Determining the Cascade of Passive Scalar Variance in the Lower Stratosphere

Erik Lindborg*
Department of Mechanics, Royal Institute of Technology, S-100 44 Stockholm, Sweden

John Y. N. Cho
Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 30 May 2000)

Using aircraft data from 7630 commercial flights, we determine the flux of temperature and ozone variance from large to small scales in the lower stratosphere. The relation that we use for this purpose is a form of the classical Yaglom relation [A. M. Yaglom, Dokl. Akad. Nauk SSSR **69**, 743 (1949)] for the third-order scalar-velocity structure function. We find that this function is negative and that it depends linearly on separation distance in the mesoscale range for temperature as well as ozone.

PACS numbers: 92.60.Ek, 47.27.Te

A central concept in the theory of nonlinear dynamics in the cascade, described in the famous poem by Richardson [1]: "Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls, and so on to viscosity." The cascade concept was developed further by Kolmogorov [2,3] to predict the shape of the kinetic energy spectrum of three-dimensional hydrodynamic turbulence. The only parameter that can determine the turbulent kinetic energy spectrum $E(k)$ is the strength of the cascade, i.e., the rate, Π, at which energy is transferred from large to small scales, also called the spectral energy flux. This flux must also be equal to the mean dissipation ϵ of kinetic energy into heat. From dimensional analysis we thus obtain

$$E(k) = C\epsilon^{2/3}k^{-5/3}, \quad (1)$$

where C is a dimensionless constant.

Yaglom [4], Obukhov [5], and Corrsin [6] developed the same type of theory for a passive scalar θ in a turbulent hydrodynamic field. By similar arguments, the spectrum $F(k)$ of the passive scalar variance is obtained as

$$F(k) = C_\theta \epsilon_\theta \epsilon^{-1/3}k^{-5/3}, \quad (2)$$

where C_θ is a dimensionless constant and ϵ_θ is the rate at which half of the passive scalar variance is smoothed out at the length scale of molecular diffusion.

Kinetic energy spectra, as well as spectra of temperature and ozone [7–9] from the upper troposphere and lower stratosphere, often exhibit a rather broad range with a $k^{-5/3}$ dependence in the mesoscale region of length scales ~10–500 km. In Fig. 1 the kinetic energy spectra and temperature spectra are reproduced from Nastrom and Gage [7]. Although these spectra cannot, in a simple way, be explained by classical three-dimensional turbulence, concepts from turbulence theory may be crucial to reach an understanding of the physics behind them. Two hypotheses have been put forward to explain the shape of the spectra. First, there is the hypothesis [10,11] that the $k^{-5/3}$ range can be explained by internal gravity waves. According to this hypothesis long gravity waves break down to shorter waves in a continuous chain, resulting in a positive energy flux from large to small scales, very much in the same way as three-dimensional Kolmogorov turbulence. The only parameter that can determine the spectrum is the energy flux and from dimensional considerations we obtain the $k^{-5/3}$ spectrum. Second, there is the hypothesis [12,13] that the $k^{-5/3}$ spectrum is the spectrum of two-dimensional turbulence with a negative energy flux, i.e., a flux from small to large scales, in accordance with Kraichnan's theory of two-dimensional turbulence [14,15].

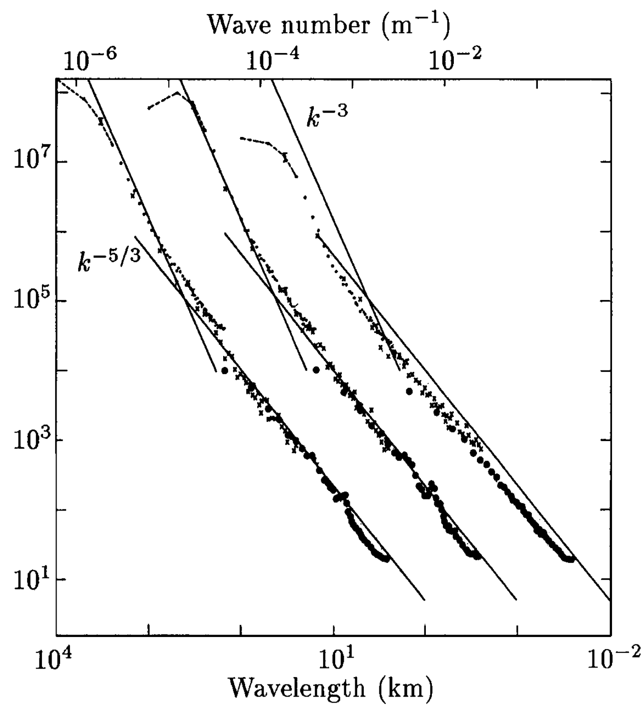

FIG. 1. From left to right: variance power spectra of zonal wind, meridional wind ($m^3 s^{-2}$), and potential temperature (K^2 m) near the tropopause from Global Atmospheric Sampling Program aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively. Reproduced from [7].

Lindborg [16] (hereafter L99) pointed out that Kolmogorov's [3] classical four-fifth law for the third-order velocity structure function may provide the key to an experimental determination of the flux of kinetic energy. This law is a consequence of a very special balance in the governing dynamical equations. The balance can be written as [17]

$$\nabla_r \cdot \langle \delta \mathbf{u} \delta \mathbf{u} \cdot \delta \mathbf{u} \rangle = -4\Pi , \quad (3)$$

where $\delta \mathbf{u} = \mathbf{u}' - \mathbf{u}$ is the difference between the velocities at two points separated by the vector $\mathbf{r}$ and $\langle \rangle$ is an ensemble average. In the three-dimensional case, integration of (3) gives the four-fifth law [3,17] for the third-order longitudinal structure function. In the two-dimensional, or quasi-two-dimensional, case, integration of (3) yields

$$\langle \delta u_L \delta u_L \delta u_L \rangle + \langle \delta u_L \delta u_T \delta u_T \rangle = -2\Pi r , \quad (4)$$

where L indicates the longitudinal direction, i.e., the direction of $\mathbf{r}$, and T indicates a transverse direction, i.e., a direction perpendicular to $\mathbf{r}$. It can be argued [16,18] that (4) should hold both in the case when Π is positive (forward cascade) and in the case when Π is negative (inverse cascade). Using data from the Measurement of Ozone and Water Vapor by Airbus In-Service Aircraft (MOZAIC) program [19], Cho and Lindborg [18,20] calculated the left-hand side of (4) and showed that there is a positive flux of kinetic energy, $\Pi \approx 6 \times 10^{-5}$ m^2/s^3, in the lower stratosphere.

For a passive scalar θ advected by a turbulent velocity field, Yaglom [4] derived a relation similar to (3),

$$\nabla_r \cdot \langle \delta \mathbf{u} \delta \theta \delta \theta \rangle = -4\epsilon_\theta . \quad (5)$$

In the three-dimensional case, integration of (5) yields the four-third law,

$$\langle \delta u_L \delta \theta \delta \theta \rangle = -\frac{4}{3} \epsilon_\theta r , \quad (6)$$

which has been experimentally confirmed in several studies, e.g., in an atmospheric boundary layer by Zhu *et al.* [21] and in wind tunnel grid turbulence by Mydlarski and Warhaft [22]. In both these studies temperature served as the passive scalar. In the atmospheric boundary layer the typical length scale where (6) holds is $r \sim 1-10$ m, while in the wind tunnel, $r \sim 1-10$ cm. In the two-dimensional, or quasi-two-dimensional, case, integration of (5) yields

$$\langle \delta u_L \delta \theta \delta \theta \rangle = -2\epsilon_\theta r . \quad (7)$$

In a highly stratified fluid with a strong degree of system rotation, we find it reasonable to apply the two-dimensional, rather than the three-dimensional, form of the Yaglom relation.

Using aircraft data from 7630 commercial flights in the MOZAIC program, we have calculated the third-order velocity-scalar structure function (7) for temperature and ozone. Temperature, of course, cannot be treated as a passive scalar for large-scale dynamics, since the temperature gradient is a main cause of winds on a global scale. However, at small scales and mesoscales it is a reasonable assumption that temperature fluctuations are mainly determined by advective forces and thus can be treated as a passive scalar. To calculate the structure functions we closely followed the procedure outlined in L99. However, there were some differences such as the following. First, our data consisted of a somewhat expanded MOZAIC set (August 1994 to December 1997, 7630 flights), whereas L99 used August 1994 to April 1997 with 5754 flights. Second, in order to avoid variability caused by sudden changes in altitude, we specified that the pairs of points used for the structure function computations lie within the same standard flight level. There were five flight levels from 9.4 to 11.8 km, each spaced 600 m apart. The adherence of the aircraft to the standard levels was quite strict [23]. Third, we used the ozone measurement to differentiate between the troposphere and the stratosphere. If the ozone levels associated with the pair of data points were both under 100 parts per billion by volume (ppbv), then the result was binned as tropospheric. If the ozone concentrations for the data-point pair were both over 200 ppbv, then the result was classified as stratospheric. If one point was in the troposphere and the other point was in the stratosphere, the result was not included in either group. The two thresholds were chosen from the ozone concentration histograms for MOZAIC cruise levels [23]. There were few points in the stratosphere equatorward of 30°, since the tropical tropopause was usually above the flight altitude. For a schematic of the flight route distributions, see Cho *et al.* [24].

Note also that because the latitude-longitude coordinates for the Air France aircraft data were purposely degraded to about a 1-min resolution (a result of pilot union concerns), we linearly interpolated the position to the original 4-s resolution. This was also done by L99 but was not stated in the paper. The data from the other airlines (Sabena, Lufthansa, and Austrian Airlines) were not altered.

Following L99 we used a set of 140 separation distances between 2 and 2510 km. For every data point the other end point was found by searching through the rest of the flight for the point coming closest to the specified r. If that distance was not within 1 km of the specified r, then the velocity difference was not computed. The separation distance r was calculated using the latitude-longitude coordinates, the altitudes, and an elliptical polar model of the Earth radius, $R = AB/(1 - B\cos\phi)$, where $A = 1.895\,946\,0 \times 10^6$ km, $B = 3.352\,812\,9 \times 10^{-3}$, and ϕ is latitude. The separation distance was taken to be the arclength, not the straight-line distance.

The calculated third-order structure function in the upper troposphere contained a lot of scatter. The only firm conclusion one could draw on the basis of this calculation was that the function is definitely negative in the mesoscale range. Therefore, we present only the stratospheric results, which showed good convergence. The number of data-point pairs averaged to obtain the following results were enormous: nearly 6 000 000 pairs at $r = 2$ km, over 4 000 000 at $r \sim 100$ km, and dropping off more rapidly at the largest scales to just under 800 000 at $r \sim 2500$ km. Calculations based on subsets of the data gave similar results to the ones presented. In Fig. 2 the compensated third-order temperature-velocity structure function $\langle \delta u_L \delta T \delta T \rangle/(2r)$ is plotted. There is a fairly clean flat linear range, $r \sim 20-400$ km, which is the range corresponding to the observed $k^{-5/3}$ spectrum. Using the two-dimensional Yaglom relation (7) a linear fit in this range gives

$$\epsilon_T = 3.8 \times 10^{-6}\ \mathrm{K}^2/\mathrm{s} \tag{8}$$

for the flux, alternatively the dissipation, of half of the temperature variance. This mean value falls well within the wide range of ϵ_T recently estimated for small-scale three-dimensional turbulence in the lower stratosphere from episodic radar observations [25]. In Fig. 3, we have plotted the compensated third-order ozone-velocity structure function $\langle \delta u_L \delta O_3 \delta O_3 \rangle/(2r)$. The shape of the function is very similar to the corresponding temperature curve, although the flat linear behavior is not as clean. A linear fit gives

$$\epsilon_{O_3} = 3.5 \times 10^{-3}\ \mathrm{ppbv}^2/\mathrm{s} \tag{9}$$

for the flux, alternatively the dissipation, of half of the ozone variance. Both curves show a rather abrupt change of sign at $r \sim 500-1000$ km, which is approximately the Rossby deformation radius. The same type of behavior was previously observed for the third-order velocity structure function [20]. We shall not try to explain this, but only point out, first, that the Coriolis force is probably crucial for the understanding of this change of sign, and second, that the assumption of statistical homogeneity, which the Yaglom relation rests on, can be questioned at scales of the order of 1000 km.

The question now is whether the observed ozone and temperature spectra are compatible with the Obukhov-Corrsin scaling (2). In this study we have calculated structure functions, rather than spectra. In the structure function formulation, the Obukhov-Corrsin relation can be written as

$$\langle \delta\theta\delta\theta \rangle = C'_\theta \epsilon_\theta \epsilon^{-1/3} r^{2/3}. \tag{10}$$

If $F(k)$ is the one-dimensional variance spectrum, the relation between the two constants is $C_\theta = 0.24 C'_\theta$. In Fig. 4 we have plotted the second-order structure functions of temperature and ozone, normalized by $\epsilon_T \epsilon^{-1/3}$ and $\epsilon_{O_3} \epsilon^{-1/3}$. We have used $\epsilon = 6.0 \times 10^{-5}\ \mathrm{m}^3/\mathrm{s}^3$, which was the overall mean value computed by Cho and Lindborg [18,20]. As can be seen, there is only an approximate agreement with an $r^{2/3}$ law for the second-order structure functions. The curves are not as clean as the corresponding $k^{-5/3}$ spectra found in previous studies, indicating that the mesoscale dynamics may not be entirely determined by the cascade process. There may be other physical effects that are more or less filtered away, when the Fourier spectra are calculated. However, the curves are reasonably close to $r^{2/3}$. Moreover, they fall reasonably close to each other when the Obukhov-Corrsin scaling is applied. From the plot the constant C_θ can be estimated to be $\sim$2.2–5, which is reasonably close to unity. Using the temperature spectrum in Fig. 1 and the same values of ϵ_T and ϵ, we estimate the same constant to be $C_\theta \approx 4$.

In conclusion, we have calculated the third-order scalar-velocity structure functions for temperature and ozone in the lower stratosphere. In the mesoscale range they both

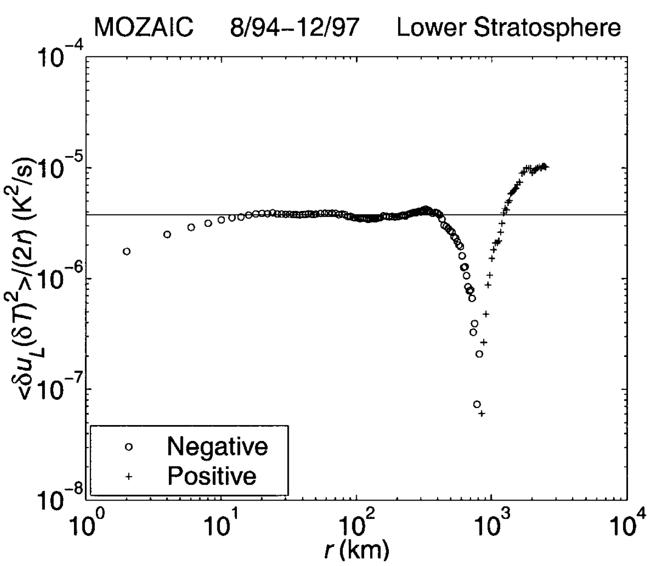

FIG. 2. $\langle \delta u_L \delta T \delta T \rangle/(2r)$ vs r. Solid line: best linear fit, $20 < r < 400$ km.

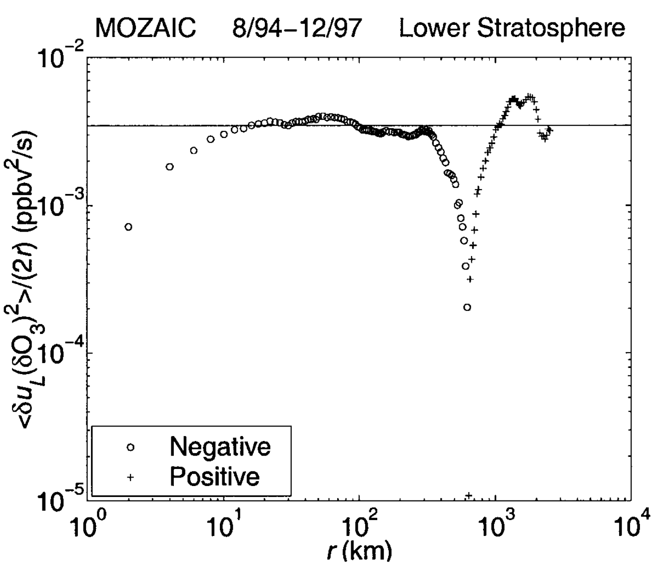

FIG. 3. $\langle \delta u_L \delta O_3 \delta O_3 \rangle/(2r)$ vs r. Solid line: best linear fit, $20 < r < 400$ km.

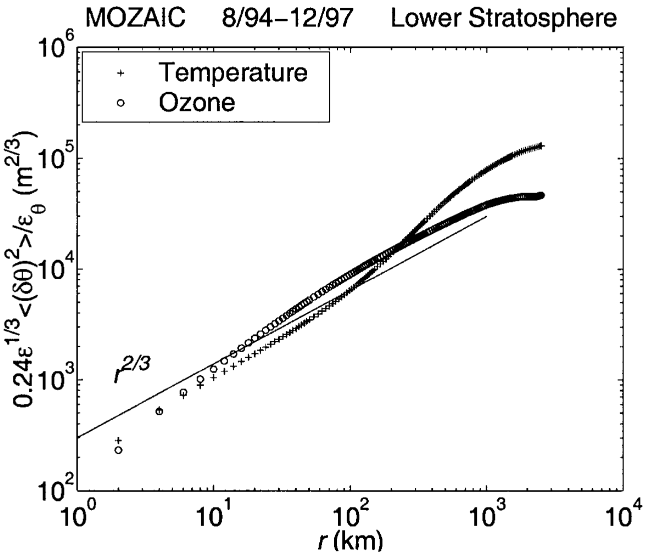

FIG. 4. Second-order structure functions of temperature and ozone vs r. The curves are multiplied by $0.24\epsilon^{1/3}/\epsilon_\theta$, where ϵ_θ is the determined dissipation of passive scalar variance. Solid line: $r^{2/3}$.

show a negative linear dependence of separation distance r, in accordance with the Yaglom relation. The stratospheric mesoscale length scales are several orders of magnitude larger than the length scales for which the Yaglom relation has previously been verified. In an approximate sense, the second-order structure functions were found to be consistent with the Obukhov-Corrsin scaling. Higher-order structure functions of ozone have been studied and compared with intermittency models by Cho *et al.* [26]. In this study we have demonstrated that there is, indeed, a cascade of passive scalar variance in the stratosphere, and we have also determined its strength. By using the two-dimensional Yaglom relation, we could determine the dissipation, or destruction, of temperature and ozone variances, by molecular diffusion. To be in possession of realistic values of these parameters, as well as the dissipation of mean kinetic energy, can be very valuable when stratospheric dynamics is to be modeled. As for the exact physical mechanism behind the mesoscale cascade, we have made no assumption. That no such assumption is needed is, in fact, the strength of the Yaglom and Kolmogorov relations. They are derived directly from the dynamical equations, using only very general assumptions. Wherever there is a cascade driven by nonlinear advective forces, these relations must hold in one form or another. One hypothesis [10,11] that is worth investigating further is that the physical mechanism behind the cascade is long gravity waves breaking down to shorter gravity waves through nonlinear interactions. Whether this is, in fact, the true answer, future modeling, observations, and numerical simulations will tell us.

We especially thank Dr. Valérie Thouret and Dr. Alain Marenco for the use of their MOZAIC program data. The KTH work was supported by the Swedish Research Council for Engineering Sciences. The MIT work was funded by NASA Grant No. NAG1-2306. MOZAIC was funded in part by the European Communities (DG XII-C, DG XII-D) with strong support form Airbus Industrie and its partners, Air France, Lufthansa, Austrian Airlines, and Sabena.

*Electronic address: erikl@mech.kth.se

[1] L. F. Richardson, *Weather Prediction by Numerical Process* (Cambridge University Press, Cambridge, England, 1922).
[2] A. N. Kolmogorov, C. R. Acad. Sci. URSS **30**, 301 (1941).
[3] A. N. Kolmogorov, C. R. Acad. Sci. URSS **32**, 19 (1941).
[4] A. M. Yaglom, Dokl. Akad. Nauk SSSR **69**, 743 (1949).
[5] A. M. Obukhov, Izv. Akad. Nauk SSSR Geogr. i Geofiz. **13**, 58 (1949).
[6] S. Corrsin, J. Appl. Phys. **22**, 469 (1951).
[7] G. D. Nastrom and K. S. Gage, J. Atmos. Sci. **42**, 950 (1985).
[8] G. D. Nastrom, W. H. Jasperson, and K. S. Gage, J. Geophys. Res. **91**, 13 201 (1986).
[9] J. Y. N. Cho, Y. Zhu, R. E. Newell, B. E. Anderson, J. D. Barrick, G. L. Gregory, G. W. Sachse, M. A. Carroll, and G. M. Albercook, J. Geophys. Res. **104**, 5697 (1999).
[10] E. M. Dewan, Science **204**, 832 (1979).
[11] E. Dewan, J. Geophys. Res. **102**, 29 799 (1997).
[12] K. S. Gage, J. Atmos. Sci. **36**, 1950 (1979).
[13] D. K. Lilly, J. Atmos. Sci. **40**, 749 (1983).
[14] R. H. Kraichnan, Phys. Fluids **10**, 1417 (1967).
[15] R. H. Kraichnan, J. Fluid Mech. **47**, 525 (1970).
[16] E. Lindborg, J. Fluid Mech. **388**, 259 (1999).
[17] U. Frisch, *Turbulence. The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, England, 1995).
[18] E. Lindborg and J. Y. N. Cho, "Horizontal Velocity Structure Functions in the Upper Troposphere and Lower Stratosphere: 2. Theoretical Considerations," J. Geophys. Res. (to be published).
[19] A. Marenco *et al.,* J. Geophys. Res. **103**, 25 631 (1998).
[20] J. Y. N. Cho and E. Lindborg, "Horizontal Velocity Structure Functions in the Upper Troposphere and Lower Stratosphere: 1. Observations," J. Geophys. Res. (to be published).
[21] Y. Zhu, R. A. Antonia, and I. Hosokawa, Phys. Fluids **7**, 1637 (1995).
[22] L. Mydlarski and Z. Warhaft, J. Fluid Mech. **358**, 135 (1998).
[23] V. Thouret, A. Marenco, P. Nédélec, and C. Grouhel, J. Geophys. Res. **103**, 25 653 (1998).
[24] J. Y. N. Cho, R. E. Newell, V. Thouret, A. Marenco, and H. Smit, EOS Trans. AGU **80**, 377 (1999).
[25] J. Dole and R. Wilson, Geophys. Res. Lett. **27**, 2625 (2000).
[26] J. Y. N. Cho, V. Thouret, R. E. Newell, and A. Marenco, "Isentropic Scaling Analysis of Ozone in the Upper Troposphere and Lower Stratosphere," J. Geophys. Res. (to be published).