The Speed and Lifetime of Cosmic-Ray Muons

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We investigate the speed and lifetime of cosmic-ray muons. The speed of cosmic-ray muons was determined by measuring time-of-flight between parallel scintillator paddles for various separations. The lifetime was determined by measuring the delay between capture and decay of muons in a plastic scintillator block. The most probable speed of muons was found to be $\beta = 1.01 \pm 0.02$ as compared with a book value of about $\beta = 0.994 \pm 0.005$. The mean lifetime was found to be $(2.22 \pm 0.04)\,\mu\mathrm{s}$ as compared with a book value of $2.197\,034(21)\,\mu\mathrm{s}$. Relativistic kinematics was found to give a much better fit, than Newtonian kinematics, between our experimental results and existing data on the energies and momenta of cosmic-ray muons.

I. THEORY

I.1. Muons

Muons are unstable, deeply penetrating, negatively charged elementary particles with a relatively long mean lifetime.[1] Energetic cosmic rays striking the earth's atmosphere approximately 15 km above the surface provide an ample source of highly energetic muons for this experiment.[2]

I.2. Time Dilation and Decay

Unstable energetic particles provide a means of testing relativistic kinematics against Newtonian kinematics; by measuring the mean lifetime τ_0 of a particle at rest (in the lab), it is possible to predict the relativistic correction to their mean lifetime $\tau = \tau_0\gamma$ where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. The probability of survival of such a particle traveling a distance d is then predicted by relativistic kinematics to be $\tau^{-1}e^{-d/(v\tau)}$; Newtonian kinematics predicts $\tau_0^{-1}e^{-d/(v\tau)}$. We measured the speed and intensity of cosmic-ray muons at approximately sea level, and compared the predicted intensity of muons at various elevations to data taken by Rossi in [3].

I.3. Speed and Momentum

Relativistic dynamics predicts a momentum-speed relationship of $p = \gamma m v$; Newtonian mechanics predicts the relationship $p = mv$. We measured the speed of cosmic-ray muons by measuring time-of-flight over various distances. Momentum data, taken by Wilson in [4] and depicted graphically by Rossi in [3], along with the textbook value for muon mass permitted a comparison between the relativistic and Newtonian relationships.

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II. LIFETIME

II.1. Methodology and Experimental Setup

In order to measure the mean lifetime of muons in the lab, we stopped cosmic-ray muons in a plastic scintillator block, as shown in Figure 1; interactions with the dense plastic material caused the muons to deposit energy into the scintillator.

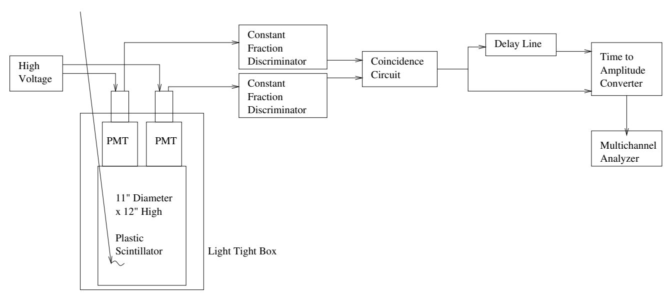


FIG. 1. Experimental setup diagram taken from [2].

When enough energy was deposited within a short period of time, the scintillator flashed, and the photomultiplier tubes (PMTs) amplified this signal (henceforth called the arrival signal). The flux rate of muons was sufficiently small that we expected only a single muon to be present (and stopped) by the detector at any particular time.

When enough energy was deposited by a particular muon, it came to a halt in the scintillator. Such muons decayed some time later, and the PMTs amplified this signal, too (henceforth called the decay signal).

In order to reduce the noise (false positives) of the PMTs, we piped the signal first through constant fraction discriminators (CFDs) and then into a coincidence circuit. Unfortunately, we neglected to measure the individual count rates of the PMTs, making a theoretical prediction of remaining noise impossible.

Because the arrival signal and the decay signal were expected to be extremely close together, relative to the expected spacing between signals from different muons, we were able to determine the time between arrival and decay using a time to amplitude converter (TAC) which measured the time interval between a time starting shortly after any signal and ending at the next signal. The delay between the incoming signal and the start time (on the order of a few nanoseconds) was judged to be insignificant relative to the decay time (on the order of a few microseconds). A multichannel analyzer (MCA) was then used to record the decay times.

Because the probability distribution for decay is time-translation invariant (the probability that a muon will decay at a time t from now is alway $\tau^{-1}e^{-t/\tau}$, assuming that it has not yet decayed), we may infer the mean lifetime from the decay-time curve, despite the fact that we didn't measure a creation event.

II.2. Data Analysis

In order to use the data from the MCA, we needed to find the conversion between channel number and time. We connected a pulse generator to the TAC, using an interval of 0.16 μs. Figure 2 shows the resulting time calibration, which gave (0.0199 ± 0.0001) µs per channel.

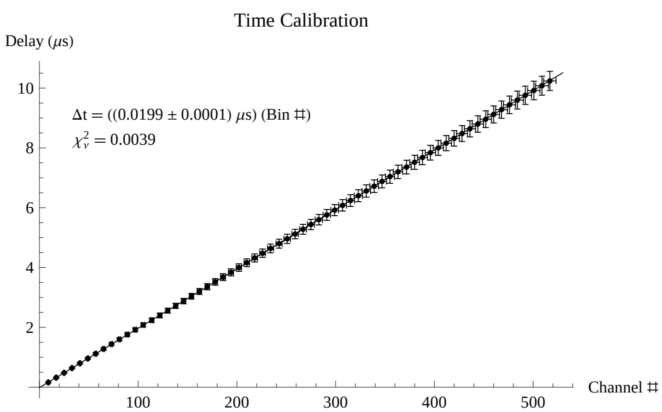
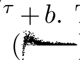


FIG. 2. Time calibration plot of delay vs. channel number. The uncertainty comes from ± 0.5 in the last decimal place on the pulse generator, which read 0.16 μs, from an assumed uncertainty of ± 0.5 channels, and from Poisson error bars on the counts; the error is obviously vastly overestimated. Channel numbers are a weighted average of the channels making up a spike.

We took muon lifetime data for approximately 18 hours. After dropping the channels which corresponded to a time that was too short for the TAC to deal with (most of which had zero counts), the lifetime data were fit to $ae^{-t/\tau} + b$. The residuals were found to be strongly systematic () with $\chi^2_\nu = 60$. Since the noise, assumed to be constant over the time interval that we measured, comes out to be $b = -0.000\,04 \pm 0.000\,09$ counts, which is negligible. I hypothesized that the systematicity in the residuals was due to ± 0 being a terrible estimate of the uncertainty for 0 counts; dropping the zeros and fitting the revised data to $ae^{-t/\tau}$ gives Figure 3; $\tau = (2.22 \pm 0.04)$ µs with $\chi^2_\nu = 1.0$, which is a remarkably good fit. The textbook value is $2.197\,034(21)$ µs.

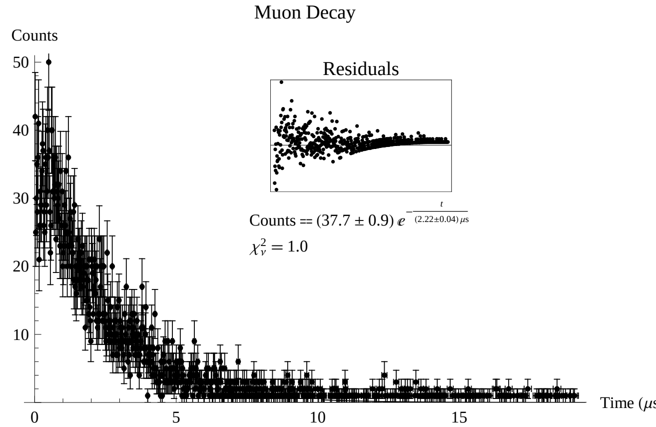


FIG. 3. Plot of muon count vs. lifetime, without any zero counts. Vertical error bars are Poisson; horizontal error bars are due to uncertainty in the time calibration and channel number (assumed to be ± 0.5 channels).

III. SPEED

III.1. Methodology and Experimental Setup

In order to measure the most probable speed of muons, we measured the time-of-flight of muons traveling various known distances between 15 cm and 350 cm. We recorded incidence events of muons in two parallel scintillator paddles, as shown in Figure 4. The events were separated from noise via CFDs. Because the time scales were so short and the precise characteristics of the cables and equipment was unknown, a delay was added to the bottom signal (via a length of RG-58 cable) so that the TAC would not cut off the lower end (shorter times) of the delay spectrum.

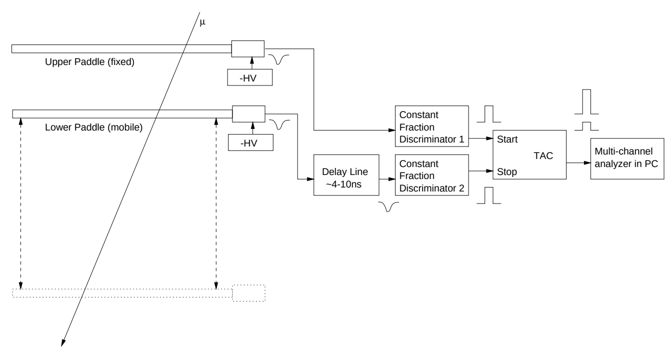


FIG. 4. Experimental setup diagram taken from [2]. A muon which strikes both paddles causes a signal to get through both CFDs. The delay line delays the later signal enough to get picked up by the TAC, which feeds in to the MCA.

The unknown time-delay characteristics of the circuits prevented us from determining an absolute channel-to-time conversion. Instead, the speed was found as the slope of a linear fit for distance between paddles vs. peak time delays to a line.

III.2. Data Analysis

In order to use the data from the MCA, we again needed to find the conversion between channel number and time. We connected a pulse generator to the TAC, using an interval of 10 ns. Figure 5 shows the resulting time calibration, which gave (0.0199 ± 0.0001) μs per channel.

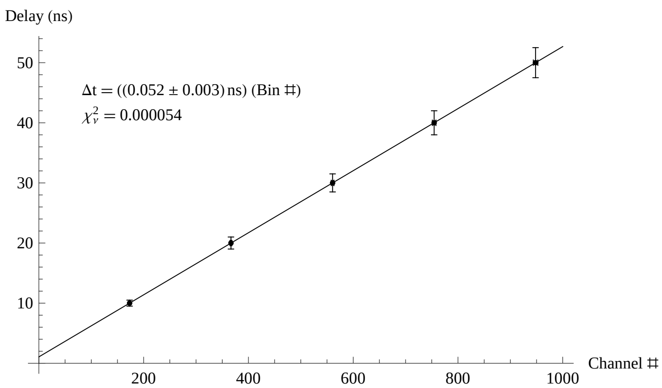


FIG. 5. Time calibration plot of delay vs. channel number. The uncertainty comes from ±0.5 in the last decimal place on the pulse generator, which read 10. ns, from an assumed uncertainty of ±0.5 channels, and from Poisson error bars on the counts; the error is obviously vastly overestimated. Channel numbers are a weighted average of the channels making up a spike.

We took data at nine different paddle separations ranging from 20 cm to 332 cm, for various lengths for time. Fitting this data was particularly troublesome, as I was unable to come up with a working theoretical model to fit the data to. The data are not normal, not Poisson, not β-distributed, not γ-distributed, not Maxwell-Boltzmann, not Maxwell-Juttner (the relativistic version of Maxwell-Boltzmann), and I was unable to construct a reasonable distribution based on the underlying physics, starting from the assumption that cosmic-ray energy is distributed according to $P(E) \sim E^{-\alpha}$ for some $\alpha \approx 2$, as suggested by [5] and [6]. Lacking space and inclination to show all my failed attempts at fitting, see Figure 6 for how the data fail to be normal for an example. I found that ignoring the uncertainties on the data and fitting to a normal distribution using least-squares gave acceptable fit parameters for all data sets. Lack of space prevents me from showing more than one fit: Figure 7.

The most probable speeds were fit to a line, as shown in Figure 8 ($\chi^2_\nu = 160$). The first three data points don't match up well to the line, likely due to the larger mean slant distance. Dropping them results in Figure 8.

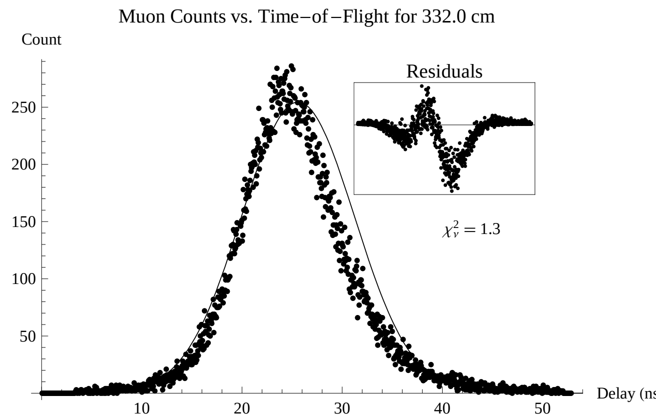


FIG. 6. Fit of time of flight to a normal distribution for a paddle separation of 332.0 cm. Error bars are not shown so as to emphasize badness-of-fit; the fit curve would not be visible if error bars were shown. Skewness likely results from some combination of the underlying cosmic-ray energy distribution, the skewness towards longer distances of the distribution of slant distances.

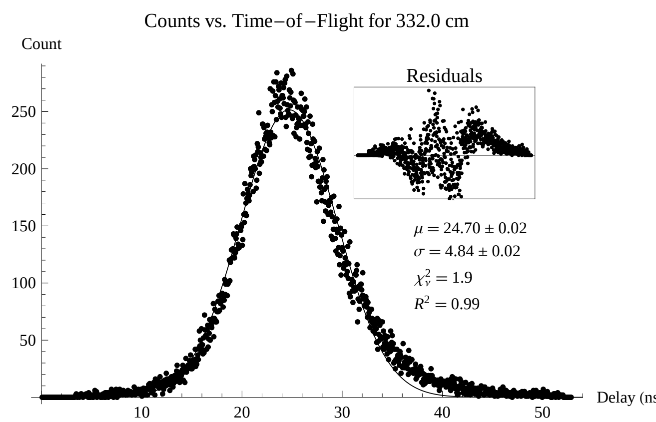


FIG. 7. Fit of time of flight to a normal distribution for a paddle separation of 332.0 cm. χ^2_ν is calculated with vertical Poisson error bars and horizontal uncertainties from the channel-to-time conversion, after dropping the zero-count data points.

This gives $\beta = 1.01 \pm 0.02$ ($\chi^2_\nu = 6.9$); the value suggested by [2] is that corresponding to 1 GeV / c, or, using the textbook value of $m_\mu = 105.658\,371\,5(35)$ MeV ([7]), $\beta = 0.994 \pm 0.005$, assuming an uncertainty of ±0.5 GeV / c on the momentum. Note that Newtonian kinematics predicts that muons with this energy travel at $\beta = 10 \pm 5$, which is well above what we measure.

Is there sufficient reason to drop the first three data points? If we assume that muons took the longest path (from one corner to the opposite one) rather than the shortest path (straight down), then we get Figure 9. So it is clearly possible, given the right distribution of muons as a function of angle, that the first three data points will be moved onto the line. However, running a Monte Carlo simulation using $P(\phi) \propto \cos^2\phi$ (details in section A) gives almost no change in data. My guess would be that

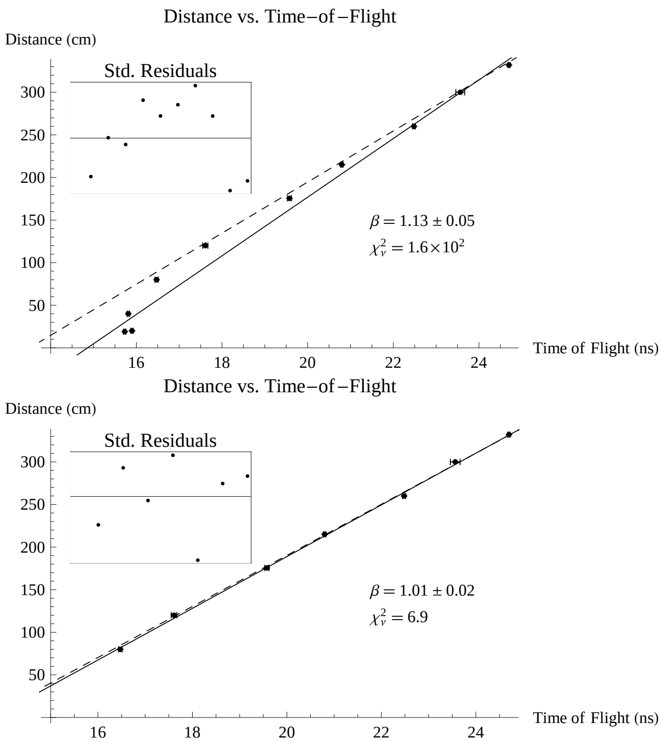


FIG. 8. Linear fits for muon speed. The dotted line has slope c, and was arbitrarily chosen to intersect with the fit line at $t = 24$ ns. The top plot shows the fit to all the data; the bottom plot shows the fit after dropping the first three data points.

either I did the simulation wrong or that muons are not distributed according to $P(\phi) \propto \cos^2\phi$ (or that the correlation between the energy and the angle is significant).

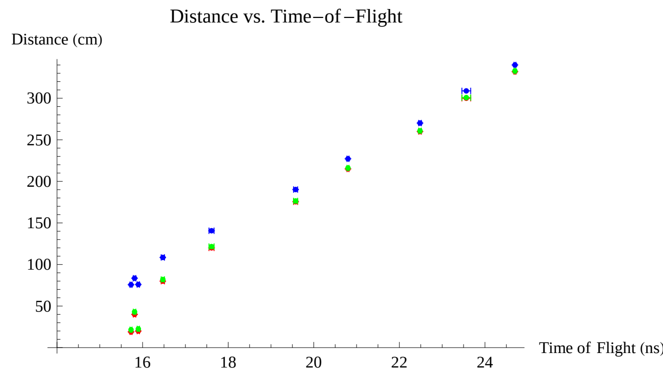


FIG. 9. The lower, red data (partially hidden by green data) are the original measurements. The higher, blue data are the corrected data assuming the muons take the longest path. The intermediate, green data are the data corrected via Monte Carlo simulation on the assumption that the muon angles are distributed according to $P(\phi) \propto \cos^2\phi$.

IV. CONCLUSIONS

The data we took, combined with others' measurements of muon muon momentum, and muon mass, strongly support relativistic kinematics over Newtonian kinematics. The most probable speed of muons was found to be $\beta = 1.01 \pm 0.02$; the relativistic prediction is $\beta = 0.994 \pm 0.005$ the classical prediction is $\beta = 10 \pm 5$. The mean lifetime was found to be $(2.22 \pm 0.04)\,\mu s$ as compared with a book value of $2.197\,034(21)\,\mu s$.

The distribution of muon times-of-flight calls in to question the validity of the claim (in [2]) that muons are distributed according to $\cos^2\phi$, or, alternatively, suggests a correlation between energy and angle so that larger angles correlate with very high energies. Determination of the underlying distribution of muon time-of-flights for a given separation is left to future investigation.

[1] "Muon," http://en.wikipedia.org/wiki/Muon (2011).
[2] MIT Department of Physics, "The speed and decay of cosmic-ray muons: Experiments in relativistic kinematics - the universal speed limit and time dilation," (2011).
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Appendix A: Monte Carlo Correction

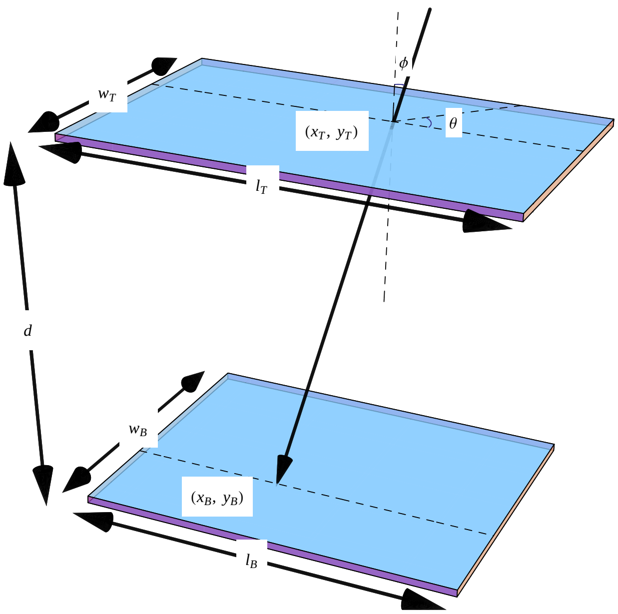

FIG. 10. A muon strikes the upper and lower paddles at azimuthal angle ϕ and planar angle θ.

Consider the setup shown in Figure 10. Give x_T, y_T, d, θ, and ϕ, we may calculate x_B and y_B as

$$(x_B, y_B) = (x_T, y_T) - \frac{d}{\cos(\phi)} (\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi)) \, .$$

We have that the slant distance $D = d/\cos\phi$. I generate $\theta \in (-\pi, \pi)$, $\phi \in (0, \pi/2)$, $x_T \in \left(-\frac{1}{2}l_T, \frac{1}{2}l_T\right)$, and $y_T \in \left(-\frac{1}{2}w_T, \frac{1}{2}w_T\right)$ uniformly at random, independently of one another. I then calculate (x_B, y_B), and throw out the point if (x_B, y_B) does not lie on the bottom paddle. I take the weighted average of the resulting values, weighting each value $\frac{d}{\cos\phi}$ by $\cos^2\phi$. The resulting plots are Figure 12 and Figure 11.

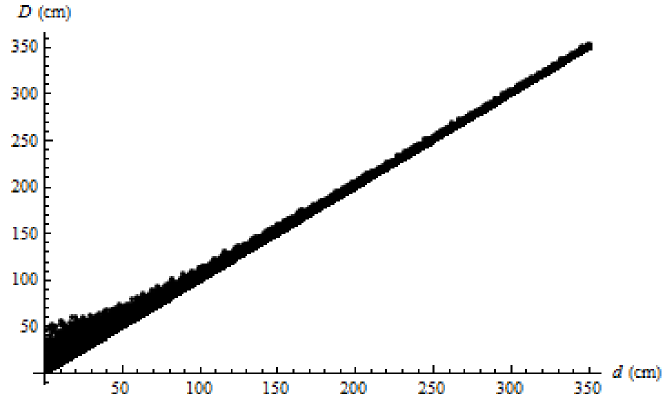

FIG. 11. A plot of many values of slant length D vs. separation d.

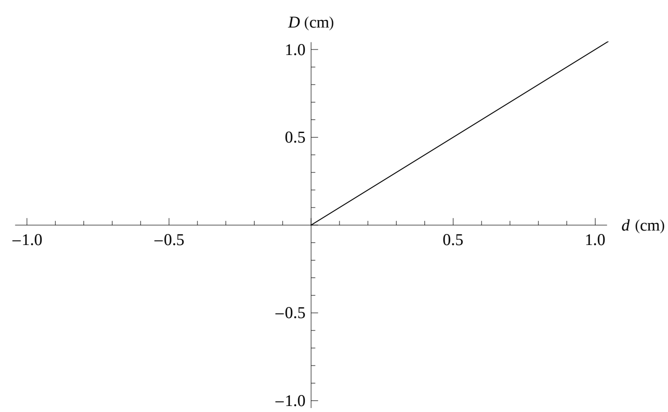

FIG. 12. A plot of the weighted average of slant lengths D vs. separation d.