

Signaling Game-based Misbehavior Inspection in V2I-enabled Highway Operations

Manxi Wu, Li Jin, Saurabh Amin, and Patrick Jaillet

Abstract—Vehicle-to-Infrastructure (V2I) communications are increasingly supporting highway operations such as electronic toll collection, carpooling, and vehicle platooning. In this paper we study the incentives of strategic misbehavior by individual vehicles who can exploit the security vulnerabilities in V2I communications and impact the highway operations. We consider a V2I-enabled highway segment facing two classes of vehicles (agent populations), each with an authorized access to one server (subset of lanes). Vehicles are strategic in that they can misreport their class (type) to the system operator and get unauthorized access to the server dedicated to the other class. This misbehavior causes a congestion externality on the compliant vehicles, and thus, needs to be deterred. We focus on an environment where the operator is able to inspect the vehicles for misbehavior. The inspection is costly and successful detection incurs a fine on the misbehaving vehicle. We formulate a signaling game to study the strategic interaction between the vehicle classes and the operator. Our equilibrium analysis provides conditions on the cost parameters that govern the vehicles’ incentive to misbehave, and determine the operator’s optimal inspection strategy.

Index Terms—Cyber-physical Systems Security, Asymmetric Information Games, Smart Highway Systems, Crime Deterrence.

I. INTRODUCTION

Vehicle-to-Infrastructure (V2I) and Vehicle-to-Vehicle (V2V) communications are commonly regarded as an integral feature of smart highway systems [1], [2]. With the projected growth of V2I and V2V capabilities, it is expected that they will support important operations such as safety-preserving maneuvers (overtaking), lane management, intersection control, etc., and also enable traffic management with connected/autonomous vehicles [3], [4], [5]. These applications typically require the presence of road-side units (RSUs) that are capable of receiving messages from individual vehicles (i.e., their on-board units (OBUs)), authenticating these messages, and providing relevant information to neighboring vehicles and/or actuators (e.g., traffic signals). This message exchange is typically supported by the Dedicated Short

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Range Communications (DSRC) technology, and it enables the RSU to gather information such as vehicle identifier, vehicle class, and safety-related data. In recent years, security concerns have been identified in this context [6], [7], [8]. The prior work has focused on the identification of cyber-security vulnerabilities and the design of defense solutions. However, an aspect that has received relatively little attention is the modeling of strategic misbehavior by vehicles that can directly impact the highway operations.

In this paper, we focus on a generic setting of lane management operation enabled by V2I communications, and develop a model of strategic misbehavior using a signaling game formulation. To better understand the setting, consider a highway segment with a downstream bottleneck; see Fig. 1. The highway is equipped with a RSU and all incoming vehicles have OBUs. The highway section has two classes of lanes: the high-priority lanes are meant to serve the travelers with preferential access to the system, and the low-priority (or general purpose) lanes are meant to serve all other travelers. We consider the two sets of lanes as parallel servers. The RSU receives and authenticates the messages from the incoming vehicles. A vehicle is provided access to the high-priority server if the RSU is able to authenticate its message and adjudge it to be a vehicle belonging to the high-priority class. Due to the presence of congestion externalities, the travel cost incurred in accessing each server increases with the aggregate number of vehicles routed through that server. We assume that the priority classes are pre-established using well-known economic principles [9].

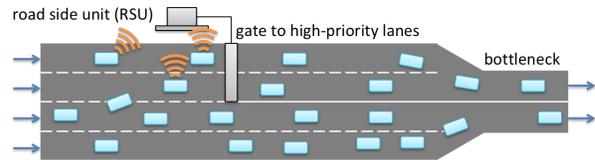


Fig. 1: A highway with V2I-based lane management operations.

The main feature that we consider in the abovementioned setting is the ability of travelers to manipulate the communication between their vehicles OBU and the RSU, so that they can access the server that does not correspond to their vehicle class. Such an attack can be realized if the vehicles (henceforth referred to as “agents”) of certain traffic class can misreport their identity information to the RSU. We consider this type of attack as an instance of *strategic misbehavior*, which needs to be deterred by the system operator because it can result in additional congestion externalities on the

other travelers. Technologically, deterrence can be facilitated by further inspecting the messages for their integrity, checking the identity information for example, using video cameras, number plate recognition, or manual inspection, and imposing suitable fines on successful detection [10], [11]. To make our model realistic, we make the following assumptions: (i) The operator has incomplete information about the priority class of incoming vehicles, in that the true class of vehicle can be known only after inspection (which is costly); (ii) Each misbehaving agent incurs a technological cost for manipulating its message, and is subject to a fine if inspected. These assumptions naturally lead us to pose our model as a signaling game [12].

In our game, the agents are non-atomic and each agent has private information about its type; i.e., each agent privately knows whether she is a high- or low-priority agent. The operator has the technological capability of message collection (via the RSU), inspection, and fine imposition. We say that an agent misbehaves if she sends a signal that is different from her true type and is thus provided access to the lane (server) that does not correspond to her true type. For the sake of generality, we consider that each type of agents can misbehave by sending the signal of the other type. All agents are subject to inspection by the operator who observes the reported types, but does not know the true type until she inspects. A misbehaving agent incurs a non-negative technological cost, and if detected, is charged a non-negative fine. Furthermore, we impose a natural assumption that the technological cost and the fine for the low-priority agent are strictly positive.

The equilibrium concept that we use is the Perfect Bayesian Equilibrium (PBE) [13], [14]. In the PBE, (a) the players satisfy sequential rationality, and (b) the operator's belief is consistent with the prior distribution of agent types and the agents' strategy according to the Bayes' rule. The specific features that distinguish our game from classical models of signaling games are: non-atomic agent populations, and congestion externality imposed by an agent on other agents accessing the same server. Under mild assumptions on the cost functions of both servers, we provide a complete characterization of the PBE for our signaling game.

In particular, we show that in equilibrium (i) a high priority agent does not have the incentive to misbehave for gaining access to the low-priority server; (ii) not every low-priority agent misbehaves. Moreover, we distinguish two regimes based on how the technological cost of misbehavior compares with the maximum gain from misbehavior (evaluated as the difference in travel costs of two servers when no agent misbehaves). In the first regime, the misbehavior is completely deterred as the technological cost is high and the operator does not inspect any agent. In the second regime, the low-priority agents misbehave with a positive probability. The inspection strategy of the operator in second regime can be further distinguished by sub-regimes that correspond to zero, partial, and complete inspection.

Our equilibrium analysis can be used to study the effects

of fraction of each priority class, the inspection cost, and the fine. First, for given technological cost of misbehavior and cost of inspection, the equilibrium misbehavior rate (and the hence the rate of inspection) decreases as the fraction of high-priority agents increases; Second, fine can be effective for decreasing misbehavior rate even when the inspection cost is high (relative to the cost of misbehavior), but may not achieve complete deterrence; Third, if the fine is sufficiently high, then there is no need to inspect all agents in equilibrium. These insights can be relevant for the design and deployment of inspection technologies to achieve higher security of V2I-enabled highway operations. Finally, we illustrate these effects in the specific setting of Electronic Toll Collection (ETC), where the two servers are modeled as M/M/1 queueing systems, and the fraction of high-priority travelers (and the associated toll that they need to pay for priority access) is exogenously known.

II. MODELING MISBEHAVIOR IN V2I-BASED HIGHWAY OPERATIONS

In this section, we consider a simple model of lane management operations on a highway section equipped with vehicle-to-infrastructure (V2I) communications capability, and discuss the misbehavior that can arise in this setting.

Suppose that the highway system faces a fixed traffic demand comprised of two types of agent populations: a high priority type, denoted h , and a low priority type, denoted l . The fraction of type h agents is $\theta \in (0, 1)$, and the fraction of type l agents is $1 - \theta$. Throughout this article, we assume that θ is exogenous and independent of potential misbehavior (see Remark 1 below). There are two sets of lanes on the highway, H and L, which we model as two parallel servers; see Fig. 2. In the absence of any misbehavior, server H only serves type h agents, and server L only serves type l agents.

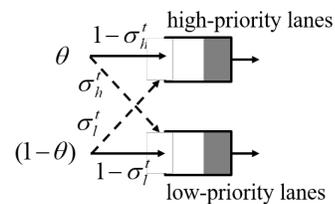


Fig. 2: A highway with two servers (sets of lanes). Each server is accessed by an authorized agent population (solid arrows), and is also subject to potential misbehavior by the other population (dashed arrows).

Remark 1: Admittedly, the assumption of fixed fractions θ precludes us from considering situations where the agent populations would choose their priority type (routing behavior) in anticipation of the potential misbehavior that they may face. Our analysis identifies the effect of θ on the misbehavior rate.

Given any fraction of type h travelers, θ , the *travel cost* (or queueing delay) on the H (resp. L) server is denoted as c_H^θ (resp. c_L^θ). In general, c_H^θ (resp. c_L^θ) increases with the aggregate demand of agents using the server H (resp. L). In our setting, to reduce his/her travel cost, an agent

may take the server not for his type, which we consider as *misbehavior*. We use σ_l^t (resp. σ_h^t) to denote the fraction of l (resp. h) agents that misbehave. Let a generic misbehavior strategy $\sigma^t = (\sigma_h^t, \sigma_l^t)$. Since the fraction of misbehaving agents impacts the demand of agents using each server, we can use the notations $c_H^\theta(\sigma^t)$ (resp. $c_L^\theta(\sigma^t)$) to denote the cost of server H (resp. L) when the fraction of h type is θ , and the strategy is σ^t . Our analysis holds for any travel cost functions as long as the following assumptions are satisfied:

- (A1) $c_H^\theta(0, 0) < \infty$, $c_L^\theta(0, 0) < \infty$, $c_H^\theta(0, 0) < c_L^\theta(0, 0)$, and $c_H^\theta(0, 1) > c_L^\theta(0, 1)$.
- (A2) $c_H^\theta(\sigma_h^t, \sigma_l^t)$ (resp. $c_L^\theta(\sigma_h^t, \sigma_l^t)$) decreases in σ_h^t (resp. σ_l^t), and increases in σ_l^t (resp. σ_h^t).
- (A3) $c_H^\theta(\sigma_h^t, \sigma_l^t)$ increases in θ . $c_L^\theta(\sigma_h^t, \sigma_l^t)$ decreases in θ .

(A1) ensures that if no agent misbehaves, then the travel costs on both servers are bounded, and the travel cost on the high-priority server H is smaller. However, if every type l agent misbehaves, then the entire traffic demand is routed through the high-priority server H, and there is no traffic on the low-priority server L. In such a case, it is reasonable to assume that the travel time on H is greater than that on L. (A2) implies that misbehaving agents impose a congestion externality on other travelers. (A3) implies that as the fraction of h type increases, the cost of H lane increases, and the cost of L lane decreases. Note that these assumptions are not restrictive since they can be satisfied as long as travel cost functions are finite and increasing in the demand, and H lane is prioritized in comparison to the L lane.

III. SIGNALING GAME FOR MISBEHAVIOR INSPECTION

In this section, we model the strategic interaction between the agent populations that are prone to misbehavior and the system operator who can decide to inspect them based on the received messages. We consider that the agents are capable of compromising the integrity of messages sent to the RSU in order to obtain access to the server that does not correspond to their true type. The operator cannot be known an agent's type with certainty unless he inspects on the agent. This information asymmetry between the agents and the operator naturally leads to a signaling game formulation.

In the game, agents are modeled as a set of continuous players. The cost of misbehavior is non-negative for each type h agent, denoted $p_h^t \in \mathbb{R}_{\geq 0}$, and strictly positive for each type l agent, denoted $p_l^t \in \mathbb{R}_{> 0}$, since the travel cost on server H is smaller than that on server L.¹

As mentioned before, the operator (defender), denoted as d , does not know the type of each agent, but can observe the agent's signal, i.e. the server taken by the agent. The signal space is the set of servers $\{H, L\}$, and we say that a type l (resp. type h) agent misbehaves if she chooses the server H (resp. L). The operator forms a belief of the true type given the observed signal. We denote $\beta(H) \triangleq (\beta(h|H), \beta(l|H))$ (resp. $\beta(L) \triangleq (\beta(h|L), \beta(l|L))$) as the operator's belief given the signal H (resp. L), where $\beta(h|H)$ and $\beta(l|H)$ (resp.

¹Note that the assumption is consistent with our model of the highway system, with server H as a high priority server.

$\beta(h|L)$ and $\beta(l|L)$) are the posterior probabilities that an agent on the server H (resp. L) is in fact a type h and l agent, respectively. Based on the signal and the belief, the operator chooses to inspect an agent, denoted I, or not to inspect, denoted N. The inspection incurs a positive cost on the operator, denoted $p^d \in \mathbb{R}_{> 0}$. We denote σ_H^d (resp. σ_L^d) as the probability of inspecting an agent on the server H (resp. L). The operator's strategy is $\sigma^d \triangleq (\sigma_H^d, \sigma_L^d)$, and the strategy profile is $\sigma \triangleq (\sigma^t, \sigma^d)$. Furthermore, for simplicity, we assume that if an agent misbehaves, and if he is inspected, then the misbehavior is detected with probability 1. Note that this is without loss of generality, since the inspection rate can be alternatively interpreted as the rate of successful detection. A fine $F_h \in \mathbb{R}_{\geq 0}$ (resp. $F_l \in \mathbb{R}_{\geq 0}$) is charged to the type h (resp. l) agent if misbehavior is detected.

We are now ready to discuss the utility functions of the agents and the operator. The utility of each agent is the summation of three parts: (i) $-c_H^\theta(\sigma^t)$ (resp. $-c_L^\theta(\sigma^t)$), which is the travel cost if the agent chooses the server H (resp. L); (ii) $-p_h^t$ (resp. $-p_l^t$), which is the technology cost of misbehavior for a type h (resp. l) agent; (iii) $-F_h$ ($-F_l$), which is the fine if the misbehavior is detected upon inspection of a type h (resp. l) agent. The utility of the operator is the summation of two parts: (i) $-p^d$, which is the inspection cost; (ii) F_h (resp. F_l), which is collected fine when the misbehavior of a type h (resp. l) agent is detected.

The game is played in three steps as follows (see Fig. 3): First, the type of each agent is chosen by the fictitious player "Nature" according to the probability distribution $\Pr(h) = \theta$ and $\Pr(l) = 1 - \theta$; Second, agents send the signal (choose the server) according to strategy σ^t based on their type; Third, the operator observes the signal. The belief β is updated based on observed signal. The operator then chooses to inspect or not according to σ^d . All the game parameters are common knowledge, except that each agent privately knows his type.

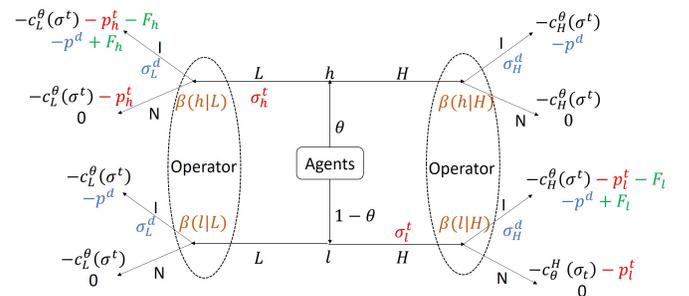


Fig. 3: Game Tree with the agent's utility (top) and the operator's utility (bottom) indicated at each leaf node.

Given strategy profile σ , we denote the expected utility of type h agents choosing the server H (resp. L) as $\mathbb{E}_\sigma[U_h^t(H)]$ (resp. $\mathbb{E}_\sigma[U_h^t(L)]$). The expected utility for type l agents are similarly denoted as $\mathbb{E}_\sigma[U_l^t(H)]$ and $\mathbb{E}_\sigma[U_l^t(L)]$, respectively. The expected utilities of agents can be written as follows:

$$\mathbb{E}_\sigma[U_h^t(\text{H})] = -c_H^\theta(\sigma^t), \quad \mathbb{E}_\sigma[U_h^t(\text{L})] = -c_L^\theta(\sigma^t) - p_h^t - F_h \sigma_L^d, \quad (1a)$$

$$\mathbb{E}_\sigma[U_l^t(\text{H})] = -c_H^\theta(\sigma^t) - p_l^t - F_l \sigma_H^d, \quad \mathbb{E}_\sigma[U_l^t(\text{L})] = -c_L^\theta(\sigma^t). \quad (1b)$$

Given any strategy profile σ and any belief β , the expected utility of the operator when observing signal H (resp. L) is denoted as $\mathbb{E}_\sigma[U_H^d|\beta]$ (resp. $\mathbb{E}_\sigma[U_L^d|\beta]$), which can be written as follows:

$$\mathbb{E}_\sigma[U_H^d|\beta] = (-p^d + F_l \beta(l|\text{H})) \sigma_H^d, \quad (2a)$$

$$\mathbb{E}_\sigma[U_L^d|\beta] = (-p^d + F_h \beta(h|\text{L})) \sigma_L^d. \quad (2b)$$

Note that in all the cost functions (1a)-(1b) and (2a)-(2b), the travel cost function is the expected travel time multiply with the value of time, and hence can be added with the fine and the misbehavior/inspection costs.

The equilibrium concept in this game is the perfect Bayesian equilibrium (PBE), see [13]:

Definition 1: A pair (σ^*, β^*) of a strategy profile σ^* and a belief assessment β^* is said to be a PBE if (σ^*, β^*) satisfies both sequential rationality and consistency:

- *Sequential rationality:* (i) The servers that are used by each type of agents incur the highest expected utility:

$$\sigma_h^{t*} > 0 \Rightarrow \mathbb{E}_{\sigma^*}[U_h^t(\text{L})] \geq \mathbb{E}_{\sigma^*}[U_h^t(\text{H})], \quad (3a)$$

$$\sigma_h^{t*} < 1 \Rightarrow \mathbb{E}_{\sigma^*}[U_h^t(\text{L})] \leq \mathbb{E}_{\sigma^*}[U_h^t(\text{H})], \quad (3b)$$

$$\sigma_l^{t*} > 0 \Rightarrow \mathbb{E}_{\sigma^*}[U_l^t(\text{H})] \geq \mathbb{E}_{\sigma^*}[U_l^t(\text{L})], \quad (3c)$$

$$\sigma_l^{t*} < 1 \Rightarrow \mathbb{E}_{\sigma^*}[U_l^t(\text{H})] \leq \mathbb{E}_{\sigma^*}[U_l^t(\text{L})]. \quad (3d)$$

- (ii) The operator maximizes expected utility:

$$\sigma_H^{d*} = \arg \max_{\sigma_H^d \in [0,1]} \mathbb{E}_\sigma[U_H^d|\beta^*], \quad \sigma_L^{d*} = \arg \max_{\sigma_L^d \in [0,1]} \mathbb{E}_\sigma[U_L^d|\beta^*] \quad (4)$$

- *Consistency:* β^* is updated according to the agent's strategy σ^* and Bayes' rule:

$$\beta^*(h|\text{H}) = \frac{\theta(1 - \sigma_h^{t*})}{\theta(1 - \sigma_h^{t*}) + (1 - \theta)\sigma_l^{t*}}, \quad (5a)$$

$$\beta^*(h|\text{L}) = \frac{\theta\sigma_h^{t*}}{\theta\sigma_h^{t*} + (1 - \theta)(1 - \sigma_l^{t*})}, \quad (5b)$$

and $\beta^*(l|\text{H}) = 1 - \beta^*(h|\text{H})$, $\beta^*(l|\text{L}) = 1 - \beta^*(h|\text{L})$.

We offer two remarks to explain this definition. First, with regard to the sequential rationality of agents (modeled as a continuous player set), the rationality constraints (3a) and (3c) ensure that if the agents of a given type misbehave with positive probability in equilibrium, then the expected utility in choosing to access the other server is no less than the expected utility in choosing to access the server corresponding to their own type. On the other hand, (3b) and (3d) constraints ensure that if agents use the server for their true type with positive probability in equilibrium, then the utility of choosing the other server is not strictly higher.

Second, the consistency of beliefs requires that the operator's updated belief of each type given the received signal is consistent with the prior distribution and the agents' strategy in accordance with the Bayes' rule.

IV. EQUILIBRIUM CHARACTERIZATION

In this section, we characterize the PBE of the signaling game. In Sec. IV-A, we show three properties of PBE that are crucial for equilibrium analysis given any game parameters. In Sec. IV-B, we focus on analyzing the equilibrium regimes, where the qualitative properties of PBE are distinguished.

A. General properties of PBE

From Assumption (A1), the costs of both servers are finite when no agent misbehaves. In fact, the following lemma shows that the costs of both servers are also finite in equilibrium.

Lemma 1: In any PBE, $c_H^\theta(\sigma^{t*}) < \infty$ and $c_L^\theta(\sigma^{t*}) < \infty$. The next proposition shows that type h agents do not misbehave in equilibrium. Consequently, the operator does not inspect agents that choose to access the server L.

Proposition 1: In any PBE, (σ^*, β^*) satisfies:

$$\sigma_h^{t*} = 0, \quad \sigma_L^{d*} = 0, \quad \beta^*(l|\text{L}) = 1, \quad \beta^*(h|\text{L}) = 0.$$

In addition, the next proposition ensures that not all type l agents misbehave in equilibrium.

Proposition 2: In any PBE, $\sigma_l^{t*} < 1$.

Proposition 1 and Proposition 2 together show that both servers are used in equilibrium.² Consequently, we only need to further consider σ_l^{t*} , β^* and σ_H^{d*} . For simplicity, we abuse the notation, and henceforth use $c_H^\theta(\sigma_l^t)$ (resp. $c_L^\theta(\sigma_l^t)$) to denote the cost of server H (resp. L) when type l agents' strategy is σ_l^t , and $\sigma_h^t = 0$. Additionally, we define $\Delta c^\theta(\sigma_l^t)$ as the cost difference between L and H when the strategy of type l agents is σ_l^t :

$$\Delta c^\theta(\sigma_l^t) \triangleq c_L^\theta(\sigma_l^t) - c_H^\theta(\sigma_l^t).$$

From Assumption (A2), we know that $\Delta c^\theta(\sigma_l^t)$ must be decreasing in σ_l^t . The function $\Delta c^\theta(\sigma_l^t)$ evaluates the incentive of a type l agent to misbehave given that the fraction of misbehaving type l agents is σ_l^t .

B. Equilibrium regimes

We now focus on how the PBE changes with game parameters. From Propositions 1-2, we know that in general, there are two possible cases in equilibrium: In the first case, no agents of type l take server H, which means misbehavior is completely deterred; and in the other case, a fraction of type l agent population takes server H. Indeed, we find that there exist two equilibrium regimes, each corresponding to one of the two cases. Furthermore, the second regime (i.e. a positive fraction of type l agents take server H) can be divided into three sub-regimes depending on whether the inspection rate of the operator on server H is zero, positive or one.

Before we characterize the PBE, we first introduce the following "threshold" function of p^d :

$$\hat{\sigma}_l^t(p^d) \triangleq \frac{p^d \theta}{(1 - \theta)(F_l - p^d)}. \quad (6)$$

The next lemma provides the best response correspondence σ_H^{d*} in equilibrium.

Lemma 2: Given any PBE:

- If $\sigma_l^{t*} < \hat{\sigma}_l^t(p^d)$, then $\beta^*(l|\text{H}) < p^d/F_l$ and $\sigma_H^{d*} = 0$.
- If $\sigma_l^{t*} = \hat{\sigma}_l^t(p^d)$, then $\beta^*(l|\text{H}) = p^d/F_l$ and $\sigma_H^{d*} \in [0, 1]$.
- If $\sigma_l^{t*} > \hat{\sigma}_l^t(p^d)$, then $\beta^*(l|\text{H}) > p^d/F_l$ and $\sigma_H^{d*} = 1$.

²This result implies that "pooling" equilibrium, in which agents of different types send identical signals, does not exist in our game due to the congestion nature in the cost functions.

Lemma 2 shows that according to Bayes' rule in (5), $\hat{\sigma}_l^t(p^d)$ leads to the belief $\beta^*(l|H) = p^d/F_l$, which is the threshold belief such that the operator is indifferent between the action I and N in equilibrium. If σ_l^{t*} is higher (resp. lower) than $\hat{\sigma}_l^t(p^d)$, then the operator will inspect the agents taking the server H with probability one (resp. zero). Specifically, as the fraction of h type goes to zero (i.e. $\theta \rightarrow 0$), the threshold $\hat{\sigma}_l^t(p^d) \rightarrow 0$, which implies that the defender will always inspect with probability 1.

We next introduce the equilibrium regimes:³

- 1) In regime A , p_l^t satisfies $p_l^t > \Delta c^\theta(0)$.
- 2) In regime B , p_l^t satisfies: $p_l^t < \Delta c^\theta(0)$. There are three sub-regimes of regime B .

Subregime B_1 :

$$\left\{ (p_l^t, p^d) \mid \max\{\Delta c^\theta(\hat{\sigma}_l^t(p^d)), 0\} < p_l^t < \Delta c^\theta(0), \right\} \quad (7)$$

Subregime B_2 :

$$\left\{ (p_l^t, p^d) \mid \begin{array}{l} \max\{\Delta c^\theta(\hat{\sigma}_l^t(p^d)) - F_l, 0\} < p_l^t \\ < \max\{\Delta c^\theta(\hat{\sigma}_l^t(p^d)), 0\}, \\ p^d > 0. \end{array} \right\} \quad (8)$$

Subregime B_3 :

$$\left\{ (p_l^t, p^d) \mid \begin{array}{l} 0 < p_l^t < \max\{\Delta c^\theta(\hat{\sigma}_l^t(p^d)) - F_l, 0\}, \\ p^d > 0. \end{array} \right\} \quad (9)$$

The interpretations of regime boundaries are more straightforward once we present the PBE in each regime, and thus will be discussed after Theorem 1. Note that these regime definitions are valid; however, B_3 can be empty.

We are now ready to fully characterize the PBE.

Theorem 1: The PBE is unique in each regime, and can be written as follows:

(a) Regime A :

$$\sigma_l^{t*} = 0, \quad \sigma_H^{d*} = 0, \quad \beta^*(h|H) = 1, \quad \beta^*(l|H) = 0. \quad (10)$$

(b) Regime B :

Subregime B_1 :

$$0 < \sigma_l^{t*} < \hat{\sigma}_l^t(p^d), \quad \sigma_H^{d*} = 0, \quad (11a)$$

$$\beta^*(h|H) = \frac{\theta}{\theta + (1-\theta)\sigma_l^{t*}}, \quad \beta^*(l|H) = \frac{(1-\theta)\sigma_l^{t*}}{\theta + (1-\theta)\sigma_l^{t*}}, \quad (11b)$$

and the unique σ_l^{t*} satisfies $\Delta c^\theta(\sigma_l^{t*}) = p_l^t$.

Subregime B_2 :

$$\sigma_l^{t*} = \hat{\sigma}_l^t(p^d), \quad \sigma_H^{d*} = \frac{\Delta c^\theta(\hat{\sigma}_l^t(p^d)) - p_l^t}{F_l}, \quad (12a)$$

$$\beta^*(h|H) = \frac{F_l - p^d}{F_l}, \quad \beta^*(l|H) = \frac{p^d}{F_l}. \quad (12b)$$

Subregime B_3 :

$$\sigma_l^{t*} > \hat{\sigma}_l^t(p^d), \quad \sigma_H^{d*} = 1, \quad (13a)$$

$$\beta^*(h|H) = \frac{\theta}{\theta + (1-\theta)\sigma_l^{t*}}, \quad \beta^*(l|H) = \frac{(1-\theta)\sigma_l^{t*}}{\theta + (1-\theta)\sigma_l^{t*}}, \quad (13b)$$

and the unique σ_l^{t*} satisfies $\Delta c^\theta(\sigma_l^{t*}) = p_l^t + F_l$. ■

First, we interpret the regime boundaries:

³We only discuss generic cases, where the game parameters are in the interior of each regime.

- (i) Regimes A and B are distinguished by the threshold $\Delta c^\theta(0)$, which is the cost reduction that a type l agent enjoys by misbehaving given that all the remaining agents do not misbehave.
- (ii) Regime B is distinguished into three sub-regimes by two thresholds: $\Delta c^\theta(\hat{\sigma}_l^t(p^d))$ and $\Delta c^\theta(\hat{\sigma}_l^t(p^d)) - F_l$. For any p^d , the threshold $\Delta c^\theta(\hat{\sigma}_l^t(p^d))$ is the gain from misbehavior given that the misbehavior rate is $\hat{\sigma}_l^t(p^d)$ and the operator does not inspect at all. The threshold $\Delta c^\theta(\hat{\sigma}_l^t(p^d)) - F_l$ is the increase in utility from misbehavior given that the misbehavior rate is $\hat{\sigma}_l^t(p^d)$ and the operator inspects each agent requesting access to server H. We say that the misbehavior cost p_l^t is relatively high compared to p^d , if $p_l^t > \Delta c^\theta(\hat{\sigma}_l^t(p^d))$; relatively medium if $\Delta c^\theta(\hat{\sigma}_l^t(p^d)) - F_l < p_l^t < \Delta c^\theta(\hat{\sigma}_l^t(p^d))$, and relatively low if $p_l^t < \Delta c^\theta(\hat{\sigma}_l^t(p^d)) - F_l$.

Second, we relate the PBE strategy profiles and the conditions determining the regime boundaries:

[Regime A]: Misbehavior cost $p_l^t > \Delta c^\theta(0)$, thus misbehavior is deterred, and no inspection is needed.

[Regime B]: Misbehavior cost $p_l^t < \Delta c^\theta(0)$, thus misbehavior occurs with positive probability.

- B_1 : [Misbehavior cost is relatively high (7).] Misbehavior rate is lower than the threshold $\hat{\sigma}_l^t(p^d)$. The operator does not inspect.
- B_2 : [Misbehavior cost is relatively medium (8).] Misbehavior rate is equal to the threshold $\hat{\sigma}_l^t(p^d)$. The operator inspects a positive fraction of agents.
- B_3 : [Misbehavior cost is relatively low (9).] Misbehavior rate is higher than the threshold $\hat{\sigma}_l^t(p^d)$. The operator inspects all the agents.

Third, we summarize how PBE changes with the misbehavior and inspection costs in table I:

		A	B_1	B_2	B_3
p_l^t increases	σ_l^{t*}	—	↓	—	↓
	σ_H^{d*}	—	—	—	—
p^d increases	σ_l^{t*}	—	—	↑	—
	σ_H^{d*}	—	—	↓	—

TABLE I: Qualitative properties of PBE

Fourth, we emphasize the implications of PBE:

- The misbehavior is completely deterred only when the misbehavior cost is sufficiently high.
- In sub-regime B_2 , the belief $\beta^*(H)$ does not depend on θ . The intuition is that since in this sub-regime, both the agents and the operator use mixed strategies in equilibrium, the strategy $\hat{\sigma}_l^t(p^d)$ in (6) increases in θ to ensure that the belief $\beta^*(l|H)$ (resp. $\beta^*(h|H)$) is maintained at the threshold value p^d/F_l (resp. $1 - p^d/F_l$), which makes the operator indifferent between I and N.
- One can verify the intuitive property that the utility of the type l (resp. h) agents is non-increasing (resp. non-decreasing) in p_l^t and non-decreasing (resp. non-increasing) in p^d . Similarly, the operator's utility is non-decreasing in p_l^t and non-increasing in p^d .

- In general, the misbehavior rate σ_i^{t*} is non-increasing in p_i^t , and non-decreasing in p^d . The inspection rate σ^{d*} is non-decreasing in p_i^t , and non-increasing in p^d .

Finally, from Assumption (A3), we know that the minimal technology cost that deters misbehavior, $\Delta c^\theta(0)$, decreases in θ , and $\Delta c^\theta(\hat{\sigma}_i^t(p^d))$ also decreases in θ for any given p^d . Therefore, as θ increases, the sizes of regime *A* (no misbehavior, no inspection) and sub-regime *B*₁ (low misbehavior rate, no inspection) increase, and the sizes of the two other regimes decrease. This implies that the misbehavior rate is lower and the inspection is less needed when more agents are of type *h*. Note again that θ is defined exogenously, and is common knowledge in our work. However, if the operator does not know θ , and he over-estimates θ , then the actual inspection rate will be lower than the optimal strategy.

Moreover, as F_l increases, the size of *B*₂ increases, and the size of *B*₃ decreases or becomes empty. However, F_l has no effect on *A* and *B*₁, where inspection is not needed. This observation implies that (i) Fine is effective to decrease the misbehavior rate when the inspection cost is relatively high compared to the misbehavior cost, but cannot deter misbehavior. (ii) If the fine is higher than $\Delta c^\theta(0)$, then inspecting all agents is not needed.

V. A SIMPLE EXAMPLE OF TOLL EVASION

In this section, we apply our equilibrium results to a specific example of Electronic Toll Collection (ETC) system. In the ETC setting, server H (resp. L) represents the tolled (resp. toll-free) lanes. Type *h* are the agents that are willing to pay the toll, and type *l* are the agents that are not willing to pay. The total arrival rate of both types of agents is $\lambda = 2400$ veh/hr. The fraction $\theta = 0.3$ is the fraction of type *h* agents. Therefore, the arrival rate of type *h* (resp. *l*) agents is $\theta\lambda$ (resp. $(1 - \theta)\lambda$). The toll is collected electronically, and the access is granted to the tolled lanes to the paying agents after the RSU obtains their reported identifier. Such an operation is technologically feasible; see e.g. the European DSRC Toll Collection systems [2].

We model the highway as two parallel $M/M/1$ queuing systems, one representing the tolled lanes (H), and the other representing the toll-free lanes (L). For background on modeling highway traffic with stochastic queuing models, see [15], [16]. Both the tolled lanes and the toll-free lanes have a capacity (service rate) of 1700 veh/hr, i.e. $\mu_H = \mu_L = 1700$ veh/hr. For ease of presentation, we assume that the travel cost on a server is the product of the expected system time and the value of time $V_oT = 50$ USD/hr. The fine is $F_l = 100$ USD. Following standard results in queuing theory (see e.g. [17]), the expected cost functions are as follows:

$$c_H^\theta(\sigma) = \begin{cases} \frac{V_oT}{\mu_H - \theta\lambda(1 - \sigma_h^t) - (1 - \theta)\lambda\sigma_l^t}, & \text{if } \theta\lambda(1 - \sigma_h^t) + (1 - \theta)\lambda\sigma_l^t < \mu_H, \\ \infty, & \text{o.w.} \end{cases}$$

$$c_L^\theta(\sigma) = \begin{cases} \frac{V_oT}{\mu_L - (1 - \theta)\lambda(1 - \sigma_l^t) - \theta\lambda\sigma_h^t}, & \text{if } (1 - \theta)\lambda(1 - \sigma_l^t) + \theta\lambda\sigma_h^t < \mu_L, \\ \infty, & \text{o.w.} \end{cases}$$

We can check that the cost functions satisfy Assumptions (A1) - (A3). Fig. 4 illustrates the regimes of PBE.

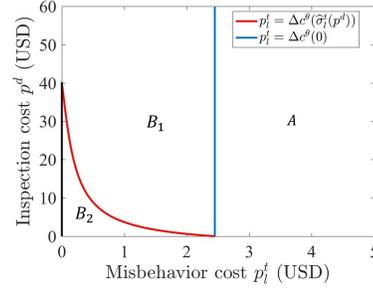


Fig. 4: PBE regimes.

In this example, the minimum p_i^t that deters misbehavior is $\Delta c^\theta(0) = 2.35$ USD. Note that this is the technology cost per signal. A device that is used to manipulate the message sent to the RSU can be expensive, but if the device is repeatedly used, the average cost can be low.

Additionally, since the fine $F_l = 100 > \Delta c^\theta(0) = 2.35$, the sub-regime *B*₃ is empty. This implies that given any p_i^t and p^d , the misbehavior rate is no higher than $\hat{\sigma}_i^t(p^d)$ (in sub-regime *B*₂), and there is no need to inspect all agents. Given parameters in *B*₂, $p_i^t = 0.5$ USD and $p^d = 5$ USD, the equilibrium misbehavior rate is $\sigma_i^{t*} = \hat{\sigma}_i^t(p^d) = 2.15\%$, and the inspection rate is $\sigma_H^{d*} = 0.34\%$.

VI. CONCLUDING REMARKS

In this article, we formulate a signaling game to study the misbehavior of two classes of agents with different priorities and the inspection strategy of the operator in a V2I-based highway system. We provide complete characterization of PBE, and study how the equilibrium structure changes with fine, costs parameters, and relative sizes of the two traffic classes.

One open question is how we can design an efficient V2I-based highway system that incorporates the inspection of misbehavior. Our analysis shows that as θ decreases, misbehavior is less likely to be deterred. This dampens the advantage of agents with high priority, and increases the need for inspection. Therefore, when designing the two-classes traffic system, the operator not only needs to consider the demand in each class and the overall efficiency in terms of the expected travel cost, but also should consider how the rate of misbehavior effects the congestion externalities in each class and the resulted inspection costs.

APPENDIX A: PROOFS OF STATEMENTS

Proof of Lemma 1. We prove by contradiction. If $c_H^\theta(\sigma^{t*}) = \infty$, the aggregate amount of agents taking server H in equilibrium must be higher than that without misbehavior. Hence, the amount of agents on server L is lower than that without misbehavior, which ensures $c_L^\theta(\sigma^{t*}) < \infty$. Given any operator's strategy $\sigma^d \in [0, 1]$, from (1), we must have $\mathbb{E}_{\sigma^*}[U_h^t(H)] < \mathbb{E}_{\sigma^*}[U_h^t(L)]$ and $\mathbb{E}_{\sigma^*}[U_l^t(H)] < \mathbb{E}_{\sigma^*}[U_l^t(L)]$. From (3b) and (3c), we have $\sigma_h^{t*} = 1$ and $\sigma_l^{t*} = 0$, i.e. no agents use server H in equilibrium. This contradicts the claim that $c_H^\theta(\sigma^{t*}) = \infty$. Analogously, we argue that $c_L^\theta(\sigma^{t*}) < \infty$. ■

Proof of Proposition 1. We first prove $\sigma_h^{t*} = 0$ by contradiction. Assume that there exists a PBE such that $\sigma_h^{t*} > 0$, i.e. there

exists a fraction of type h agents using server L. From (1a) and (3a), we must have $-c_L^\theta(\sigma^{t*}) - p_h^t \stackrel{(1a)}{\geq} \mathbb{E}_{\sigma^*}[U_h^t(L)] \stackrel{(3a)}{\geq} \mathbb{E}_{\sigma^*}[U_h^t(H)] \stackrel{(1a)}{=} -c_H^\theta(\sigma^{t*})$. Thus, $c_L^\theta(\sigma^{t*}) \leq c_H^\theta(\sigma^{t*}) - p_h^t$. Since $p_h^t \geq 0$, $c_L^\theta(\sigma^{t*}) \leq c_H^\theta(\sigma^{t*})$. From (1b), we have $\mathbb{E}_{\sigma^*}[U_l^t(L)] \stackrel{(1b)}{=} -c_L^\theta(\sigma^{t*}) \geq -c_H^\theta(\sigma^{t*}) \stackrel{(?)}{>} \mathbb{E}_{\sigma^*}[U_l^t(H)]$. Hence, from (3c), we must have $\sigma_l^{t*} = 0$, i.e. no agents of type l take server H. Additionally, since $c_L(\sigma^t)$ is increasing in σ_h^t and decreasing in σ_l^t , when $\sigma_h^{t*} > 0$ and $\sigma_l^{t*} = 0$, we have $c_L^\theta(\sigma^{t*}) > c_L^\theta(0, 0)$. Analogously, $c_H(\sigma^t)$ is increasing in σ_l^t and decreasing in σ_h^t , and thus $c_H^\theta(\sigma^{t*}) < c_H^\theta(0, 0)$. Consequently, $c_H^\theta(0, 0) > c_H^\theta(\sigma^{t*}) \geq c_L^\theta(\sigma^{t*}) > c_L^\theta(0, 0)$. However, this contradicts Assumption (A1). Therefore, we can conclude that $\sigma_h^{t*} = 0$.

Next, from (5), we can check that the belief updated by Bayes' rule satisfies $\beta^*(l|L) = 1$ and $\beta^*(h|L) = 0$.

Finally, since $\beta^*(l|L) = 1$ implies that only type l agents take server L. From (2b), the action I is strictly dominated by the action N. Hence, $\sigma_L^{d*} = 0$. ■

Proof of Proposition 2. Again we prove this claim by contradiction. Assume that $\sigma_l^{t*} = 1$, i.e. all the agents of type l uses server H. From Proposition 1, agents of type h do not use server L in equilibrium. Therefore, in PBE, no agents use server L. From Assumption (A1), we know that $\mathbb{E}_{\sigma^*}[U_l^t(L)] = -c_L^\theta(0, 1) > -c_H^\theta(0, 1) \geq \mathbb{E}_{\sigma^*}[U_l^t(H)]$, which contradicts the equilibrium condition in (3c). Hence, $\sigma_l^{t*} < 1$. ■

Proof of Lemma 2. From (5), we can check that if $\sigma_l^{t*} = \hat{\sigma}_l^t$, then $\beta^*(l|H) = p^d/F_l$. In this case, $-p^d + F_l\beta^*(l|H) = 0$, and thus any $\sigma_H^{d*} \in [0, 1]$ maximizes $\mathbb{E}_{\sigma^*}[U_H^d|\beta^*]$ in (2a). Additionally, since $\beta^*(l|H)$ increases in σ_l^{t*} , if $\sigma_l^{t*} < \hat{\sigma}_l^t$, we must have $\beta^*(l|H) < p^d/F_l$. Consequently, $-p^d + F_l\beta^*(l|H) < 0$, and from (2a) and (4), $\sigma_H^{d*} = 0$. The case for $\sigma_l^{t*} > \hat{\sigma}_l^t$ can be argued analogously. ■

Proof of Theorem 1.

- (a) In regime A, since $p_l^t > \Delta c^\theta(0)$, from (1b), we have $\mathbb{E}_{\sigma^*}[U_l^t(H)] \leq -c_H^\theta(\sigma_l^{t*}) - p_l^t < -c_L^\theta(\sigma_l^{t*}) = \mathbb{E}_{\sigma^*}[U_l^t(L)]$. Therefore, from (3c), we must have $\sigma_l^{t*} = 0$. From (5), we can check that $\beta^*(h|H) = 1$ and $\beta^*(l|H) = 0$. Following from Lemma 2, $\sigma_H^{d*} = 0$. Thus, the PBE in (10) is the unique equilibrium.
- (b) In regime B, we first prove by contradiction that $\sigma_l^{t*} \in (0, 1)$. Assume that $\sigma_l^{t*} = 0$, then from (4) and (5), β^* and σ_L^{d*} must be in (10). Then, from (1b), $\mathbb{E}_{\sigma^*}[U_l^t(L)] = -c_L^\theta(0)$. However, if type l agents deviate to choose H, the utility is $-c_H^\theta(0) - p_l^t$. Since in regime B, $p_l^t < \Delta c^\theta(0)$, type l agents has incentive to deviate to H, which contradicts $\sigma_l^{t*} = 0$. Additionally, from Proposition 2, $\sigma_l^{t*} < 1$. Therefore, in this regimes $\sigma_l^{t*} \in (0, 1)$, i.e. type l agents take both servers in equilibrium, which implies the follows:

$$\mathbb{E}_{\sigma^*}[U_l^t(L)] = \mathbb{E}_{\sigma^*}[U_l^t(H)]. \quad (15)$$

Furthermore, there are three cases for σ_H^{d*} : $\sigma_H^{d*} = 0$, $\sigma_H^{d*} \in (0, 1)$ and $\sigma_H^{d*} = 1$. It turns out that these three cases correspond to sub-regime B_1 , B_2 and B_3 , respectively.

- (B₂) $\sigma_H^{d*} \in (0, 1)$: In this case, from Lemma 2, we know that $\beta^*(l|H)$ must be p^d/F_l , and $\sigma_l^{t*} = \hat{\sigma}_l^t$ in (6) is the unique equilibrium strategy. Additionally, from (15), the operator's strategy σ_L^{d*} should satisfy $-c_L^\theta(\sigma_l^{t*}) = -c_H^\theta(\sigma_l^{t*}) - p_l^t - F_l\sigma_L^{d*}$. Thus, σ_L^{d*} is in (12a). Furthermore, it remains to be shown that σ_l^{t*} and σ_L^{d*} in (12a) are feasible strategies in this case, i.e. $0 < \sigma_l^{t*} < 1$ and $0 \leq \sigma_L^{d*} < 1$. We can check that these constraints are satisfied when p^d and p_l^t satisfy (8). Therefore, PBE in (12) is the unique equilibrium in B_2 .
- (B₁) $\sigma_H^{d*} = 0$: In this case, $\mathbb{E}_{\sigma^*}[U_l^t(H)] = -c_H^\theta(\sigma_l^{t*}) - p_l^t$. From (15), we must have $\mathbb{E}_{\sigma^*}[U_l^t(H)] = -c_H^\theta(\sigma_l^{t*}) -$

$p_l^t = -c_L^\theta(\sigma_l^{t*}) = \mathbb{E}_{\sigma^*}[U_l^t(L)]$, which leads to $\Delta c^\theta(\sigma_l^{t*}) = p_l^t$. From (5), β^* is in (11b).

We now discuss the conditions on p_l^t and p^d , under which the strategy profile in (11a) is a PBE in this case. We have argued that the condition $p_l^t < \Delta c^\theta(0)$ is needed to ensure that $\sigma_l^{t*} \in (0, 1)$. Additionally, we need to show that the operator has no incentive to deviate. From Lemma 2, as long as $\beta^*(l|H) < p^d/F_l$, the action N strictly dominates the action I. Therefore, we need $\sigma_l^{t*} < \hat{\sigma}_l^t$. Since $\Delta c^\theta(\sigma_l^t)$ decreases in σ_l^t and $\Delta c^\theta(\sigma_l^{t*}) = p_l^t$, we must have $p_l^t = \Delta c^\theta(\sigma_l^{t*}) > \Delta c^\theta(\hat{\sigma}_l^t)$, which leads to constraints in (7).

- (B₃) $\sigma_H^{d*} = 1$: In this case, from (15), we obtain $\mathbb{E}_{\sigma^*}[U_l^t(L)] = -c_L^\theta(\sigma_l^{t*}) = -c_H^\theta(\sigma_l^{t*}) - p_l^t - F_l = \mathbb{E}_{\sigma^*}[U_l^t(H)]$. Therefore, σ_l^{t*} satisfies $\Delta c^\theta(\sigma_l^{t*}) = p_l^t + F_l$. From (5), β^* is obtained from (13b). To ensure that the action I is a dominant strategy for the operator, from Lemma 2, we need $\beta^*(l|H) > p^d/F_l$, and $\sigma_l^{t*} > \hat{\sigma}_l^t$. Besides, $\Delta c^\theta(\sigma_l^t)$ decreases in σ_l^t and $\Delta c^\theta(\sigma_l^{t*}) = p_l^t + F_l$, we can conclude that $p_l^t + F_l = \Delta c^\theta(\sigma_l^{t*}) < \Delta c^\theta(\hat{\sigma}_l^t)$, i.e. p_l^t satisfies (9). ■

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