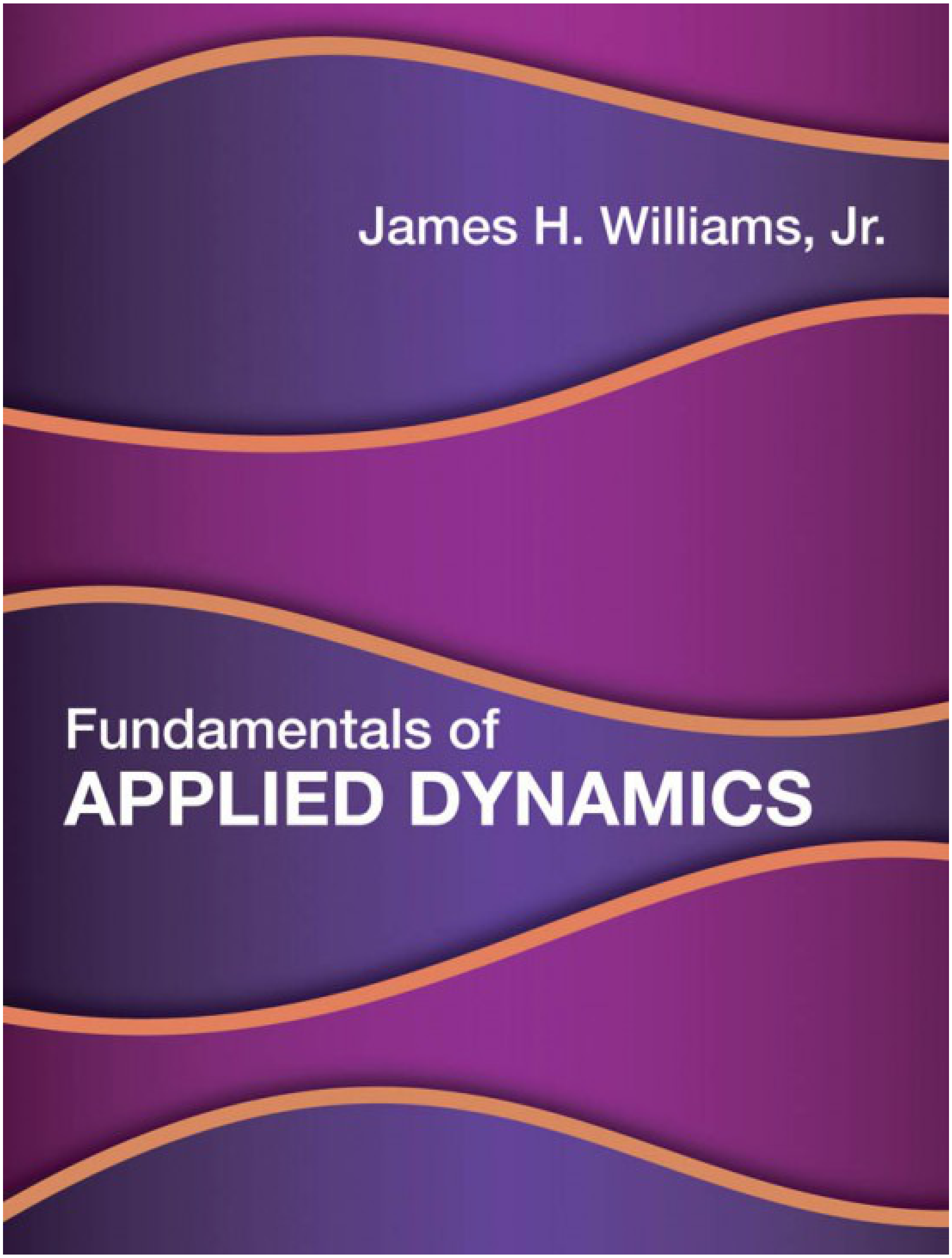
Fundamentals of Applied Dynamics

MIT Press

Wave Propagation

— An Introduction to Engineering Analyses —

MIT Press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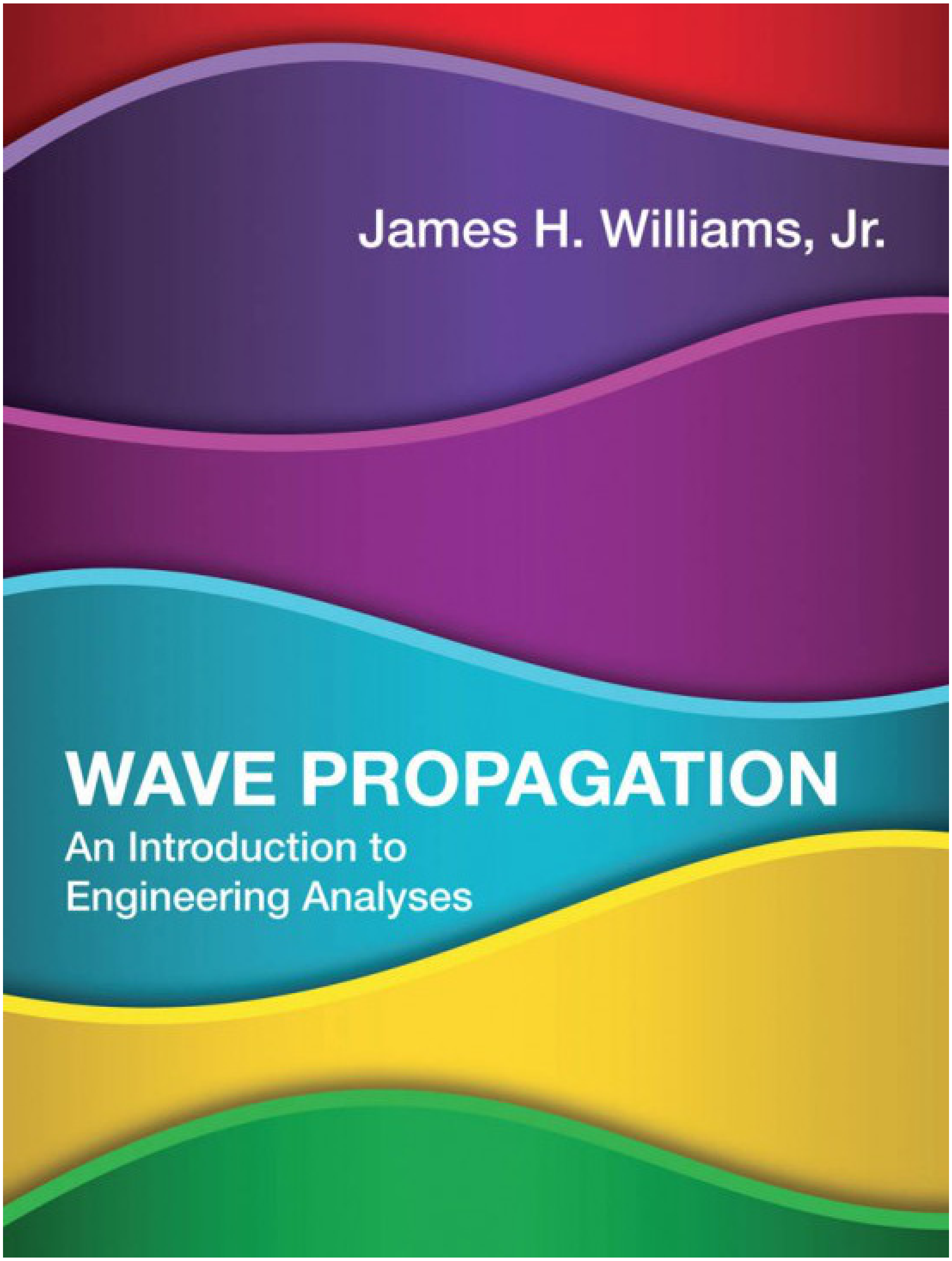
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WAVE PROPAGATION
An Introduction to
Engineering Analyses

Wave Propagation

— An Introduction to Engineering Analyses —

Sophomores → (2.001 and *2.003*) …

Style: Extended and Repetitive Expositions

200+ End-of-Chapter Problems

Chapter 1 Introduction to Wave Propagation

Vignette I

Chapter 2 The Classical Wave Equation

Vignette II and Vignette III

Chapter 3 Wave Propagation in Infinite Media

… ■ Examples … // … ♦ Examples …

Vignette IV

Chapter 4 Wave Propagation in Semi-Infinite Media

Vignette V and Vignette VI

Chapter 5 Wave Propagation in Finite Media

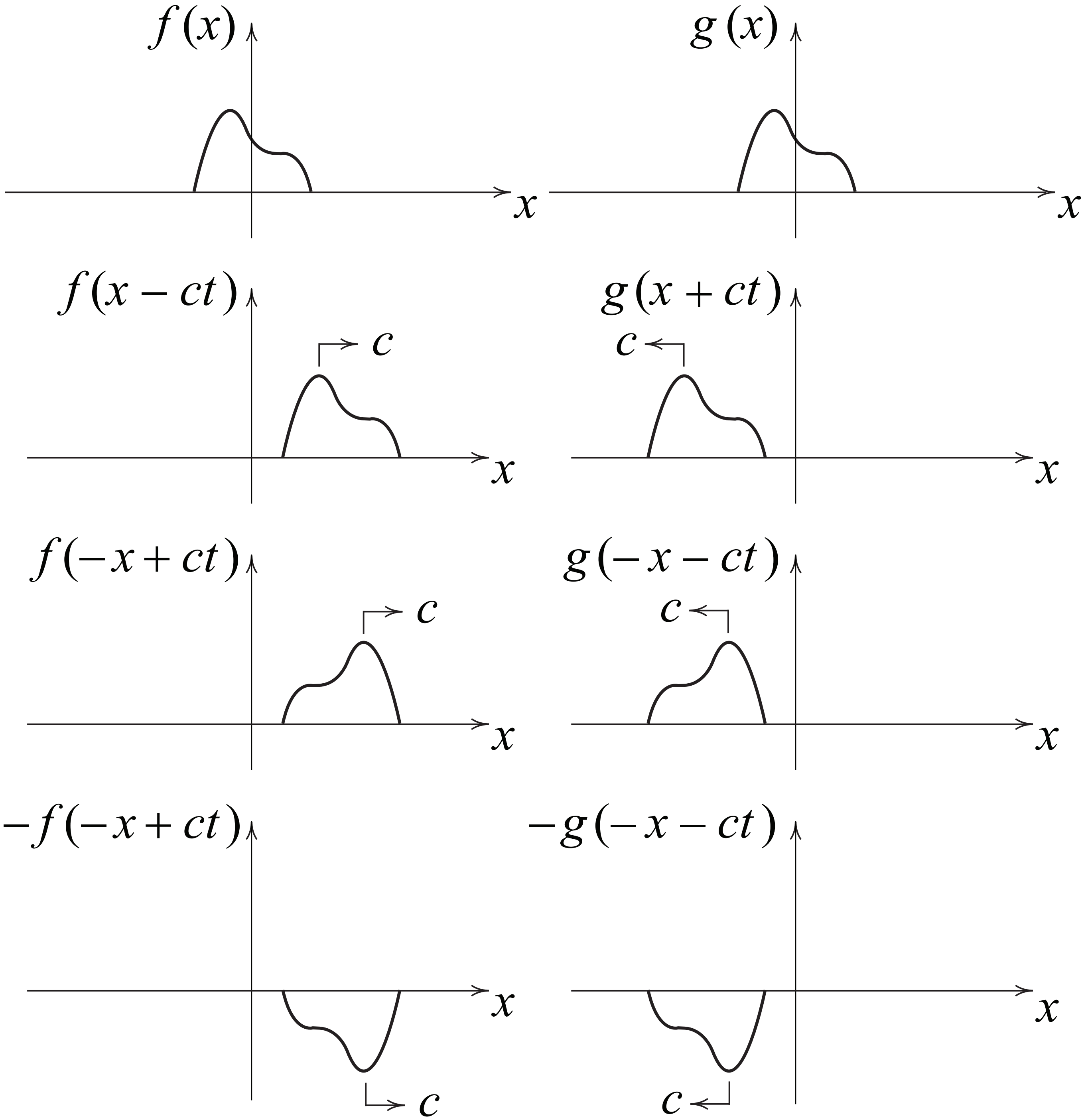
Classical Wave Equation

$$\frac{\partial^2 z(x,t)}{\partial t^2} = c^2 \frac{\partial^2 z(x,t)}{\partial x^2}$$

Wave Functions

$$z(x,t) = f(x-ct) + g(x+ct)$$

where $f(x)$ and $g(x)$ are *sample functions*.



Infinite Continua

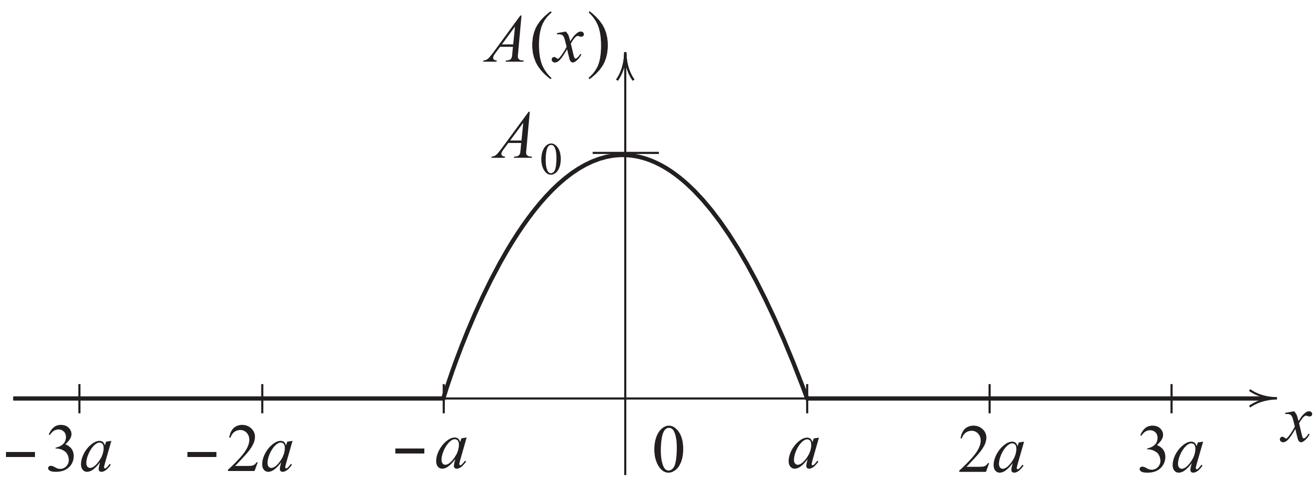
Strings   Rods   Circular Shafts
Shear Beams   Electric Transmission Lines

Initial Conditions on Infinite Systems

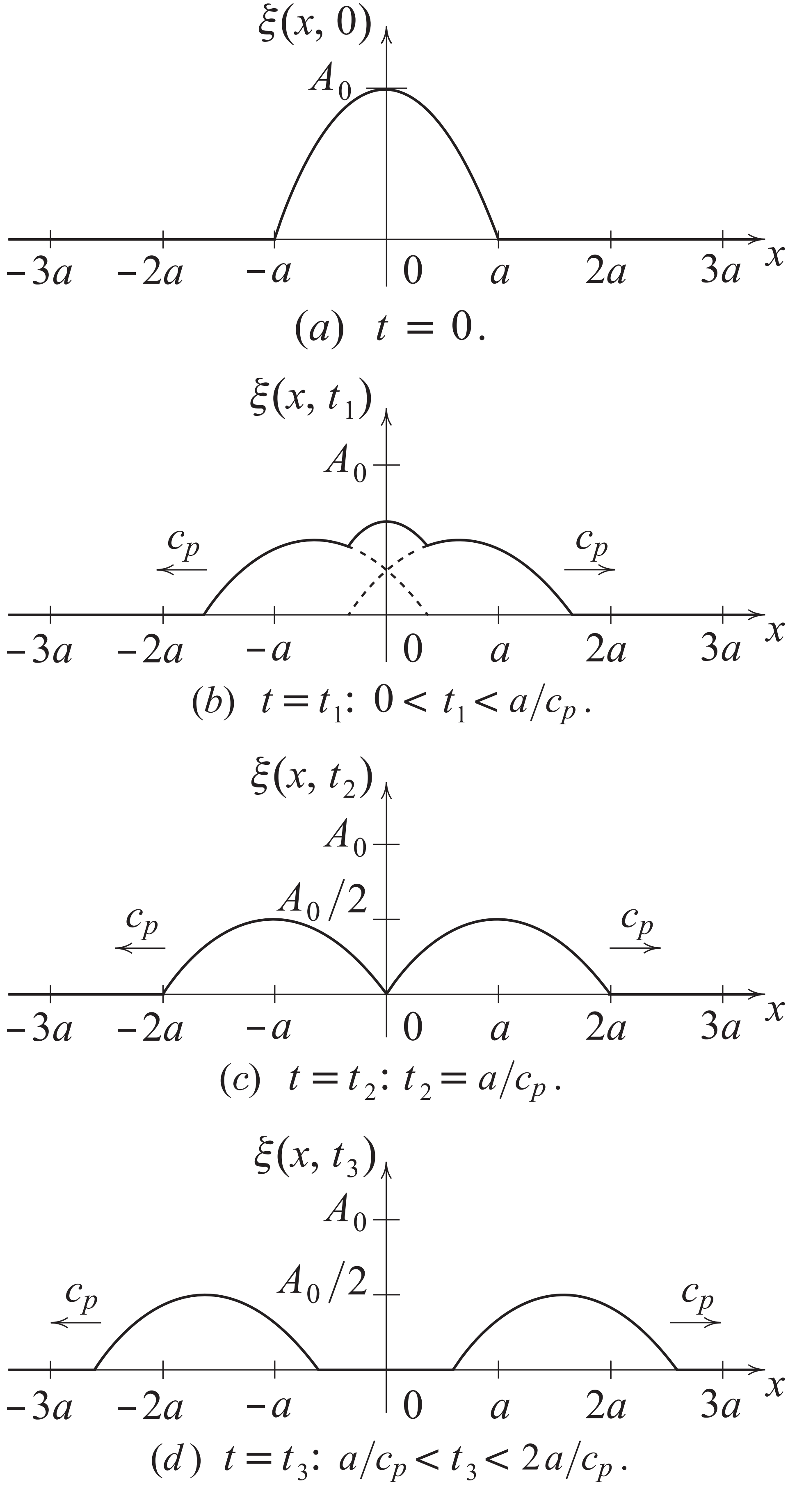
Initial conditions: $\xi(x,0) = A(x)$ and $\dot{\xi}(x,0) = 0$, where

$$A(x) = \begin{cases} A_0(1 - x^2/a^2), & |x| \le a \\ 0, & |x| > a \end{cases}$$

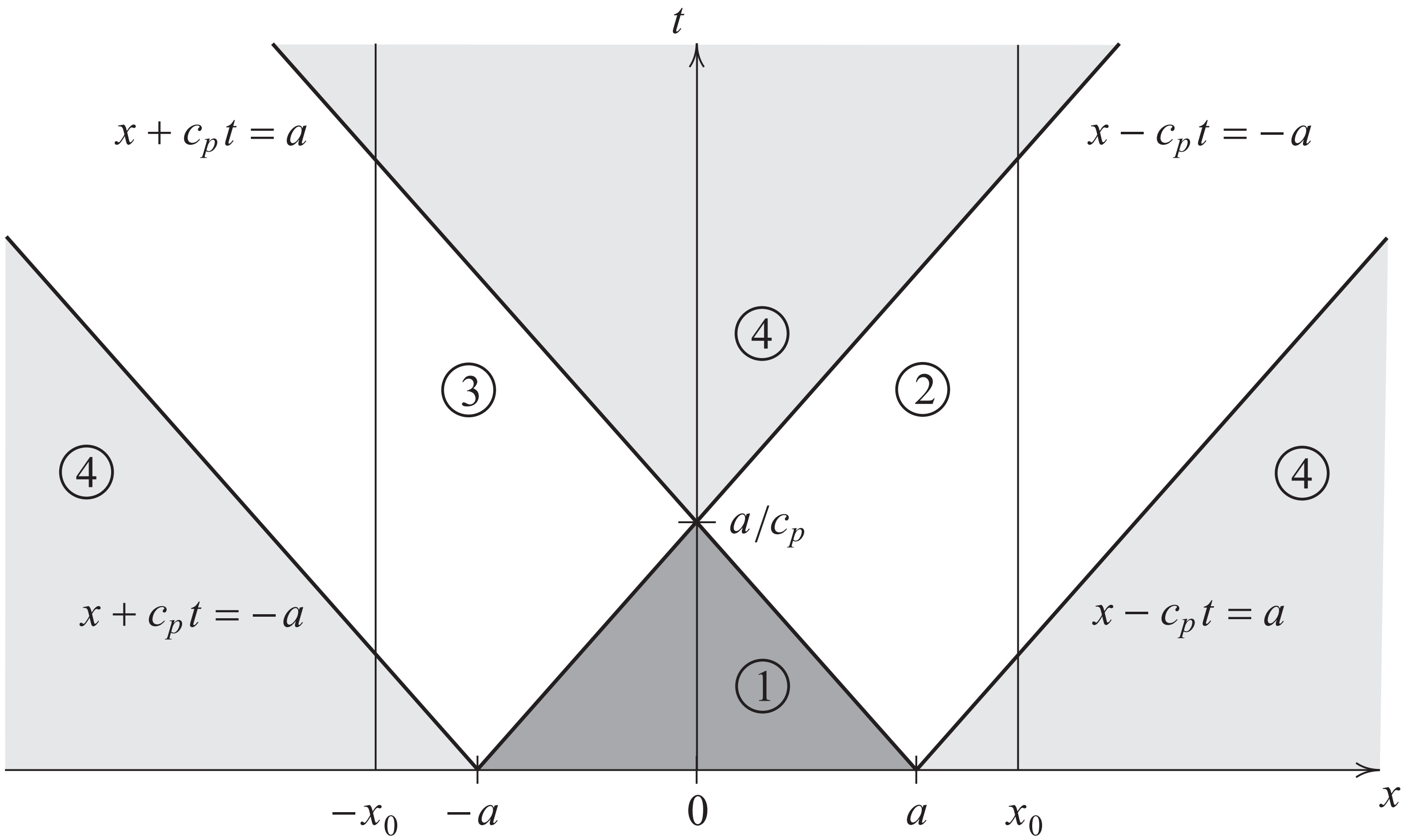
depicted as



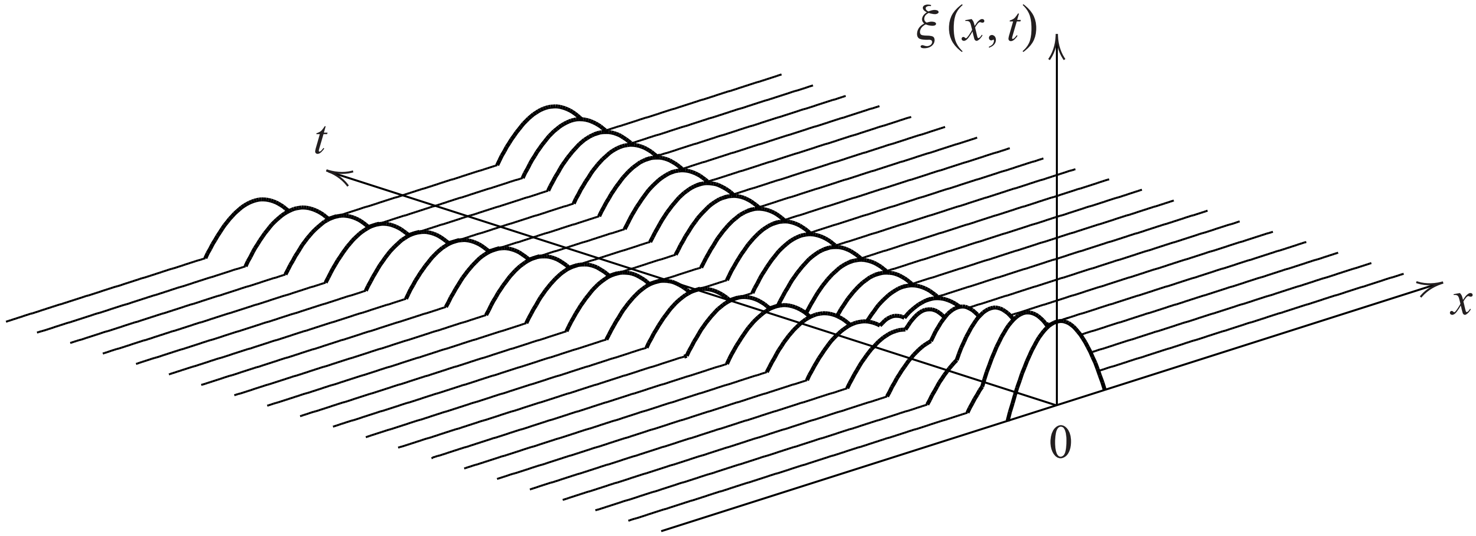
Find: $\xi(x,t)$ [and $\dot{\xi}(x,t), F(x,t)$]



Displacements of uniform rod released from rest, shown at increasing times.

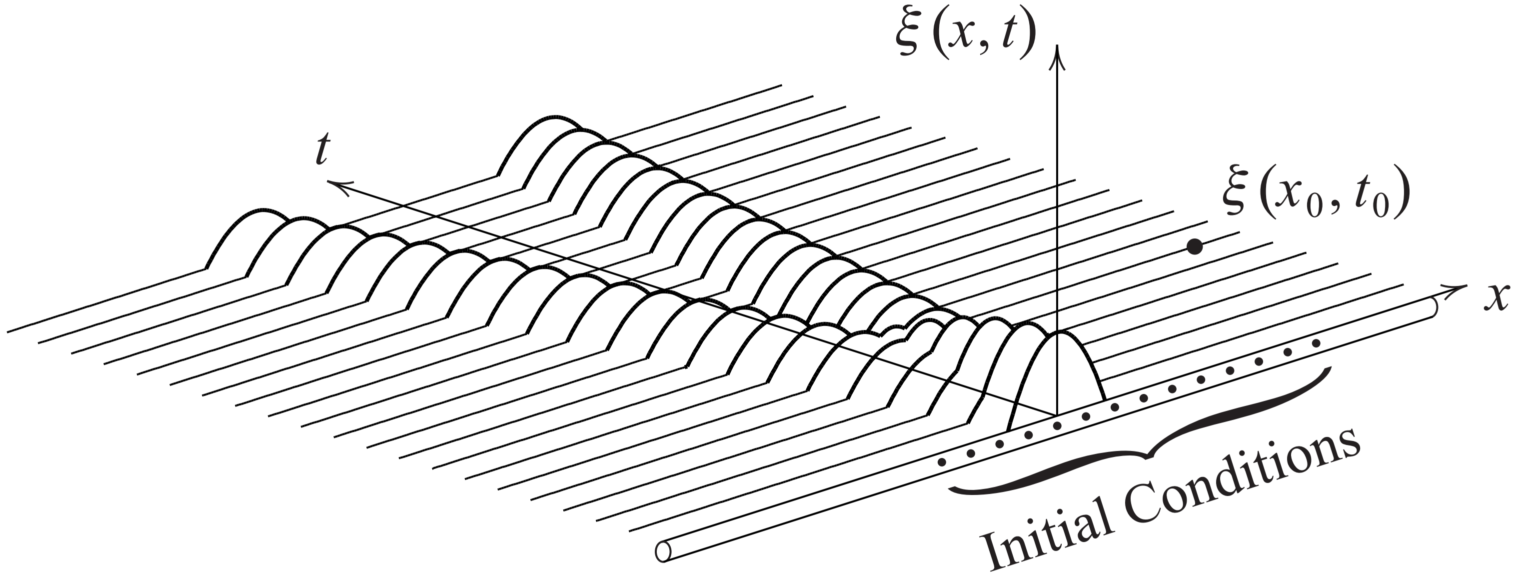


Domains 1 through 4 on *x*-*t* plane.



Three-dimensional schematic of displacement
wave propagation in rod.

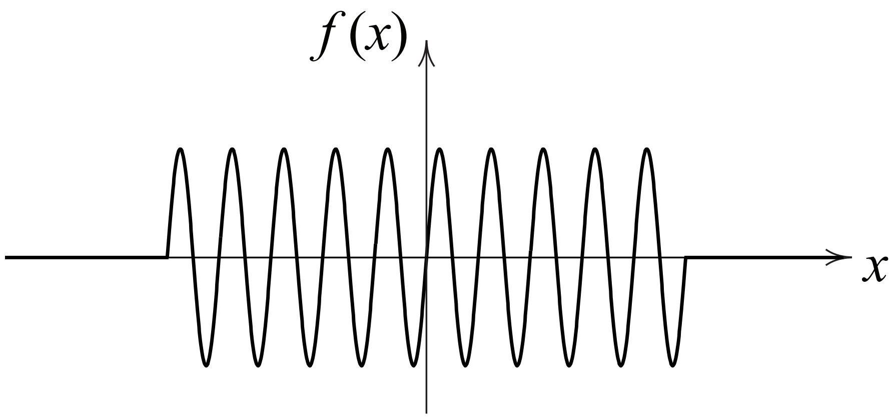
- **Domain of Dependence**



- **Time Lags**

- **Transmission of Energy by Arbitrary and Harmonic Waveforms**

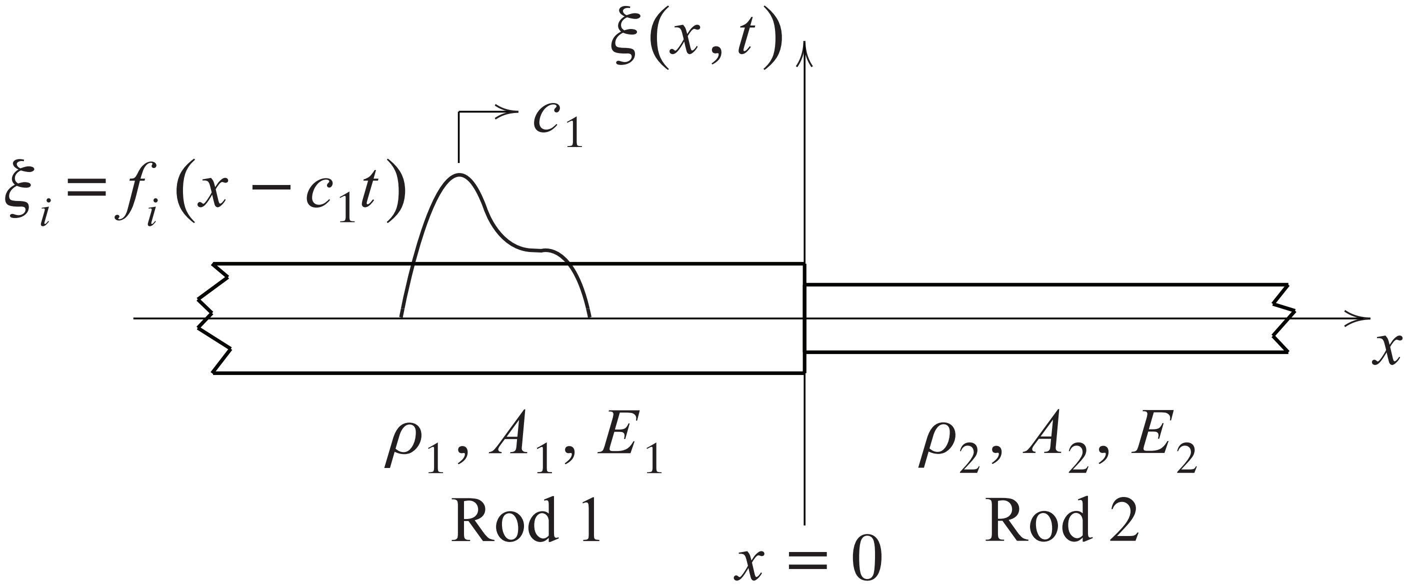
- Fourier Series
- Fourier Integral of Tone Burst Wave



- Ultrasonic Attenuation of Tone Burst Wave
- NDE of Impact-Damaged Fiber Composites
- NDE of Fatigued Fiber Composites

• Chapter 4 Wave Propagation in Semi-Infinite Media

Reflection and Transmission Coefficients at Junctions; and Reflection Coefficients at Boundaries



• Vignettes

• Chapter 5 Wave Propagation in Finite Media

One-Dimensional Wave Fields in Finite Media [Timewise Global and Point Variations]

Vignettes

I. Is There a Smallest Quantity of Energy?

$$\left[h = 6.62607015 \times 10^{-34}\,\mathrm{J}\cdot\mathrm{s}\right]$$

$$\left[\mathscr{E} = h \cdot \bar{f}\right]$$

$$\left[\mathrm{J}\cdot\mathrm{s} = (\mathrm{N}\cdot\mathrm{m})\cdot(\mathrm{s}) = (\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2})\cdot\mathrm{m}\cdot\mathrm{s} = \mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right]$$

II. Gravitational Waves & **L**aser **I**nterferometer **G**ravitational-Wave **O**bservatory (LIGO)

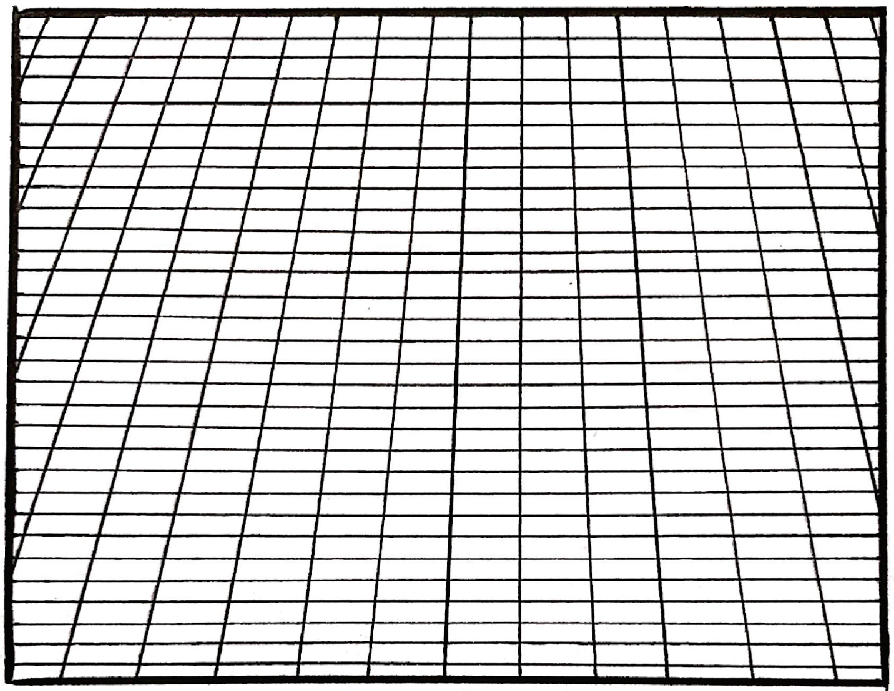
III. NDE of Composite Materials and Structures

IV. Sound Waves and Sound Channels in the Ocean

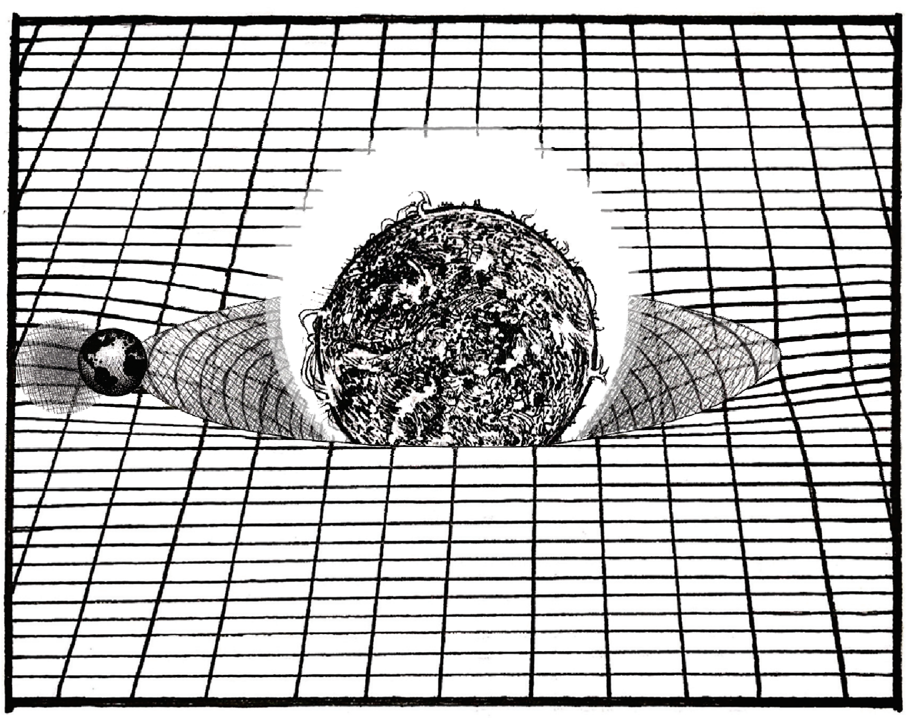
V. Domino Waves

VI. Falling Slinky

II. Gravitational Waves & LIGO

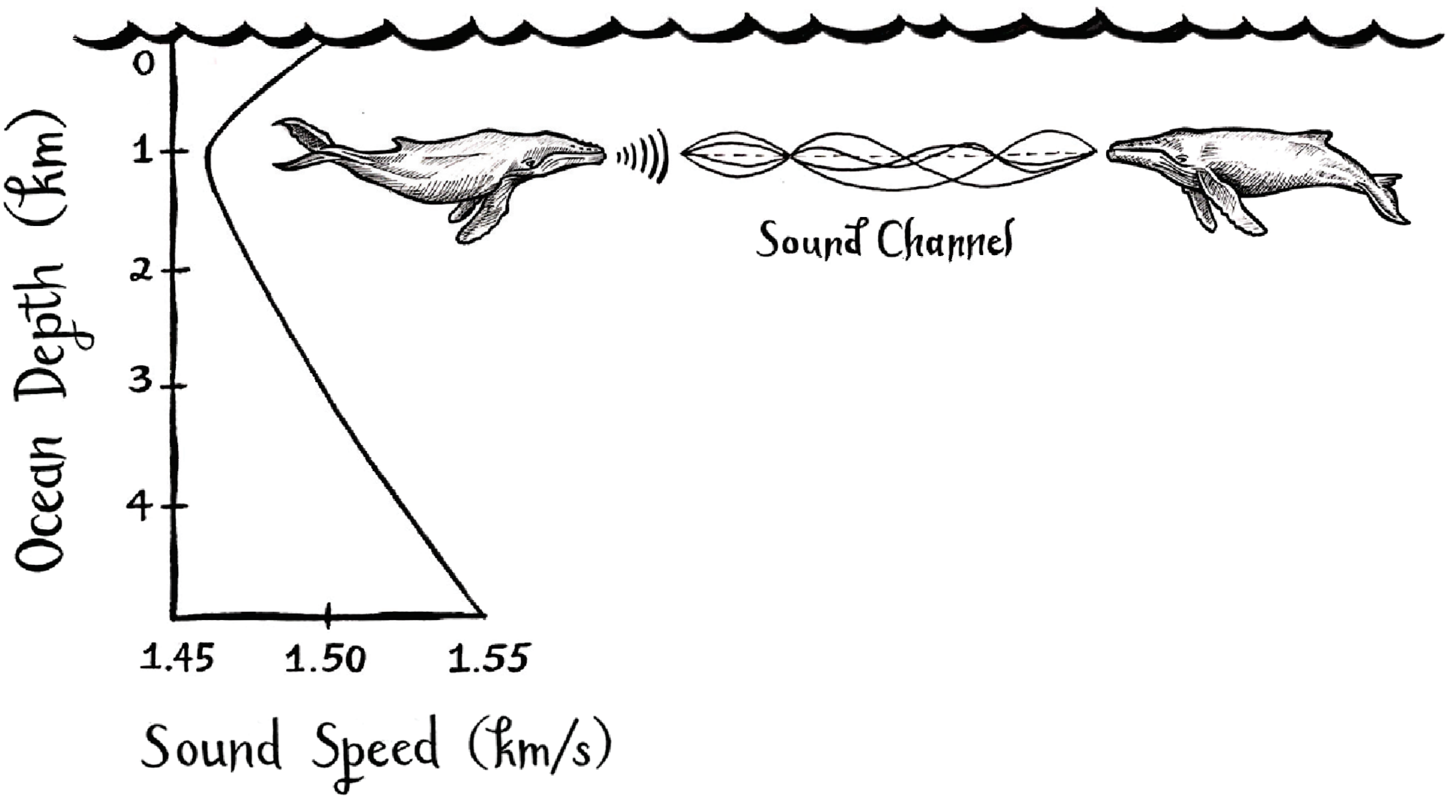


Flat space-time

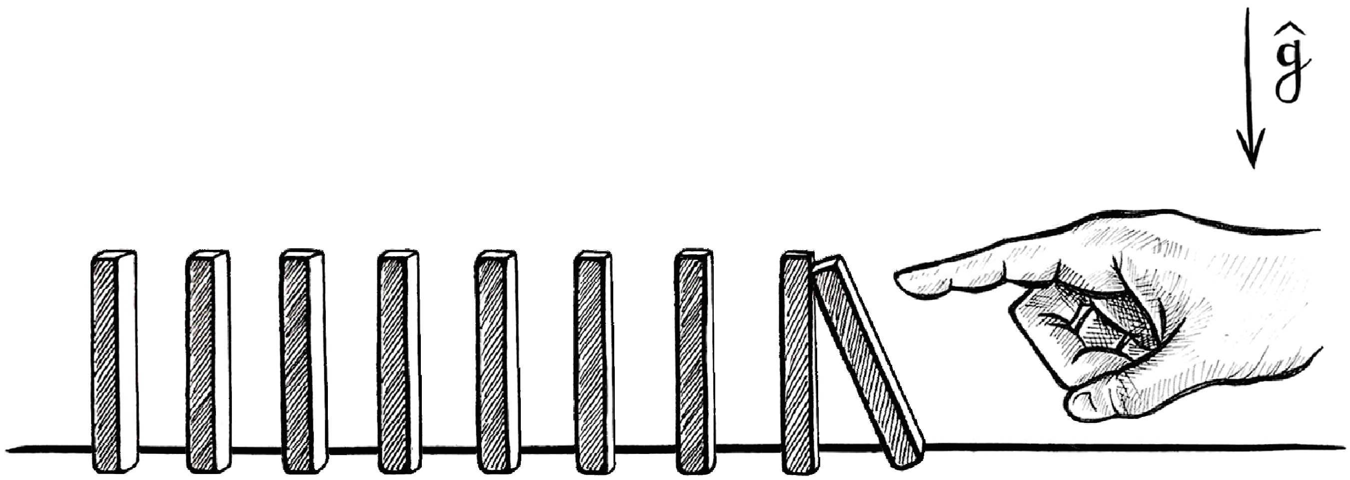


Curved space-time

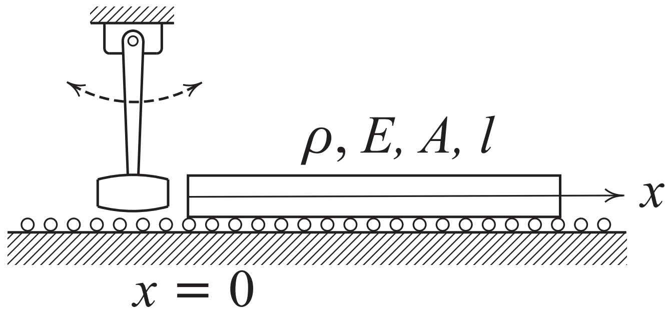
IV. Sound Waves and Sound Channels in the Ocean



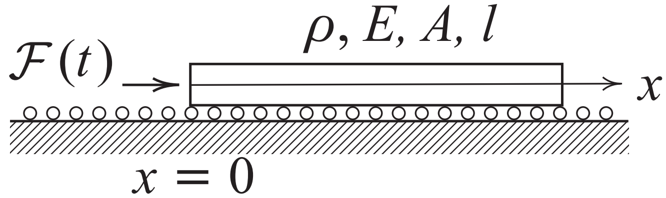
V. Domino Waves

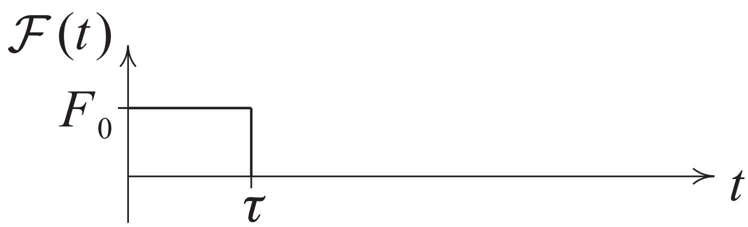


■ Field and Point Timewise Motion

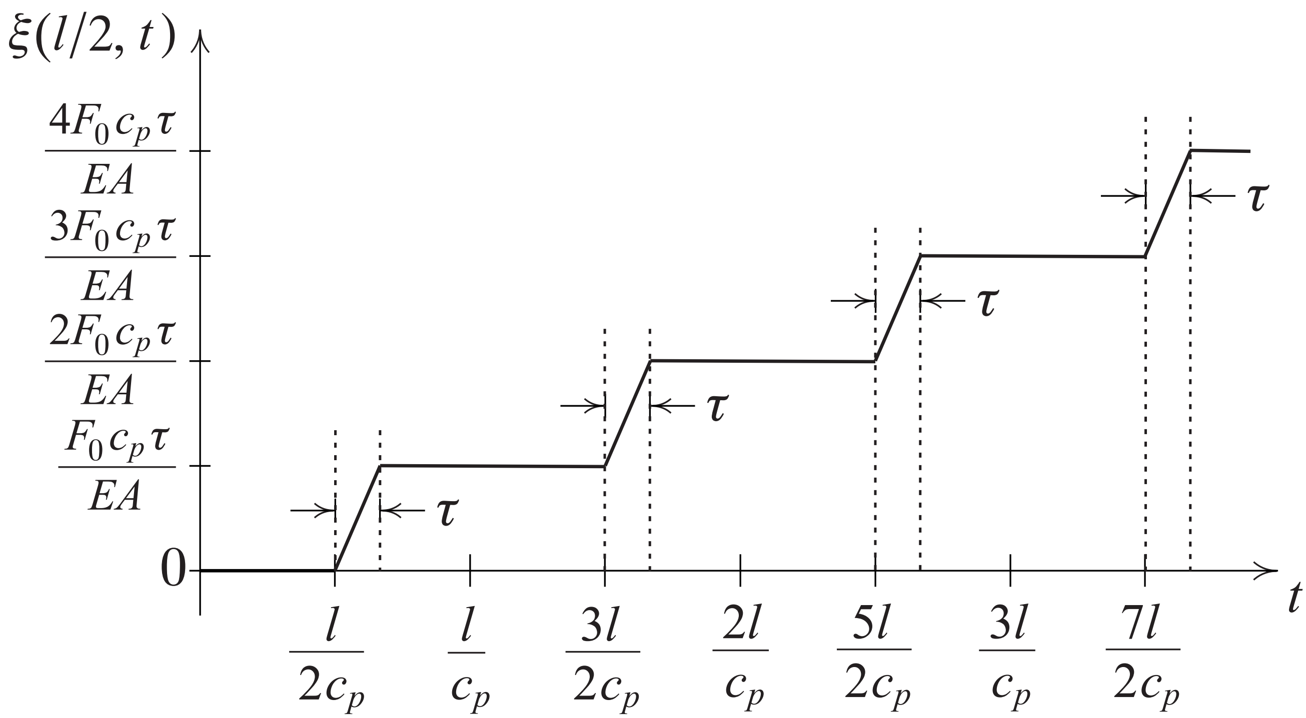


Free-free finite length rod.





Applied force $\mathcal{F}(t)$.

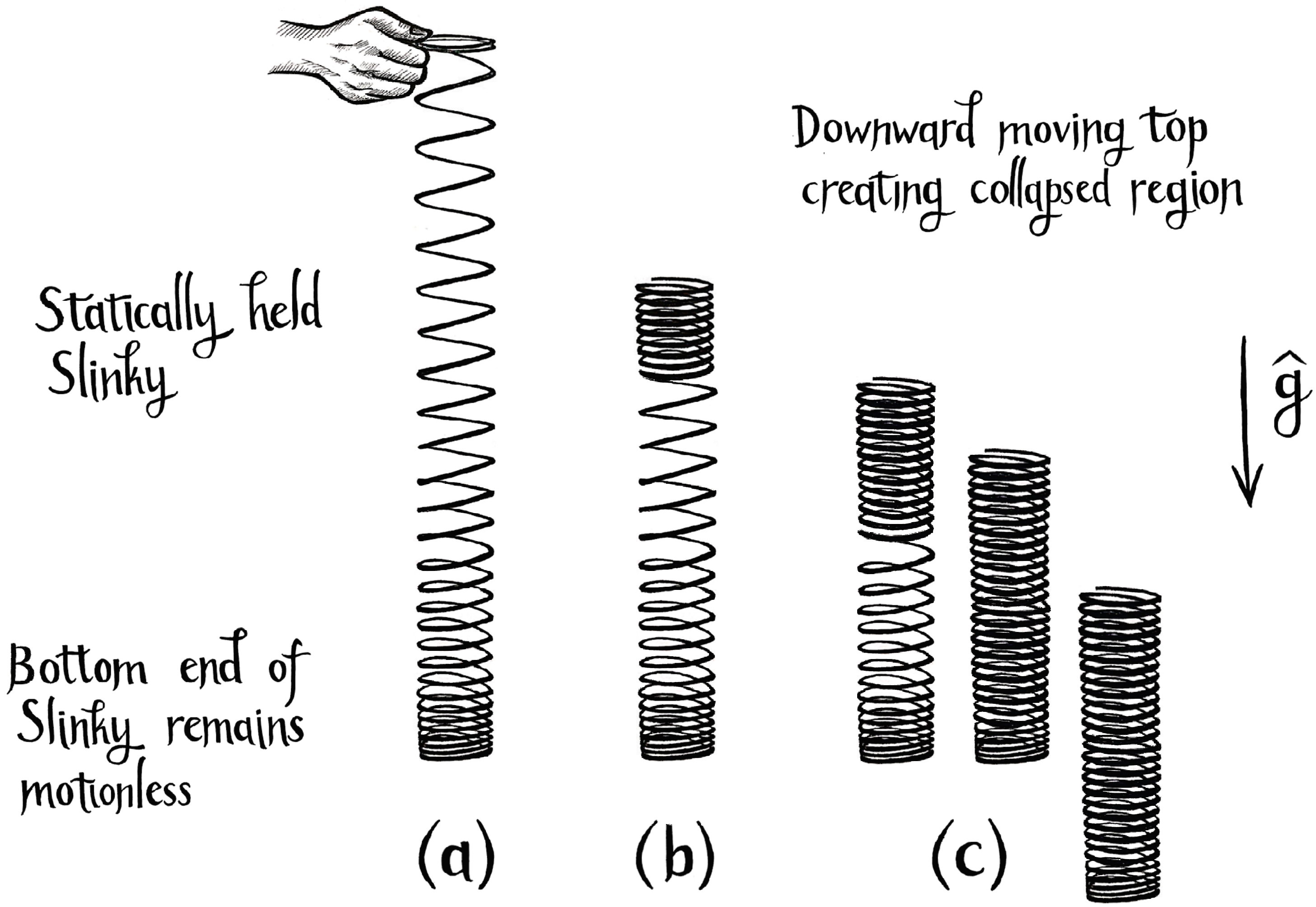


Midpoint displacement $\xi(l/2, t)$ of elastic rod.

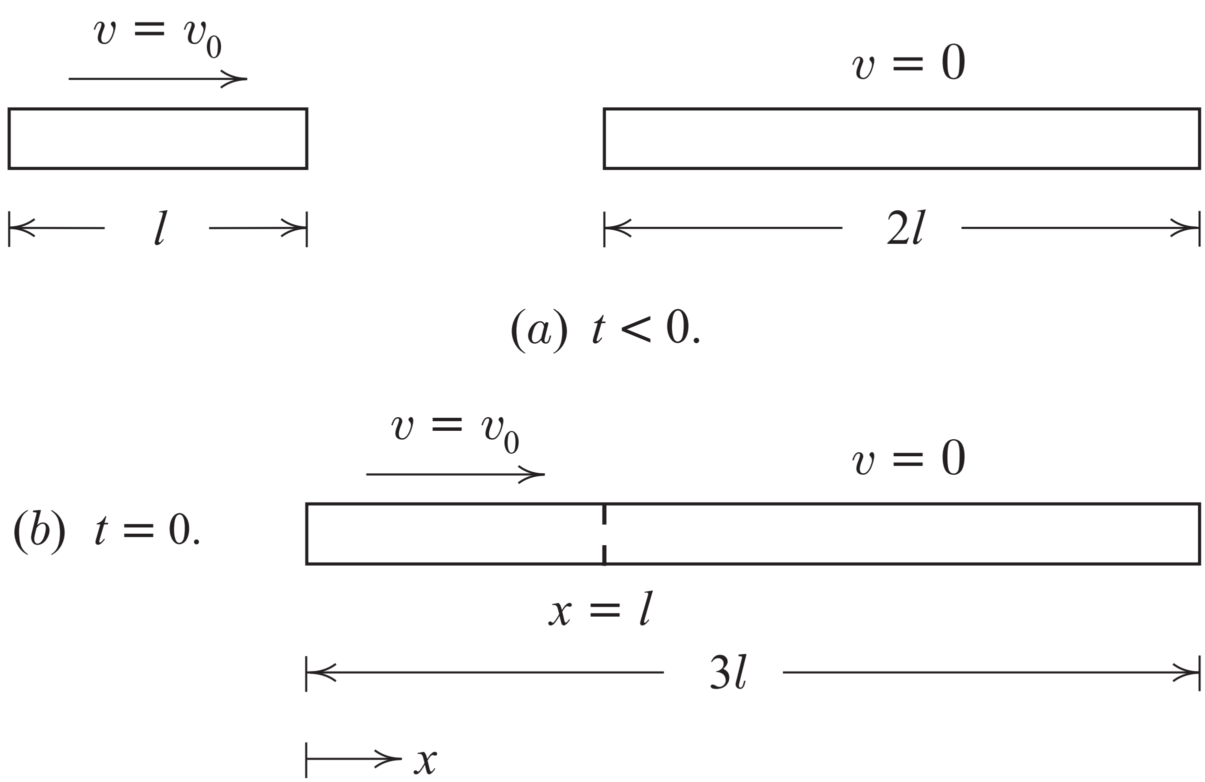
Video of Falling Slinky

https // tinyurl com / y2psyp7y
(Deactivated via this Site)

VI. Falling Slinky



Collision of Elastic Rods

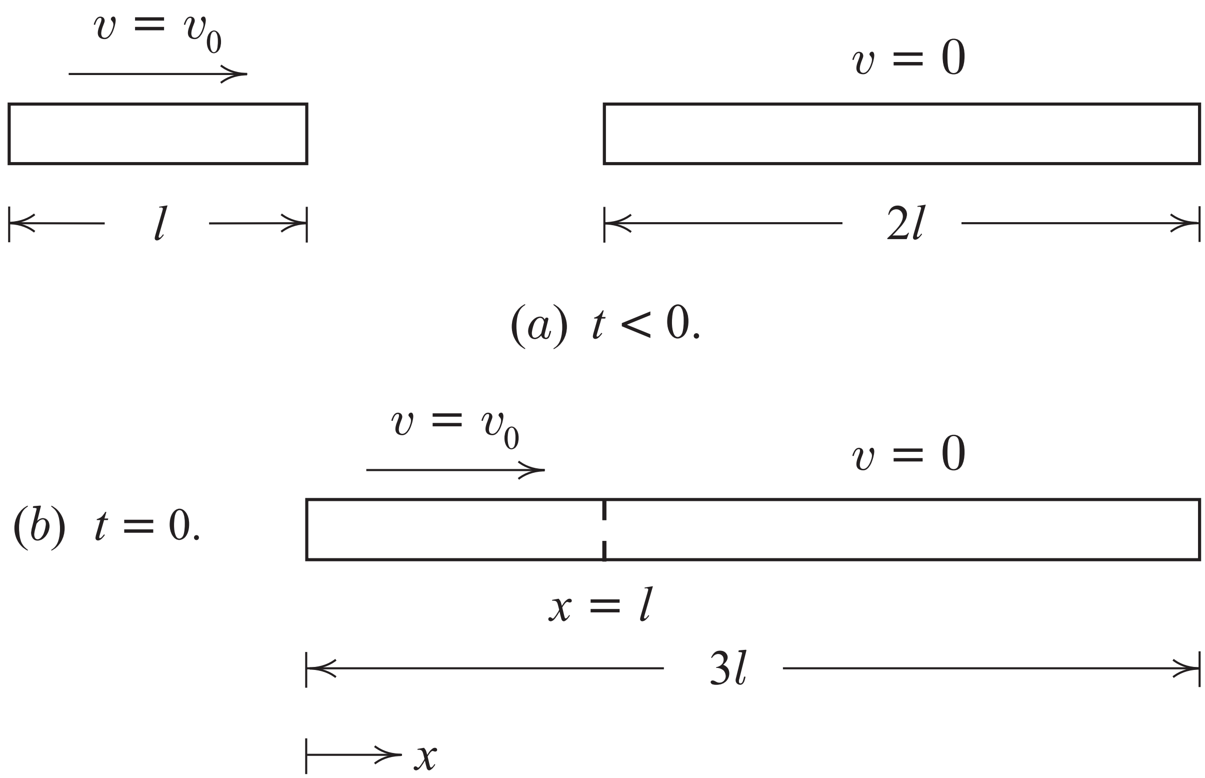


Initial Displacement: $\xi(x,0) = 0,\ 0 < x < 3l$

Initial Particle Velocity:

$$\dot{\xi}(x,0) = \begin{cases} v_0, & 0 < x < l \\ 0, & l < x < 3l \end{cases}$$

Collision of Elastic Rods



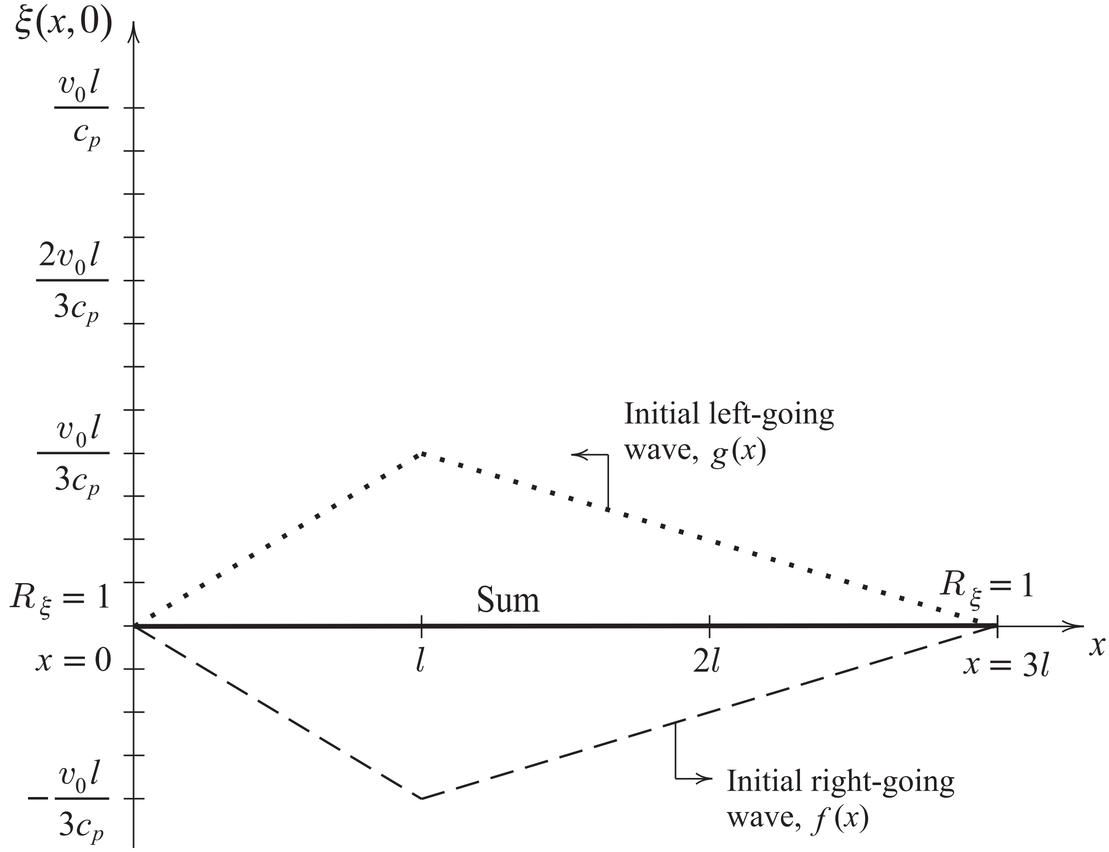
Initial Displacement: $\xi(x,0) = 0,\ 0 < x < 3l$

Initial Particle Velocity:

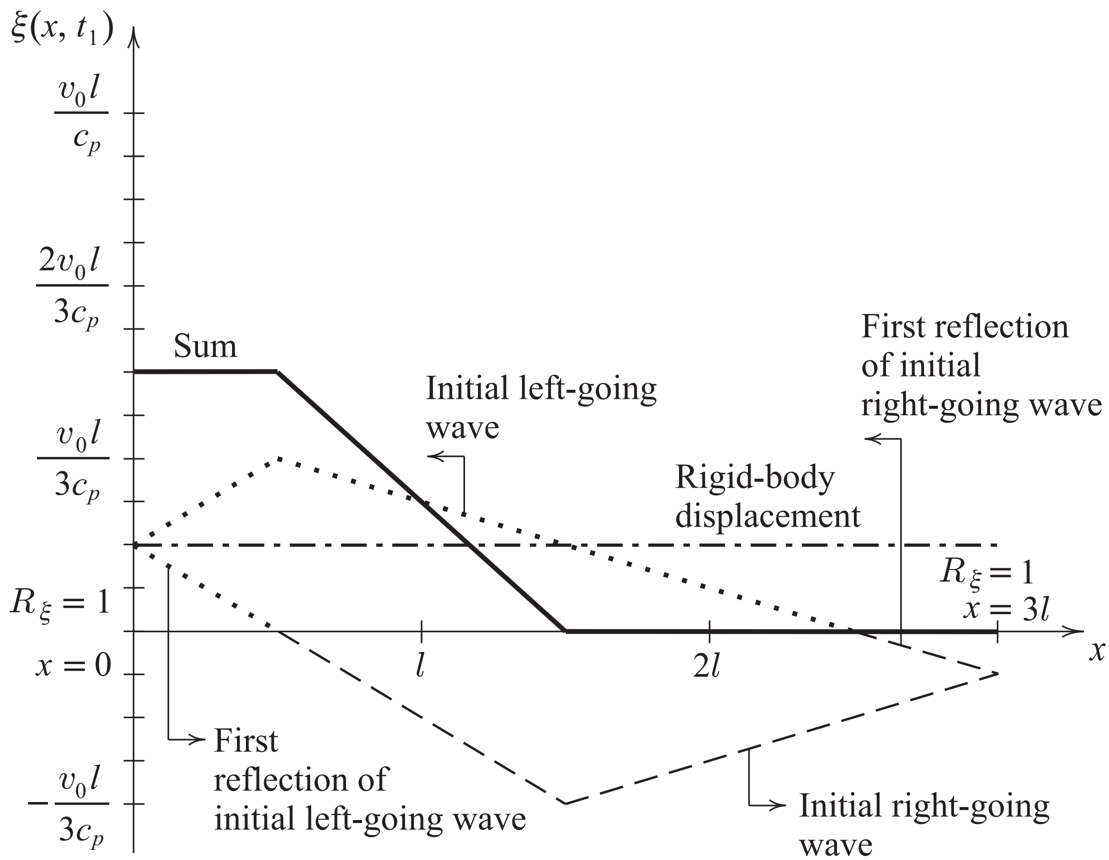
$$\dot{\xi}(x,0) = \begin{cases} v_0, & 0 < x < l \\ 0, & l < x < 3l \end{cases}$$

$$= \frac{v_0}{3} + \begin{cases} \dfrac{2v_0}{3}, & 0 < x < l \\ -\dfrac{v_0}{3}, & l < x < 3l \end{cases}$$

Rigid-body and *elastic* initial particle velocities.

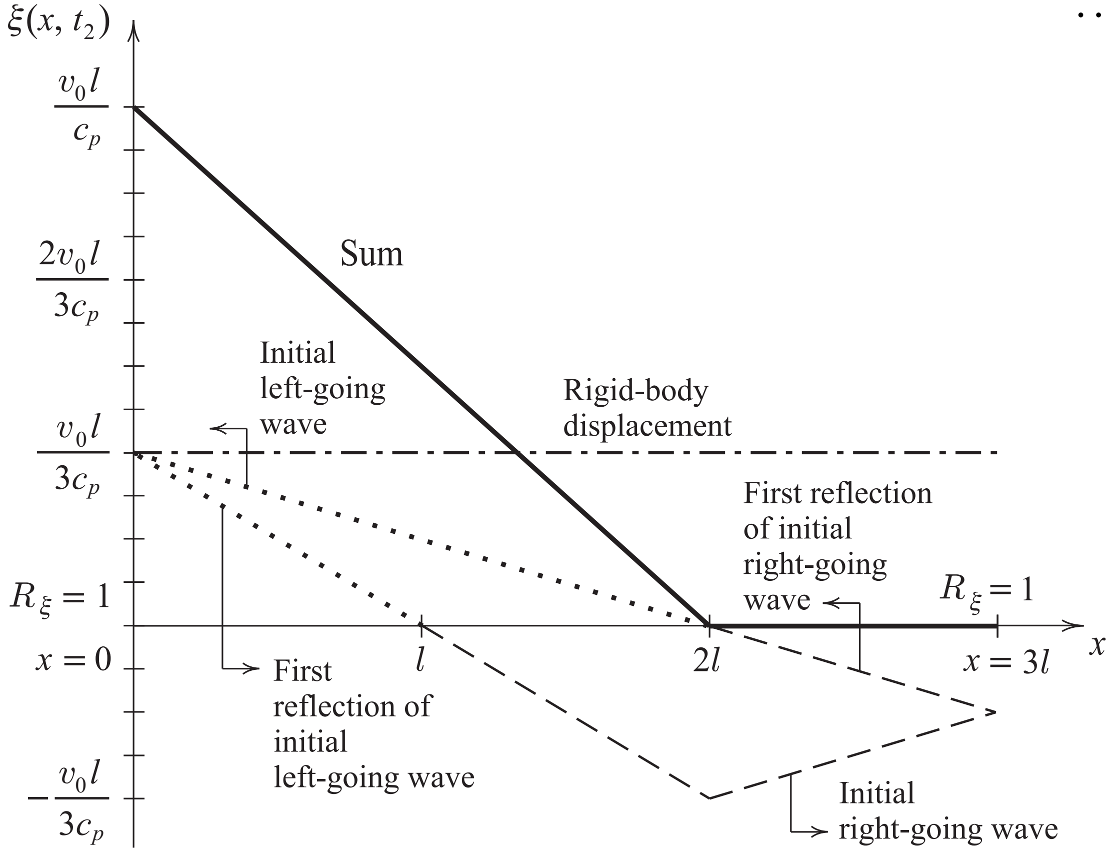


(*a*) $t = 0$.

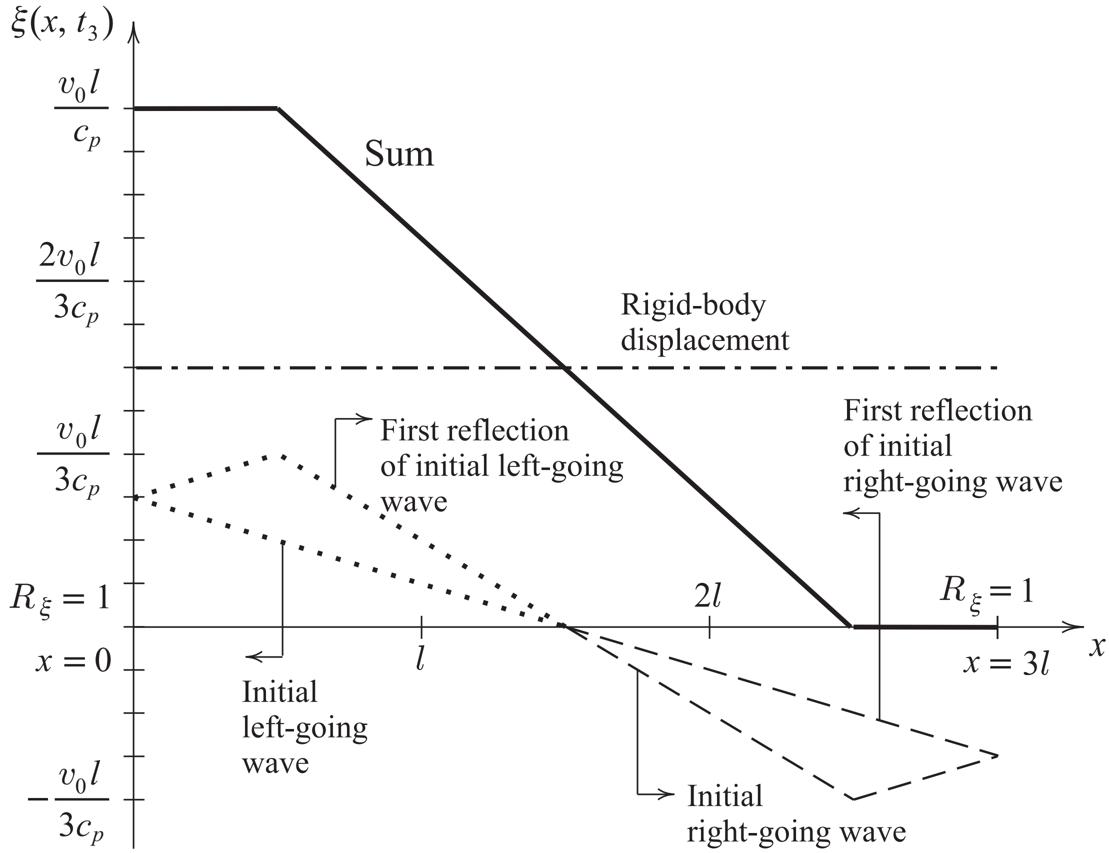


(*b*) $t = t_1 = l/2c_p$.

Displacement waves, rigid-body displacement, and their sums for elastic rod system after collision.

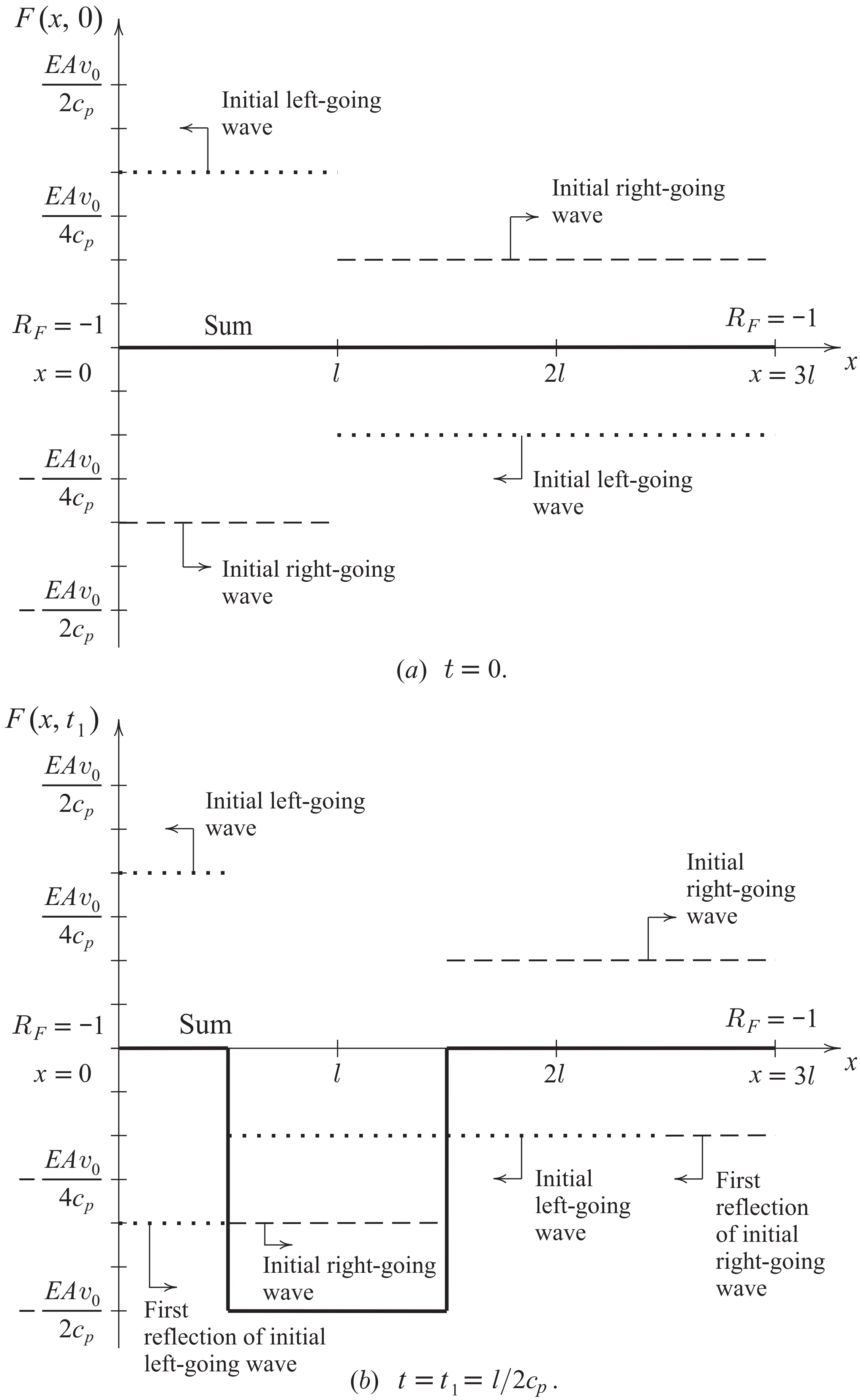


(c) $t = t_2 = l/c_p$.



(d) $t = t_3 = 3l/2c_p$.

Displacement waves, rigid-body displacement, and their sums for elastic rod system after collision (continued).



Force waves in elastic rod system after collision.

Acknowledgments

Many …

With immense gratitude to

Gang Chen

Bestowal

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&
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in the names of James H. Williams, Jr. '67
and Raymond J. Nagem '80

Dedication

To A. Neil (1964) and Jane Pappalardo,
by measure of profound and indelible devotion and love,
MIT Royalty.