Laser Doppler Anemometry [LDA]

The concept of a Doppler shift is familiar to us from the downshift in pitch that we hear as a siren moves towards and then away from us. The faster the moving source of sound, the greater the shift in frequency. This effect is also observed with light. When light is reflected from a moving object, the frequency of the scattered light is shifted by an amount proportional to the speed of the object. So, one could estimate the speed by observing the frequency shift. This is the basis for LDA. A flow is seeded with small, neutrally buoyant particles that scatter light. The particles are illuminated by a known frequency of laser light. The scattered light is detected by a photomultiplier tube (PMT), an instrument that generates a current in proportion to absorbed photon energy, and then amplifies that current. The difference between the incident and scattered light frequencies is called the Doppler shift.

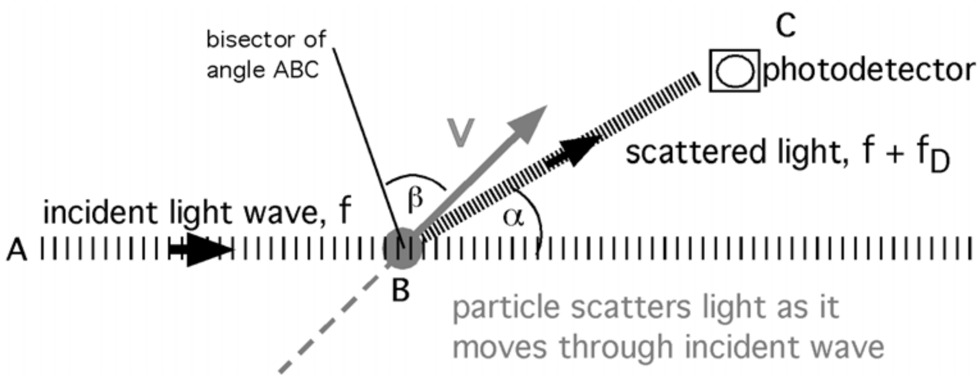

Figure 1. A particle moves through an incident light wave of frequency f and scatters light in all directions. The scattered light picked up by the photodetector will be shifted by f_D.

The Doppler shift, f_D, depends on the speed, V, and direction of the particle motion, the wavelength of the light, λ, and the orientation of the observer. The orientation of the observer is defined by the angle α between the incident light wave and the photodetector [PMT]. The direction of particle motion is defined by β, the angle between the velocity vector and the bisector of ABC (Figure 1). Then

$$(1)\quad f_D = \frac{2V}{\lambda}\cos\beta \sin\frac{\alpha}{2}.$$

A direct way to estimate f_D is to measure the incident frequency, f, and the observed frequency, f_O, and find the difference. However, the Doppler shift is a very small fraction of the incident frequency, so this results in estimating a small value from the difference of two large values, a process with a high degree of uncertainty. To improve the estimate of f_D, a method using two incident beams has been developed. In this configuration the incident beam is split into two beams of equal intensity. The beams are directed to intersect, and the point of intersection is the measurement volume. Particles that pass through the measurement volume scatter light from both beams. The frequency shift of the light scattered from each beam will be different, becaue the orientation of the two beams relative to the photodetector and relative to the particle's velocity vector are different (see Figure 2). That is, α and β defined in Figure 1 are different for the two beams.

Let the Doppler shift for the two scattered beams be f_{D1} and f_{D2}, *i.e.* the scattered beam will have frequencies $f + f_{D1}$ and $f + f_{D2}$. Because f_{D1} and f_{D2} are both much smaller than f, the scattered light waves have nearly equal frequency. When waves of equal amplitude and nearly equal frequency are superimposed, the amplitude of the resulting signal periodically rises and falls. This modulation is called a beat. The beat frequency is one half the difference between the two original frequencies. Thus, when the two bursts of scattered light are superimposed within the photodetector, the resulting signal has a beat with frequency, $|f_{D1} - f_{D2}|/2$. As shown below, the beat frequency will be the Doppler frequency we seek.

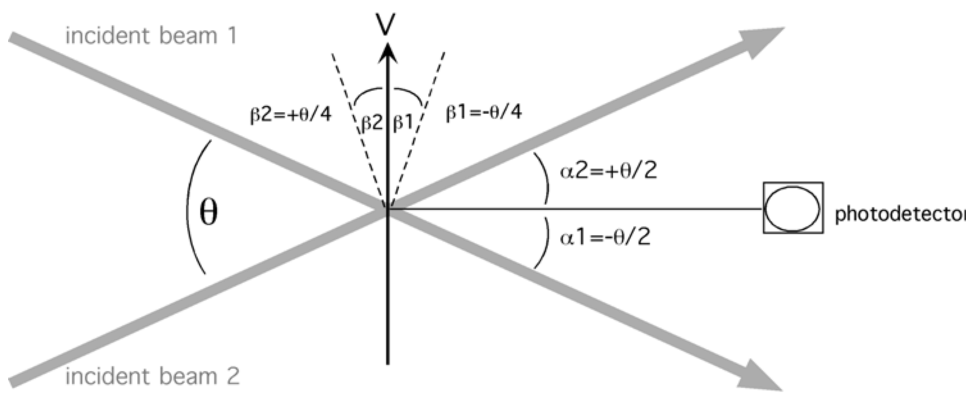

Figure 2. Beam and photodetector configuration for forward scatter, differential LDA.

As an example, consider the geometry depicted in Figure 2. Using equation (1), the frequency shift for beam 1 is $f_{D1} = (2V/\lambda)\cos(-\theta/4)\sin(-\theta/4)$, and for beam 2 is $f_{D2} = (2V/\lambda)\cos(+\theta/4)\sin(+\theta/4)$. Because sin(-a) =-sin(a) and cos(-a) = cos(a), we see that $f_{D1} = - f_{D2}$, which is consistent with the symmetry of Figure 2. The beat frequency is

$$(2) \qquad \frac{|f_{D1} - f_{D2}|}{2} = \frac{2V}{\lambda}\cos\left(\frac{\theta}{4}\right)\sin\left(\frac{\theta}{4}\right) = \frac{2V}{\lambda}\sin\left(\frac{\theta}{2}\right).$$

Comparison of the second term in (2) with the either f_{D1} or f_{D2} confirms that the beat frequency is the absolute value of the Doppler shift we seek. The same result can be found for any choice of α, β and θ, *if* V is taken as the velocity component perpendicular to bisector of beam 1 and beam 2.

Another useful way to interpret the signal recorded by the photodetector is in terms of the interference fringe pattern generated at the beam crossing. The fringe pattern, shown in Figure 3, consists of alternating zones of brightness and darkness. The fringe spacing, d_f, is the distance between sequential bright (or dark) zones.

(3) $$d_f = \frac{\lambda}{2\sin(\theta/2)}$$

As a particle crosses the fringe pattern, the intensity of the scattered light varies with the intensity of the fringes. Thus, the amplitude of the signal burst varies with time-scale d_F/V, where V is the velocity component perpendicular to the fringe pattern, *i.e*. perpendicular to the bisector of the two incoming beams. The frequency of the amplitude modulation is thus,

(4) $$\frac{V}{d_f} = \frac{2V}{\lambda}\sin(\theta/2),$$

which, by comparison with (2), is the Doppler frequency, f_D. Note that with the two-beam system, the Doppler frequency is not dependent on the position of the photodetector, as the angles α and β do not appear in (4) or (2).

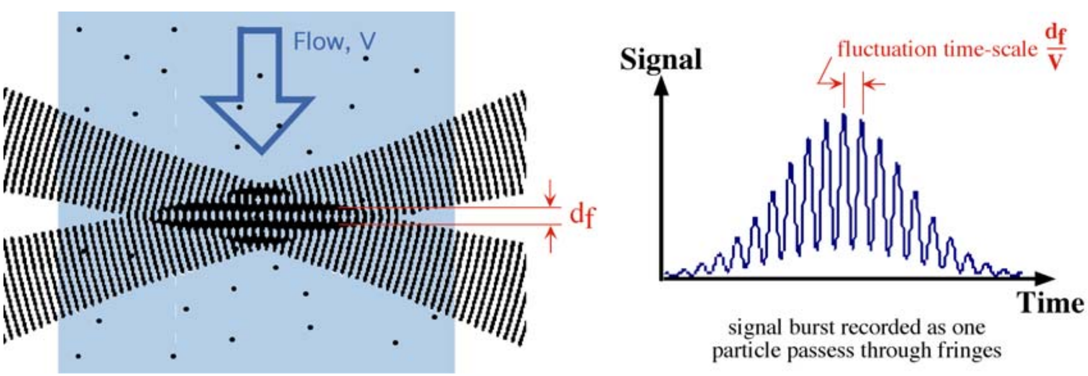

Figure 3. A fringe pattern is created at the intersection of the two incident beams. The fringe spacing is d_f. As a particle moves through the fringes, the photodetector records a signal burst whose amplitude is modulated by the fringe pattern. The frequency of the modulation is the Doppler frequency, $f_D = (2V/\lambda)\sin(\theta/2)$, where V is the velocity component perpendicular to the fringes.

Note that the Doppler frequency depends only on the magnitude of V, not the direction, *i.e*. positive and negative values of V will produce the same Doppler frequency (Figure 4). To correct for this directional ambiquity, the frequency of one of the incoming beams is shifted by a known value, f_s. This causes the fringe pattern to move at speed $V_s = f_s\, d_f$ toward the incoming unshifted beam (Figure 4). The frequency recorded by the photodector is now

(5) $$f_d = \left| f_s + \frac{2V}{\lambda}\sin(\theta/2) \right|,$$

and the sign of V is reflected in f_d. That is, a particle moving through the fringes at speed V shifts the detected frequency, f_d, up (postive V) or down (negative V) from f_s. To avoid directional ambiguity, $f_s > |(2V/\lambda)\sin(\theta/2)|$, as shown in Figure 4. Thus, to optimize the system, a different shift frequency will be required for different flow conditions. An acoustic-optical device called a Bragg cell generates the required frequency shift.

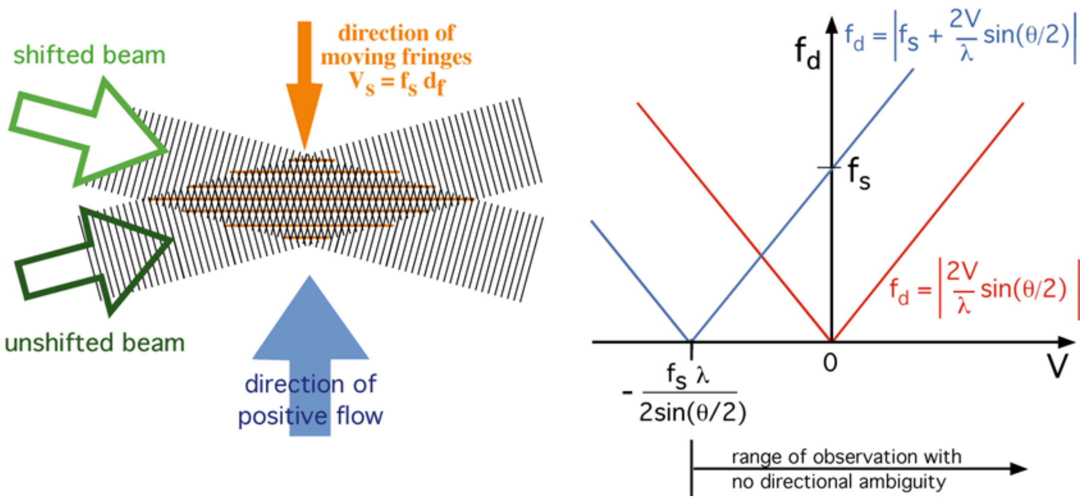

Figure 4. Removing directional ambiguity with frequency shifting. With stationary fringes the detected frequency, f_d, is the same for +V and -V (red curve), *i.e.* we cannot distinguish direction. To remove this ambiguity we add a frequency shift, f_s. Then, as shown by the blue curve, +V and -V produce unique values of f_d. The frequency shift is introduced optically by shifting one of the incident light beams by f_s. This causes the fringes in the interference pattern to move at $V_s = f_s d_f$, and shifts the frequency of the scattered light by f_s. Directional ambiguity is then removed for $V > -f_s\lambda/(2\sin(\theta/2))$.

After a burst signal (Figure 3) is detected by the PMT, three different processing schemes can be used to determine f_d. A 'counter' processor first isolates the modulation in the burst using a high-pass filter, and then counts the number of zero-crossings in a unit of time. A 'spectrum analyzer' calculates the Fourier transform of the burst signal, and then selects the peak frequency as f_d. Finally, correlation processors use a correlation algorithm to determine f_d.

As described above, a single pair of incident beams measures a single velocity component, perpendicular to the bisector of the two beams (see Figure 2). Additional components can be measured by adding additional pairs of beams that cross at the same measurement point. Each pair must have a unique wavelength so that the burst signals can be distinguished by filtering. Commonly, the blue, green, and violet lines of an argon-ion laser are used for multi-component measurements. The measurement volume, defined by the volume of intersection for all beams, is elliptical. In its longest dimension, parallel to the beam bisector, the volume can be 0.1 to 1 mm long, depending of the intersection angle θ.

The best scattering signals are obtained when the particle diameter is several times the wavelength. However, the particles must be small enough to follow the flow. The particles will scatter light in all directions, but the highest intensity of scattered light will be on the forward side (forward scatter) of the particle, *i.e*. in the direction away from the incident light. Much less light is scattered in other directions. However, positioning the PMT for forward scatter requires the PMT to be opposite to the light source. It is more convenient to use the back-scattered light, because this allows integration of transmitting and receiving optics in a single head. This is simpler for maintaining alignment among multiple velocity components.

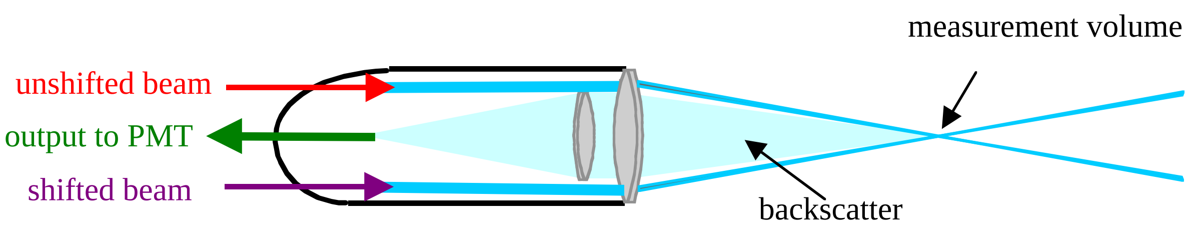

Figure 5. Transmitting and receiving optics in same probe head.

Because the arrival of each particle is random, the time between velocity estimates is random. So, the velocity is not evenly sampled in time. Before most analyses techniques can be applied to the data, *e.g*. FFT techniques, the record must be re-sampled onto an equi-spaced time grid. This, of course, contributes uncertainty to the final analysis. In addition, more samples are generated when the velocity is higher, such that an ensemble-average of the data does not reflect the true time-average. This bias is eliminated by using a time-weighted average, which weights each velocity estimate by the duration of the underlying burst. Assuming the particle concentration is uniform, this provides proportionally higher weight to lower velocity samples, which occur when samples are less frequent. Finally, LDA systems are capable of sampling at rates of 500-1000 Hz, such that conversion to an equi-sampled record of say 200-300 Hz is straight-forward. The high sampling rate and small sample volume makes LDA well suited for turbulence measurements.

For more information:

Durst, F., A. Melling, and J. Whitelaw. 1981. *Principles and Practices of Laser-Doppler Anemometry*. Academic Press.

http://www.dantecmt.com/lda/Princip/Index.html

http://www.tsi.com/fluid/products/ldv/ldv.htm