

8.08 Statistical Physics II

Magnetism

(Dated: February 28, 2011)

1. A highly simplified theory for the temperature dependence of the molar specific heat c associated with the transition from paramagnetism to ferromagnetism of ions with spins $s = 1/2$, gives a curve of the form $c = c_{max}(2T/T_0 - 1)$ for $T_0/2 < T < T_0$ and zero otherwise. What is c_{max} in terms of fundamental constants?

Solution

$$C = T \frac{\partial S}{\partial T}$$

By going from $T = 0$ to $T > T_c$

$$\Delta S = \int_0^T C \frac{dT}{T} = C_{max} a x (1 - \ln 2)$$

We have to independently calculate ΔS using the physical properties of the system. At zero temperature all spins are aligned, hence $S = 0$. At infinite temperature, spins are random, hence $S = k \ln 2^N = N k \ln 2$. Putting everything together we get

$$\frac{C_{max}}{N} = k \frac{\ln 2}{1 - \ln 2}$$

2. Consider an assembly of magnetic atoms in the absence of an external magnetic field and described by the Hamiltonian $H = -J \sum S_{jz} S_{kz}$. Treat this problem by the simple Weiss molecular-field approximation.

- (a) Calculate the behavior of the mean energy of this system in the limiting cases where $T \ll T_c$ where $T \approx T_c$. Here T_c denotes the Curie temperature.
 (b) Calculate the behavior of the heat capacity in the same three temperature limits.

Solution:

- (a) The average energy is $E = -N g \mu_0 H_i S_{jz} = -N S k T x B_s(x)$ where $B(x) = \frac{1}{J} [(J + 1/2) \coth[(J + 1/2)x] - 1/2 \coth(x/2)]$. The consistency condition is $B_s(x) = k T x / 2n J S$.
 For $T \ll T_c$ $x = 2n J S / k T$ and $E = -N S k T (2n J S / k T) = -2n N S^2 J$.
 For $T \approx T_c$ we use the approximation $\coth = 1/x + x/3 - x^3/45$, so $B_s(x) = (1/S)[(S + 1/2) \coth(S + 1/2)x - (1/2) \coth(1/2x)]$ becomes

$$B_s(x) \approx (1/3)(S + 1)x - (S^2 + S + 1/2)(S + 1)/45x^3$$

. Finally

$$x^2 = \frac{45}{S^2 + S + 1/2} [1/3 - kT/2nJS(S + 1)]$$

and

$$E = -\frac{5NS(S + 1)kT}{(S^2 + S + 1/2)} (1 - 3kT/2nJS(S + 1))$$

For $T \gg T_c$. We see that $x = 0$ at $kT = kT_c = (2/3)nJS(S + 1)$, so $E = 0$

- (b) From the above analysis $C = \partial E / \partial T = \frac{15Nk^2T}{nJ(S^2 + S + 1/2)} (1 - S(S + 1)nJ/3kT)$ for $T \approx T_c$ and zero everywhere else.

3. The magnetic susceptibility per unit volume of a magnetic solid is given by $\chi = A/(T - \theta)$, where A and θ are constants. How much does the entropy per unit volume change if, at temperature T , the magnetic field is increased from 0 to H_0 ?

Solution:

$$\Delta S = \int_0^{H_0} (\partial S / \partial H)_T dH$$

From $dE = TdS - MdH$ we get $(\partial S / \partial H)_T = (\partial M / \partial T)_H$

Since $M = \chi H$ and $\chi = A/(T - \theta)$ we get $(\partial M / \partial T)_H = -AH/(T - \theta)^2$ and

$$\Delta = - \int_0^{H_0} AH/(T - \theta)^2 = - \frac{AH_0^2}{2(T - \theta)^2}$$

4. The magnetic susceptibility per mole of a substance containing interacting atoms is given by the Curie - Weiss law, $\chi = A/(T - \theta)$, where A and θ are constants independent of temperature and magnetic field. θ is however dependent on pressure according to $\theta = \theta_0(1 + \alpha p)$. Calculate the change in molar volume of this substance when, at a fixed temperature and pressure, the magnetic field is increased from 0 to H_0 .

Solution:

$$\Delta v = \int_0^{H_0} (\partial v / \partial H)_{T,p} dH$$

From $dE = TdS - pdV - MdH$ we get $(\partial v / \partial H)_{T,p} = -(\partial M / \partial p)_{T,H}$

Since $M = \chi H$ and $\chi = A/[T - \theta_0(1 + \alpha p)]$ we get

$$(\partial M / \partial p)_{T,H} = \frac{A\theta_0\alpha H}{[T - \theta_0(1 + \alpha p)]^2}$$

and

$$\Delta v = - \int_0^{H_0} \frac{A\theta_0\alpha H}{[T - \theta_0(1 + \alpha p)]^2} = - \frac{A\theta_0\alpha}{[T - \theta_0(1 + \alpha p)]^2} \frac{H_0^2}{2}$$

5. (a) Show that for a fixed temperature, the entropy of a metal is independent of magnetic field in both its superconducting and normal states. (The magnetic susceptibility in the normal state is negligibly small).
 (b) Given the critical field curve $H = H(T)$ for a superconductor, find a general expression for the difference $C_s - C_n$.
 (c) What is the answer to part (b) at the transition temperature?

Solution:

- (a) From $dE = TdS - pdV + HdM$ we get $(\partial S / \partial H)_{V,T} = (\partial M / \partial T)_{V,H}$

In the normal state $M \approx 0$ so $(\partial S / \partial H)_{V,T} \approx 0$ In the superconducting state

$$B = H + 4\pi M/V \Rightarrow M = -VH/4$$

- (b) Since the heats capacity is given by $C = T(\partial S / \partial T)$ we have

$$S_s - S_n = \frac{V}{4\pi} H \frac{dH}{dT} \Rightarrow C_s - C_n = T \frac{\partial}{\partial T} (S_n - S_s) = \frac{VT}{4\pi} (dH/dT)^2 + \frac{VTH}{4\pi} (d^2H/dT^2)$$

- (c) At the transition temperature, the critical field is 0, so

$$C_s - C_n = \frac{VT_c}{4\pi} \left(\frac{\partial H}{\partial T} \right)^2$$