that the statistic TD is distributed as a t distribution with (N1+N2-2) degrees of freedom. This assumption is true when the population is approximately normally distributed.

Kendall and Stuart (2) pointed out that, Student's t test is a robust test, it can validly be applied to data possessing any arbitrary frequency distribution, except if the lengths of the two samples tested are unequal, then the distribution may not be skewed.

In applying Student's t test in the present study, for testing the stationarity in the mean, the series was split into two samples of equal size. The null hypothesis assumes zero for the difference between the means of the populations. The condition for the acceptance of the null hypothesis, that the series tested is a random sample from a population which is stationary in the mean, is given for a two sided test of size a by,

$$-T_{a/2} \leq TD \leq T_{a/2}$$

where

$T_{a/2}$:value from t distribution with (N1+N2-2) degrees of freedom corresponding to exceedence probability a/2.