Frontiers & Challenges in CAE Simulations

Klaus-Jürgen Bathe

CST and ECT Conferences, Sitges / Barcelona, Spain, September 2018

Content of Presentation

- Philosophy adopted in our research and developments

- Analysis of shells – the MITC3+ and MITC4+ elements

- 3D solids – the 3D-MITC8 element

- The enriched subspace iteration scheme for frequency solutions

- Direct time integration
- Multiphysics problems, FSI with EM
- Molecular structures – DNA and Proteins
- Overlapping finite elements and the “AMORE paradigm”
- Concluding remarks

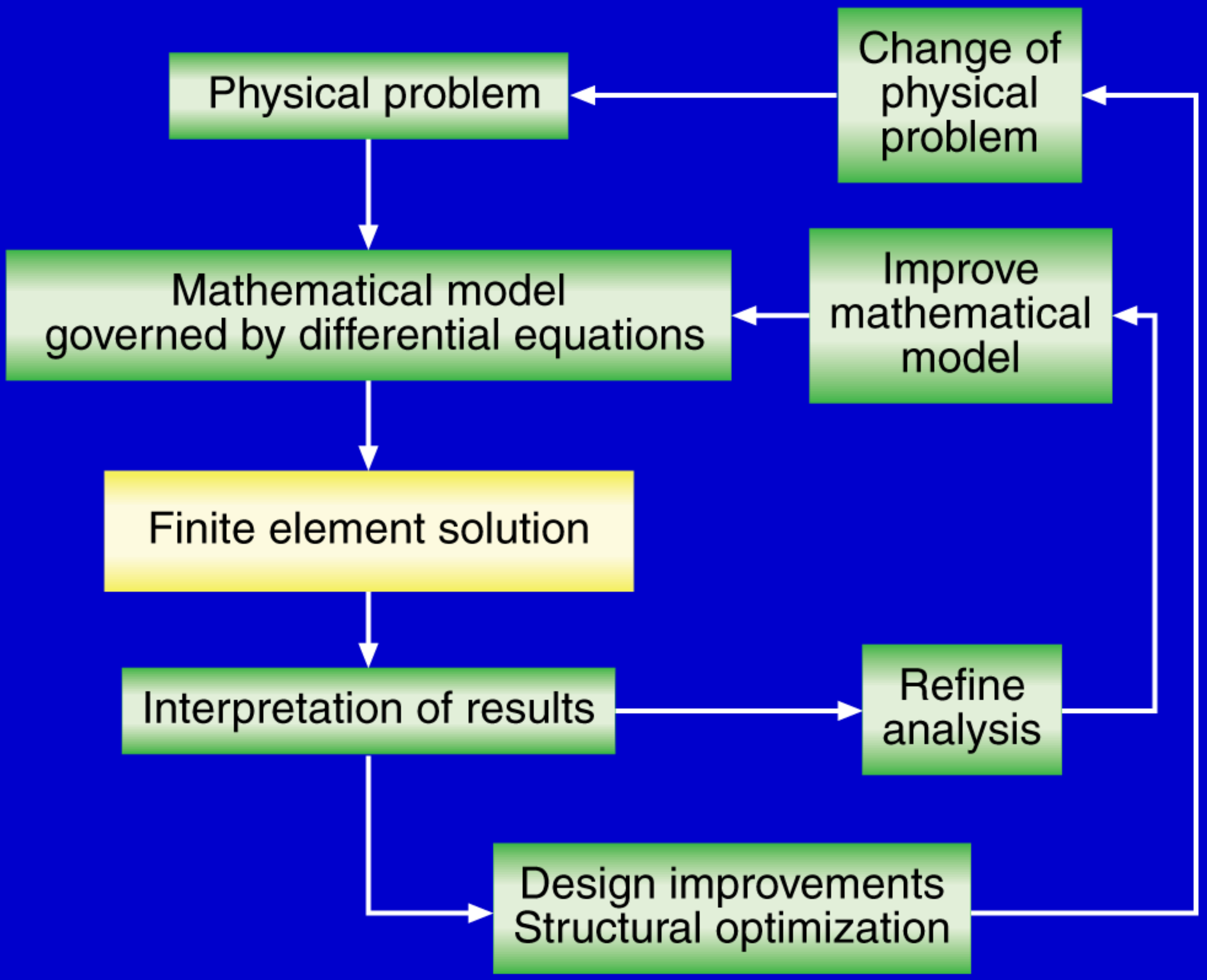
Our research and development 'philosophy':

To develop "reliable and efficient procedures" that can be used in general to --

advance simulations as practiced in industry and the sciences;

simulations of "structures" in multiphysics environments

The process of modeling for analysis



Our focus: The analysis of problems involving ---

Solids and shells, general structures, from the km-scale to the nano-scale

Multi-physics media governed by the Navier-Stokes and Maxwell's equations

The fully coupled response

The analysis of shells

"General shell elements" widely used do not include the through-the-thickness stress (and strain), e.g. the MITC4 element

But in some analyses this stress can be important --- like metal forming, contact problems

The analysis of shells:

Testing of elements

Considering the solution scheme ---

convergence should be optimal for all problems

and independent of the shell thickness

$$\|\,\mathbf{error}\,\| \approx \mathbf{c}\ \mathbf{h}^{\mathbf{k}}$$

A general mixed formulation is:

Find $U \in \mathbf{\Sigma}$ and $N \in \mathbf{\Xi}$ such that

$$\tilde{A}\left(U,V\right) + B\left(N,V\right) = F\left(V\right)$$

$$\forall\, V \in \mathbf{\Sigma}$$

$$B\left(Q,U\right) - t^2\, C\left(Q,N\right) = 0$$

$$\forall\, Q \in \mathbf{\Xi}$$

Choose $\mathbf{\Sigma}_h$ and $\mathbf{\Xi}_h$ for the finite element spaces

The inf-sup condition is on the bilinear form B !

For the discretization, we

- must have no spurious element mode, patch tests satisfied, element spatial isotropy

- must have consistency

- want optimal convergence, inf-sup condition is satisfied

Want a scheme with no (artificial numerical) stabilization factors

The question of what norm to use:

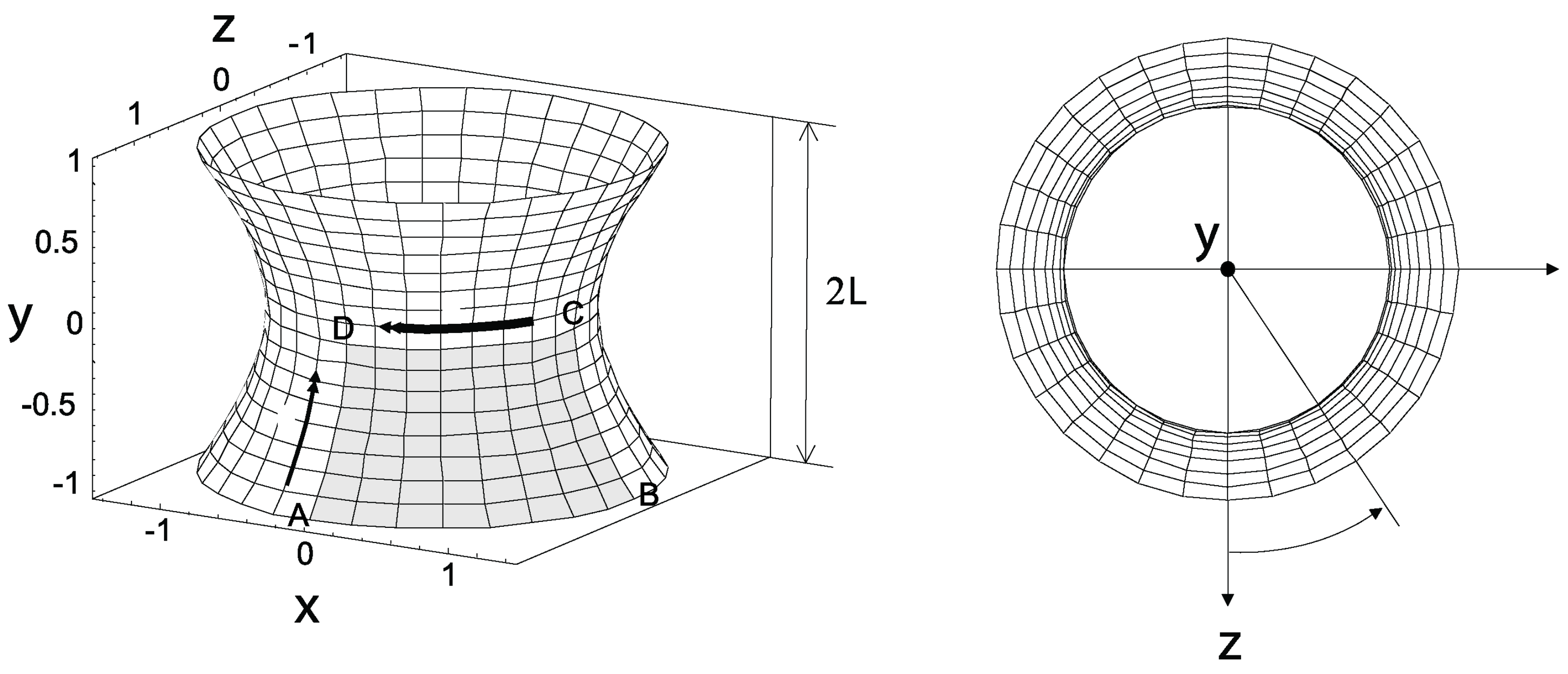
Point values are meaningless

Difference in energies is not good for mixed formulations

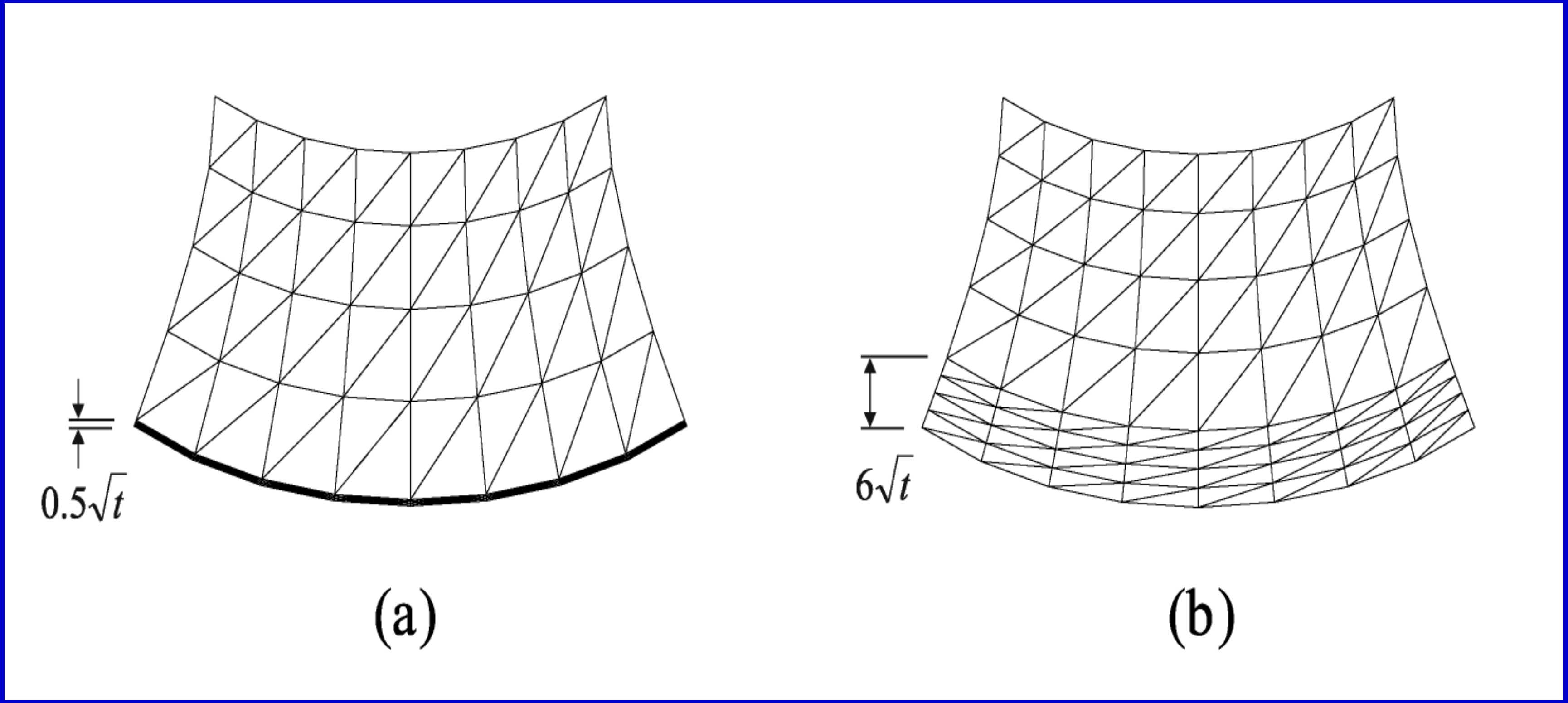
's-norm' (F Hiller, KJ Bathe) is valuable, it seems to be the only norm that can be used for all types of shell problems

$$\| \cdot \|_s^2 = \int_\Omega (\text{stress error}) \cdot (\text{strain error})\, \mathrm{d}V$$

We have used this norm extensively

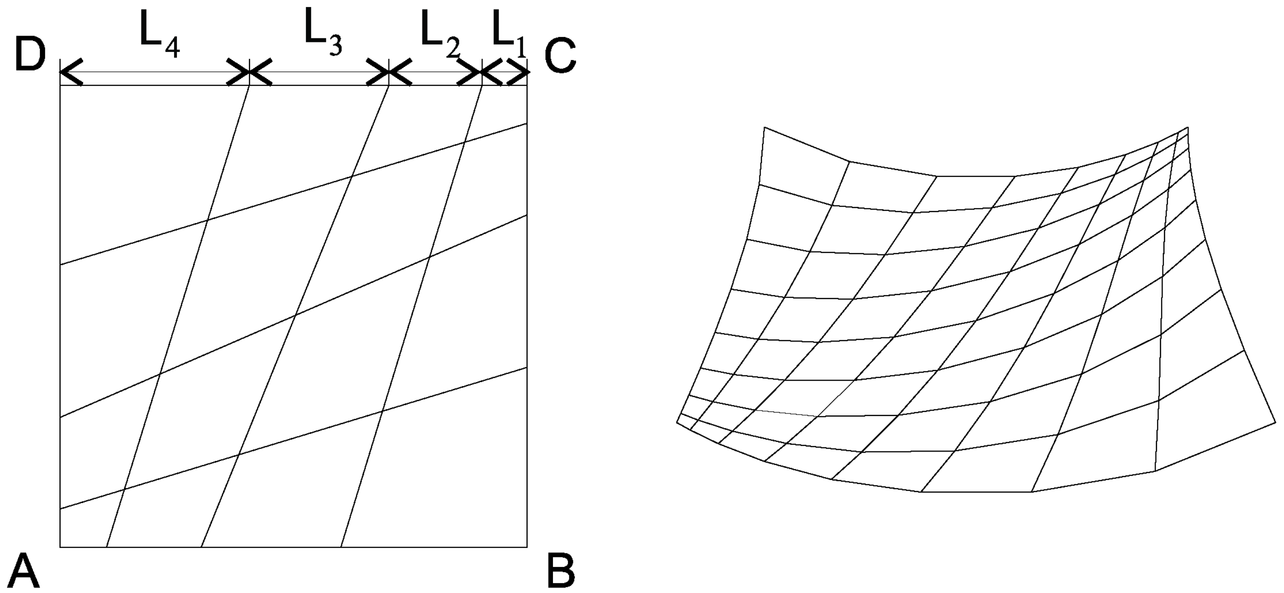


The hyperbolic shell test problem, free and fixed ends

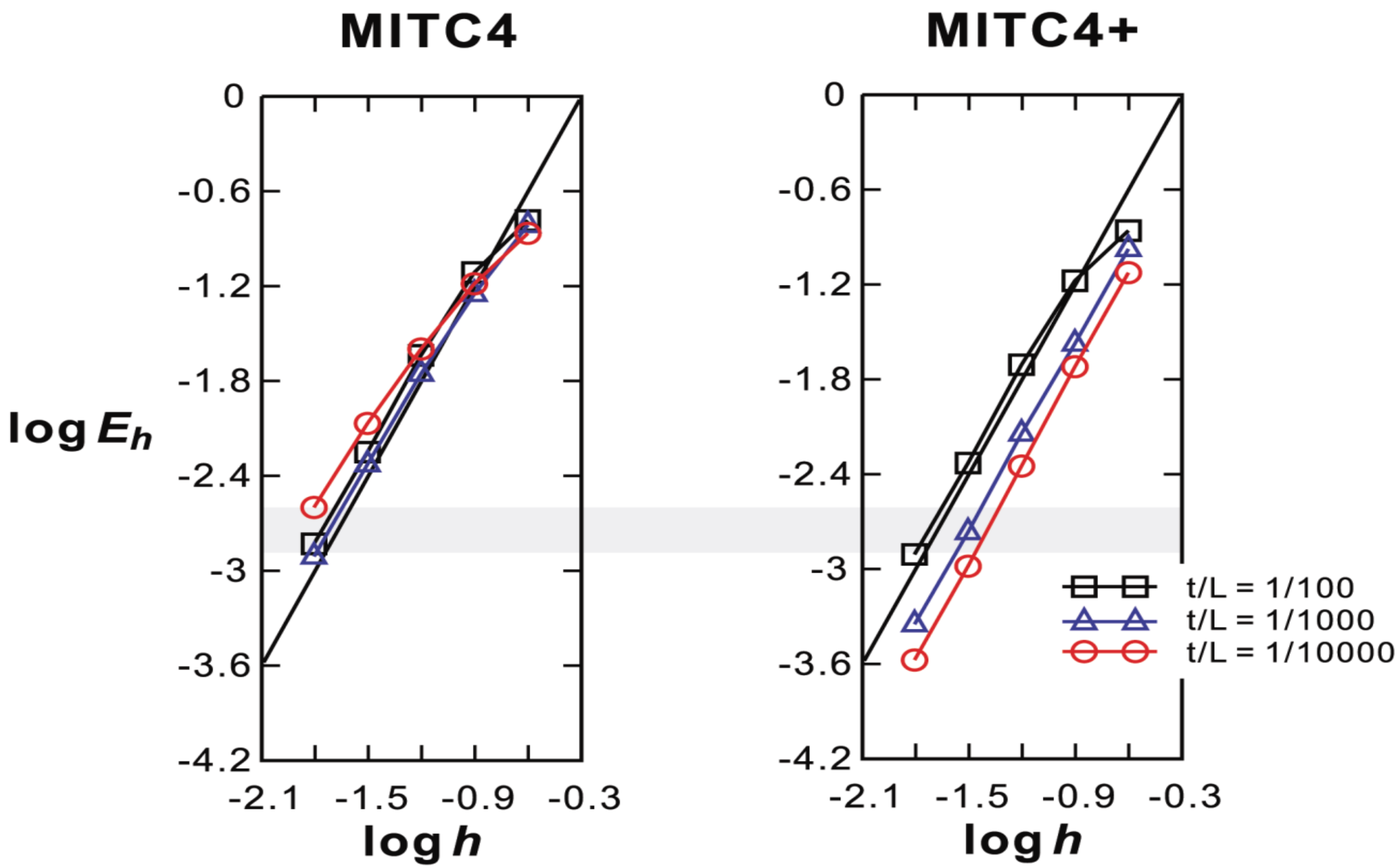


Use of 3 and 4- node shell elements, with boundary layers

Meshing used for 1/8th of the structure



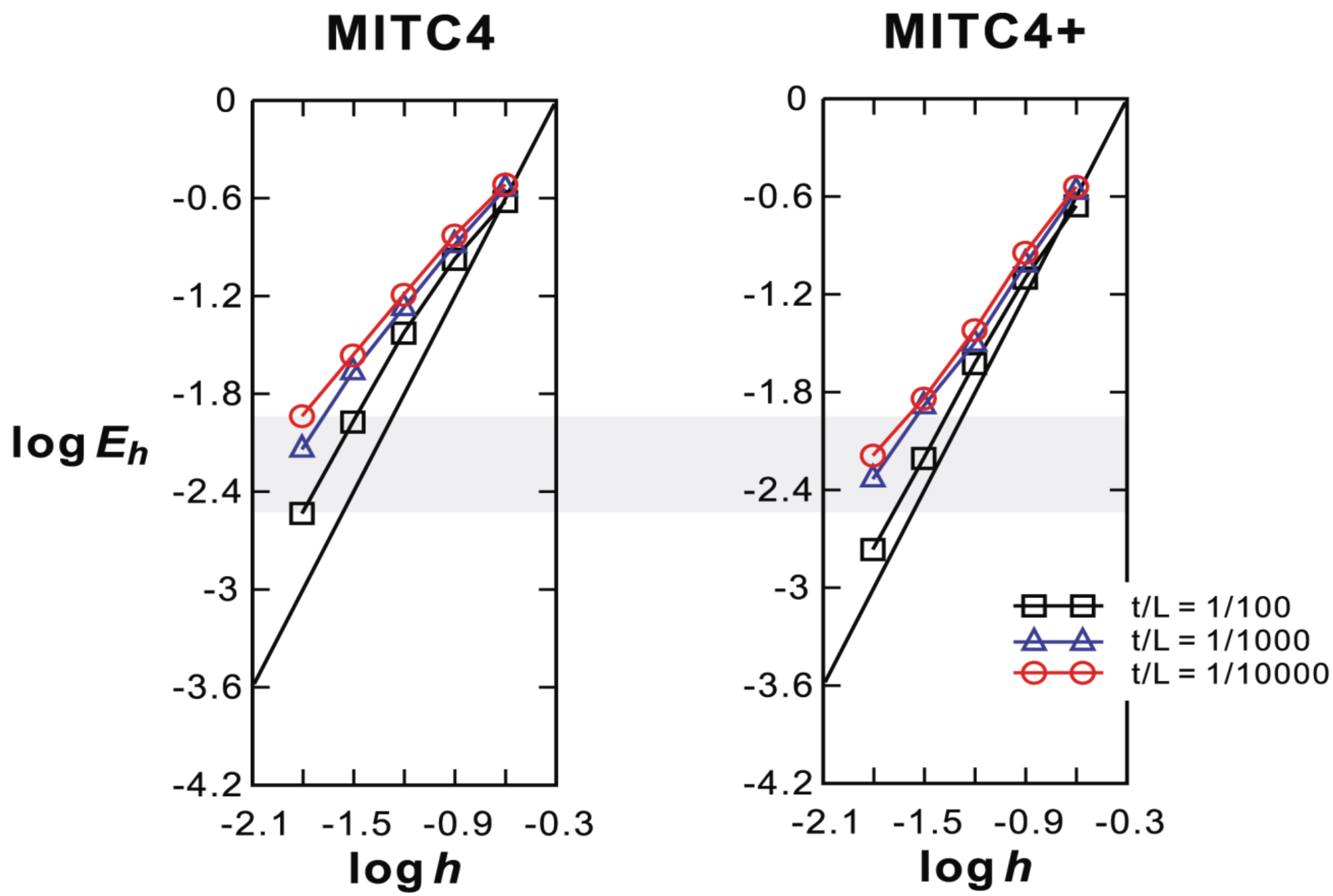
Distorted meshes used



(a)

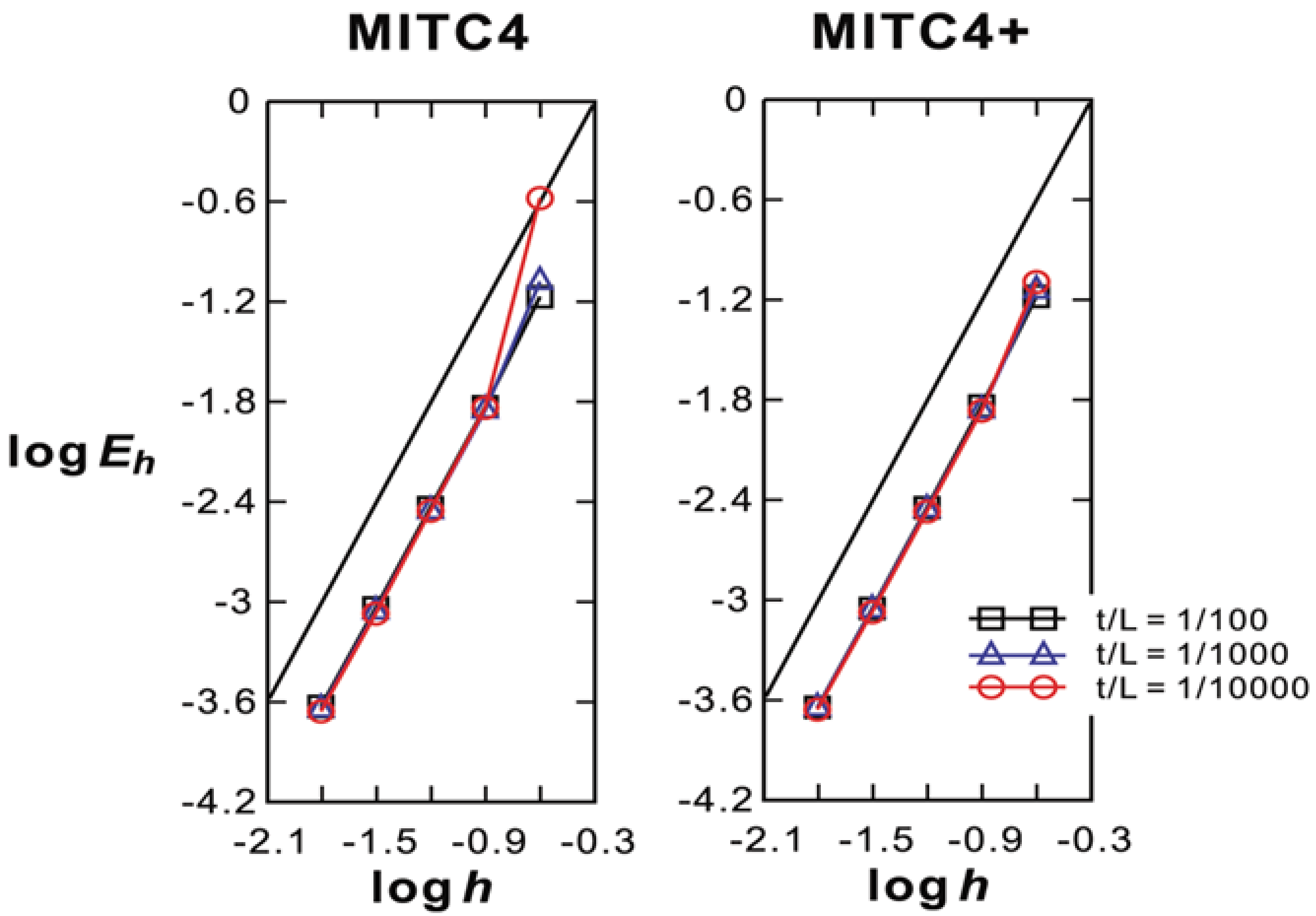
Clamped shell results; regular meshes

Y Ko, PS Lee, KJ Bathe. A new 4-node … . C & S 2017



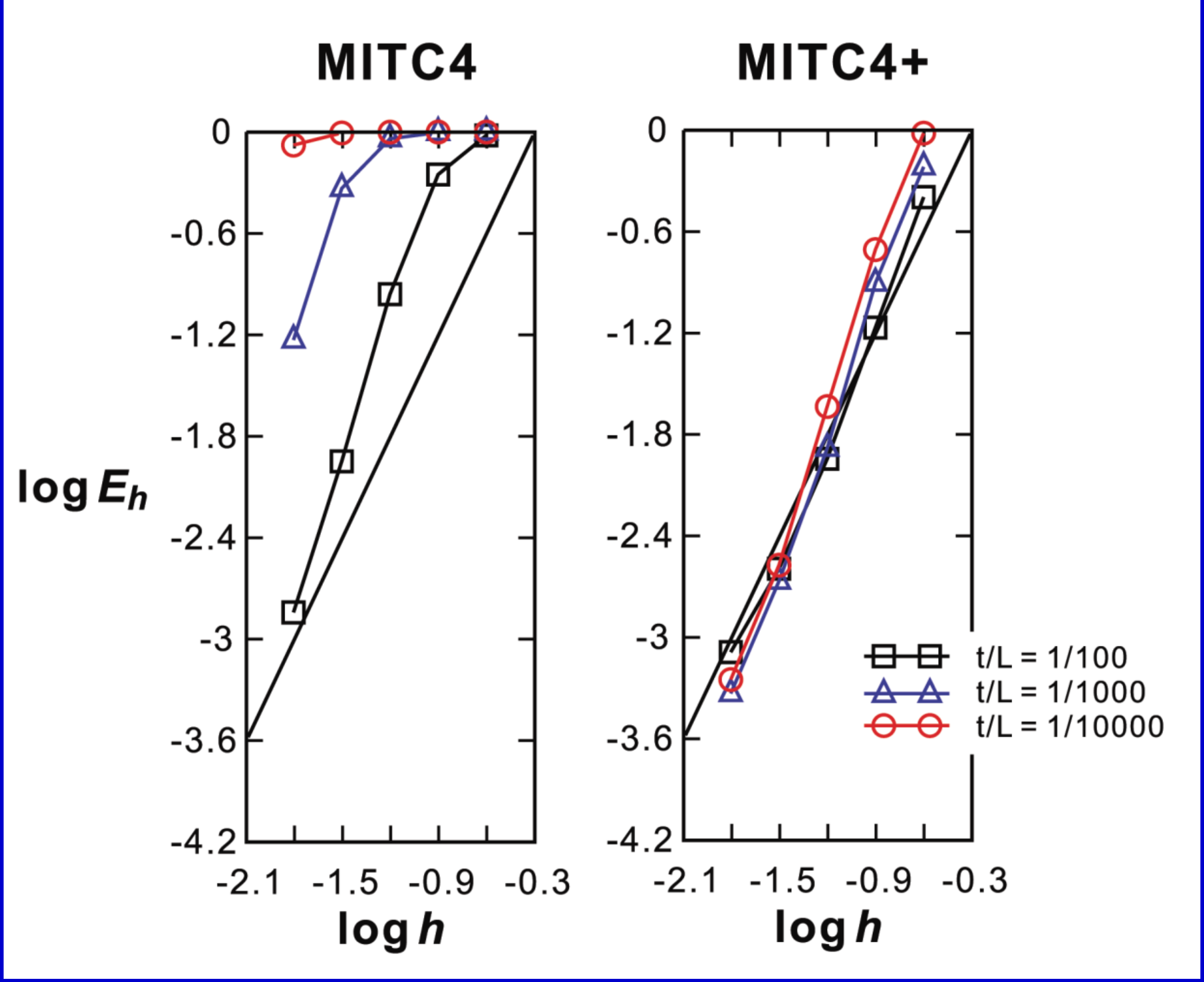
(b)

Clamped shell results; distorted meshes

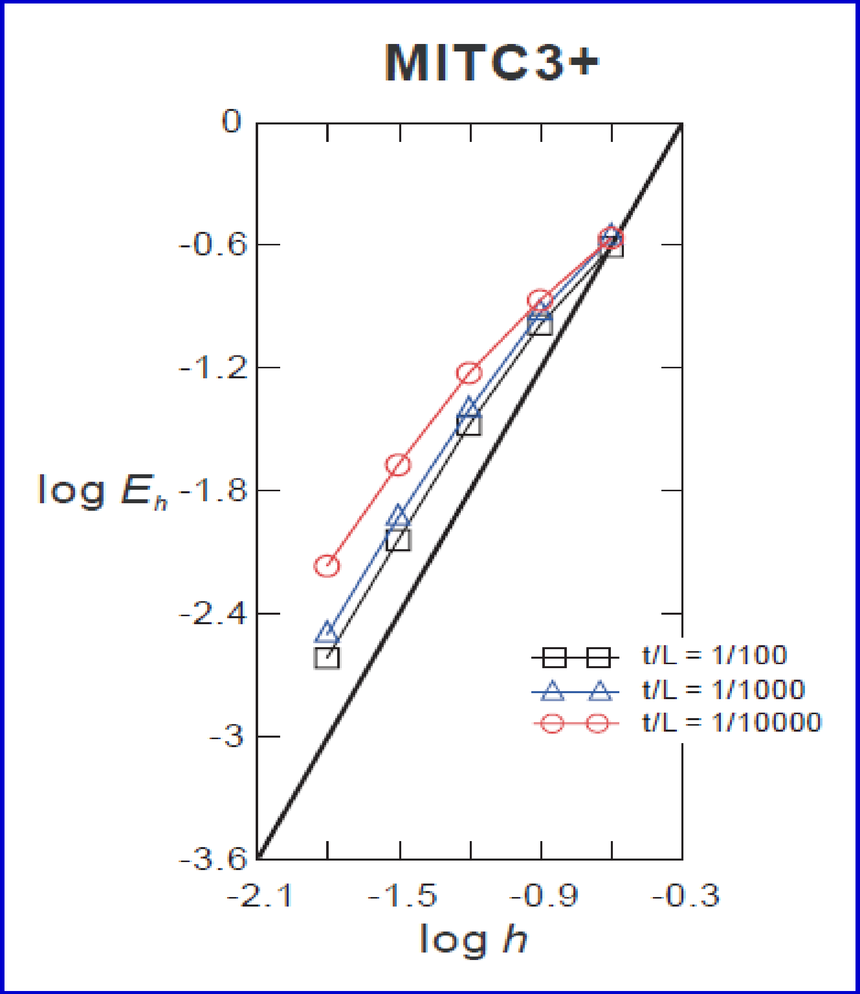


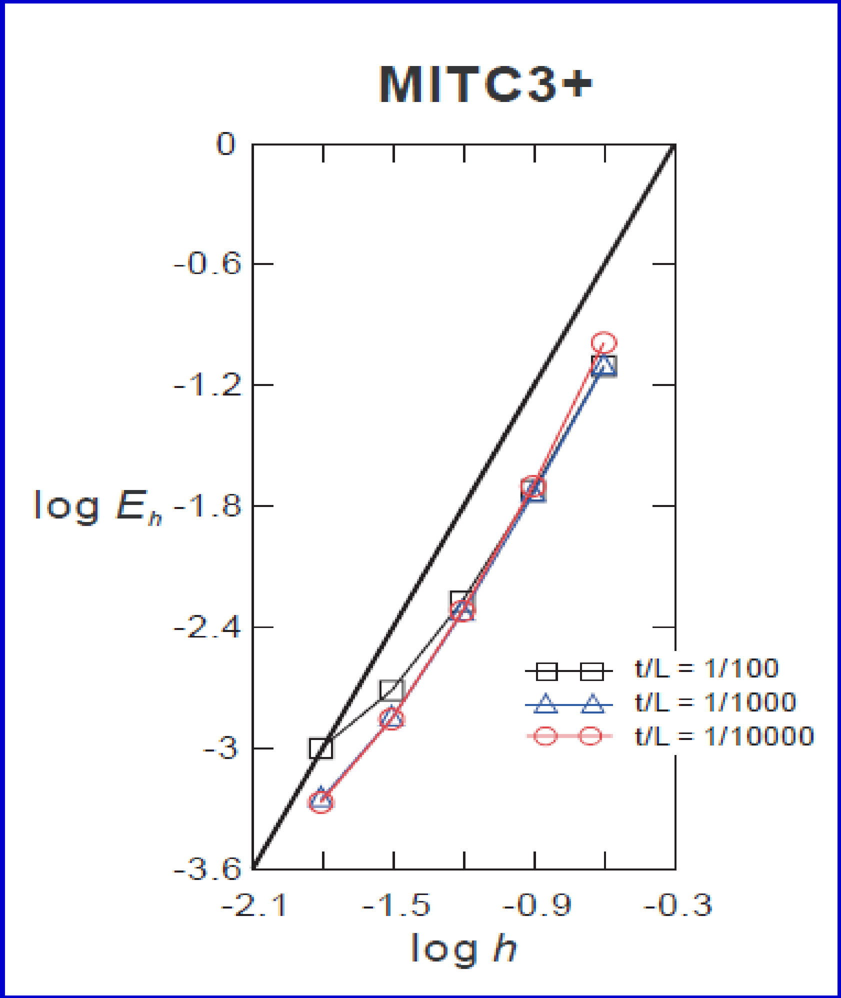
(a)

Free shell results; regular meshes



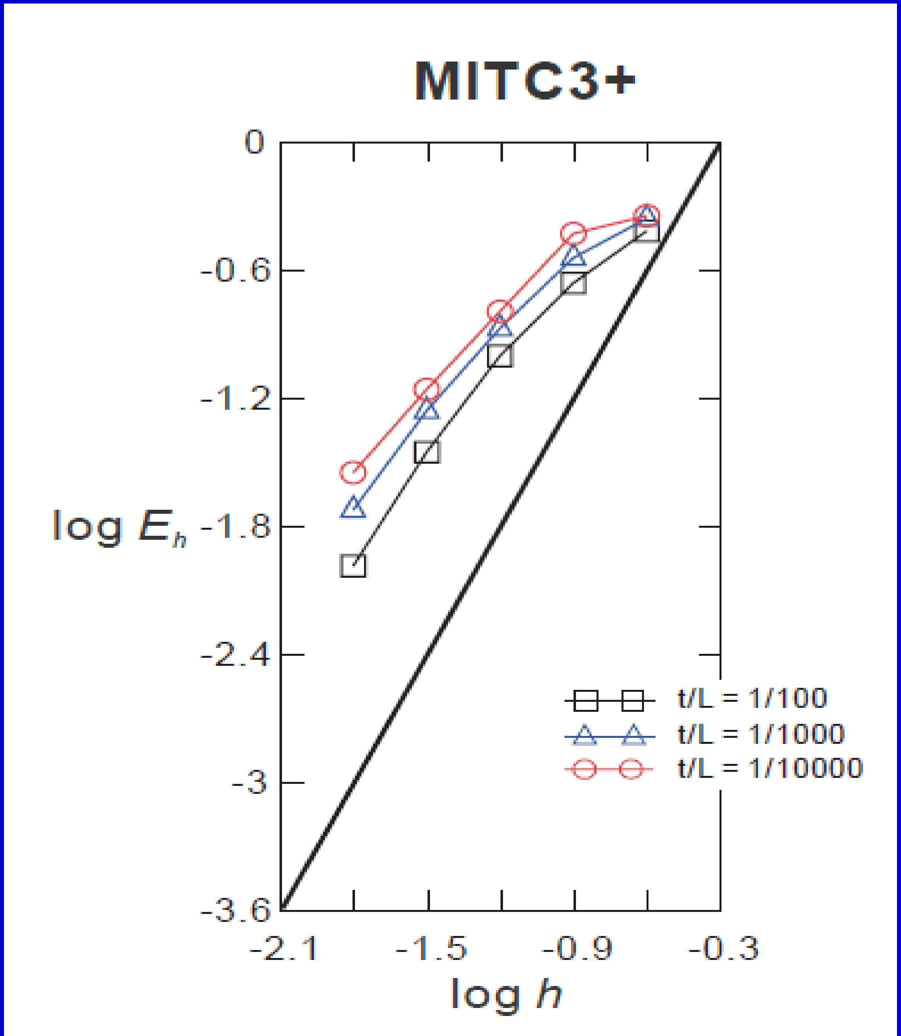
Free shell results; distorted meshes

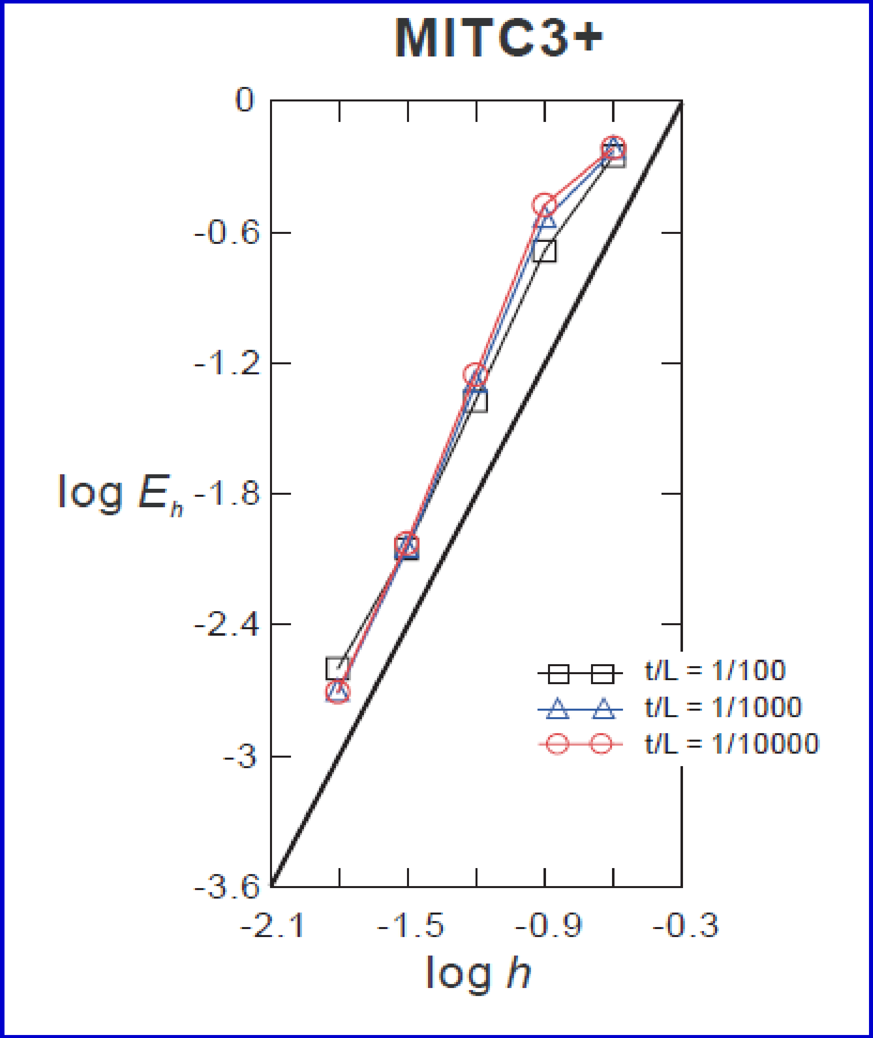




Regular meshes
left: fixed structure, right: free structure

Y Lee, PS Lee, KJ Bathe. The MITC3+ … . C & S 2014
H Jun, K Yoon, PS Lee, KJ Bathe. The MITC3+ … . CMAME 2018





Distorted meshes

left: fixed structure, right: free structure

Observations –

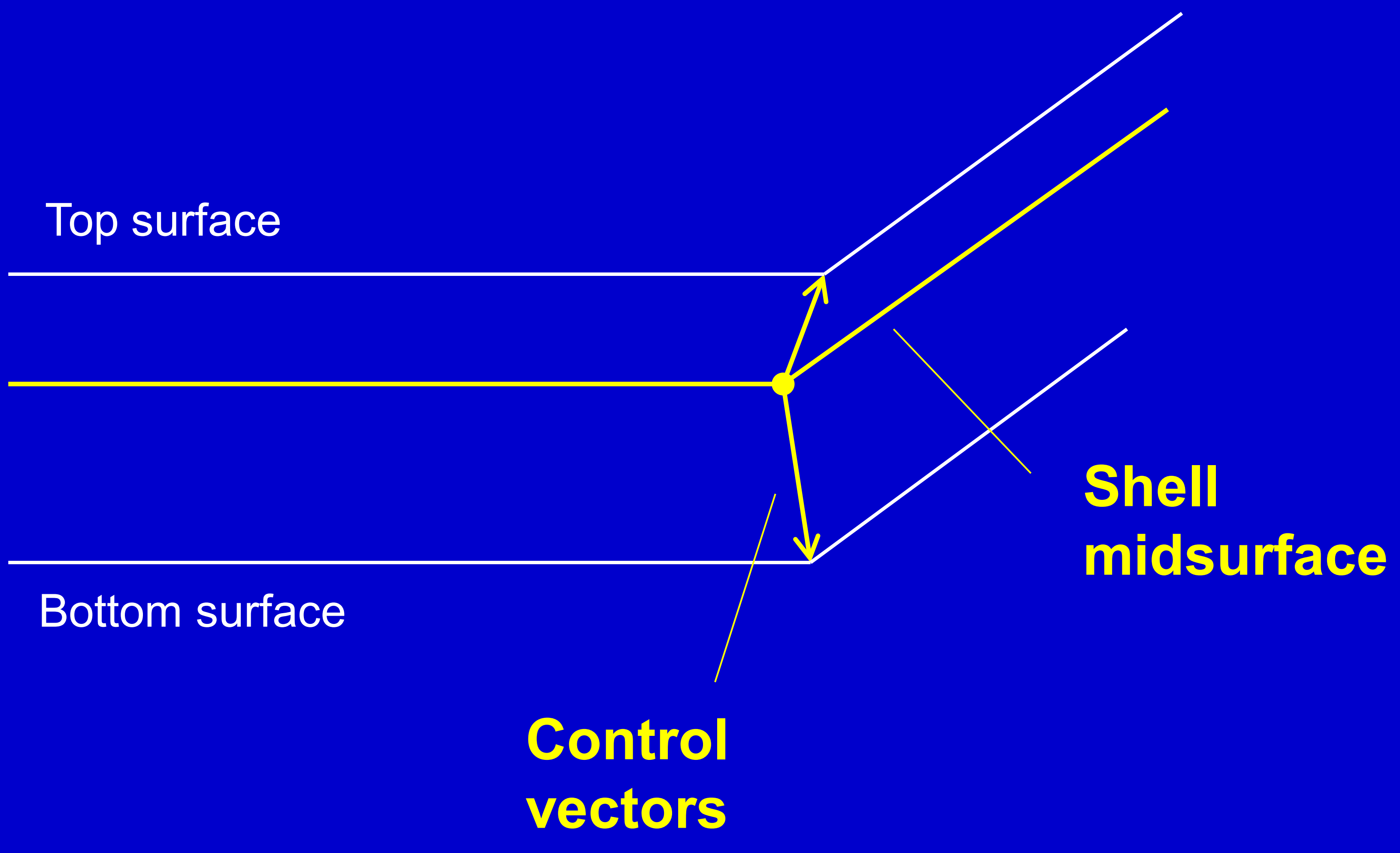
The MITC4+ and MITC3+ elements are practically optimal

but do not include the through-the-thickness stress and strain

In some analyses this stress can be important --- like in metal forming, contact problems, ….

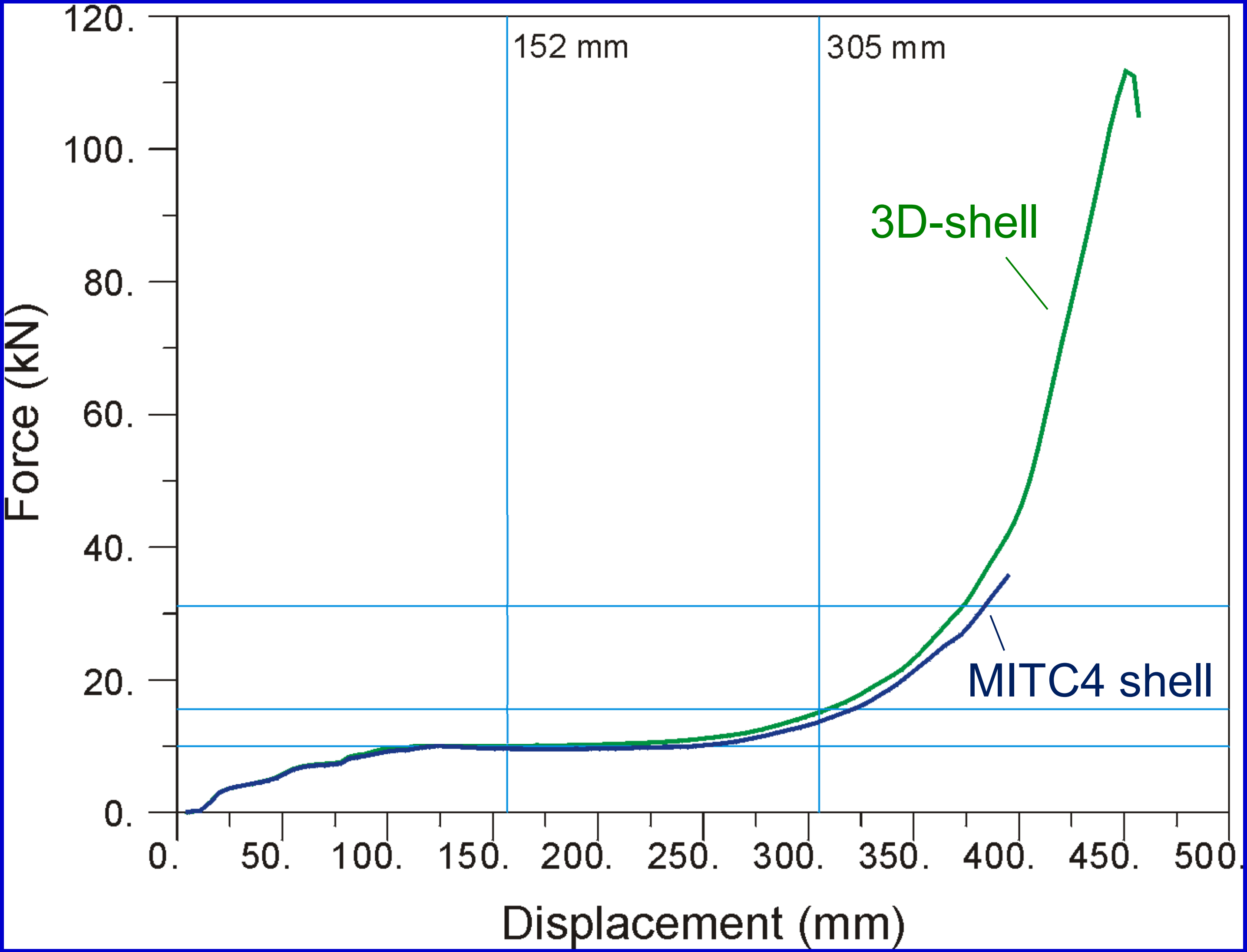
A 3D-shell element can then be effective

3D-Shell Element



Door crush problem

Door Crush Problem



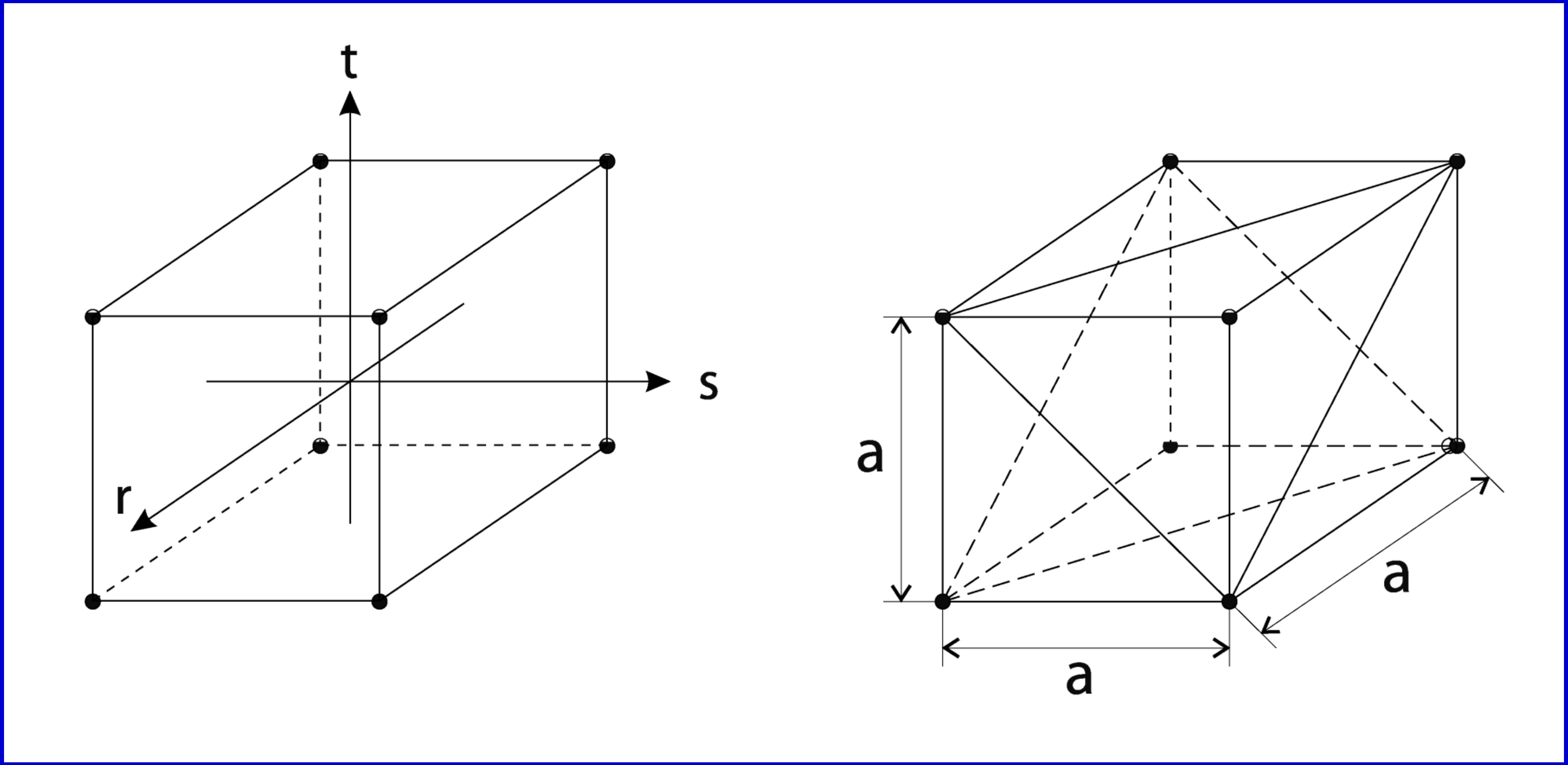
A new 3D solid element, the 3D-MITC8 element

Eight-node element using MITC technique, described by only displacement DOF

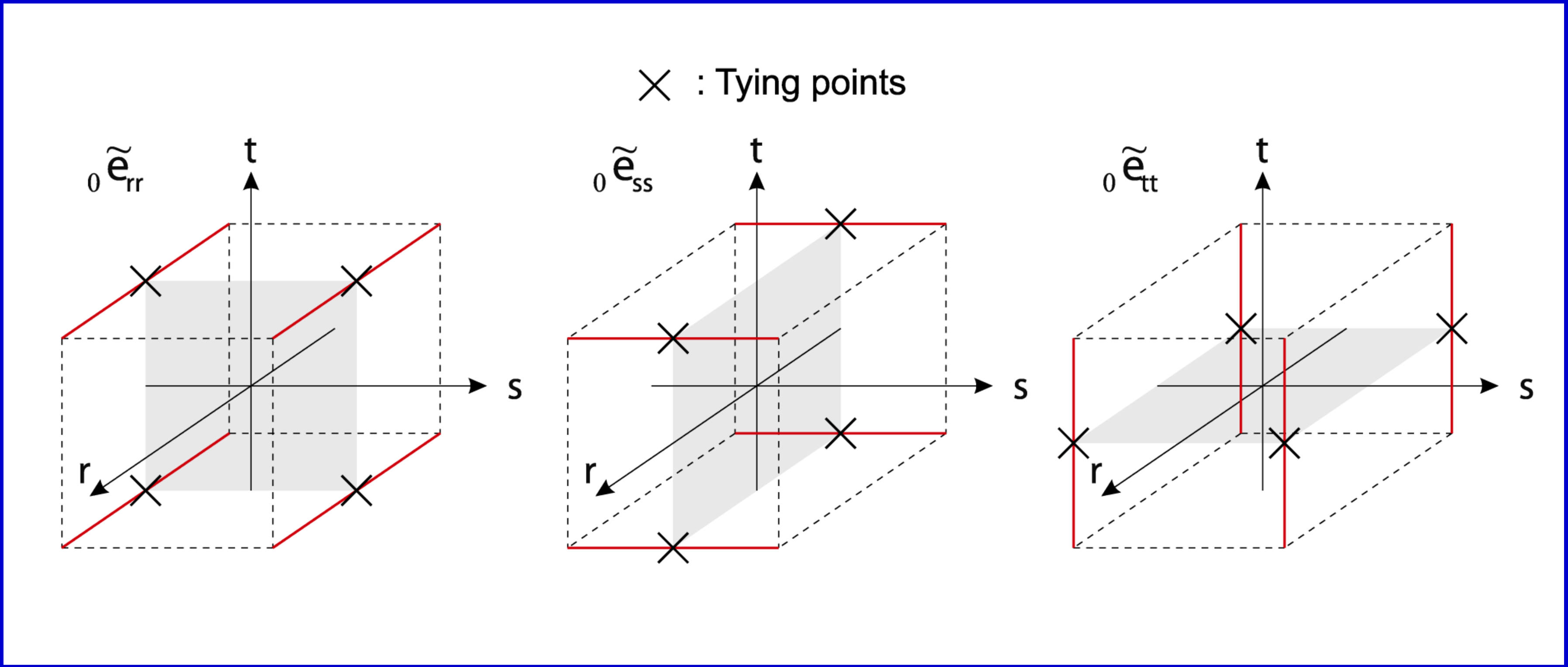
The element is isotropic, passes the patch tests, and can be used in linear and non-linear analyes

The element displays no spurious mode in nonlinear analysis as does the H8I9 element

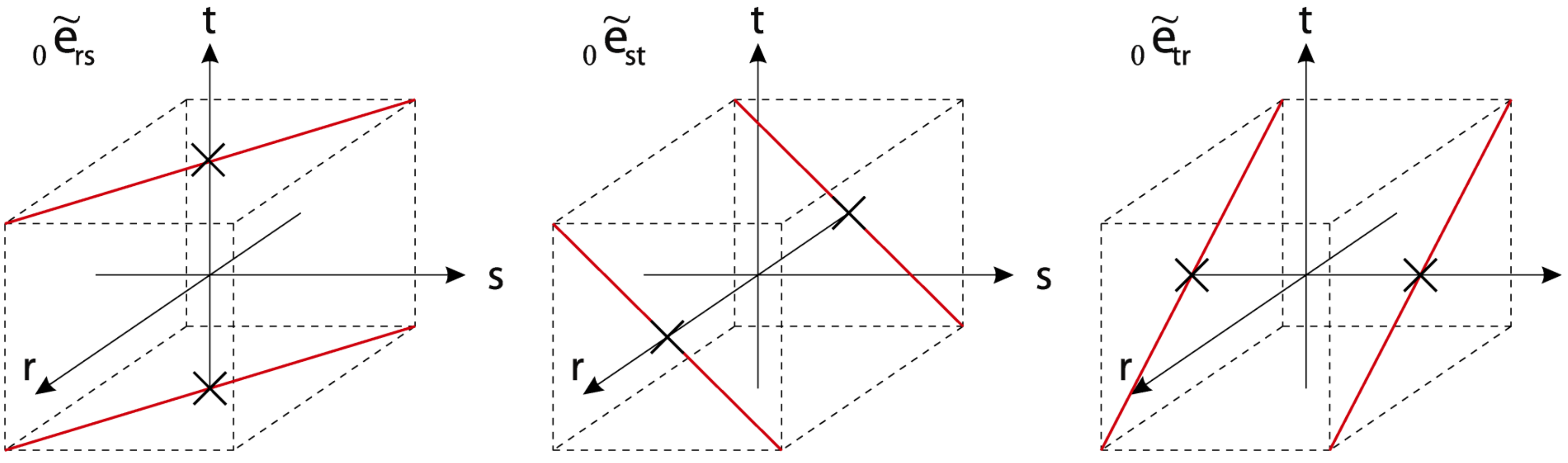
Y Ko, KJ Bathe. A new 8-node … . C & S 2018



The domain of the 8-node element idealized by a stable truss structure

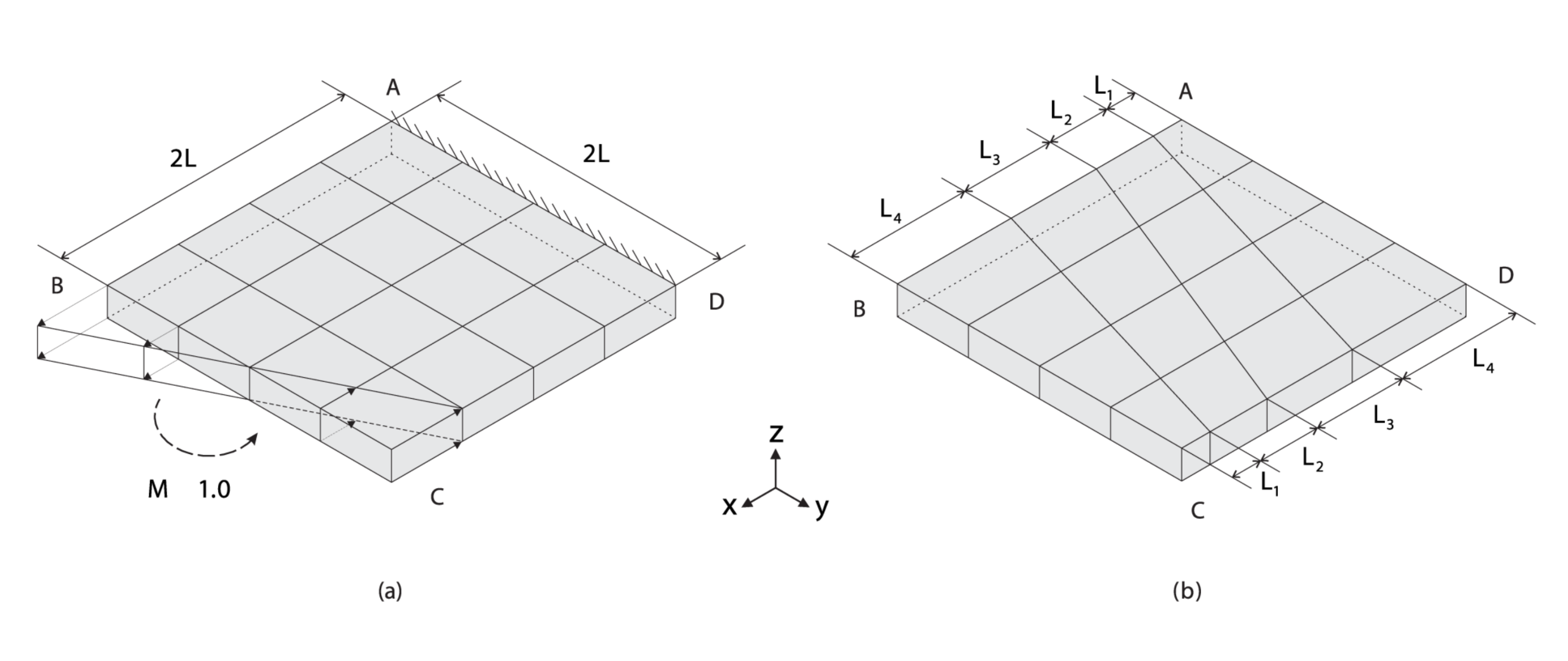


2-node truss elements at edges for normal stress components and their interpolations

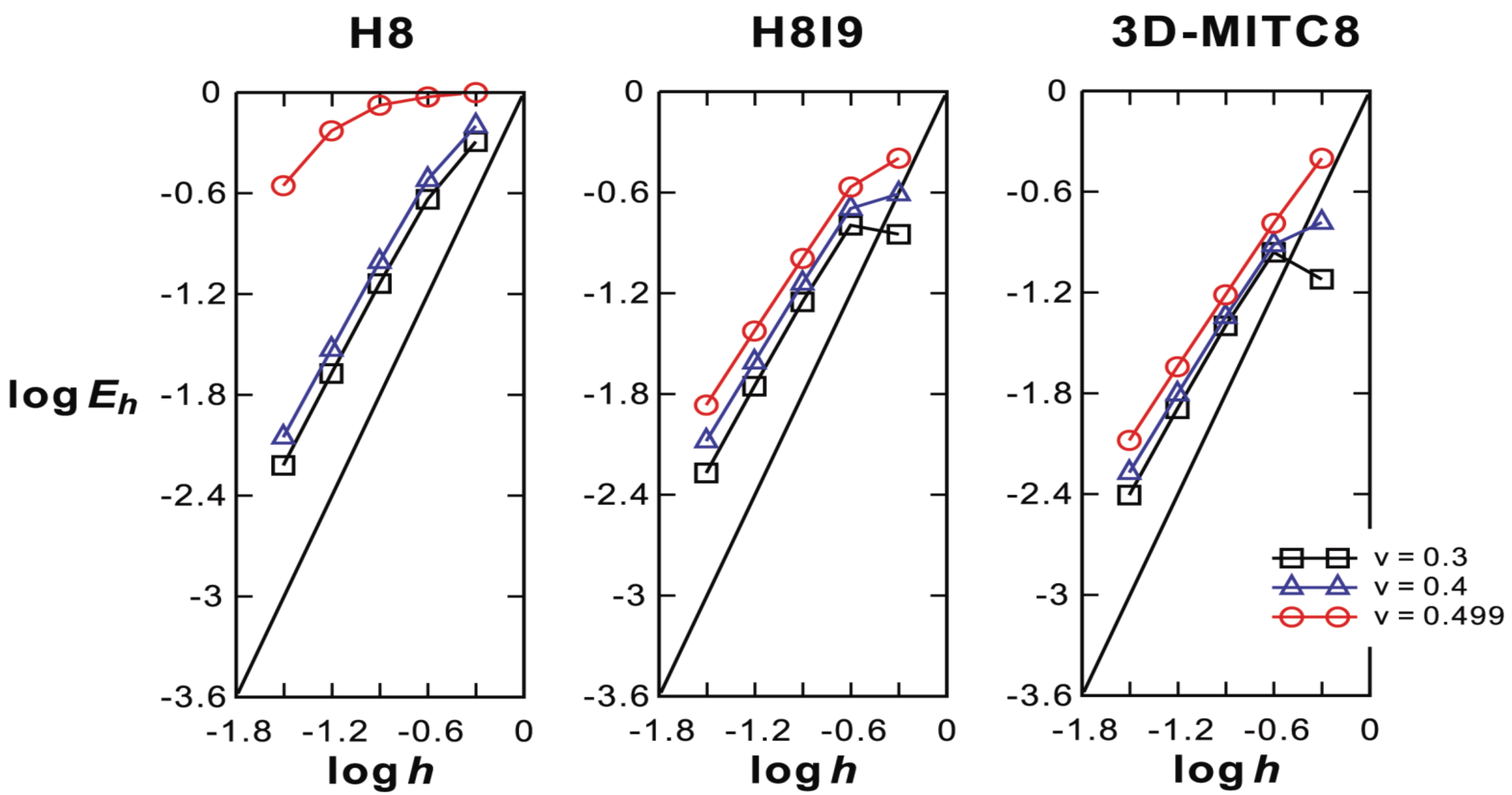


2-node diagonal truss elements for shear stress components and their interpolations

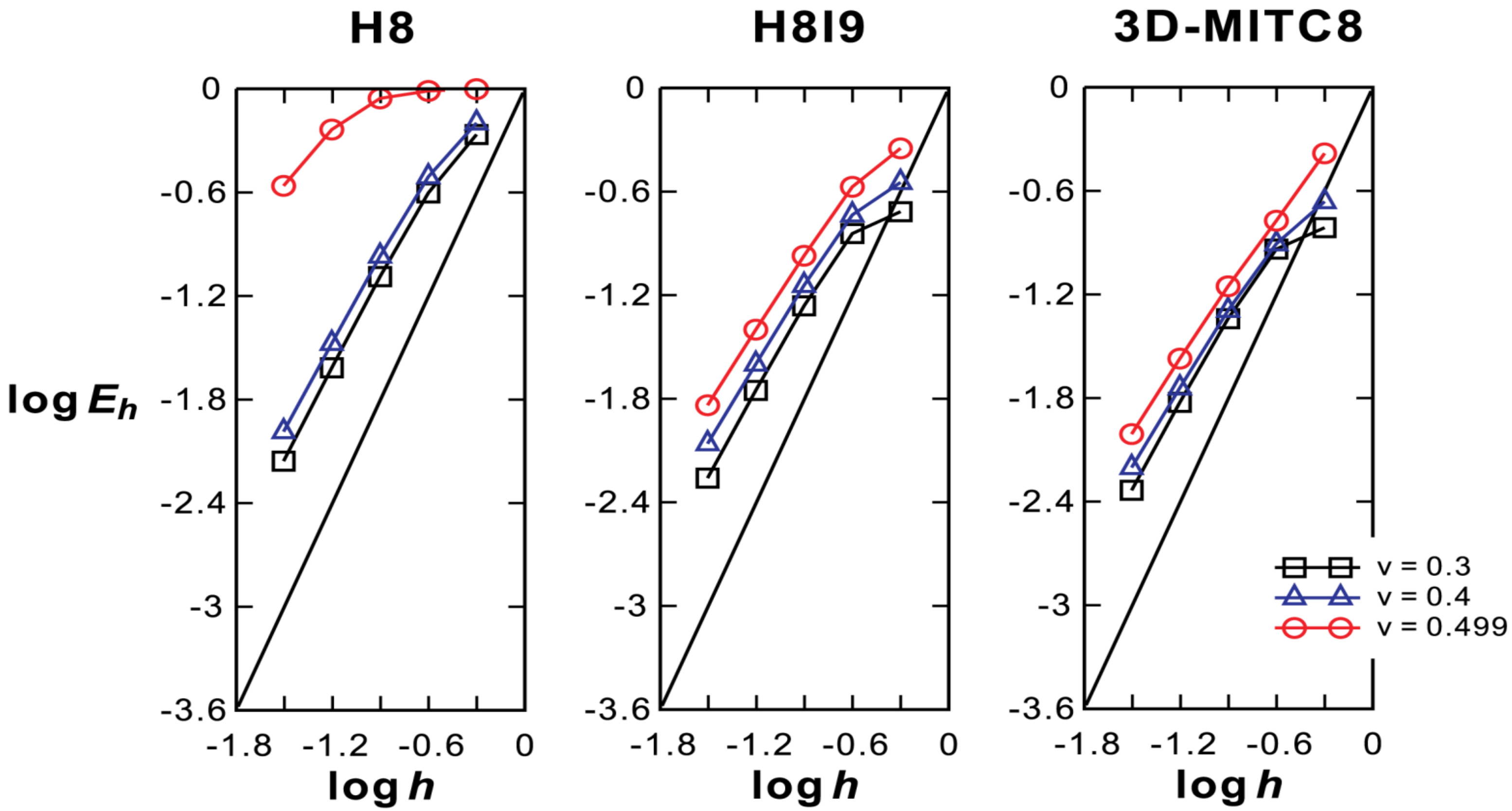
Illustrative solutions



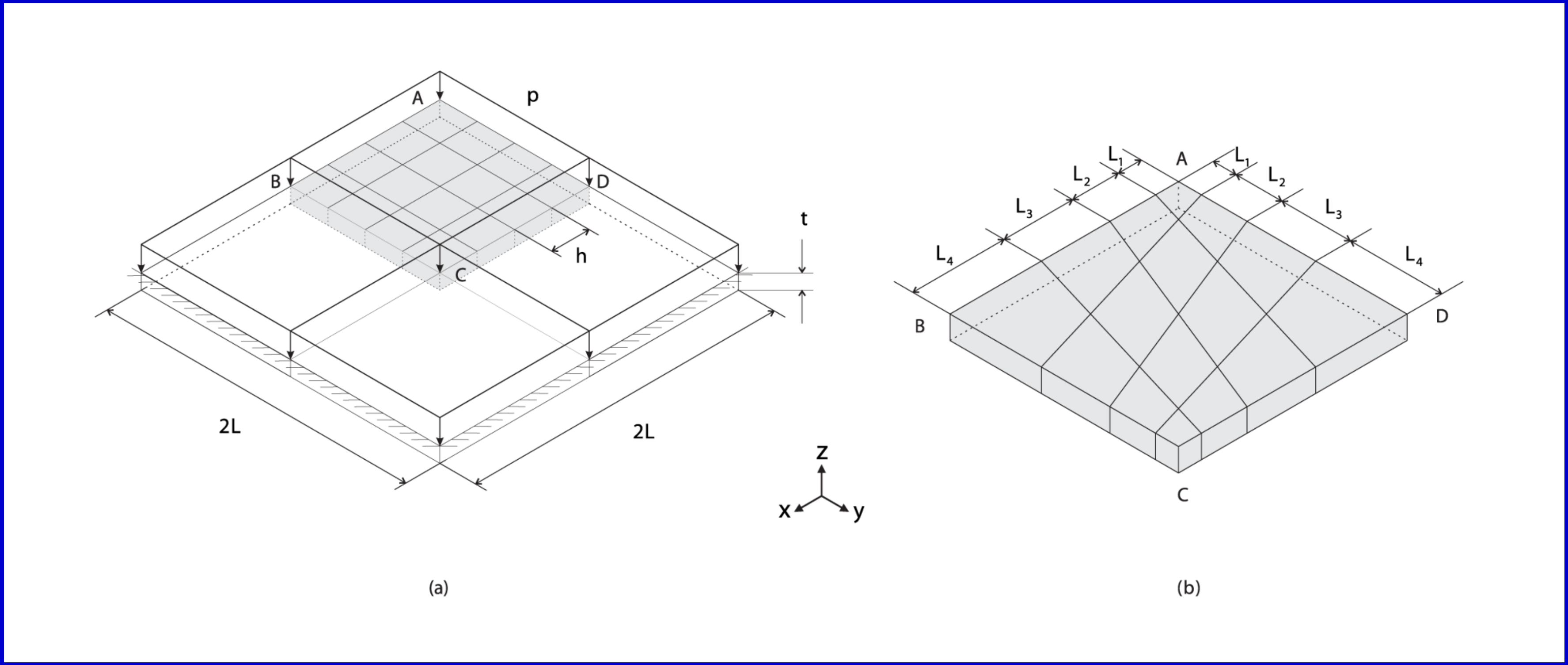
Analysis of cantilever plate subjected to in-plane moment, test for volumetric locking



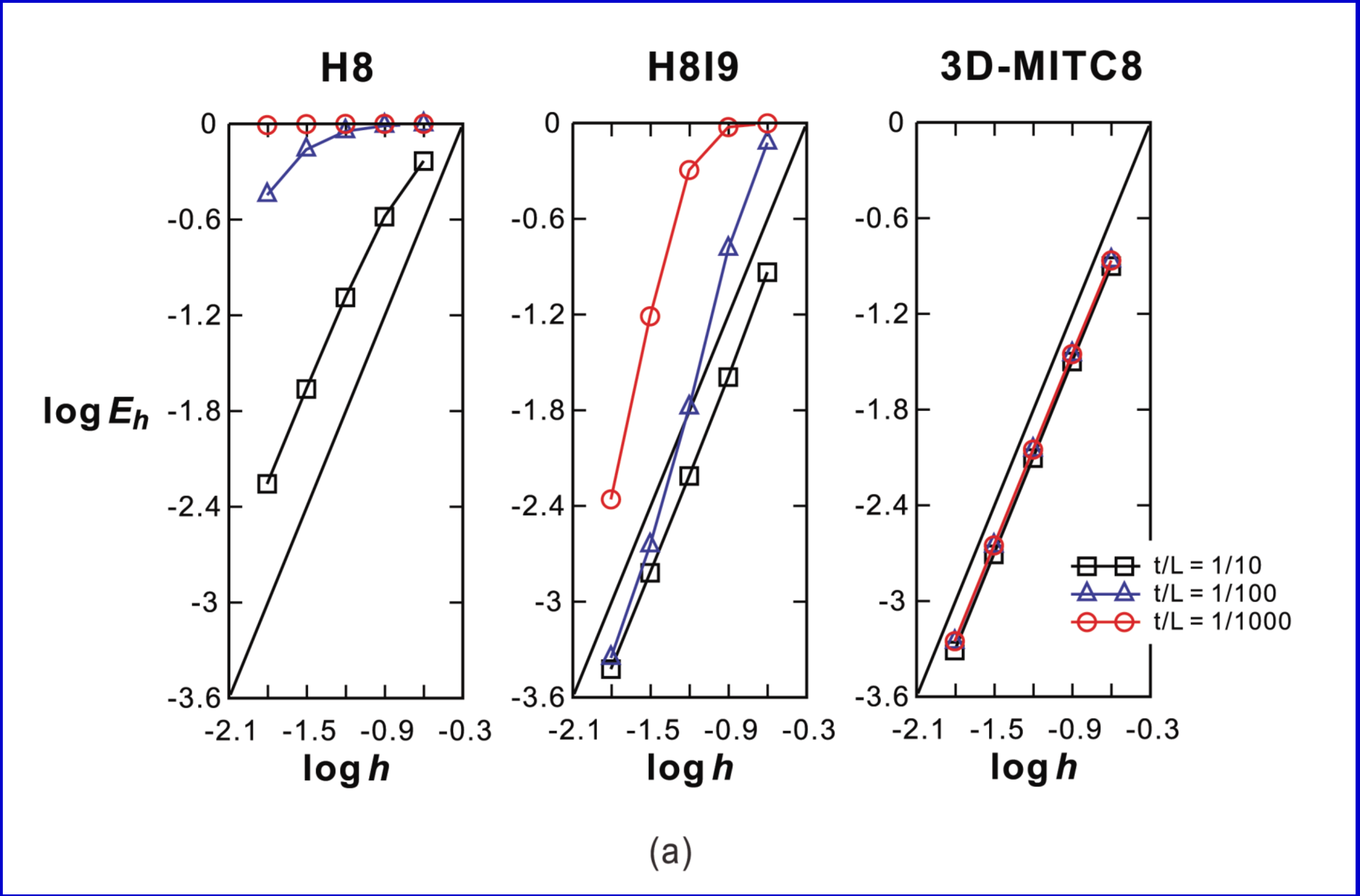
Analysis of cantilever plate, regular meshes



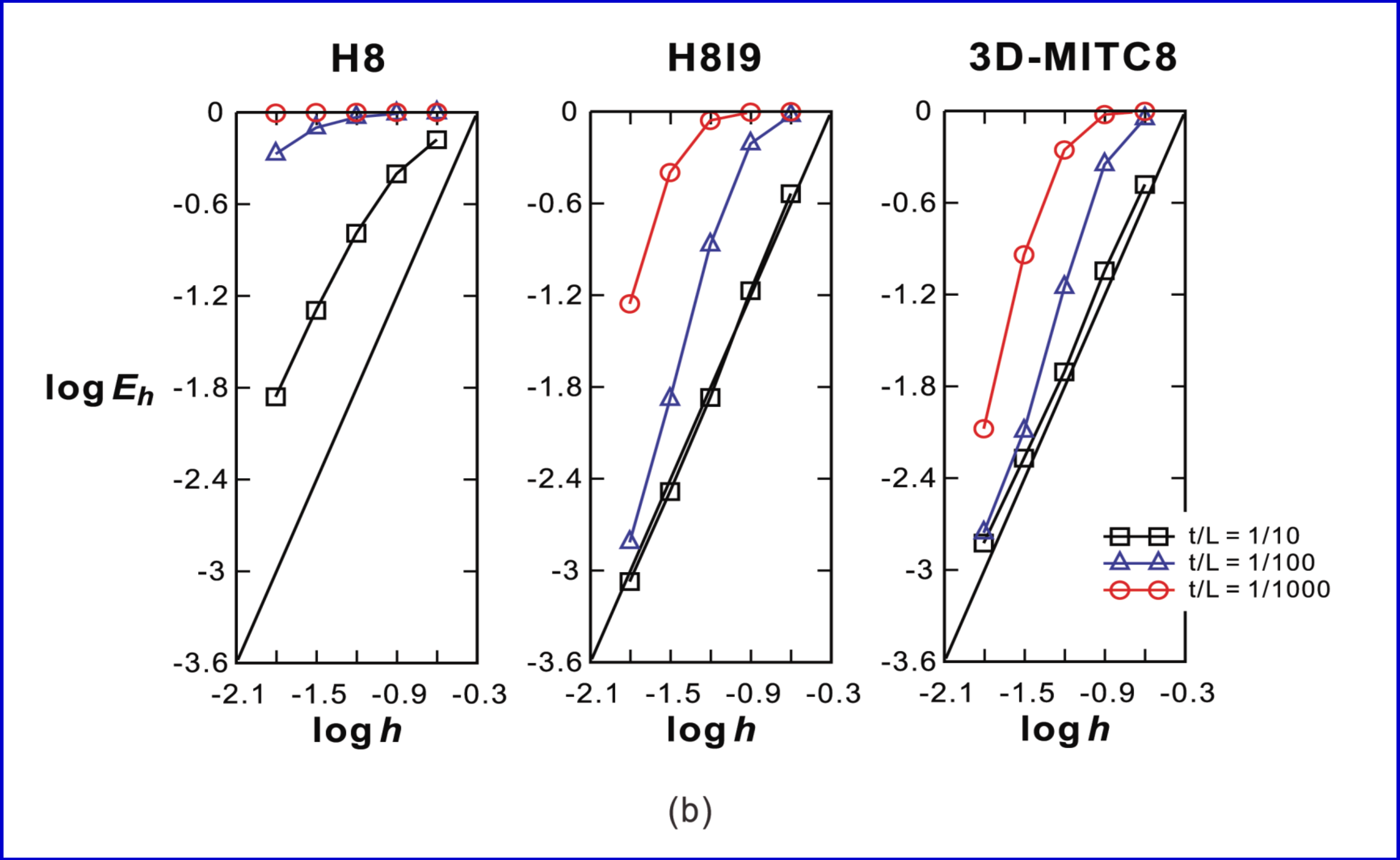
Analysis of cantilever plate, distorted meshes



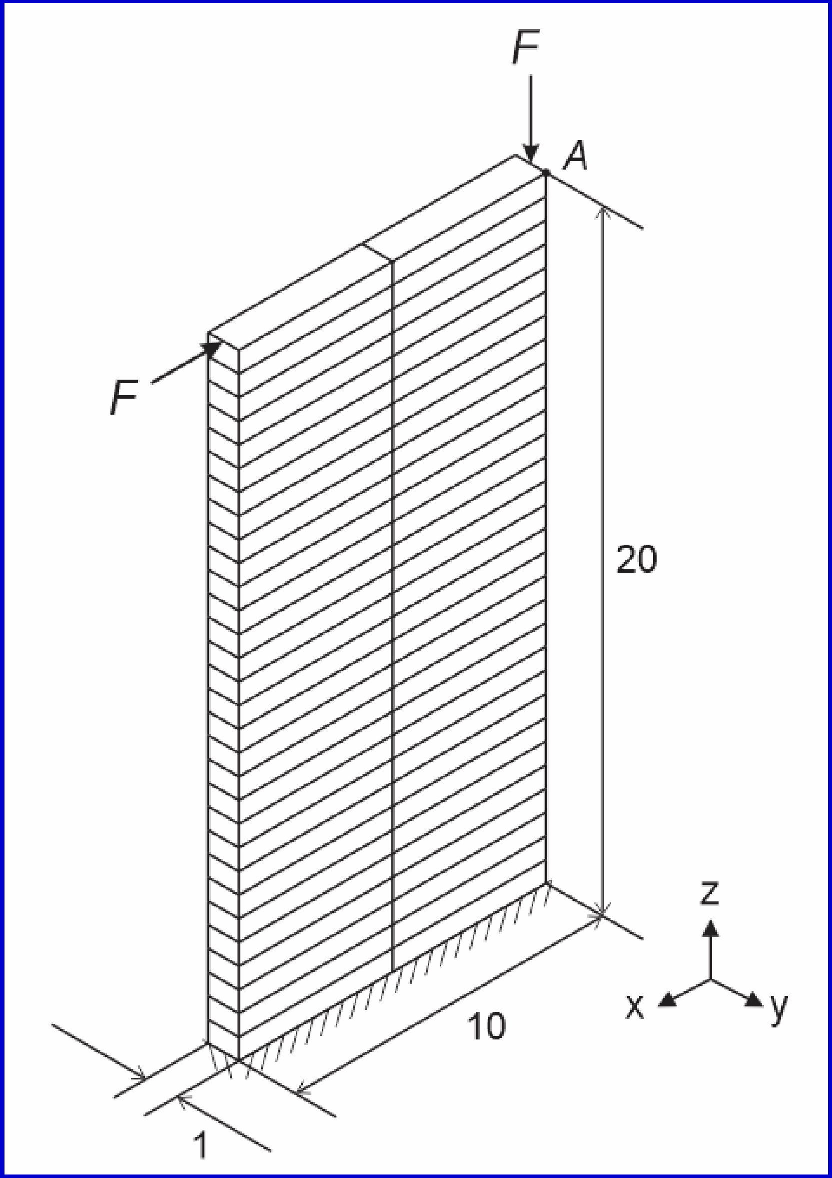
Analysis of clamped plate subjected to transverse pressure, test for shear locking



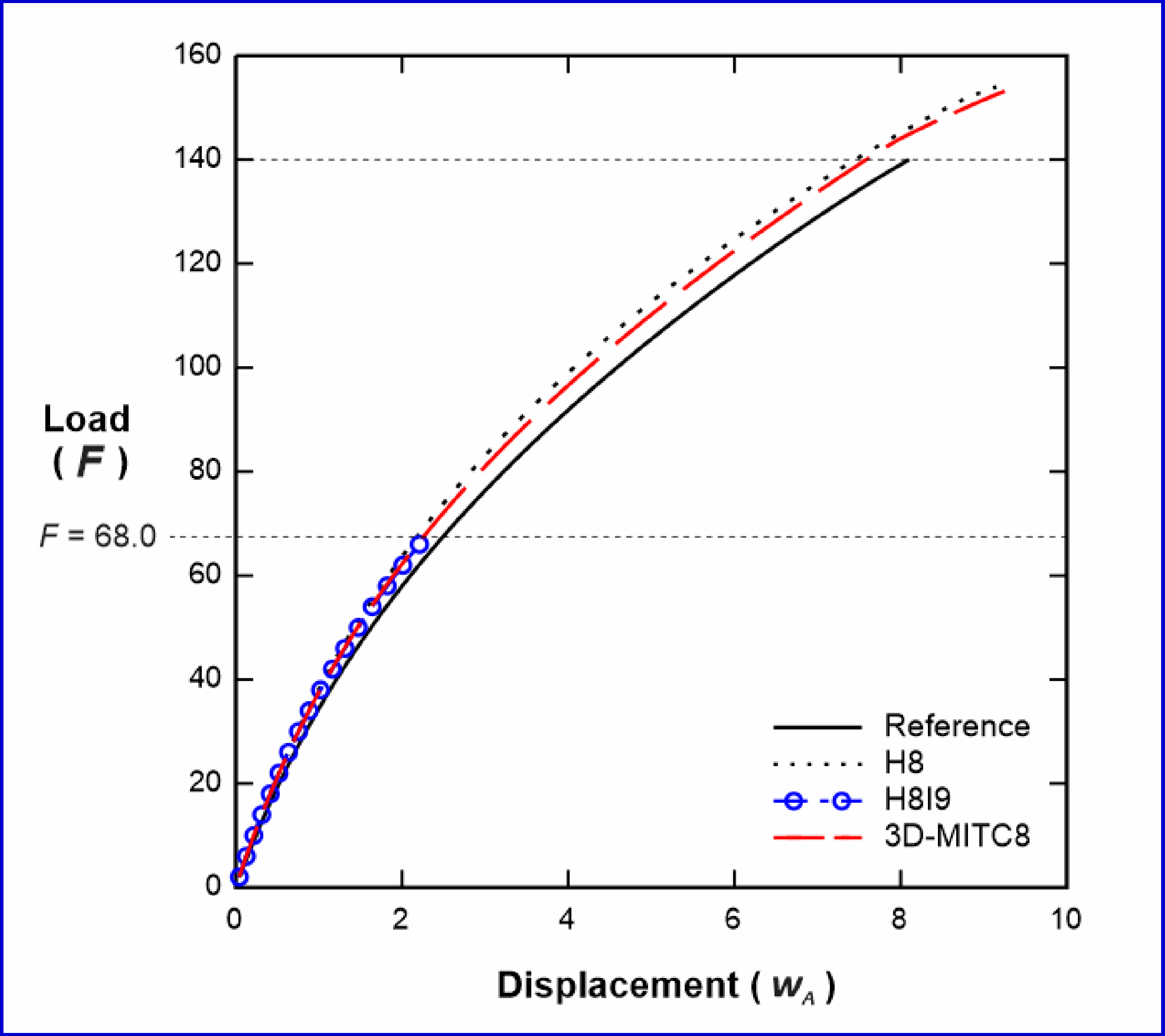
Analysis of clamped plate, regular meshes



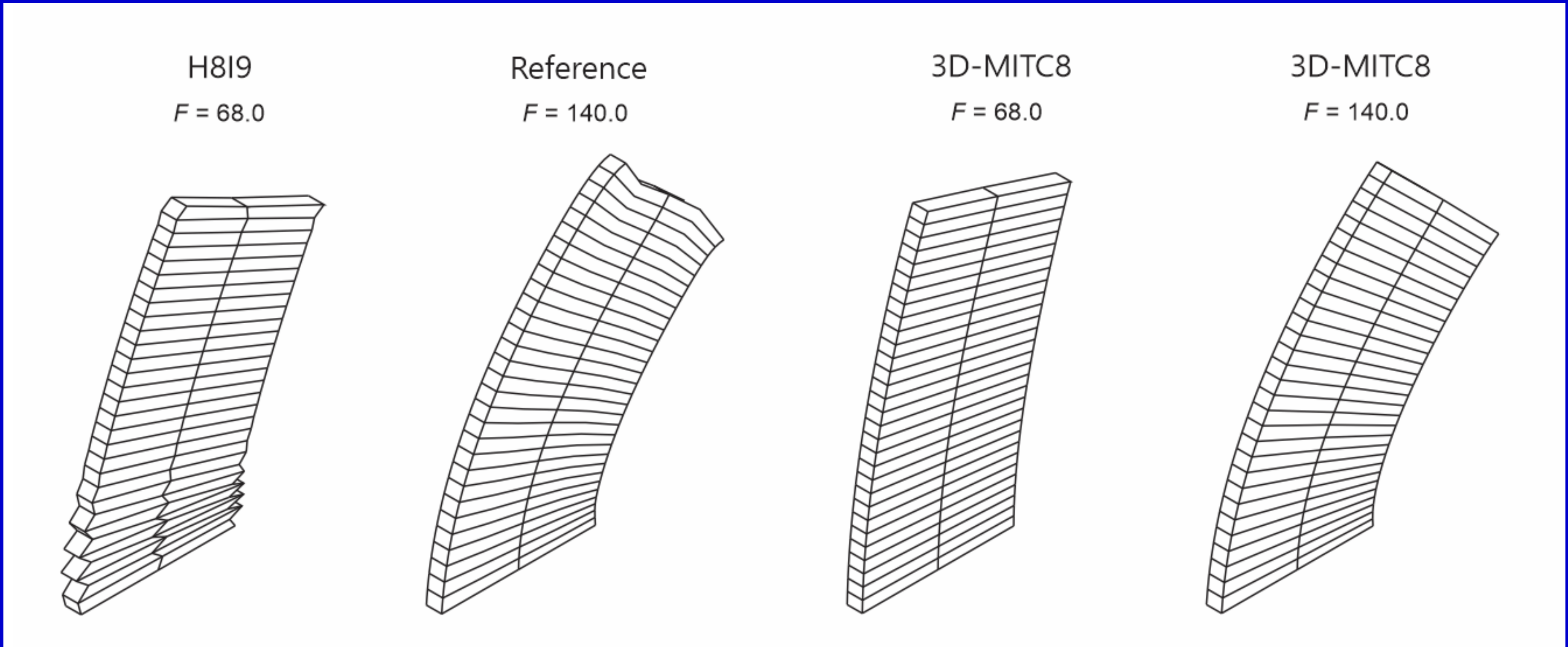
Analysis of clamped plate, distorted meshes



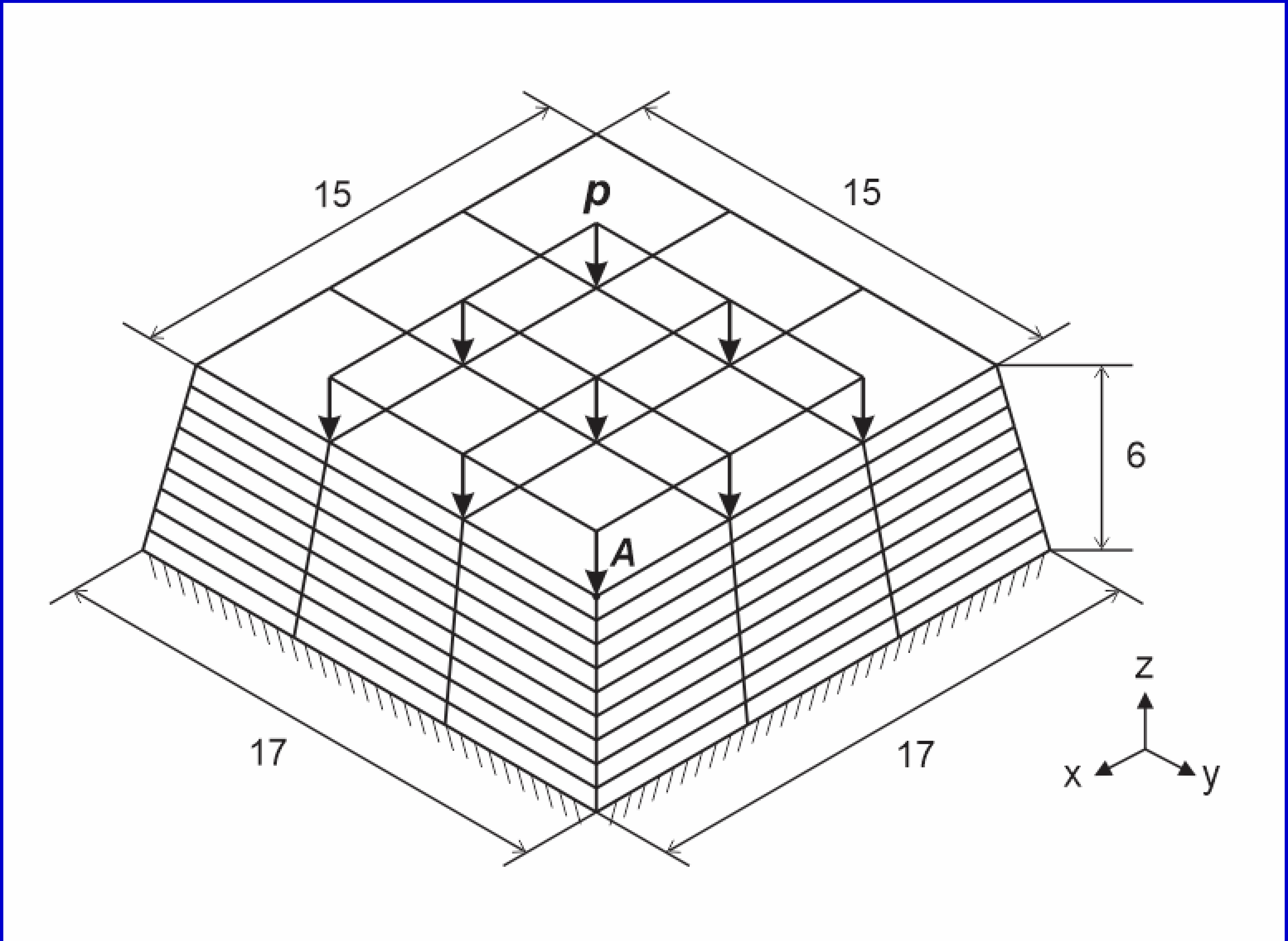
Large displacement analysis of cantilever



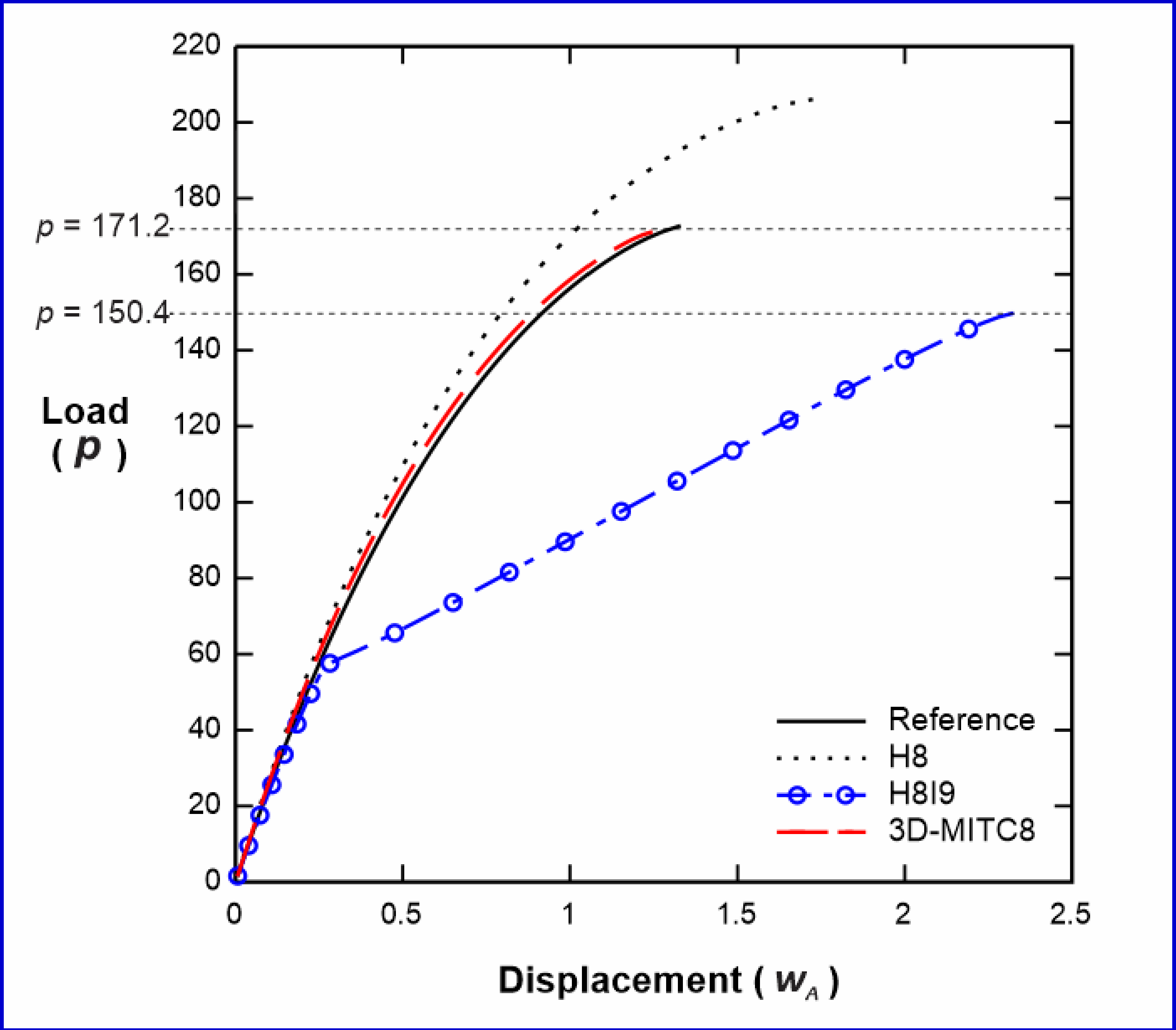
Response of cantilever



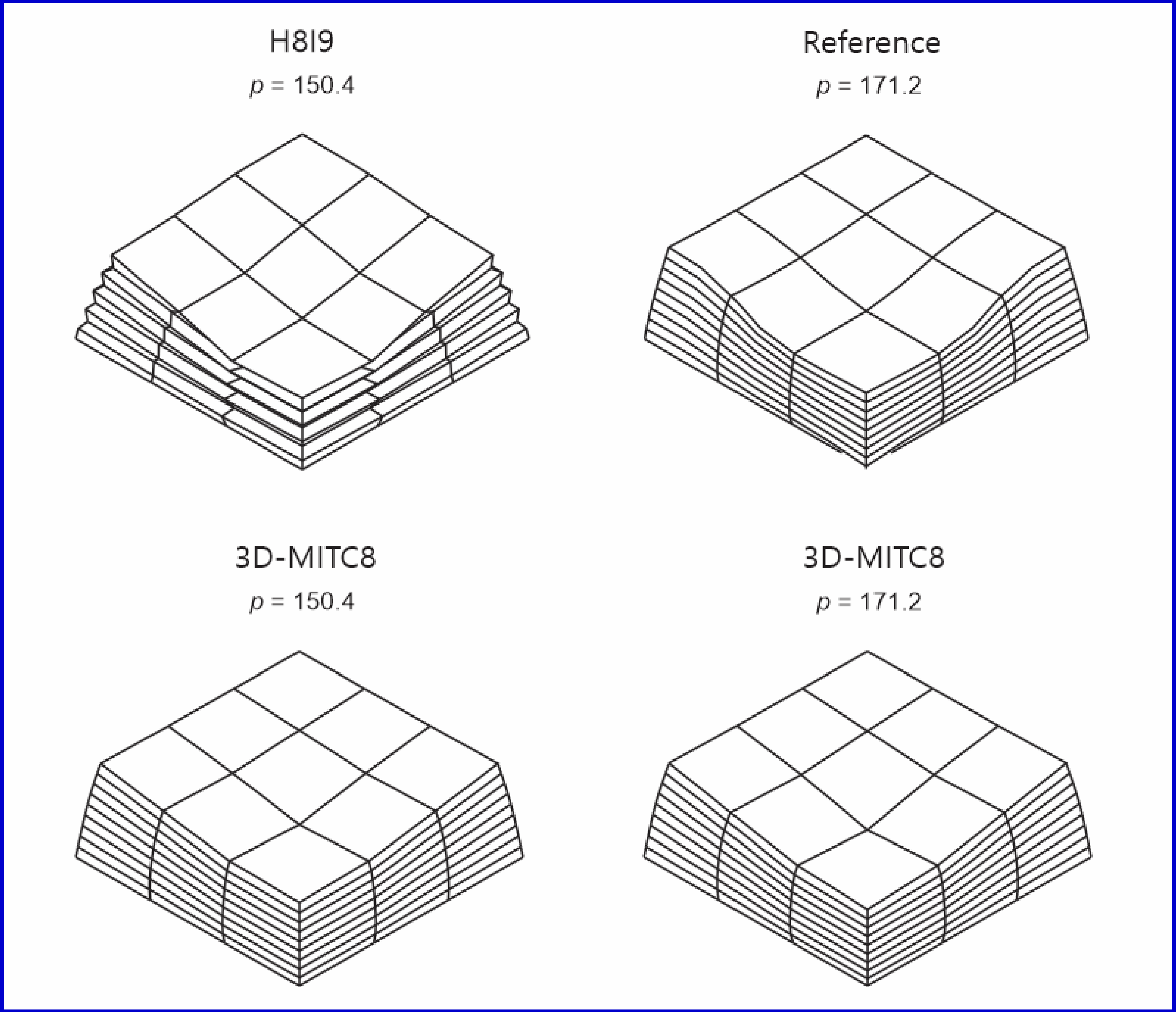
Deformed shapes of cantilever



Analysis of rubber block



Response of rubber block



Deformed shapes of rubber block

Inf-sup testing of elements

We want $\exists \gamma_A > 0$

$$\inf_{\mathbf{w}_h \in V_h} \sup_{\mathbf{v}_h \in V_h} \frac{\mathbf{w}_h{}^T \mathbf{A}_h \mathbf{v}_h}{\|\mathbf{w}_h\| \|\mathbf{v}_h\|} \geq \gamma_A$$

and find the smallest eigenvalue of

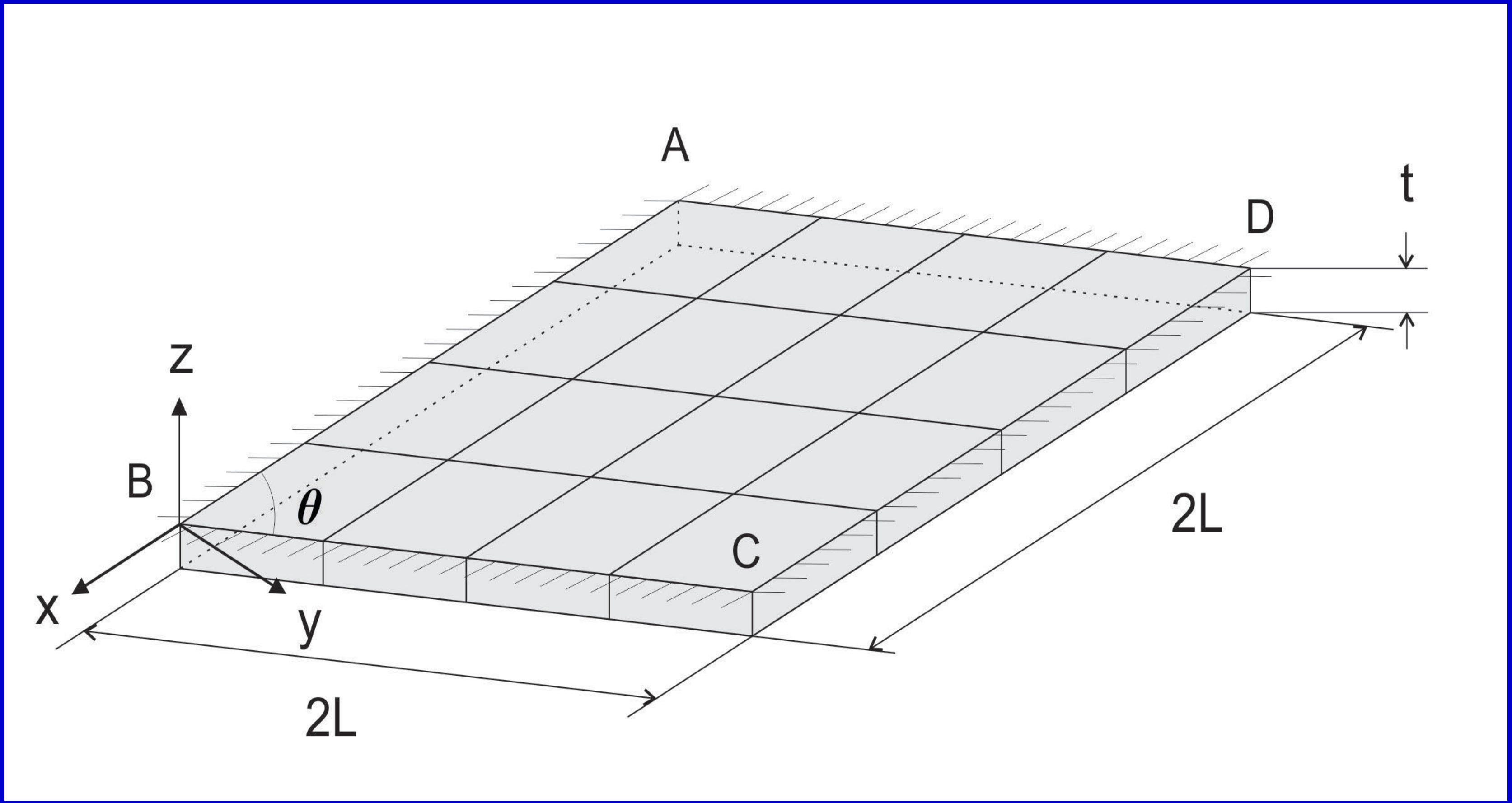
$$\mathbf{A}_h \mathbf{v}_h = \lambda_k \mathbf{G}_h \mathbf{v}_h$$

Y Ko, KJ Bathe. Inf-sup testing of … . C & S 2018

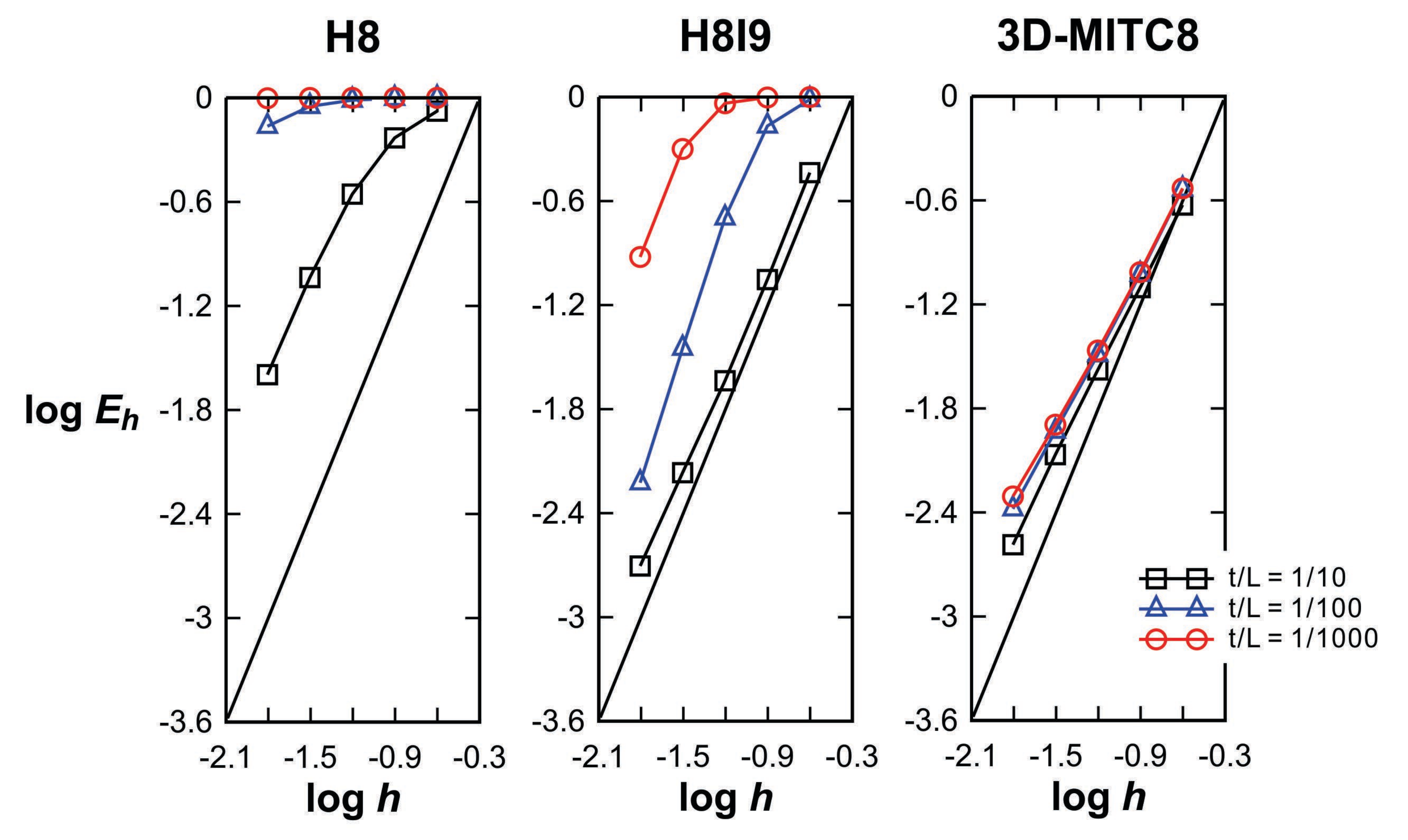
Since $\mathbf{A}_h$ is the stiffness matrix (not the matrix of the bilinear form B, which we do not have explicitly) we really test for coercivity

But we still deem to get insight into the stability of the discretization scheme

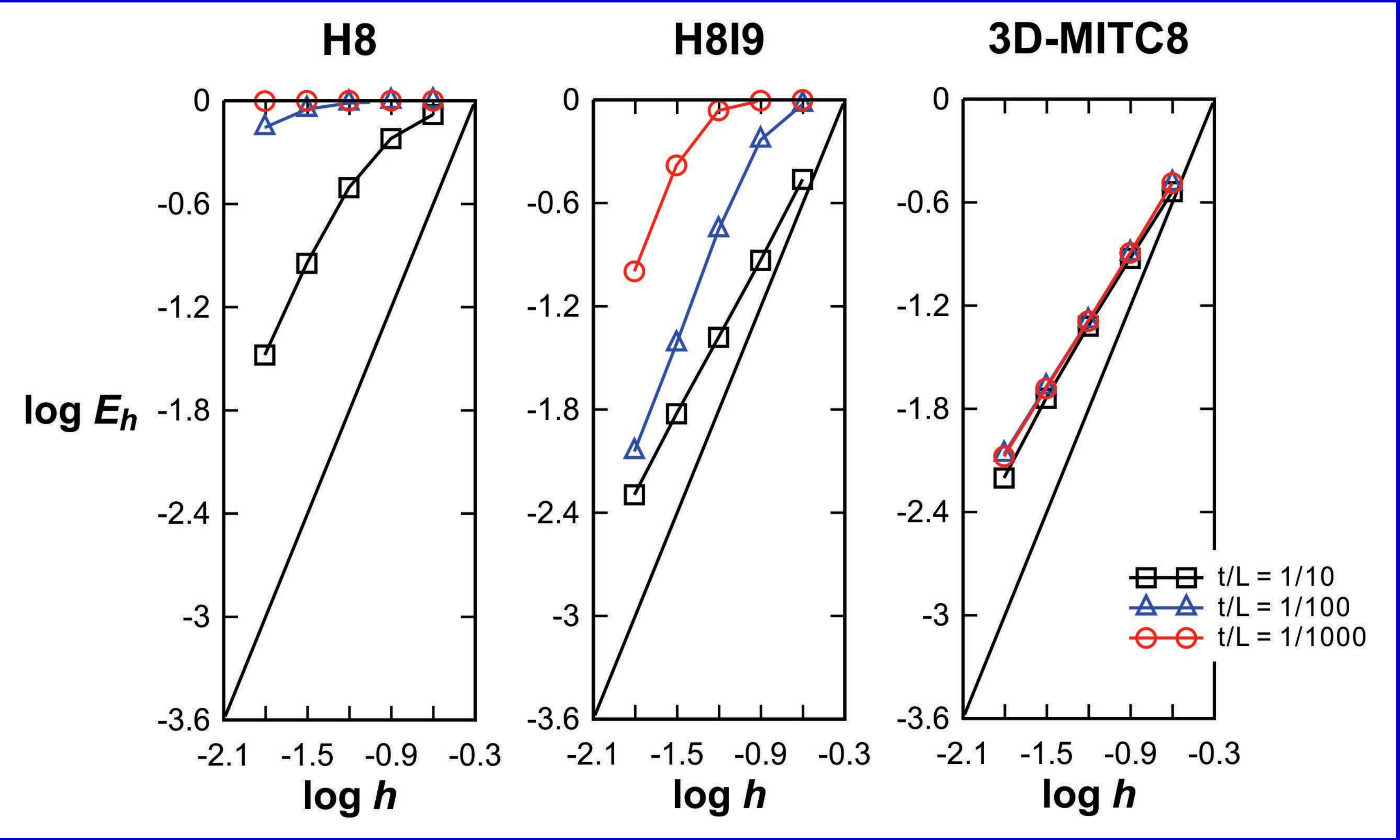
The numerical results show that indeed we do obtain good insight, but more mathematical analysis would be valuable



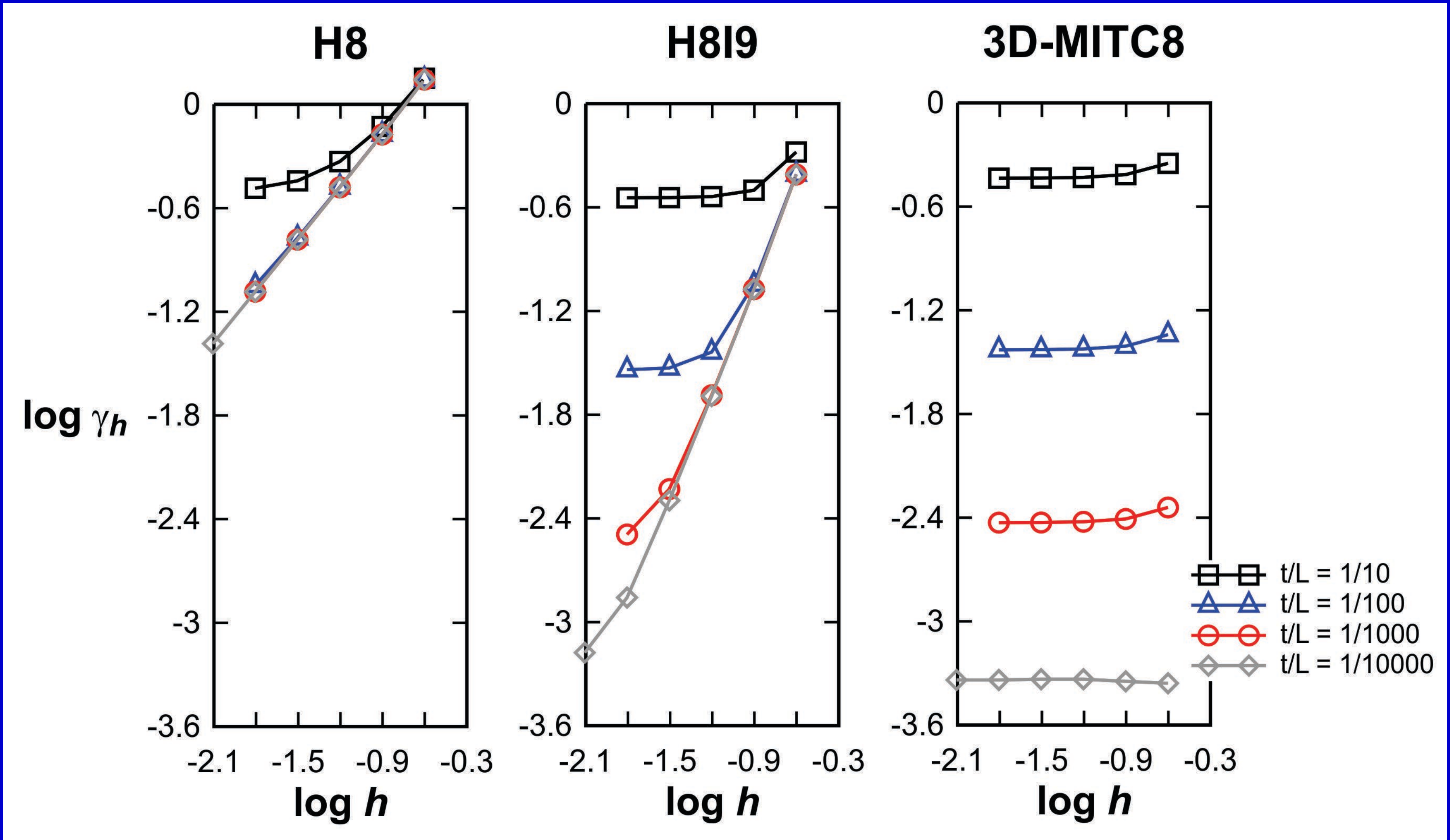
Clamped skew plate, angle of skew θ



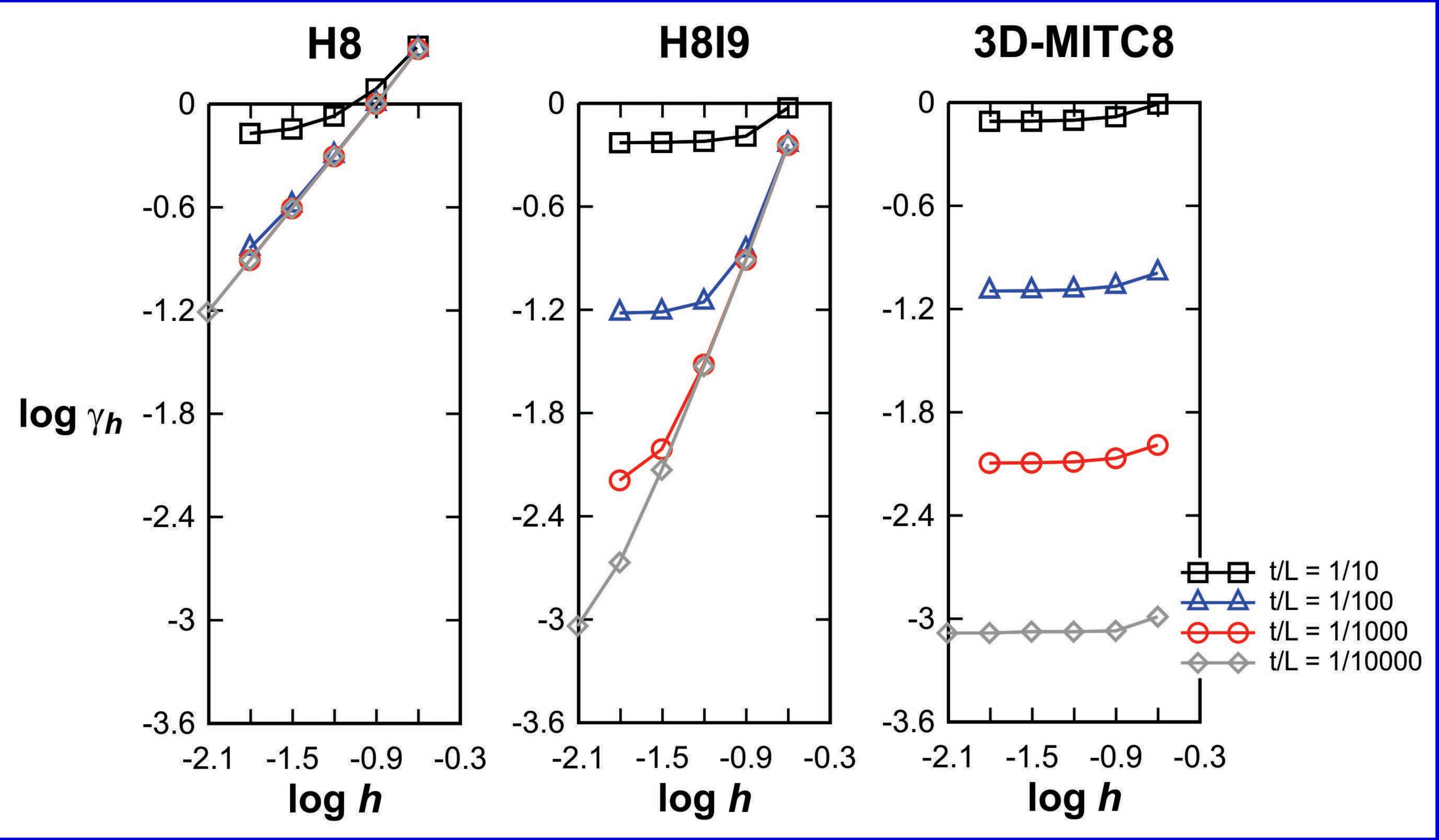
s-norm for applied uniform pressure, θ = 60 degrees



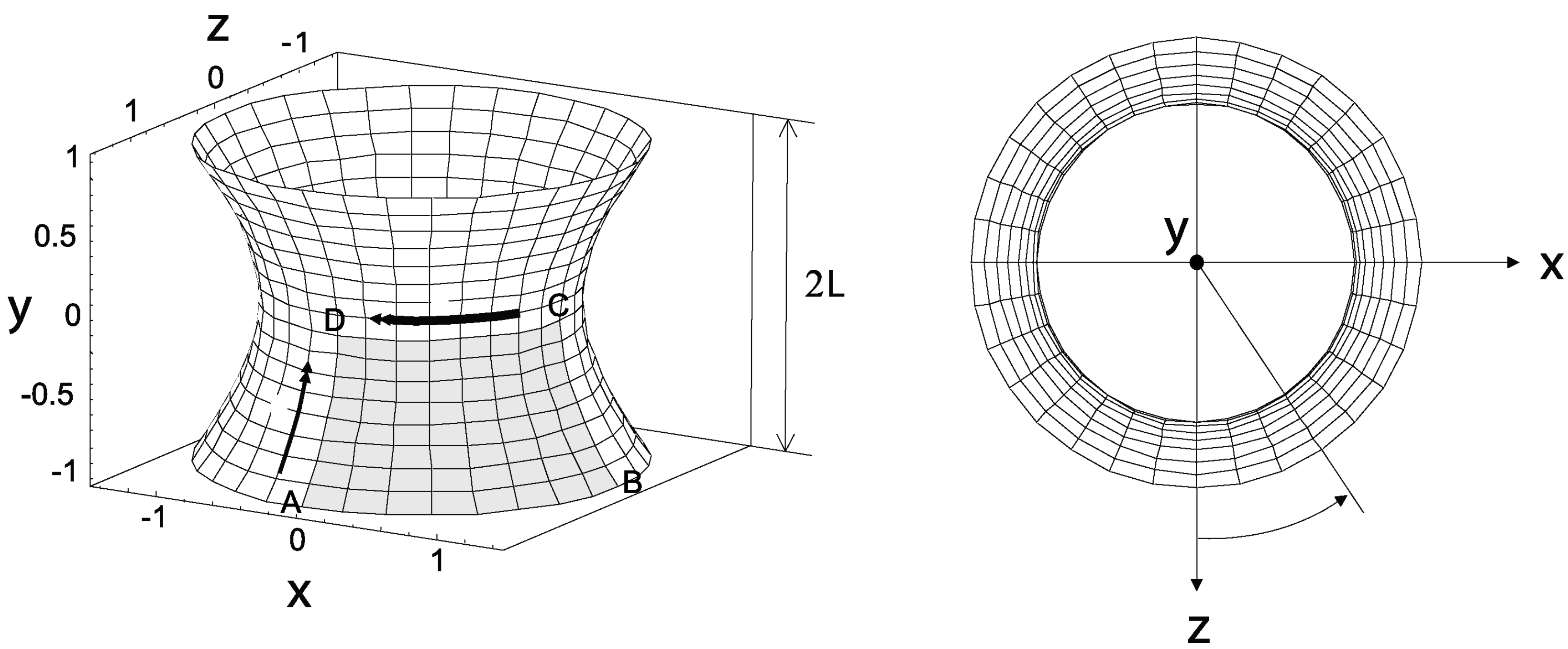
s-norm for applied uniform pressure, θ = 30 degrees



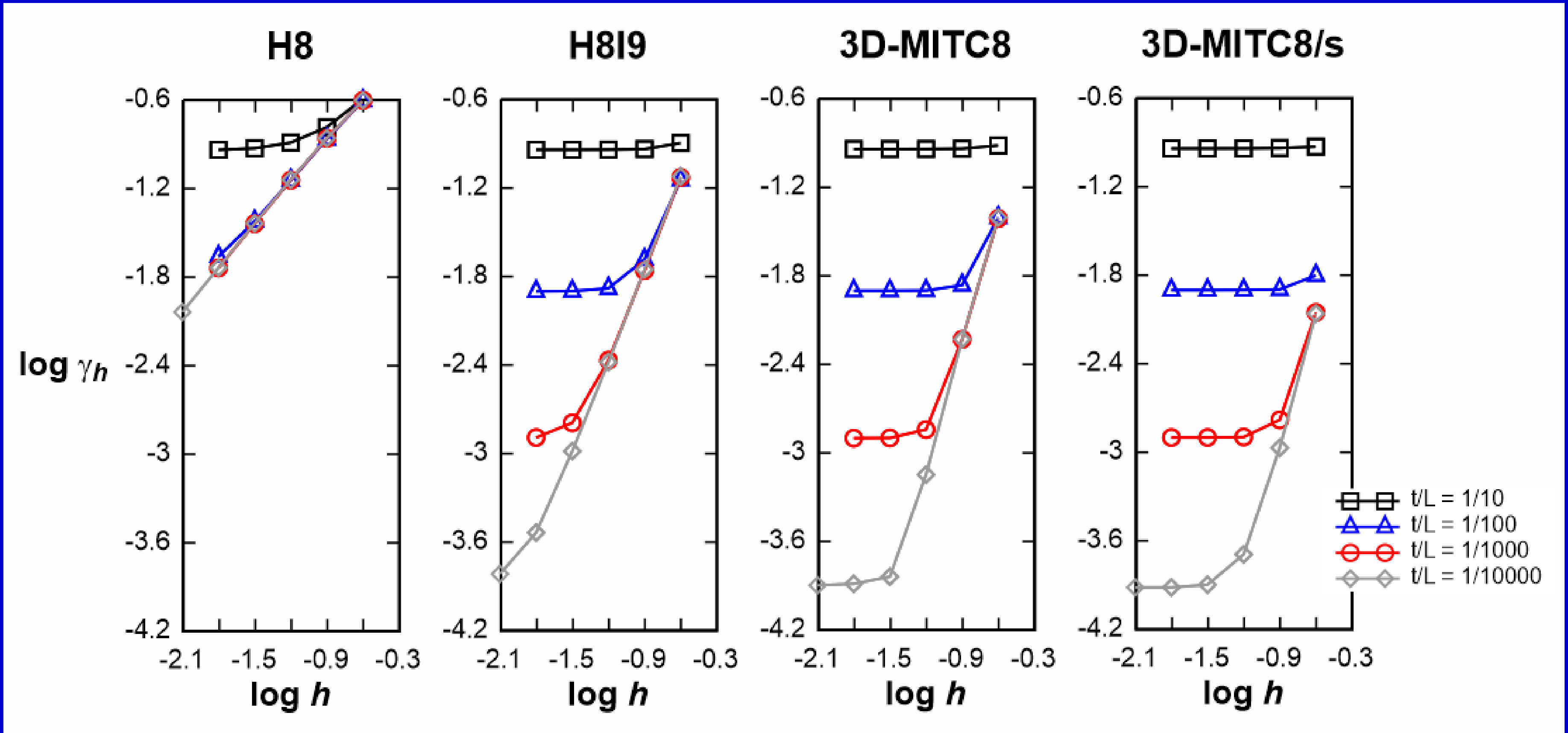
Inf-sup test, θ = 60 degrees



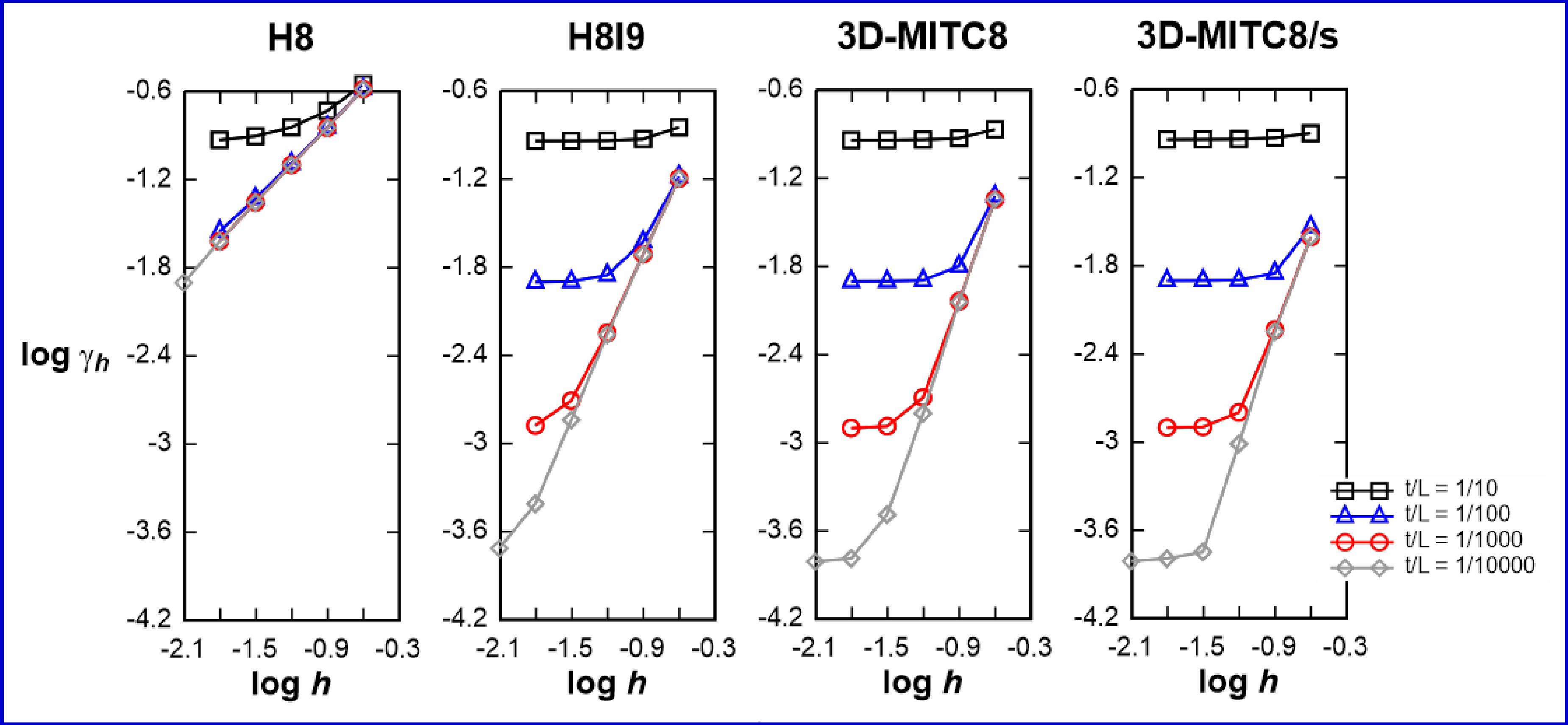
Inf-sup test, θ = 30 degrees



The hyperbolic shell test problem, free ends



Hyperbolic shell inf-sup test, regular meshes



Hyperbolic shell inf-sup test, distorted meshes

The enriched subspace iteration method

We are interested in solving:

$$\mathbf{K}\boldsymbol{\varphi}_i = \lambda_i \mathbf{M}\boldsymbol{\varphi}_i$$

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_p$$

with *p* = 100, 200, 300, …

$$\mathbf{X}_k = \left[\mathbf{\Phi}_k, \mathbf{X}_k^a, \mathbf{X}_k^b \right]$$

$$\mathbf{K}\overline{\mathbf{X}}_{k+1}^a = \mathbf{M}\mathbf{X}_k^a$$

$$\overline{\mathbf{X}}_{k+1} = \left[\mathbf{\Phi}_k, \overline{\mathbf{X}}_{k+1}^a, \overline{\mathbf{Y}}_{k+1} \right]$$

Iteration vectors

KT Kim, KJ Bathe. The Bathe subspace … . C & S 2017

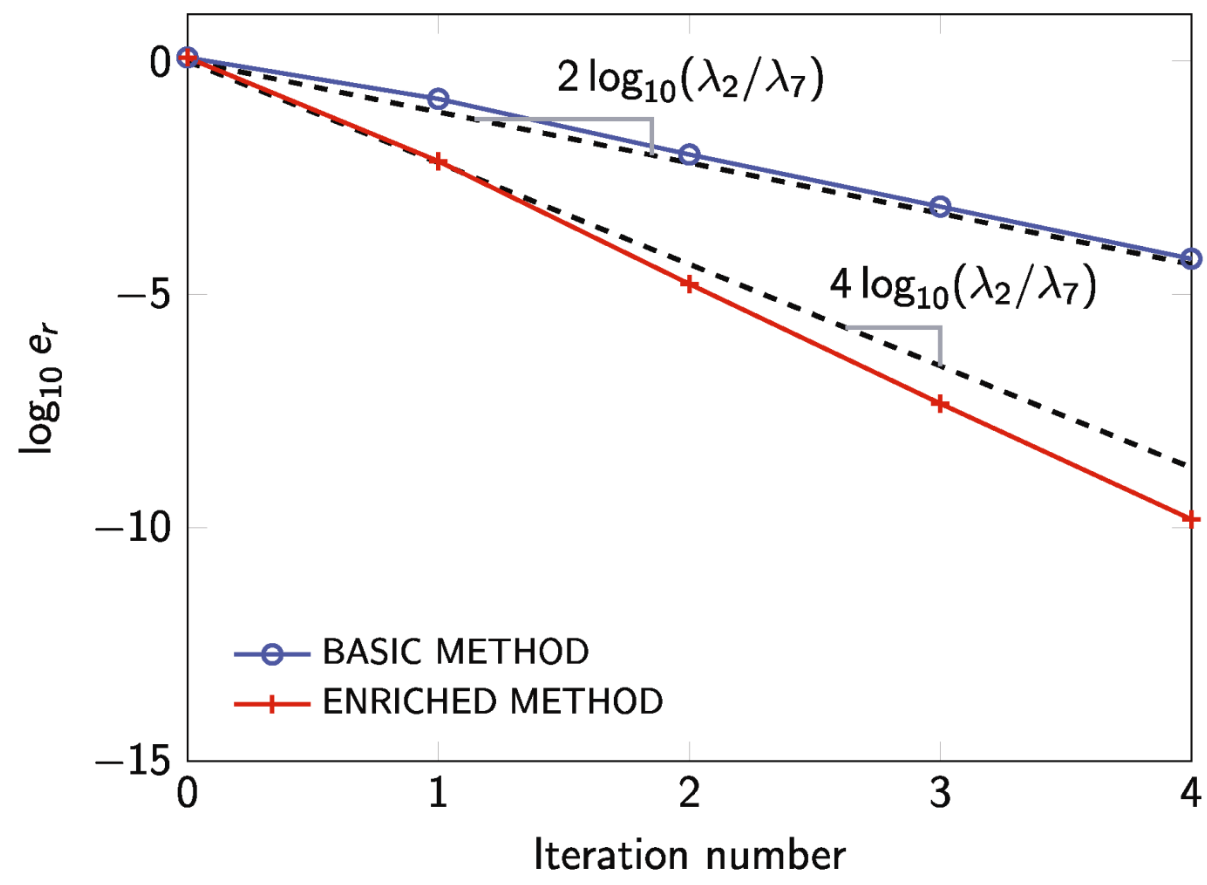
$$\mathbf{K}_{k+1} = \overline{\mathbf{X}}_{k+1}^{\mathrm{T}} \mathbf{K} \overline{\mathbf{X}}_{k+1}$$

$$\mathbf{M}_{k+1} = \overline{\mathbf{X}}_{k+1}^{\mathrm{T}} \mathbf{M} \overline{\mathbf{X}}_{k+1}$$

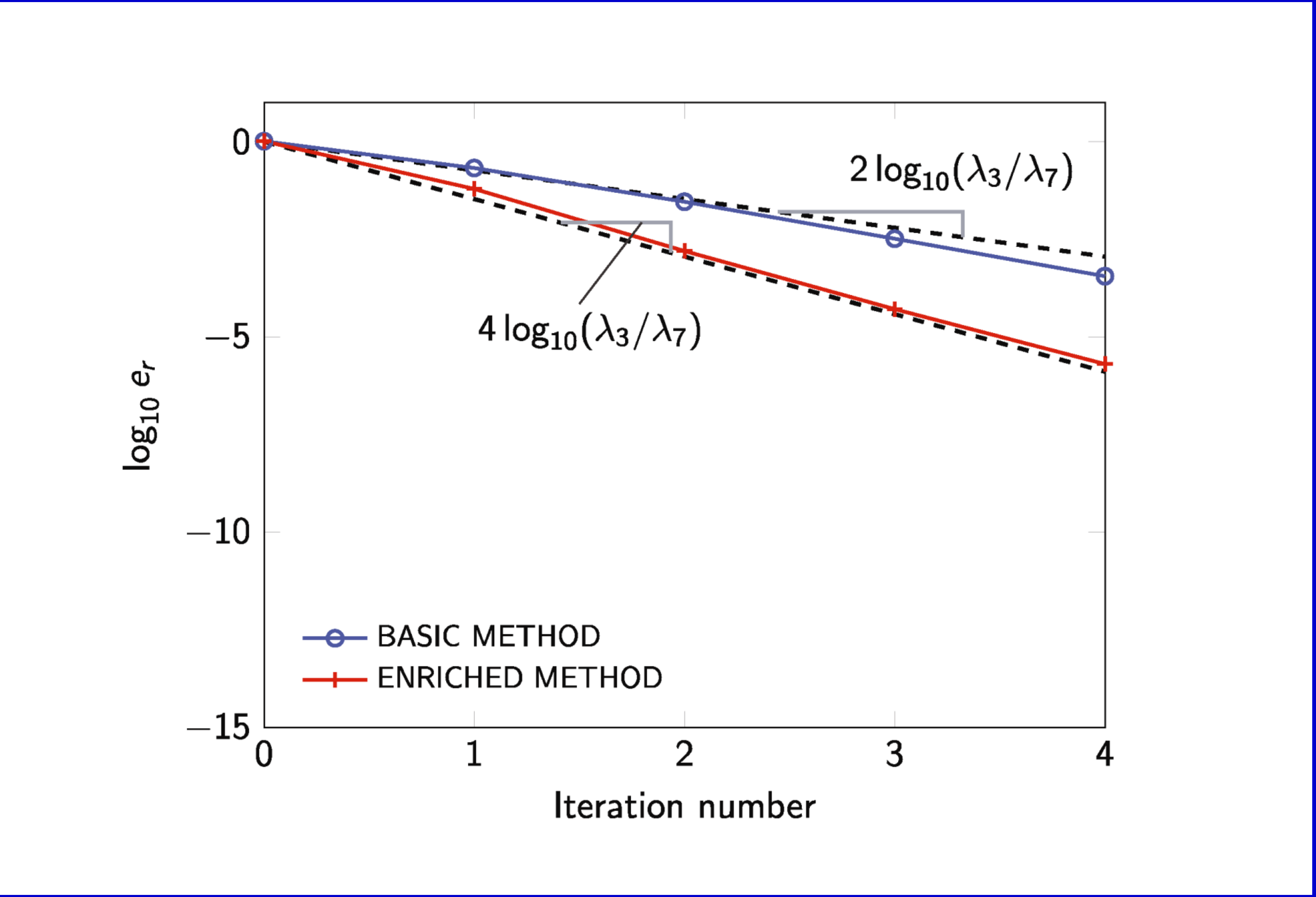
$$\mathbf{K}_{k+1} \mathbf{Q}_{k+1} = \mathbf{M}_{k+1} \mathbf{Q}_{k+1} \mathbf{\Lambda}_{k+1}$$

$$\mathbf{X}_{k+1} = \overline{\mathbf{X}}_{k+1} \mathbf{Q}_{k+1}$$

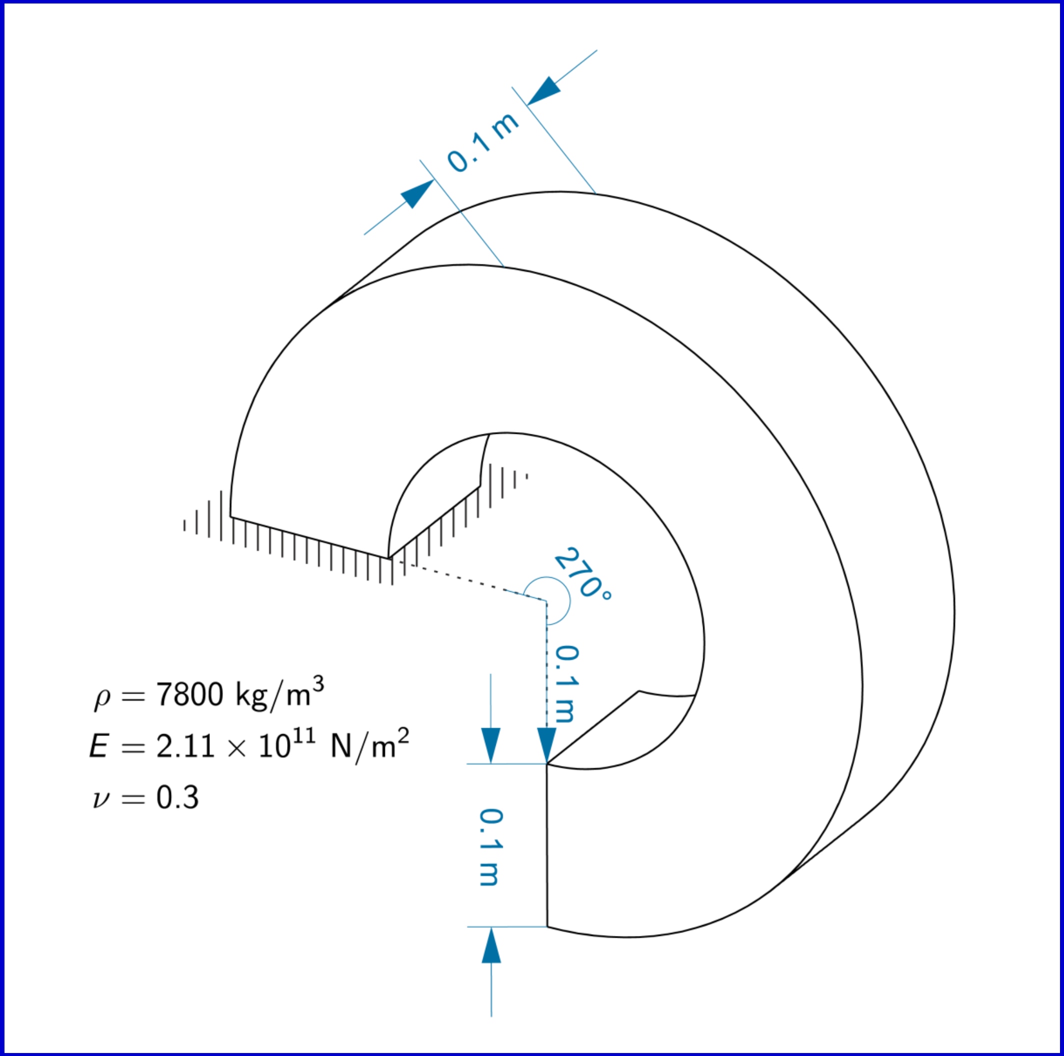
Update of iteration vectors



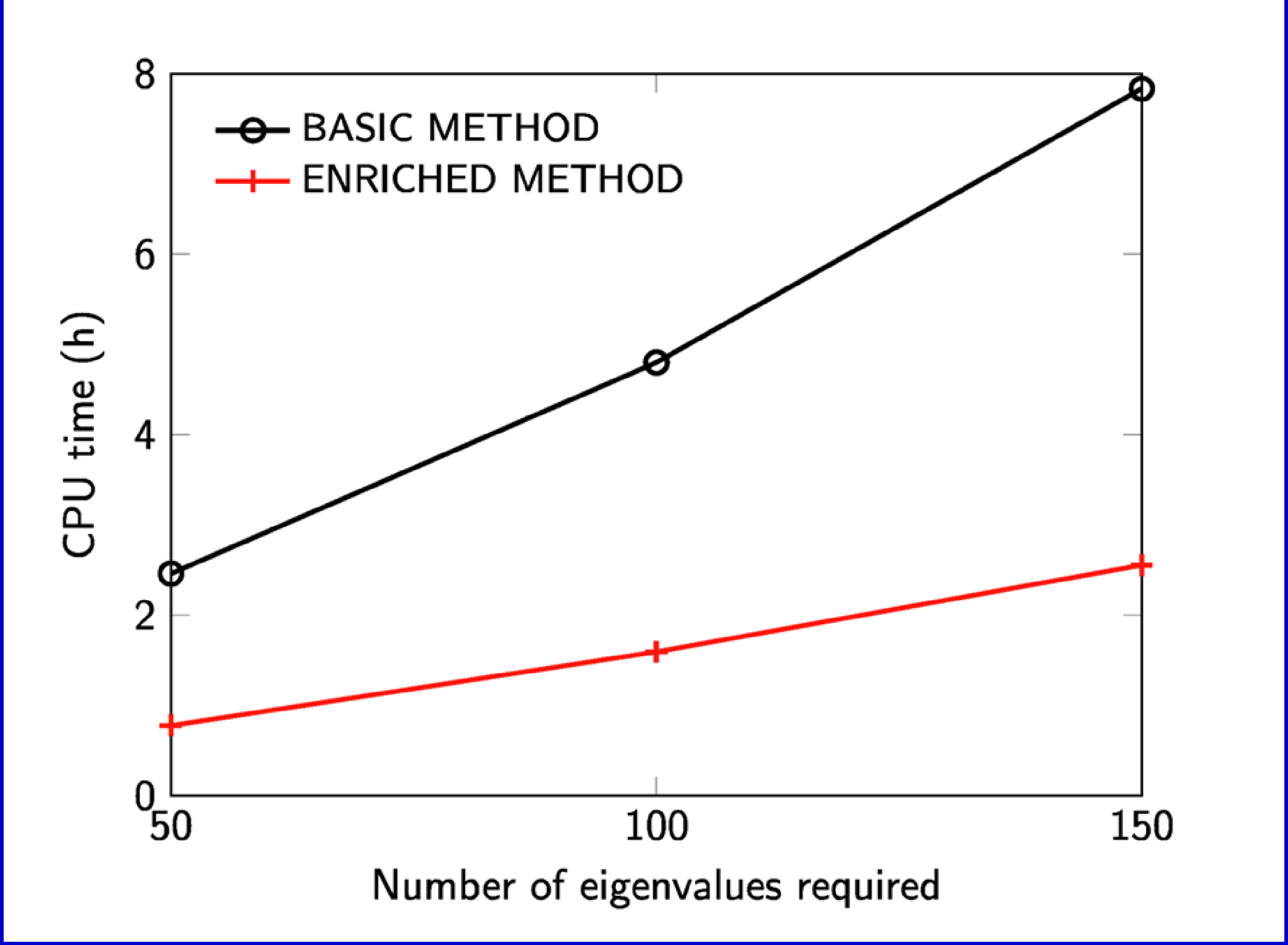
Convergence rates of eigenvalues



Convergence rates of eigenvalues



Solution of ring cantilever structure



Solution times for 3D ring structure,
~ 600,000 DOF, m ~ 1,600; 1 core m/c, 2.4 GHz

Model of exhaust manifold from Volvo, NEQ = 2,220,273, w/ contact, 16-core SMP

(on a DELL 2-proc. computer, each w/ 12 cores, Linux)

Number of freq./ vectors	Time used (min)
50	7
100	12
200	37

Almost linear increase in solution times

Model of tractor front end from John Deere, NEQ = 966,755, 16-core SMP

(on a DELL 2-proc. computer, each w/ 12 cores, Linux)

Number of freq./ vectors	Time used (min)
100	4
150	7
200	11
300	23

Note – no DMP used in solutions, although the subspace iteration parallelizes well in DMP

Time integration: implicit integration

Use 2 sub-steps per time step, with splitting ratio γ

Trapezoidal rule, case $\gamma = 0.5$

$$\mathbf{M}\,{}^{t+\Delta t/2}\ddot{\mathbf{U}} + \mathbf{C}\,{}^{t+\Delta t/2}\dot{\mathbf{U}} = {}^{t+\Delta t/2}\mathbf{R} - {}^{t+\Delta t/2}\mathbf{F}$$

$${}^{t+\Delta t/2}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \frac{\Delta t}{4}\left({}^{t}\ddot{\mathbf{U}} + {}^{t+\Delta t/2}\ddot{\mathbf{U}}\right)$$

$${}^{t+\Delta t/2}\mathbf{U} = {}^{t}\mathbf{U} + \frac{\Delta t}{4}\left({}^{t}\dot{\mathbf{U}} + {}^{t+\Delta t/2}\dot{\mathbf{U}}\right)$$

In general γ can be varied, known as Bathe method

Euler backward method:

$$\mathbf{M}\ {}^{t+\Delta t}\ddot{\mathbf{U}} + \mathbf{C}\ {}^{t+\Delta t}\dot{\mathbf{U}} = {}^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F}$$

$${}^{t+\Delta t}\dot{\mathbf{U}} = \frac{1}{\Delta t}\,{}^{t}\mathbf{U} - \frac{4}{\Delta t}\,{}^{t+\Delta t/2}\mathbf{U} + \frac{3}{\Delta t}\,{}^{t+\Delta t}\mathbf{U}$$

$${}^{t+\Delta t}\ddot{\mathbf{U}} = \frac{1}{\Delta t}\,{}^{t}\dot{\mathbf{U}} - \frac{4}{\Delta t}\,{}^{t+\Delta t/2}\dot{\mathbf{U}} + \frac{3}{\Delta t}\,{}^{t+\Delta t}\dot{\mathbf{U}}$$

Each of these "ingredients" widely known for decades, the valuable point is to use them together in one time step and study the performance of the scheme …

Properties:

-- no parameter to adjust, simply the time step has to be sufficiently small for accuracy

-- solves in nonlinear analysis when the TR fails

-- excellent accuracy characteristics

Effective in the analysis of problems in structural dynamics and wave propagations ---
but some researchers want to adjust the AD, PE

Various endeavors have been published

Effect of time step splitting --

- In linear analysis the use of $\gamma = (2 - \sqrt{2})$ means just one coefficient matrix (at max AD, min PE but almost the same as for the case $\gamma = 0.5$)

- In nonlinear analysis, different γ values might be used but $\gamma = 0.5$ is reasonable considering the range of general nonlinear analyses

For the 1st sub-step

$$ {}^{t+\gamma\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + \left(\frac{\gamma\Delta t}{2}\right)\left({}^{t}\dot{\mathbf{U}} + {}^{t+\gamma\Delta t}\dot{\mathbf{U}}\right) $$

$$ {}^{t+\gamma\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \left(\frac{\gamma\Delta t}{2}\right)\left({}^{t}\ddot{\mathbf{U}} + {}^{t+\gamma\Delta t}\ddot{\mathbf{U}}\right) $$

and accordingly for the 2nd sub-step.

For different values of γ we obtain different PE and AD,

G Noh, KJ Bathe. Further insights … . C & S 2018

But --- using 2 important natural parameters we can work with the splitting ratio γ and the spectral radius ρ_∞ for the 2nd sub-step:

$$ {}^{t+\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + \Delta t\,(q_0\,{}^{t}\dot{\mathbf{U}} + q_1\,{}^{t+\gamma\Delta t}\dot{\mathbf{U}} + q_2\,{}^{t+\Delta t}\dot{\mathbf{U}}) $$

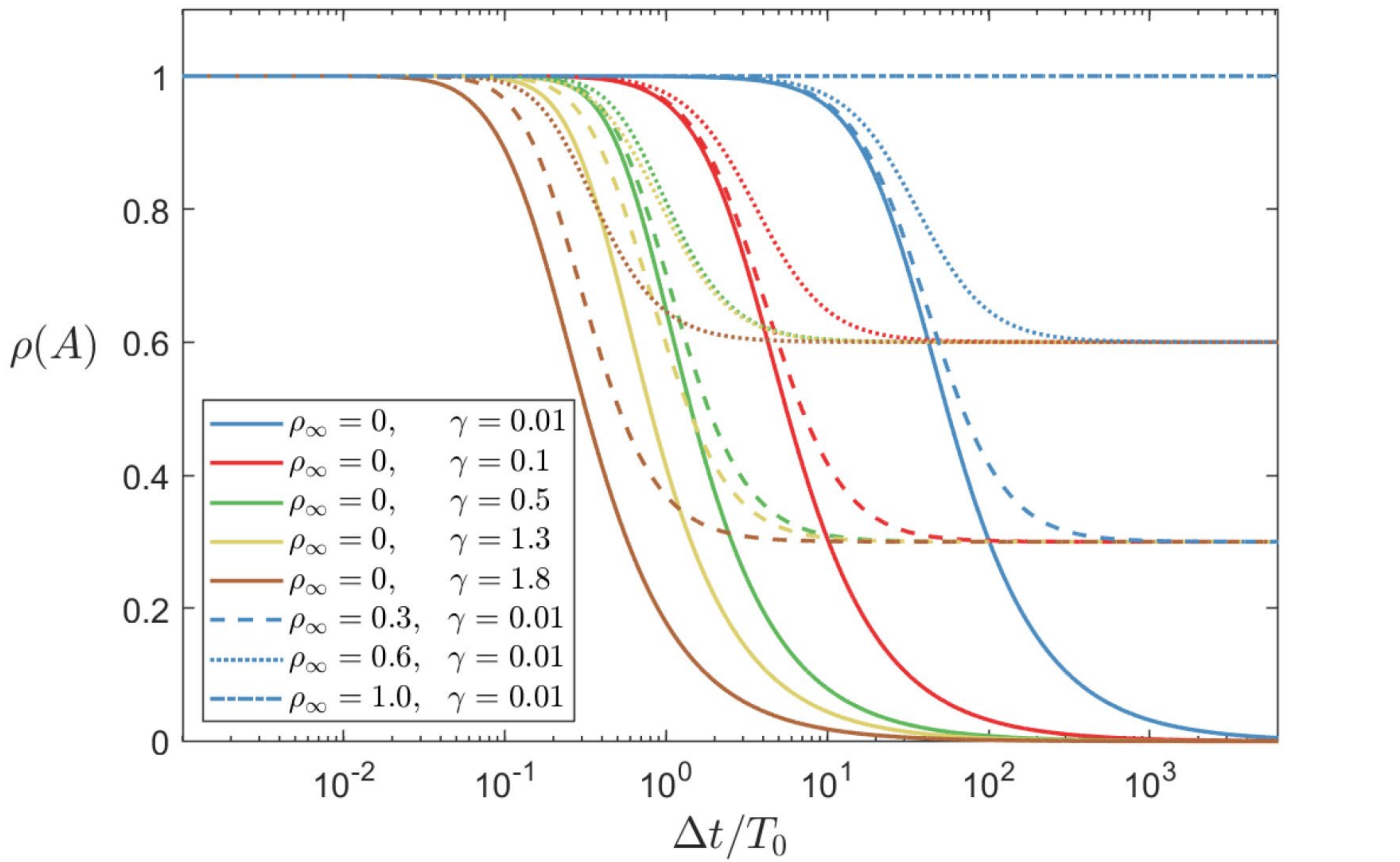
$$ {}^{t+\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \Delta t\,(q_0\,{}^{t}\ddot{\mathbf{U}} + q_1\,{}^{t+\gamma\Delta t}\ddot{\mathbf{U}} + q_2\,{}^{t+\Delta t}\ddot{\mathbf{U}}) $$

with

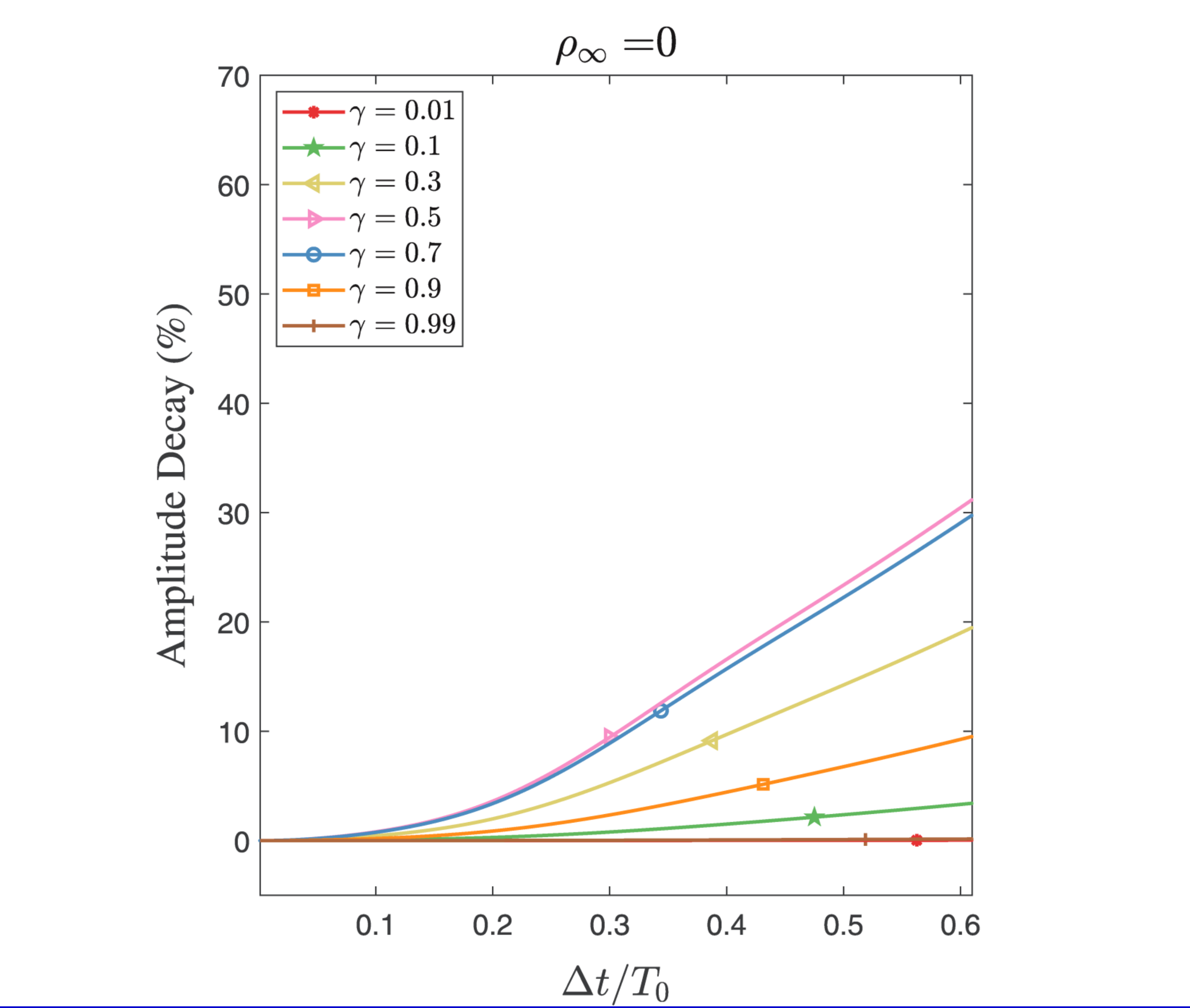
$$ q_0 = (\gamma - 1)q_1 + \frac{1}{2}; \quad q_2 = -\gamma q_1 + \frac{1}{2} $$

$$ q_1 = \frac{\rho_\infty + 1}{2\gamma(\rho_\infty - 1) + 4} \qquad \gamma \in \mathbb{R} \mid \gamma \neq 0, \gamma \neq 1 $$

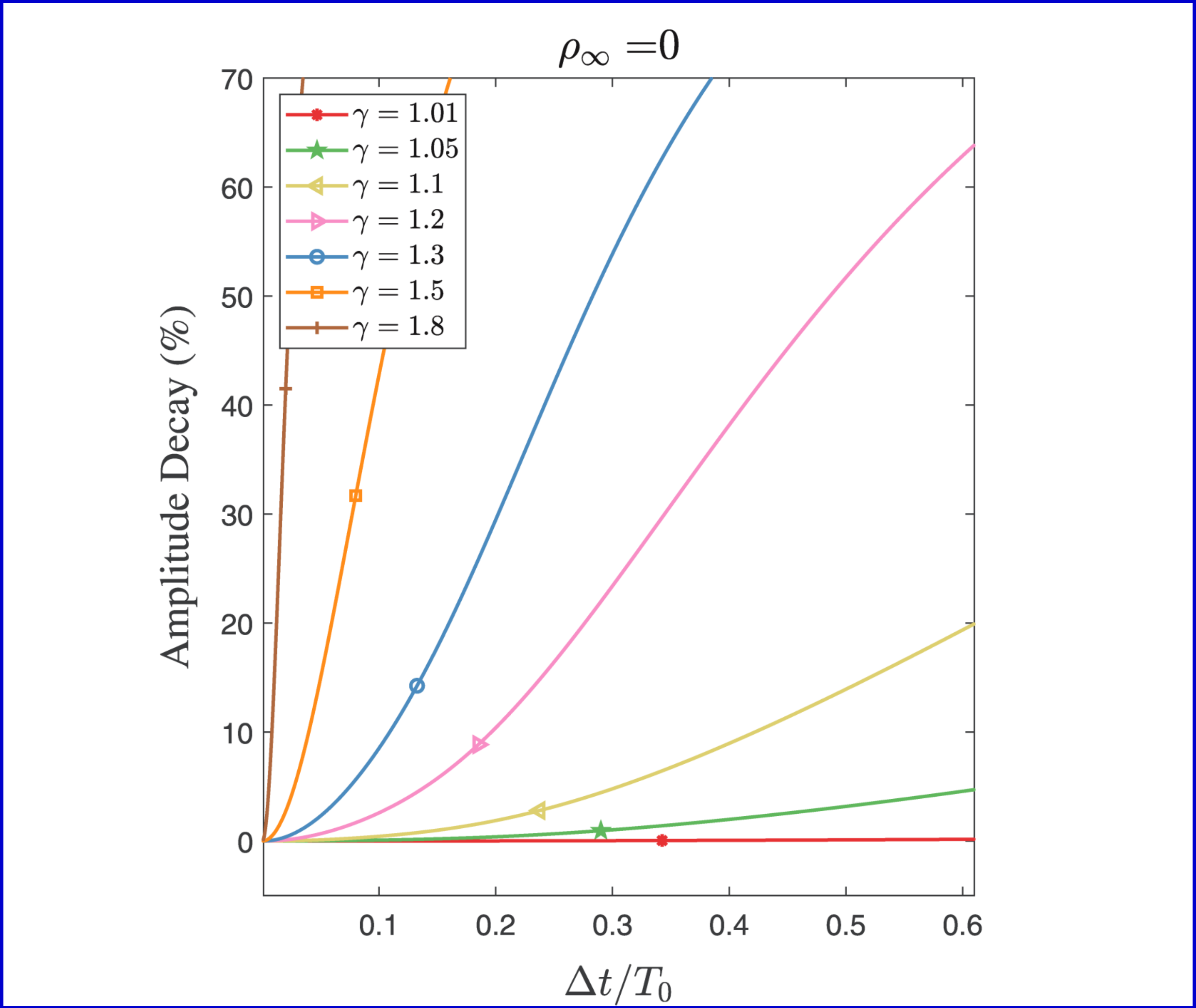
G Noh and KJ Bathe. The Bathe Time … C & S 2019



Spectral radii for different values of parameters



Amplitude decay



Amplitude decay, the case $\gamma > 1$ can be important

The scheme is the standard Bathe method when $\rho_\infty = 0$ and γ is varying

It is the TR for each sub-step when $\rho_\infty = 1$

Further, the scheme is unconditionally stable and second-order accurate

In some analyses, the use of the 2 parameters can be valuable -- but we only need to use ρ_∞ as a parameter and can use the corresponding optimal γ. Hence the method is a 1-parameter scheme using usually $\rho_\infty = 0$

Another way to proceed --

use $\gamma = 0.5$ and for the 2nd sub-step:

$$
\begin{aligned}
{}^{t+\Delta t}\mathbf{U} = \\
{}^{t}\mathbf{U} + \left(0.5\Delta t\right)\left((1-\beta_1)\,{}^{t}\dot{\mathbf{U}} + \beta_1\,{}^{t+\gamma\Delta t}\dot{\mathbf{U}}\right) \\
+\left(0.5\Delta t\right)\left((1-\beta_2)\,{}^{t+\gamma\Delta t}\dot{\mathbf{U}} + \beta_2\,{}^{t+\Delta t}\dot{\mathbf{U}}\right)
\end{aligned}
$$

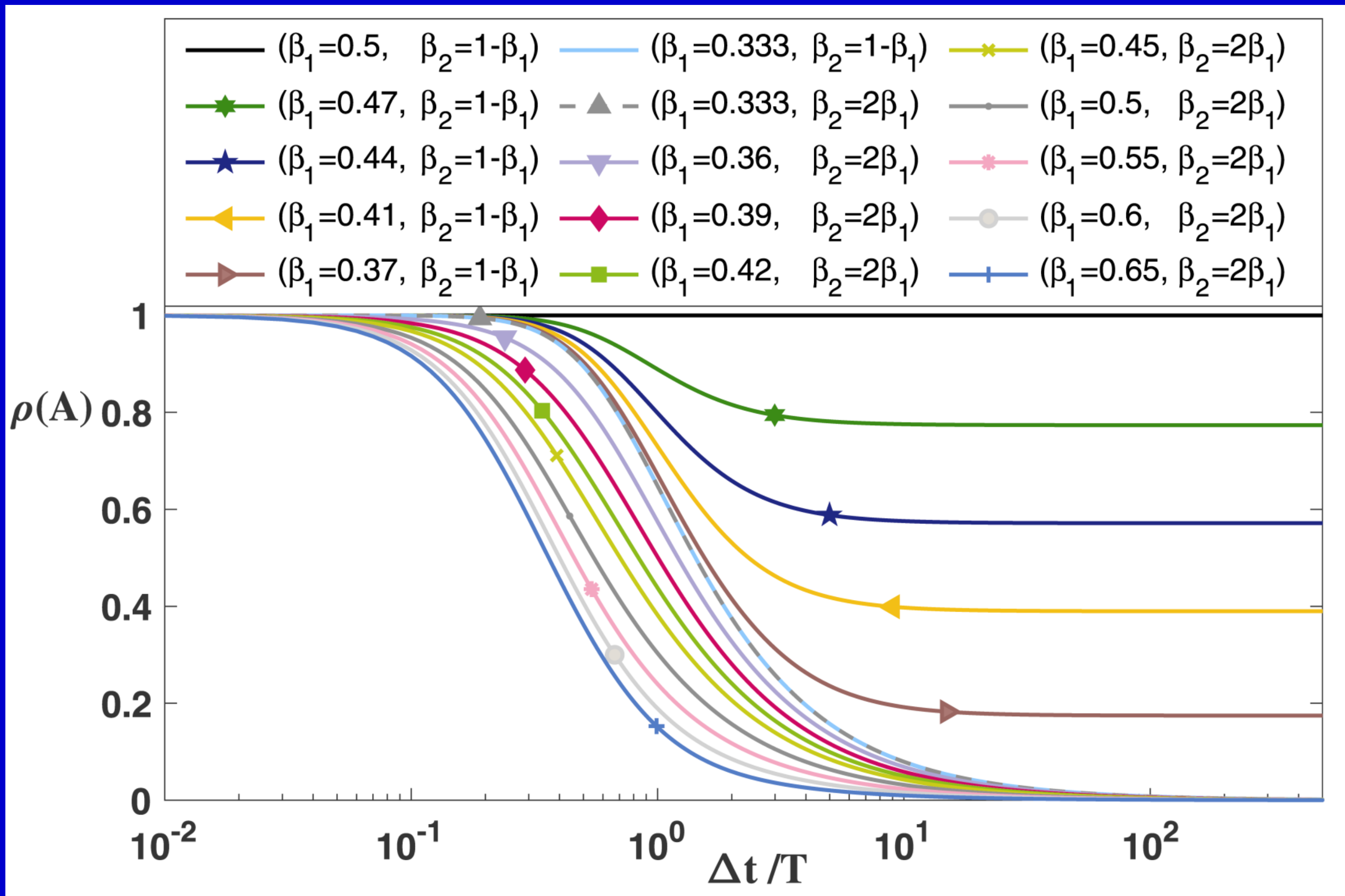
Here β_1 and β_2 are parameters to obtain different AD and PE

Assumption for the velocity --

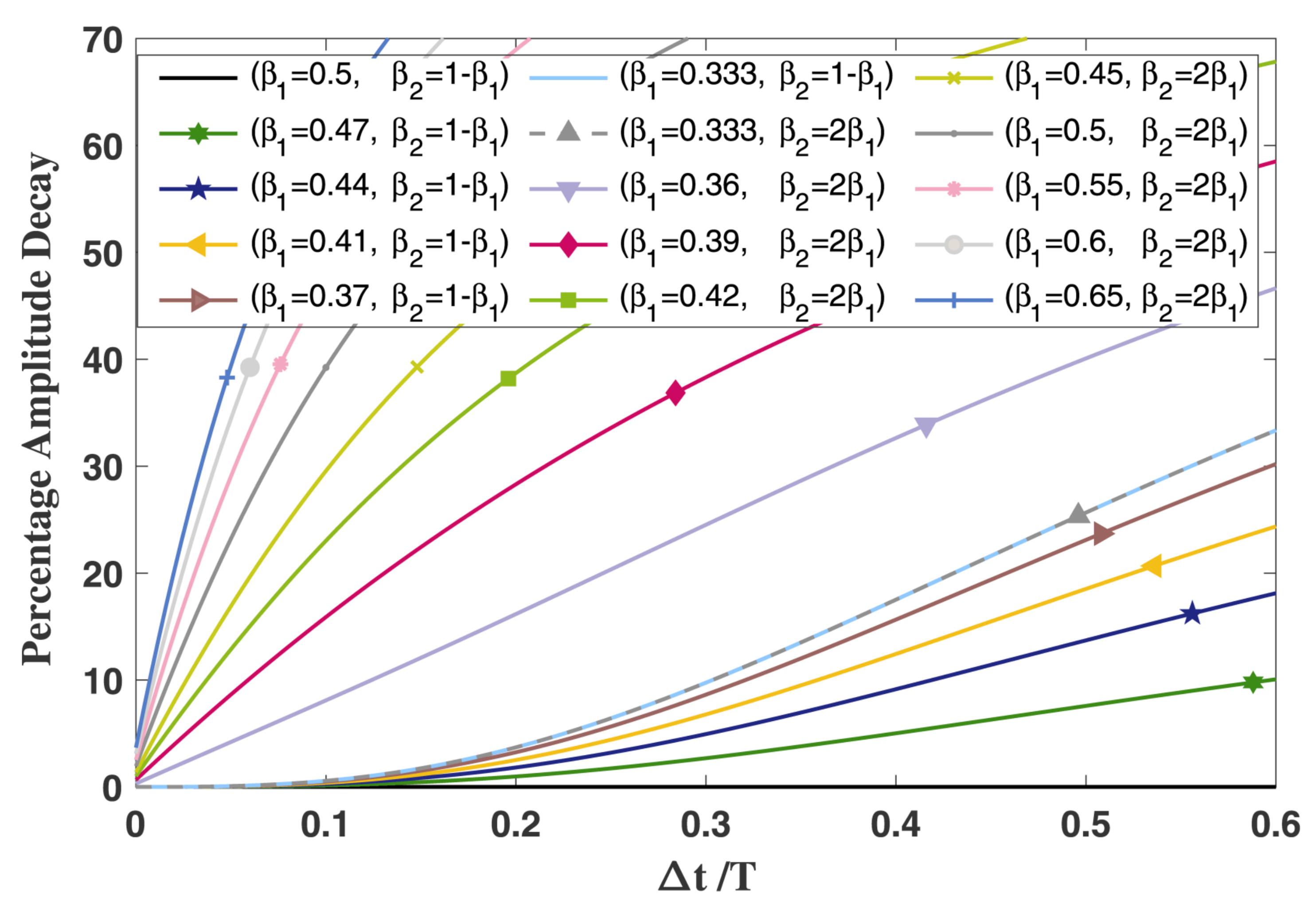
$$
{}^{t+\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + (0.5\Delta t)\left((1-\beta_1)\,{}^{t}\ddot{\mathbf{U}} + \beta_1\,{}^{t+\gamma\Delta t}\ddot{\mathbf{U}}\right) + (0.5\Delta t)\left((1-\beta_2)\,{}^{t+\gamma\Delta t}\ddot{\mathbf{U}} + \beta_2\,{}^{t+\Delta t}\ddot{\mathbf{U}}\right)
$$

Again, the β_1 and β_2 are parameters to obtain different AD and PE . This scheme is a special case of the ρ_∞ - Bathe scheme

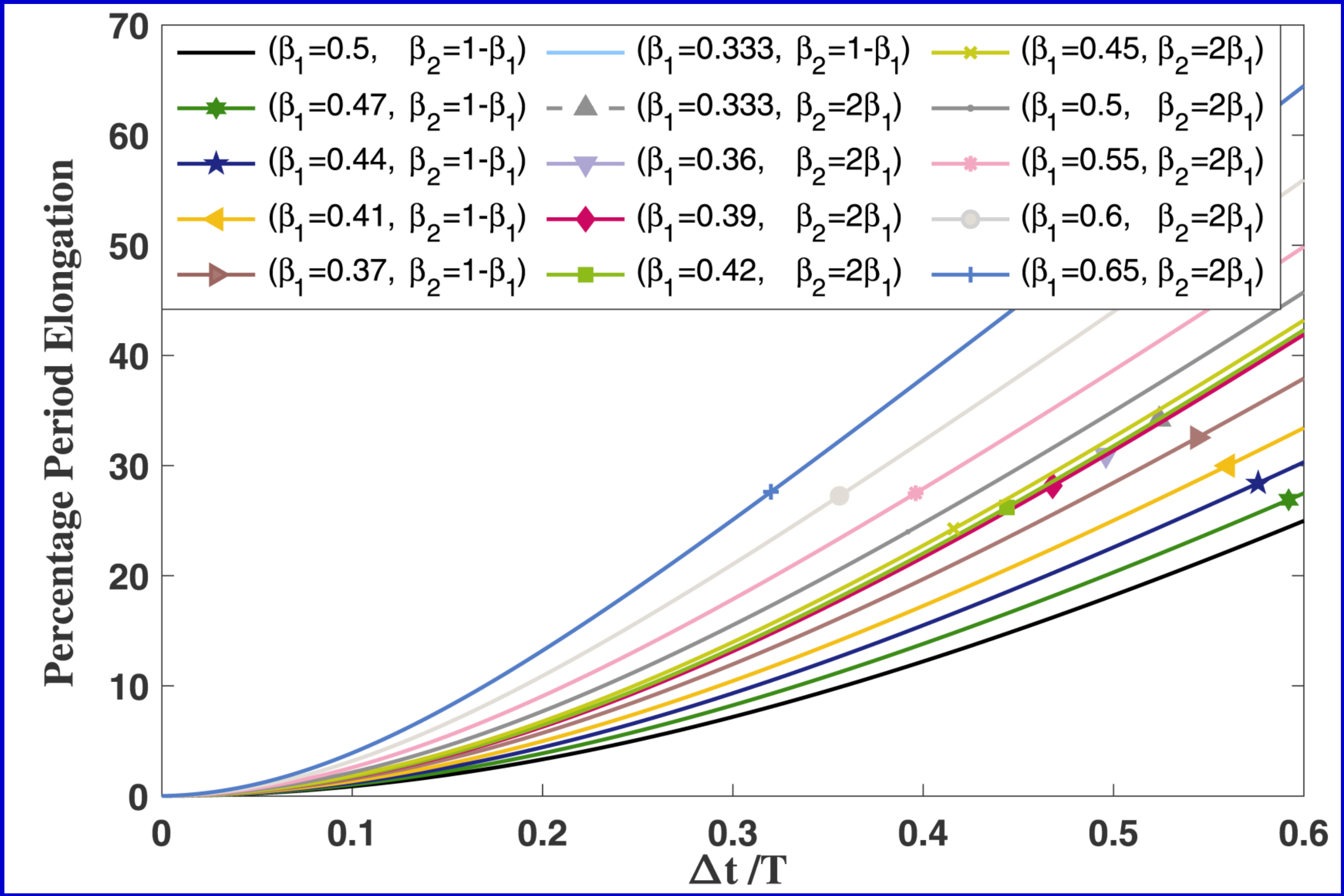
MM Malakiyeh, S Shojaee, KJ Bathe. The Bathe … . C & S 2019



Spectral radii, include TR and Bathe std. scheme

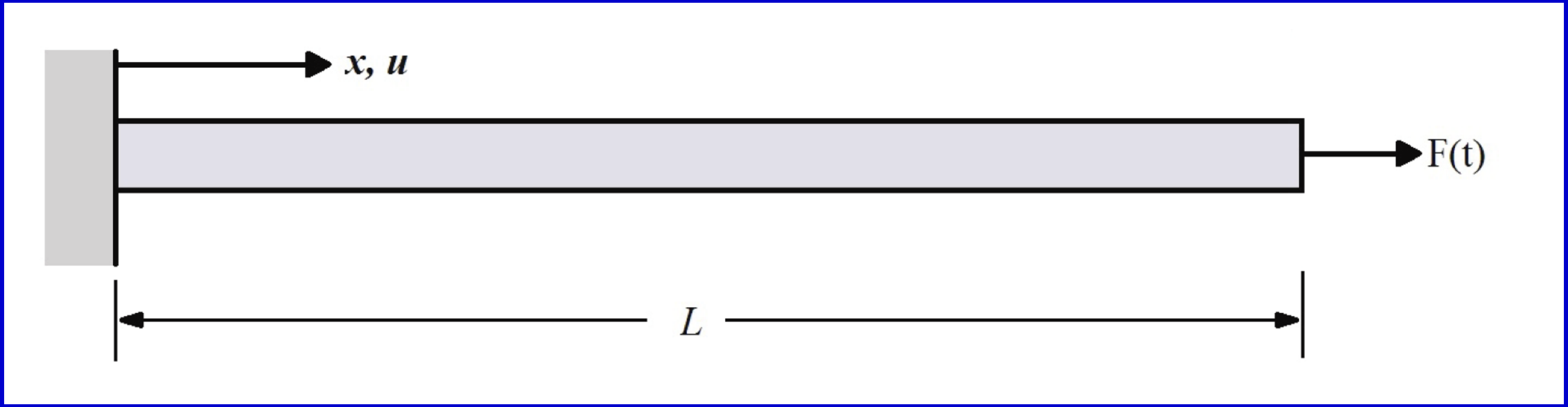


Amplitude decay (AD)

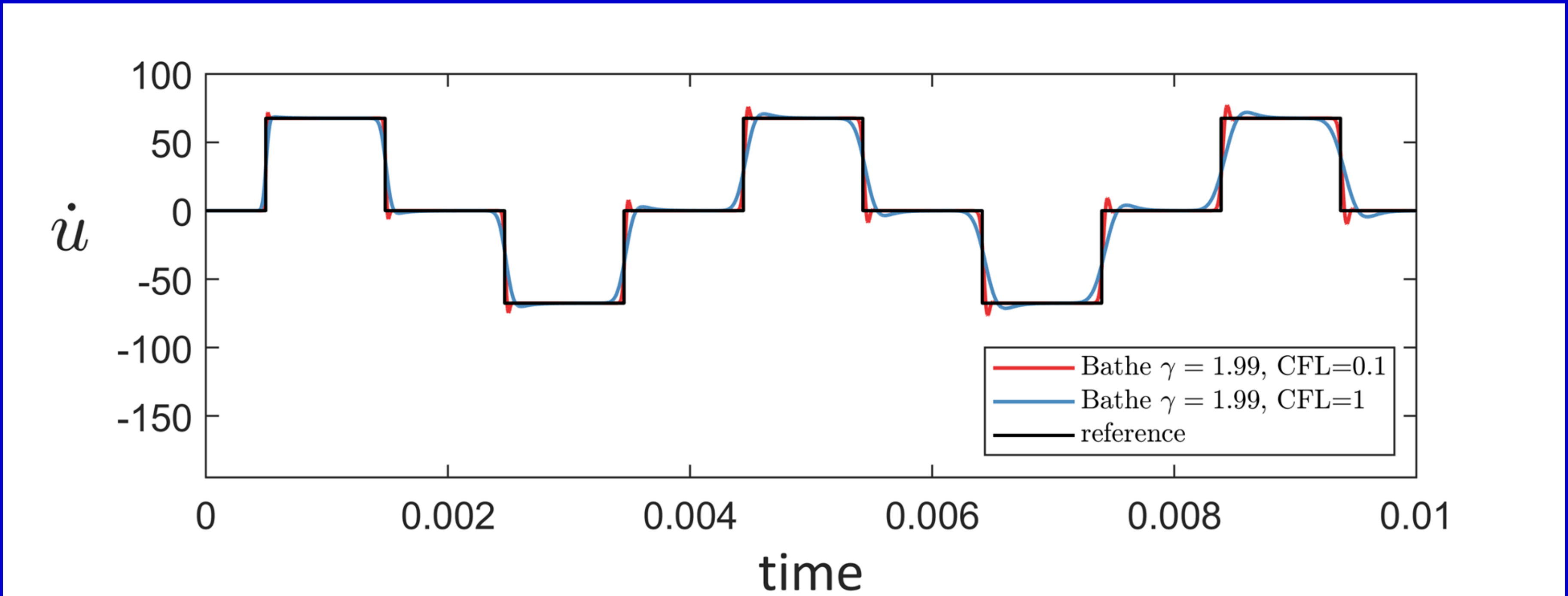


Period elongation (PE)

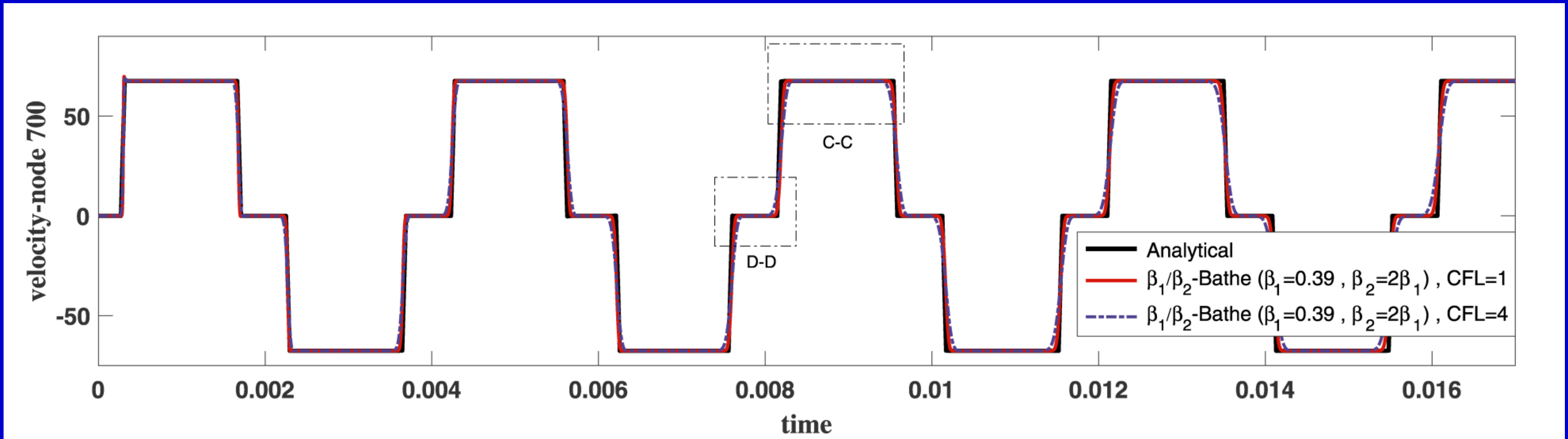
Illustrative example



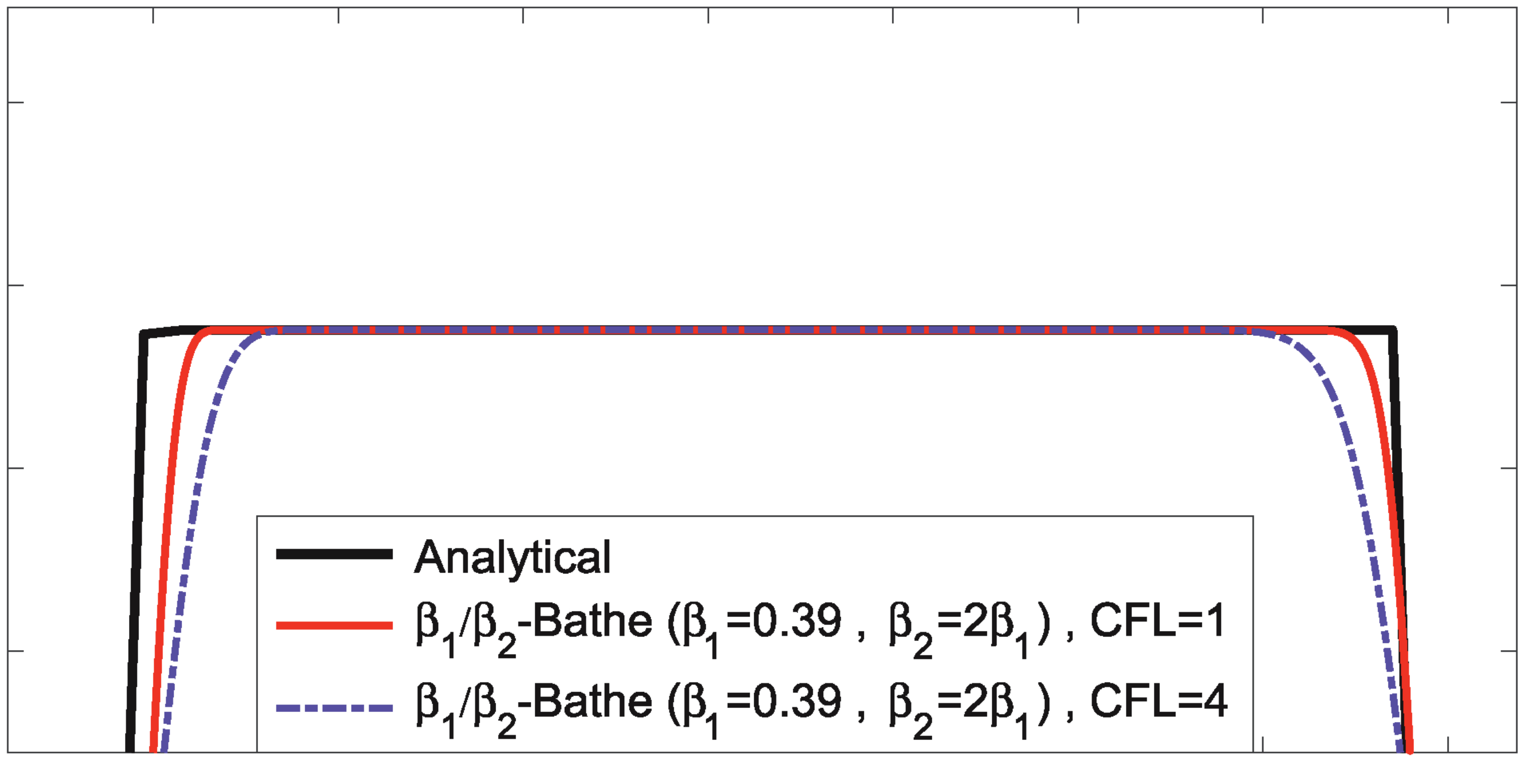
Analysis of bar subjected to step load; consider the velocity at a station x



Using the Bathe method with
γ = 1.99, CFL = 1, 0.1



Using the Bathe method with CFL=1, 4; $\beta_1 = 0.39, \beta_2 = 2\beta_1$



Detail of velocity at x = 0.7 L

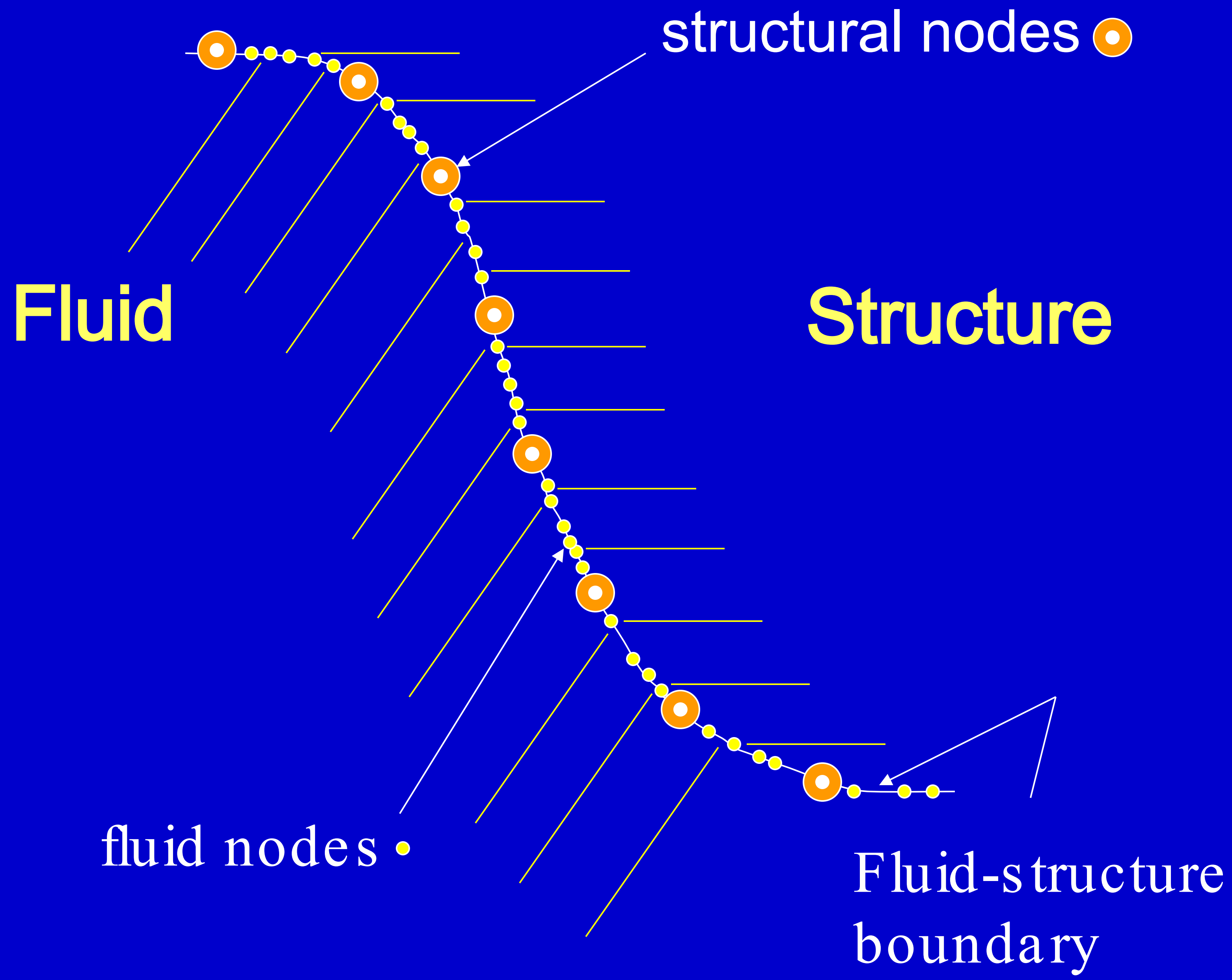
Multi-physics simulations

Fluid-structure interactions

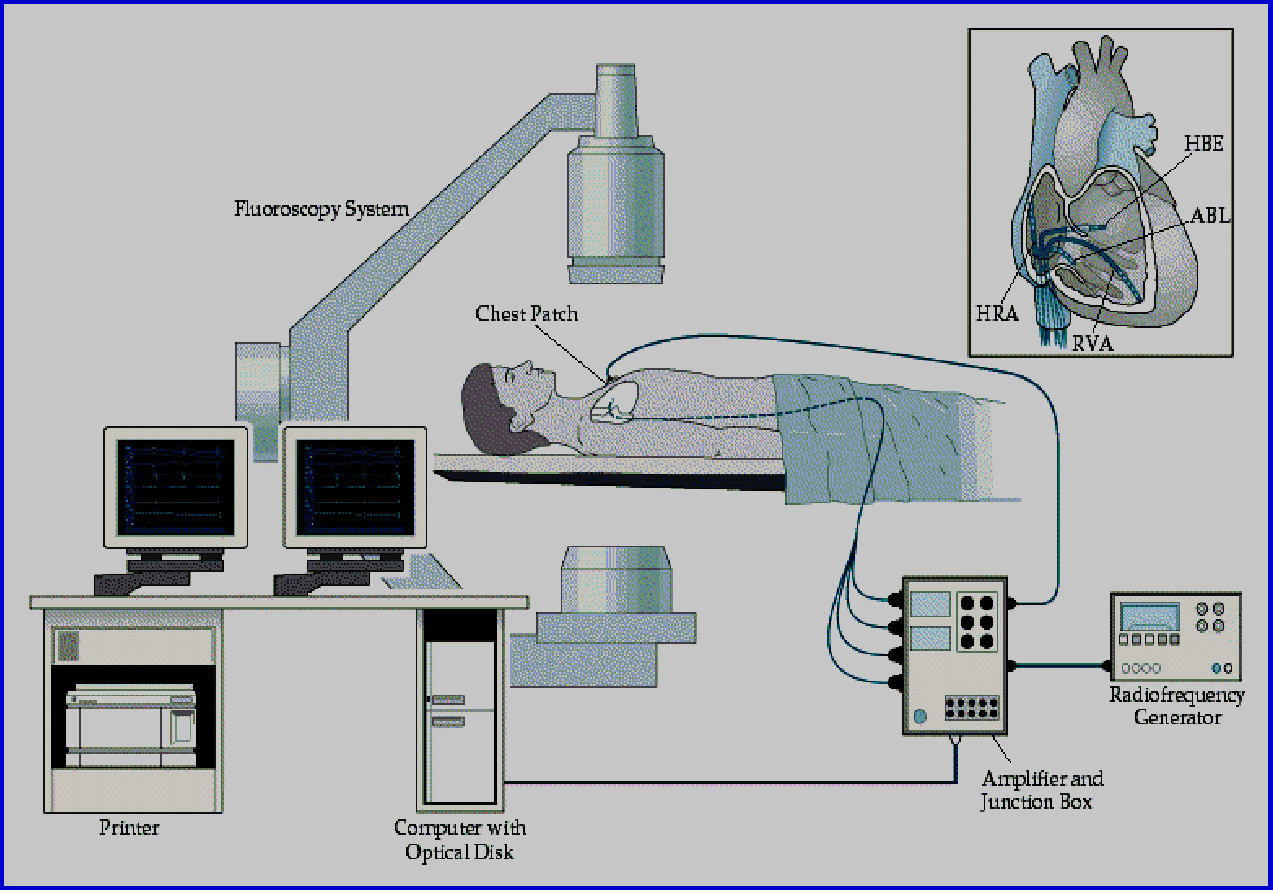
e.g. blasts, pumps, shock-absorbers, hydro-mounts, ..

Electromagnetic effects

e.g. pumps, heating, mixing, medical ablation, …

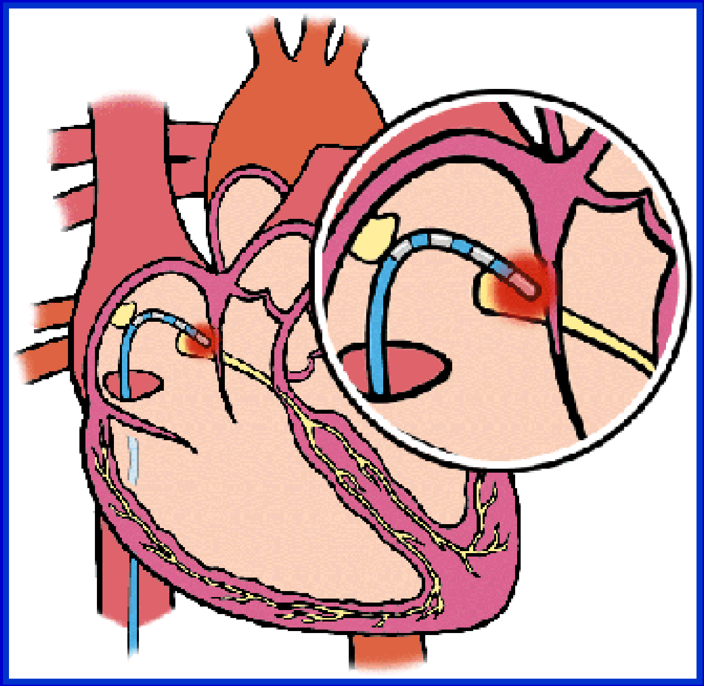

structural nodes
Fluid
Structure
fluid nodes
Fluid-structure
boundary

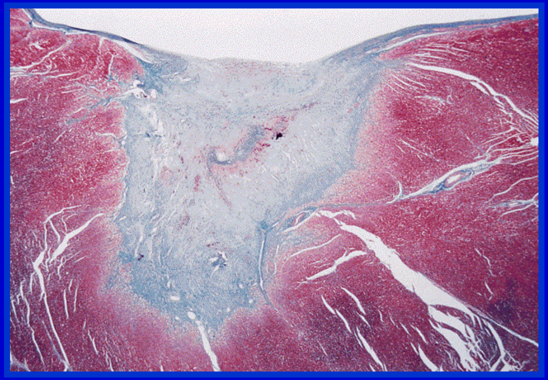
RF Ablation Lab Set Up



Radio-frequency tissue ablation

Electrode



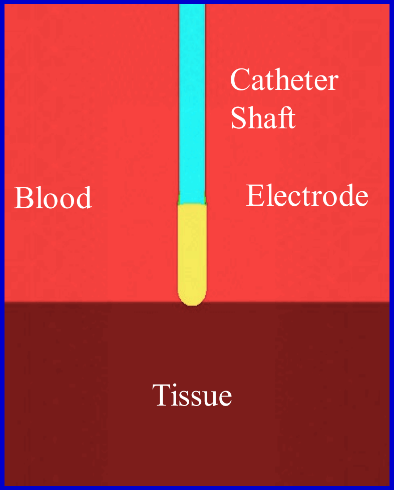


Lesion

Modeling Parameters

Conventional Catheter
Power: 38W
#Nodes: 111,205
#Elements: 667,880

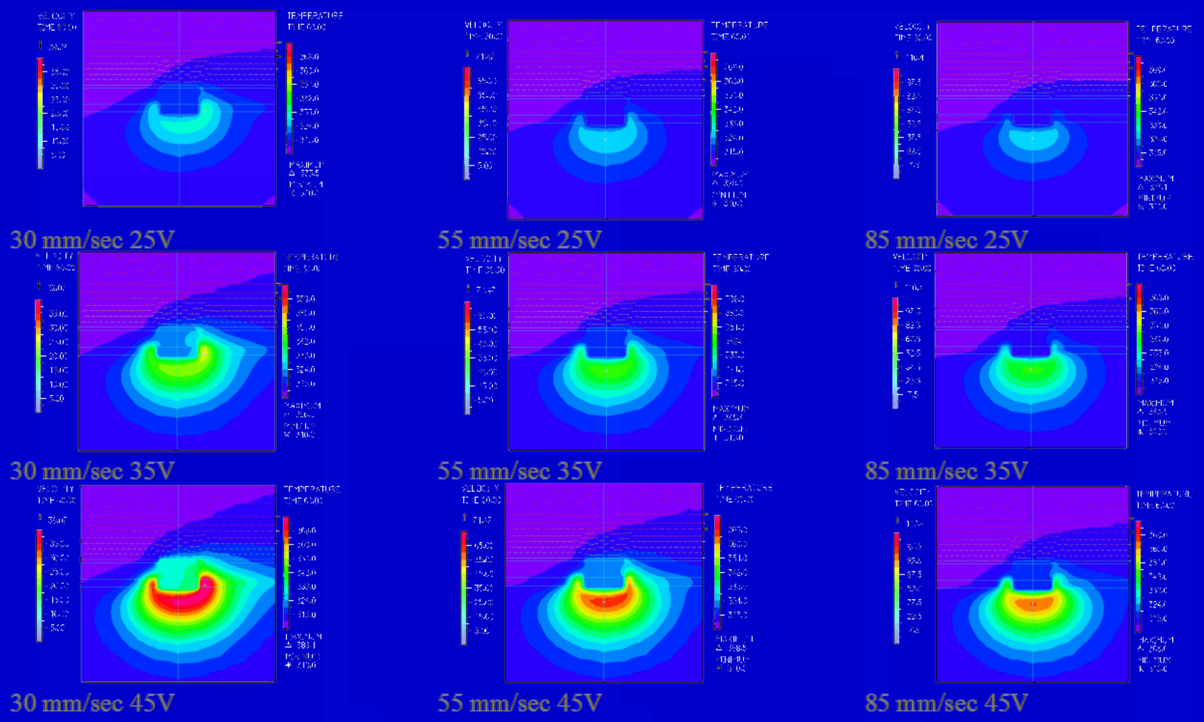
25mm High Density Mesh
Power: 74W, 91W
#Nodes: 1,405,995
#Elements: 8,627,692



Temperature/Velocity Profiles for Varying Flows

Increasing Flow

Increasing Power



A state of the art industry FSI analysis

-- A scroll compressor, 3D analysis

-- Transient response, many thousands of time steps

-- Bathe implicit time integration used, for fluid and solid

-- Solved for 5 (to 10) revolutions, using adaptive meshing for the fluid

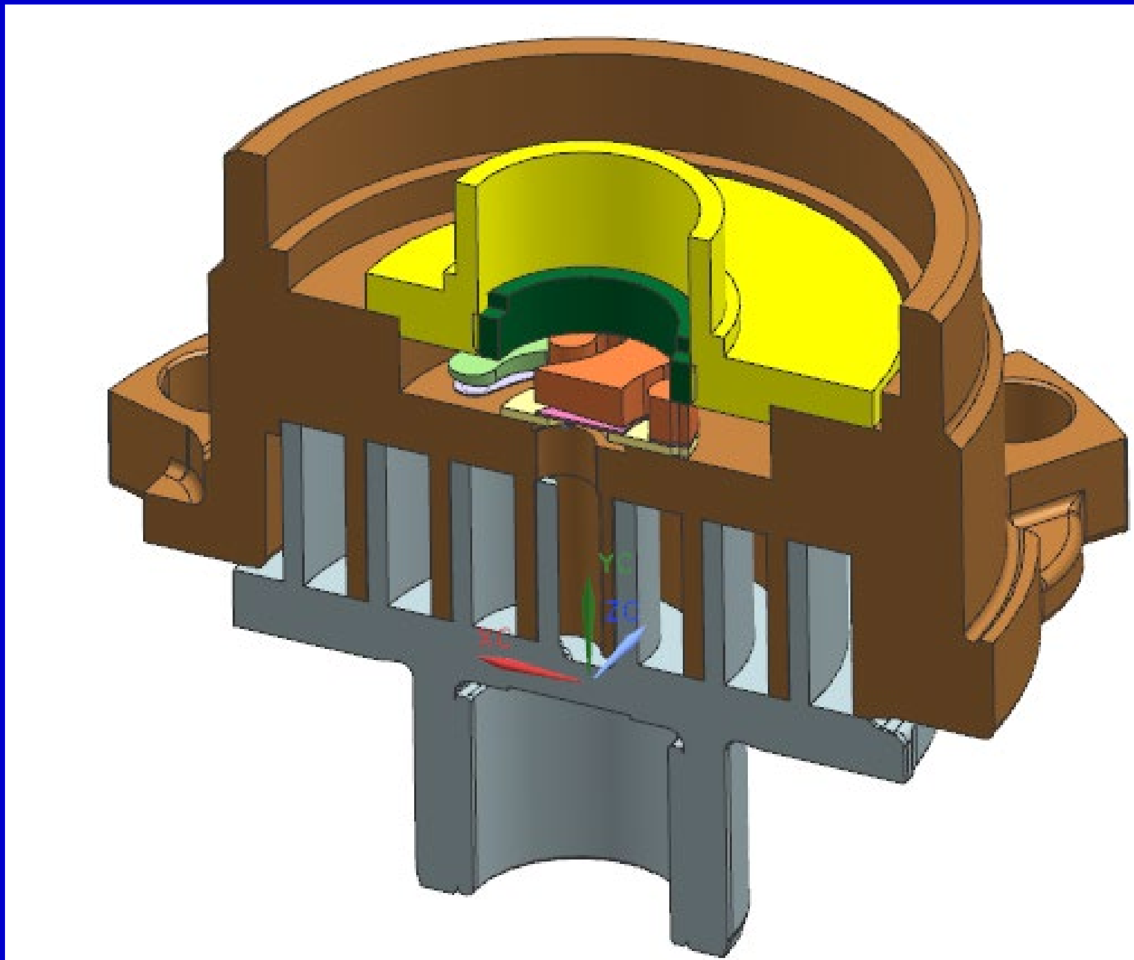
Solution obtained by Emerson Inc. using ADINA

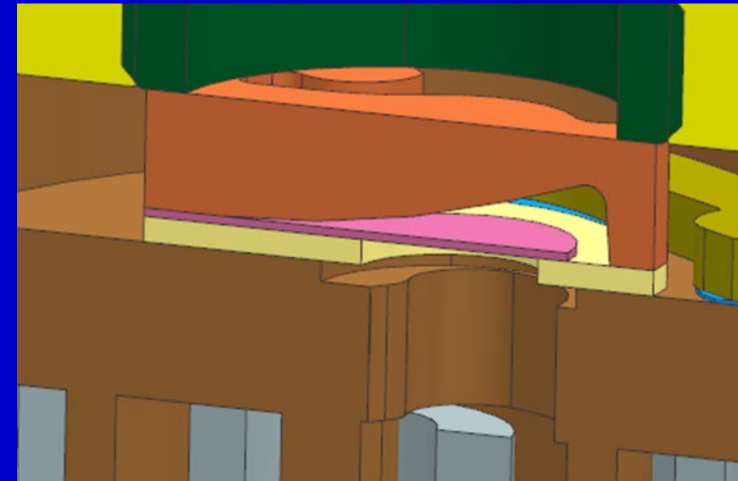
Emerson Climate Technologies
Scroll Compressor Analysis

Device for compressing air or refrigerant

Used in air conditioners, automobile superchargers, industrial refrigerators

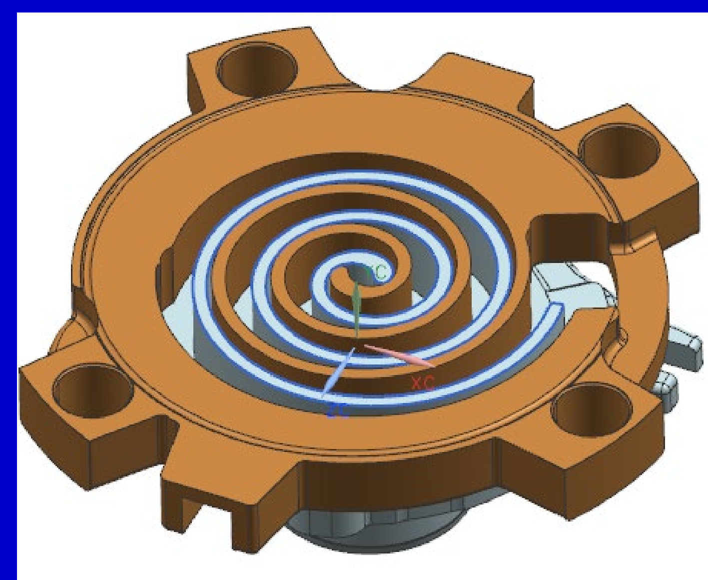
Fewer moving parts, quieter, more efficient

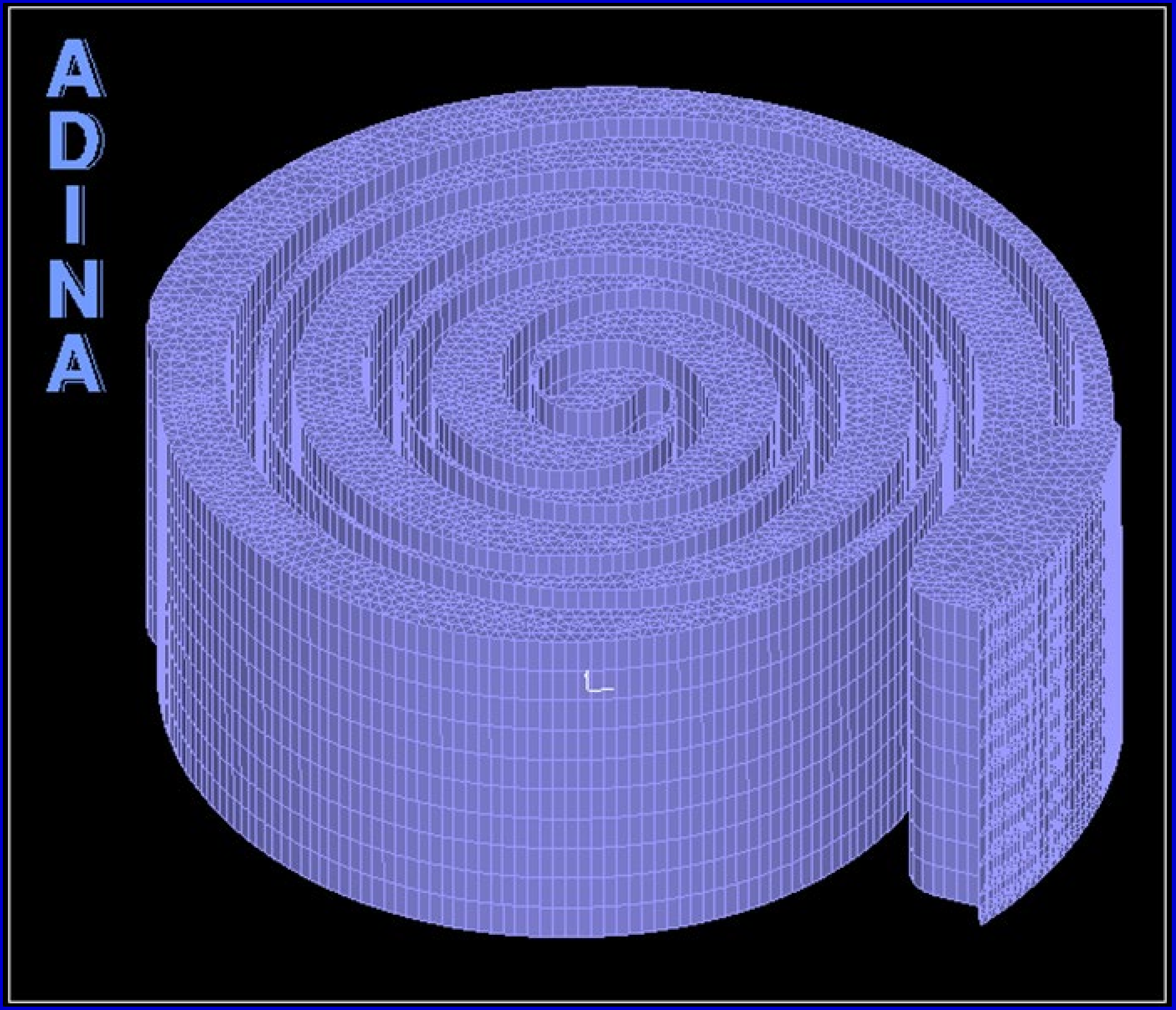




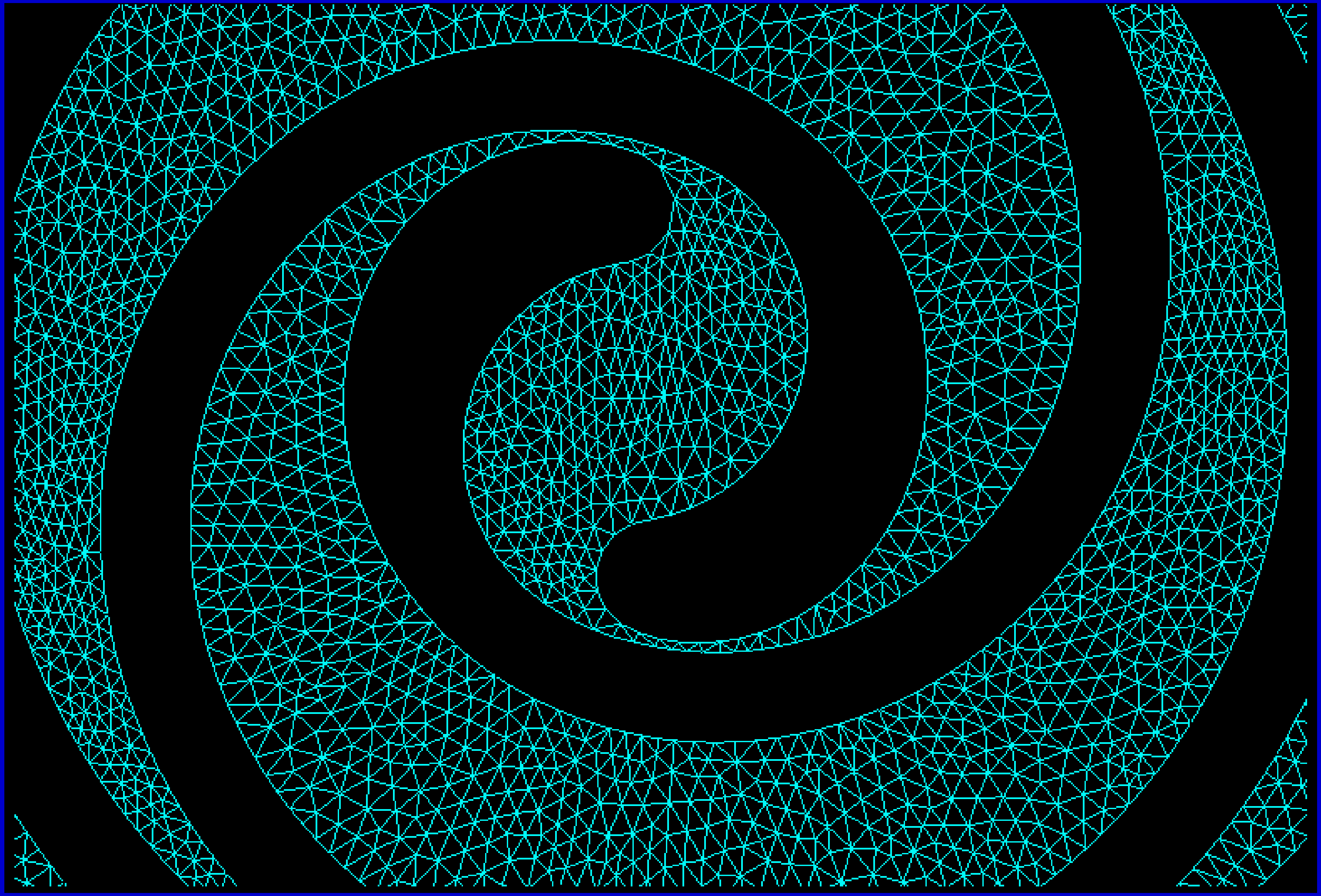
Valve

Orbiting scroll

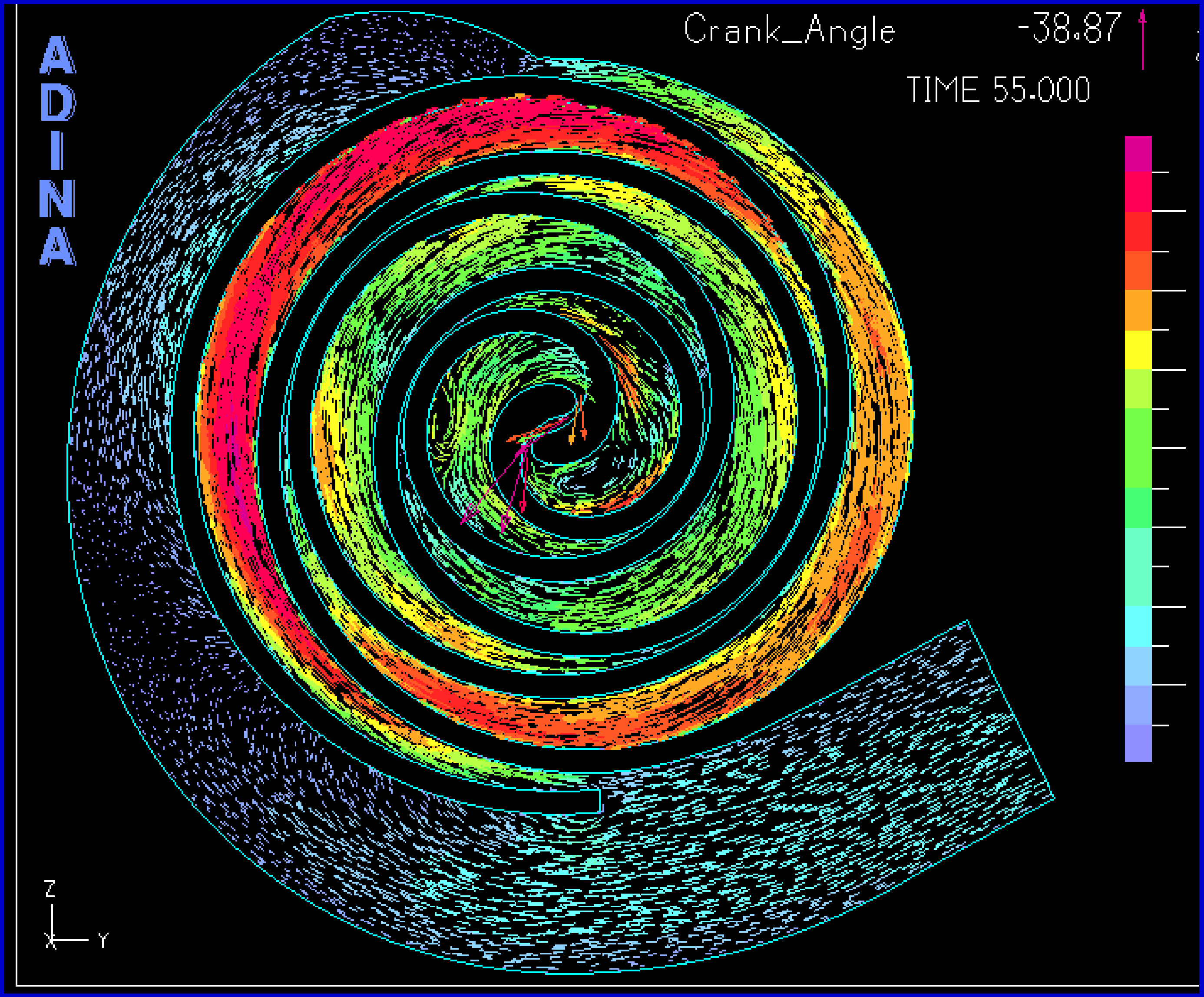




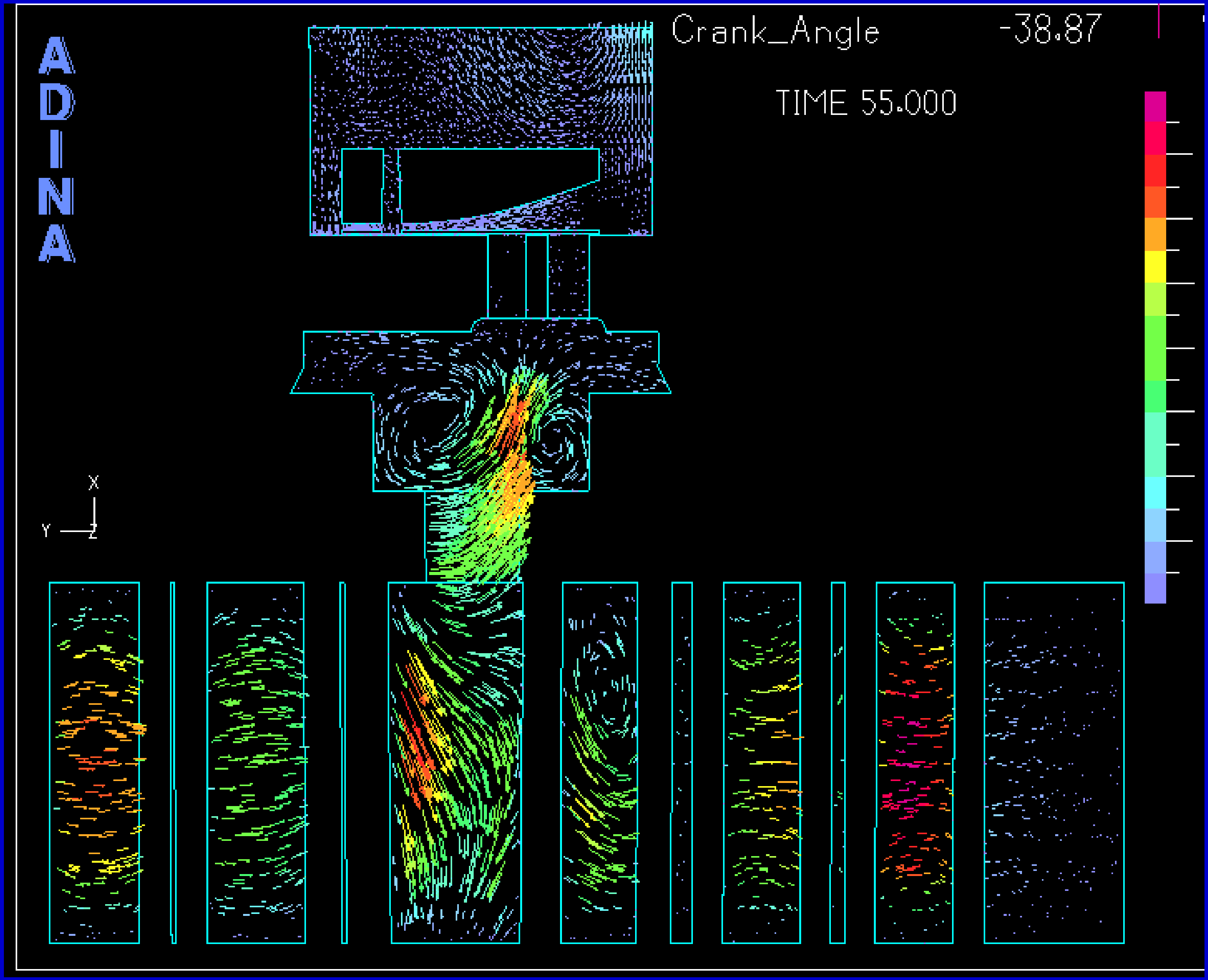
The fluid mesh used



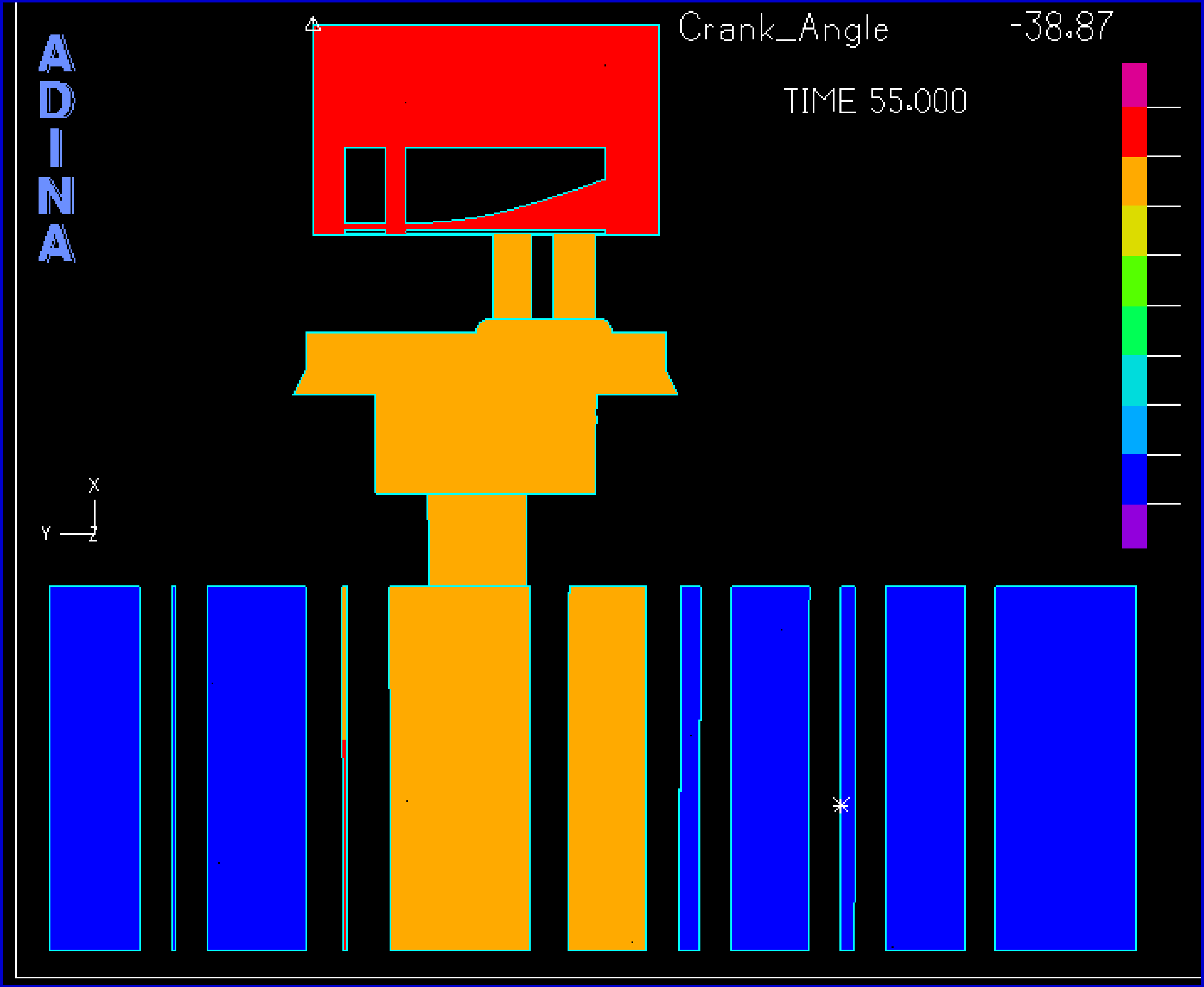
Mesh deformations in horizontal cut



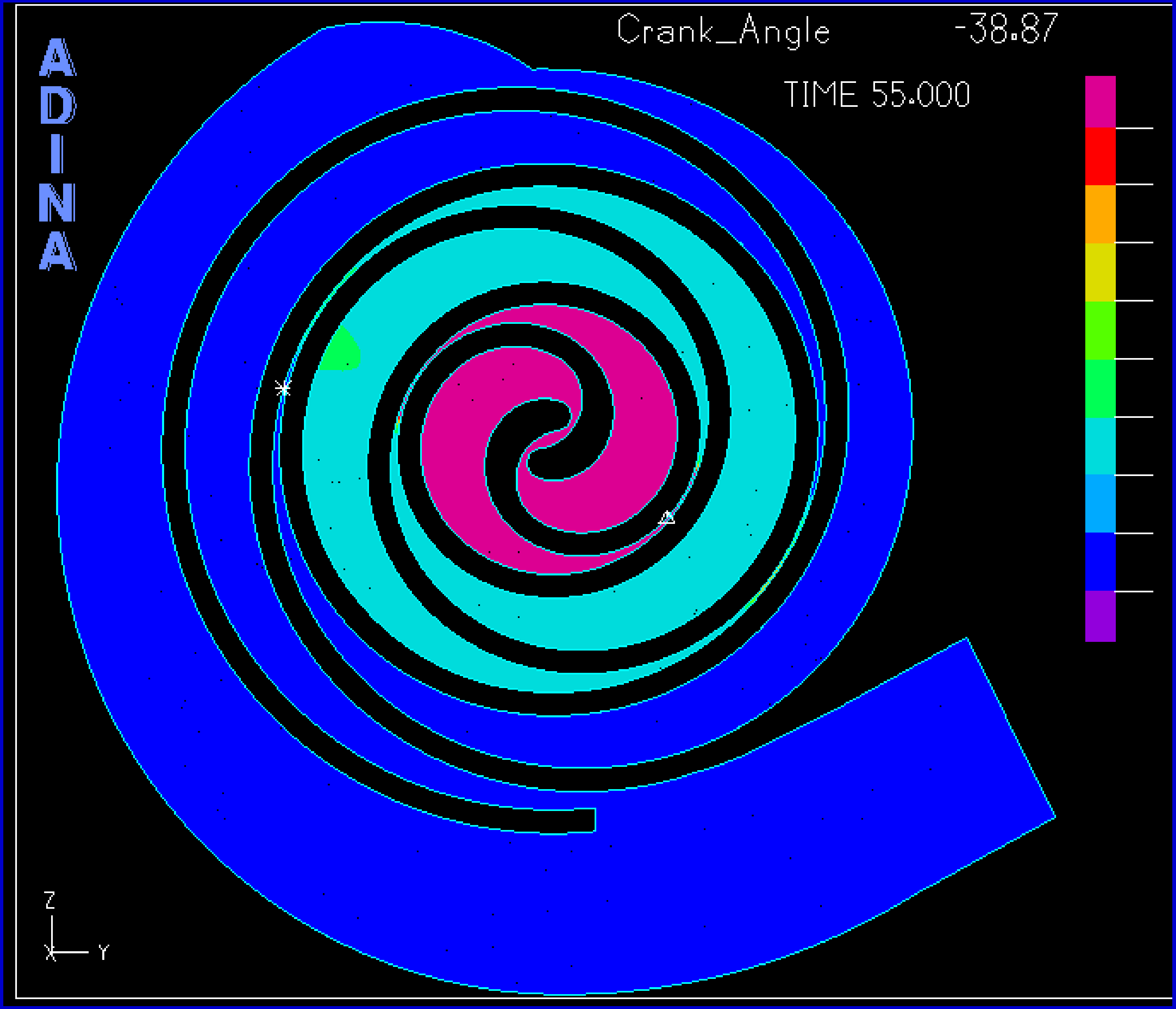
Fluid velocity in horizontal cut



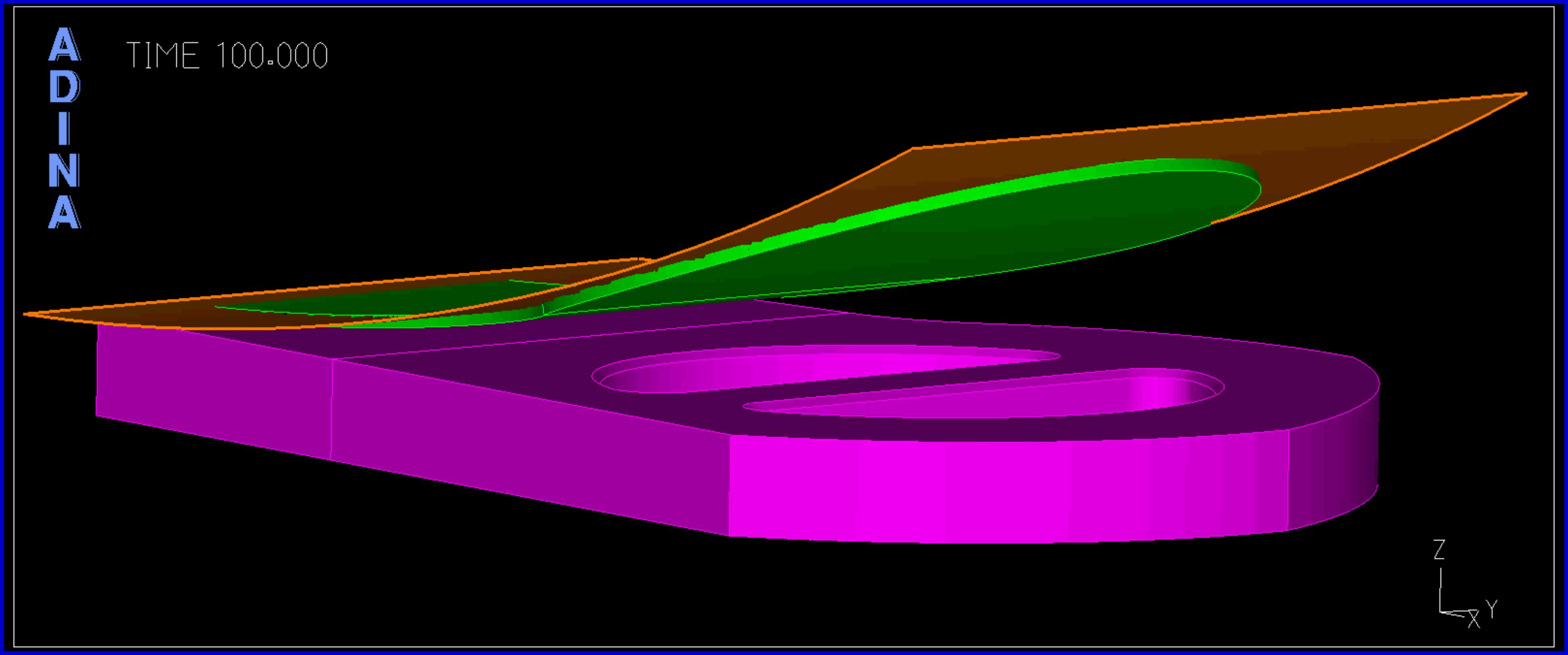
Fluid velocity in vertical cut



Pressure in vertical cut

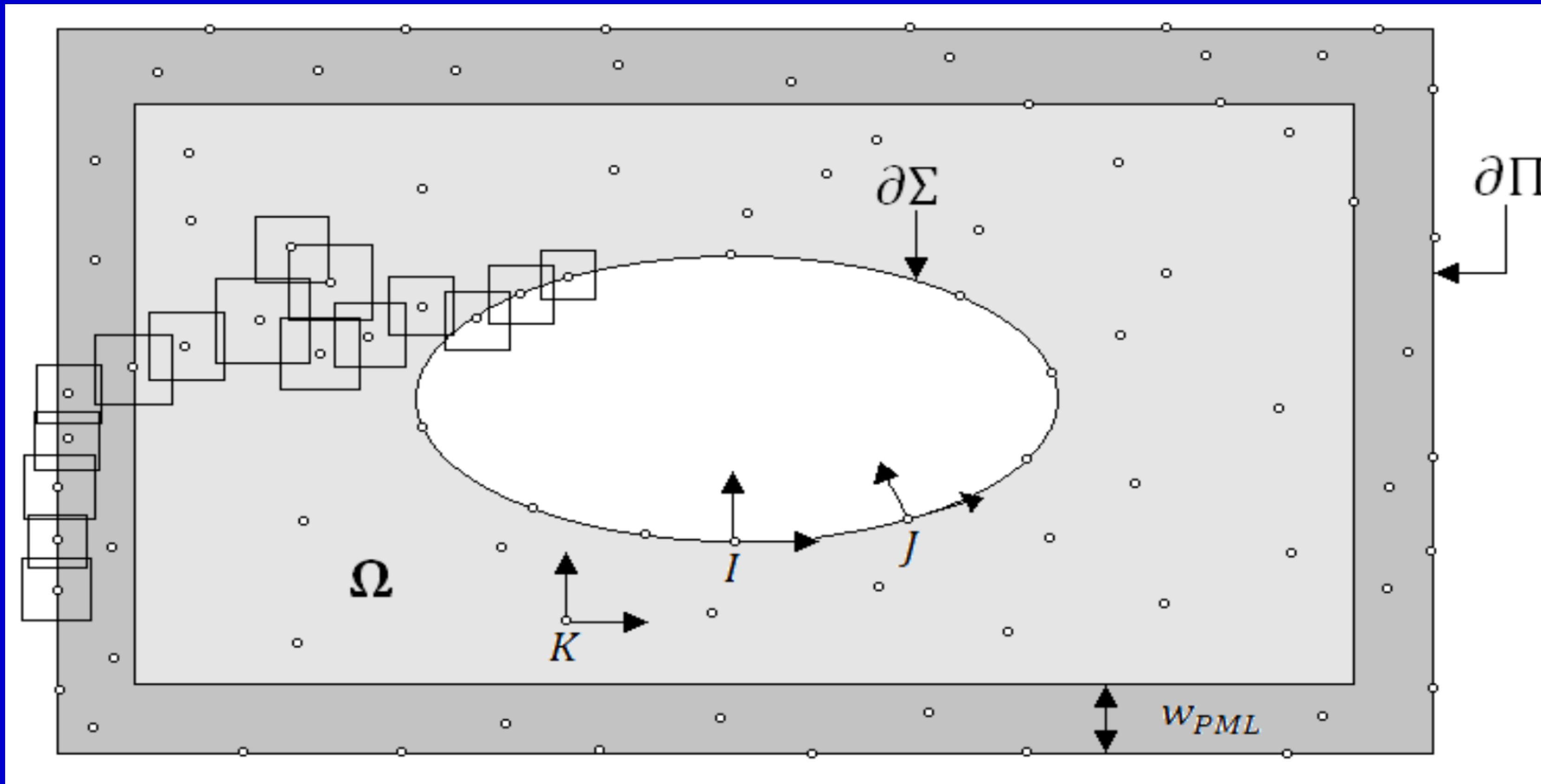


Gas pressure in horizontal cut



Cracking of valve, new design reached

EM wave scattering by mesh-free method



Like MFS and ‘overlapping elements’

W.L. Nicomedes, K.J. Bathe, F.J.S. Moreira and R.C. Mesquita.
Meshfree analysis of electromagnetic wave … . C & S 2017.

Finite element simulations at the nano-scale

Proteins and DNA structures

Finite element methods can be very effective in analyzing these structures

Basic applications in biological engineering, energy engineering, medical sciences, ….

Huge field for the future … !

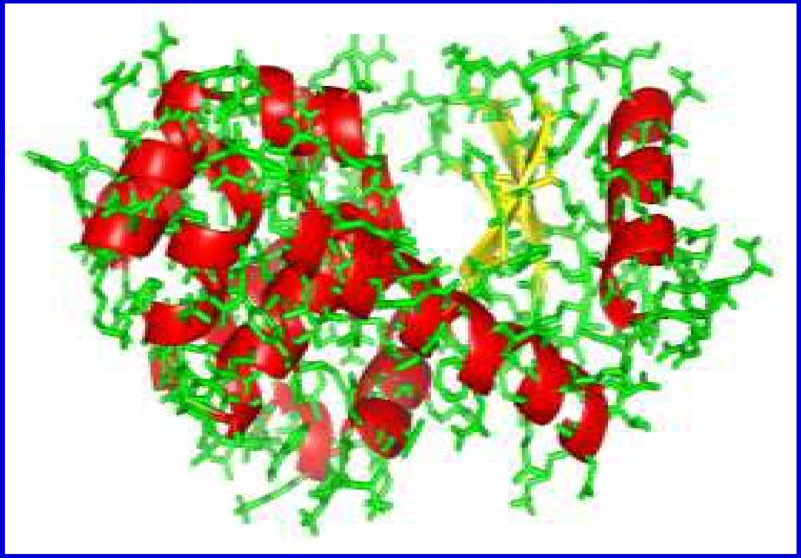
The most basic approach is to analyze DNA structures and proteins using 'Molecular Dynamics' … computationally very restrictive

Langevin or Brownian dynamics is an avenue to follow ---

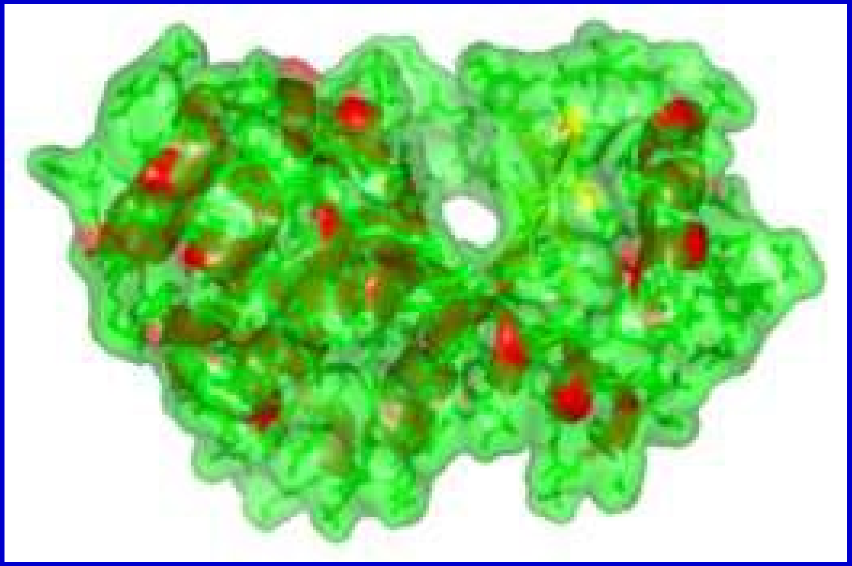
We developed finite element procedures for effective solutions,

with these procedures we can efficiently analyze even "supramolecular assemblies"

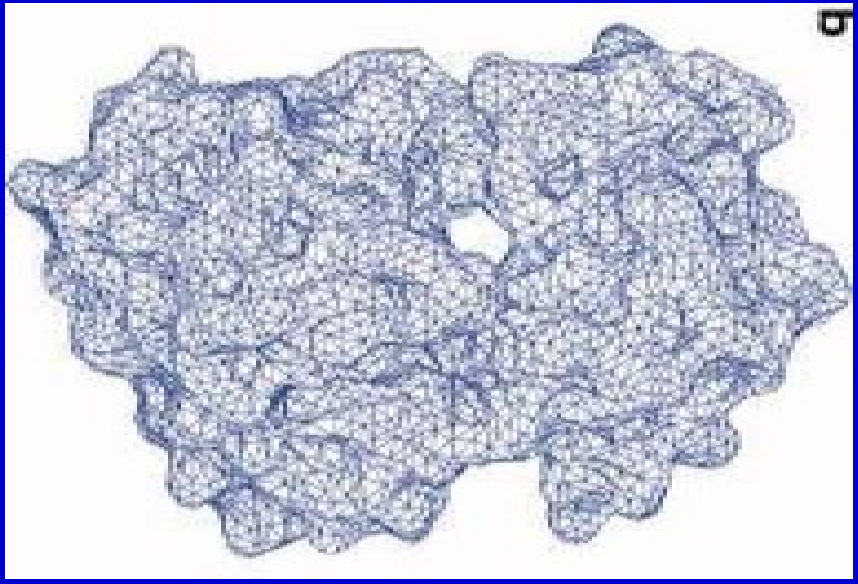
RS Sedeh, G Yun, JY Lee, KJ Bathe, DN Kim. A Framework …. . C & S 2018



Structure



Molecular surface



Finite element model of protein for frequency solutions (using the subspace iteration method) and dynamic analyses

Equations of motions (Langevin dynamics)

$$\mathbf{M\ddot{x}} + \mathbf{Z\dot{x}} + \mathbf{Kx} = \mathbf{f}(t)$$

K = protein elastic stiffness matrix

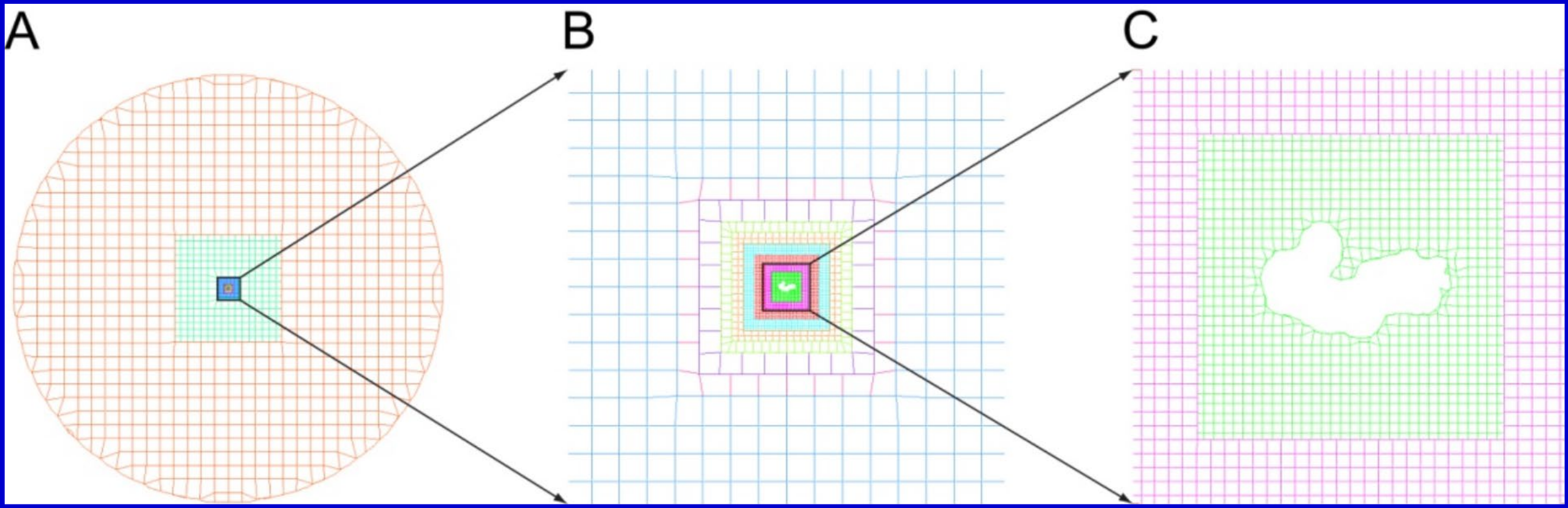
Z = solvent friction matrix

M is assumed = 0 in Brownian dynamics

f(t) = solvent-induced

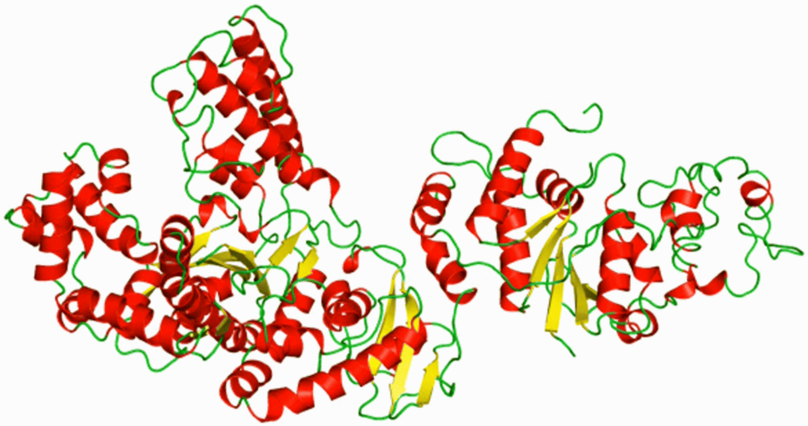
$$\left\langle \mathbf{f_i(t)} \right\rangle = 0$$

$$\left\langle \mathbf{f_i(t) \times f_j(t')} \right\rangle = 2k_B T Z_{ij} \delta(t - t')$$

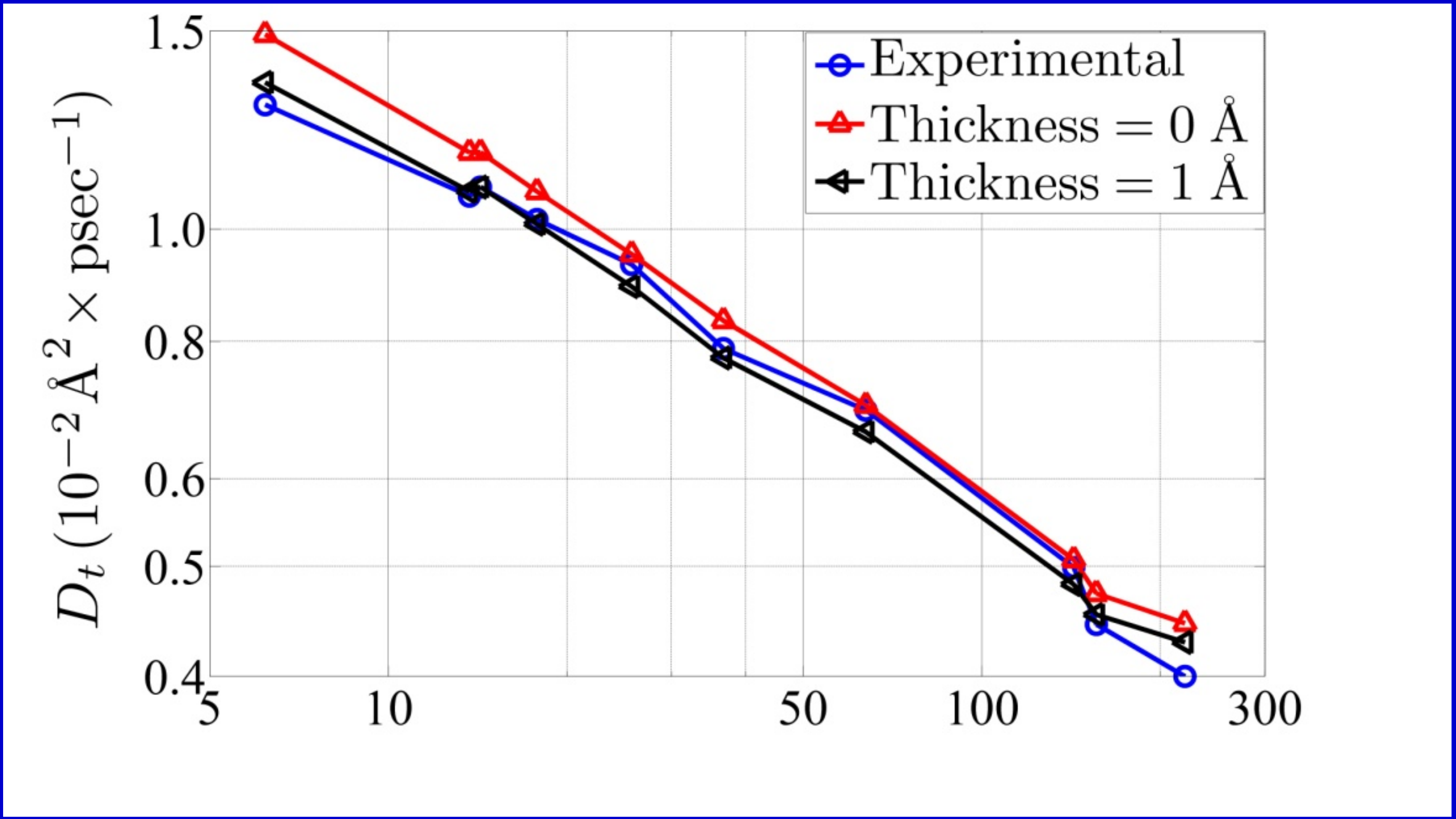


Finite element modeling, using multiple meshes glued together

The mesh between the Taq polymerase surface and the sphere surface (in cross-section)



Brownian dynamics simulation of protein



Molecular weight (kDa)

Translational diffusion coefficients of 10 proteins, including Hemoglobin, Lysozyme & Adolase

The AMORE paradigm of analysis

Automatic Meshing with Overlapping and Regular Elements

For any geometry, CAD defined or otherwise, that is, not restricted to CAD functions

-- Analysis part is immersed in a Cartesian grid

-- Boundary of given geometry is discretized

-- Cells inside the geometry are turned into finite elements, the other cells are removed

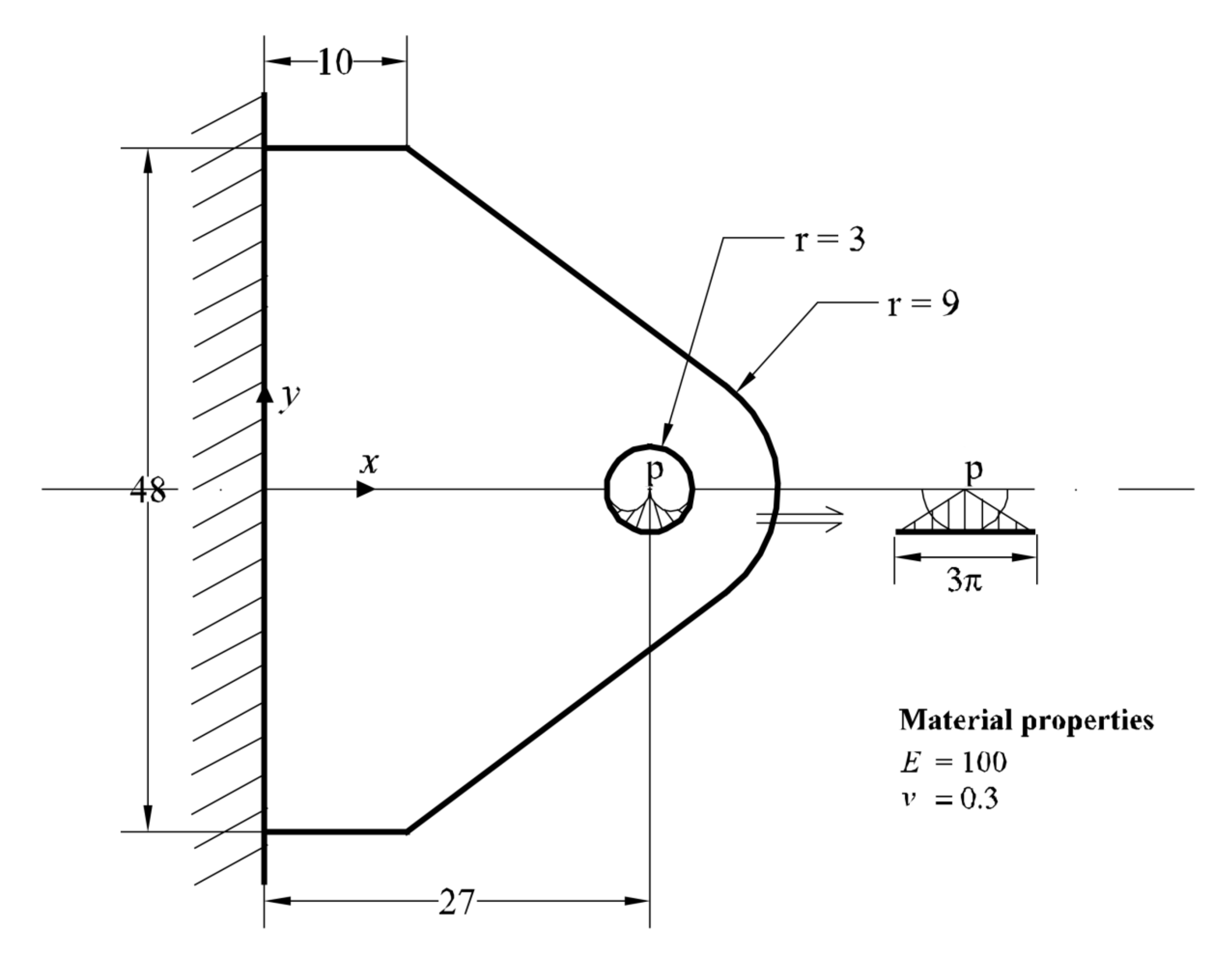
-- Overlapping elements are used to fill in the empty space

Important point: The OFE are distortion insensitive

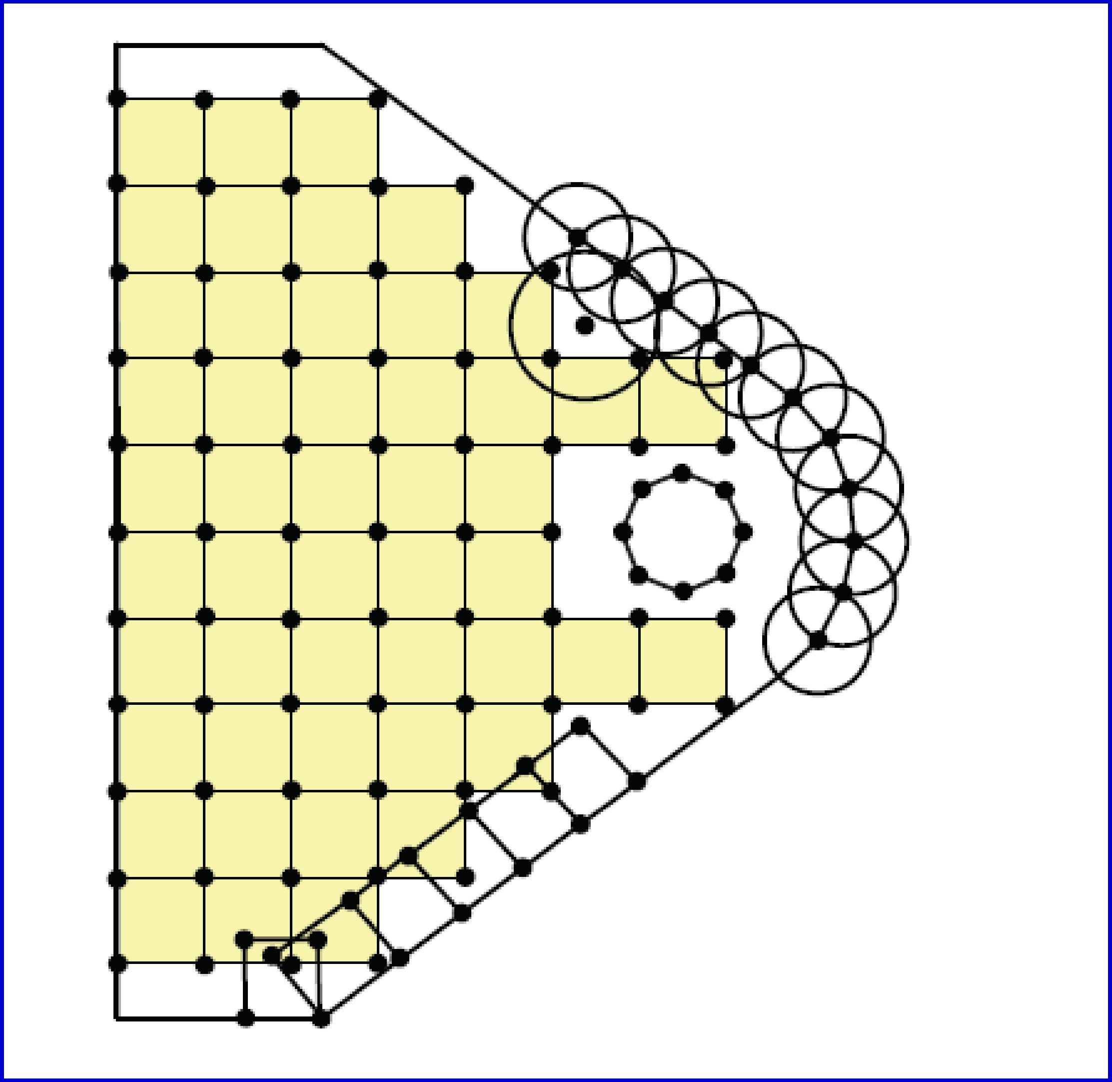
KJ Bathe. Overlapping …. . SEMC 2016

KJ Bathe, L Zhang. … A New Paradigm … . C & S 2017

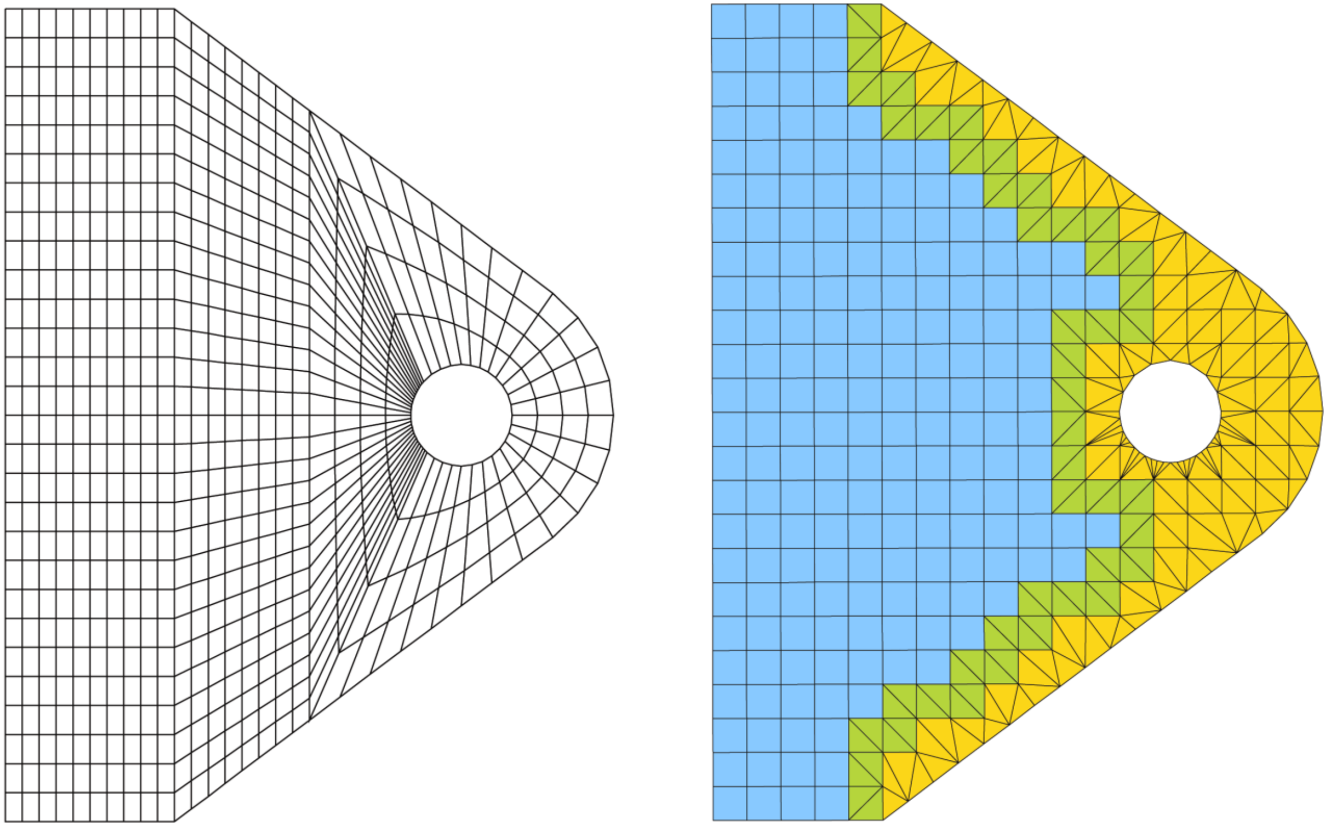
KJ Bathe. The AMORE … . Adv. in Eng. Software 2019



Example analysis of bracket

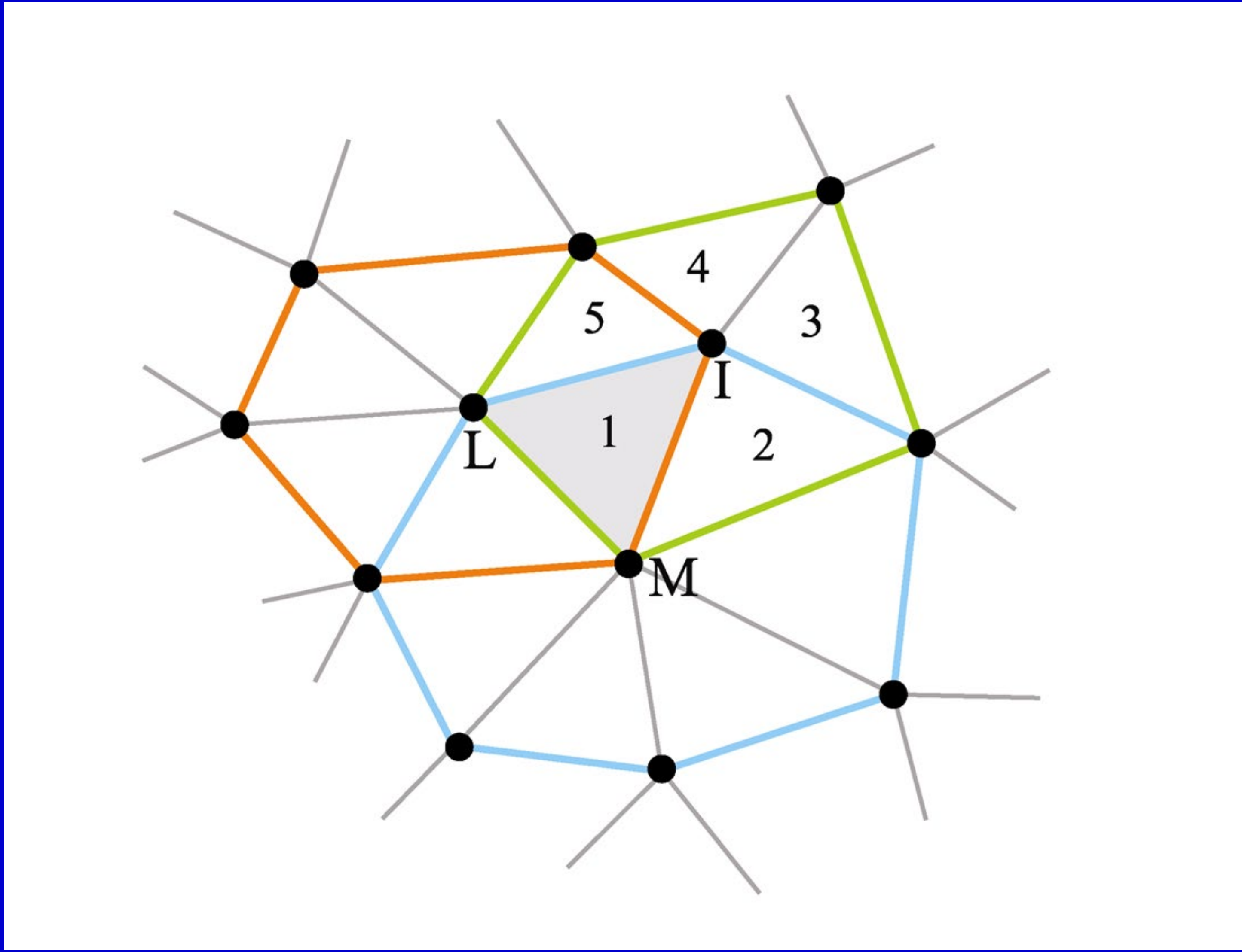


Schematic -- example analysis of bracket

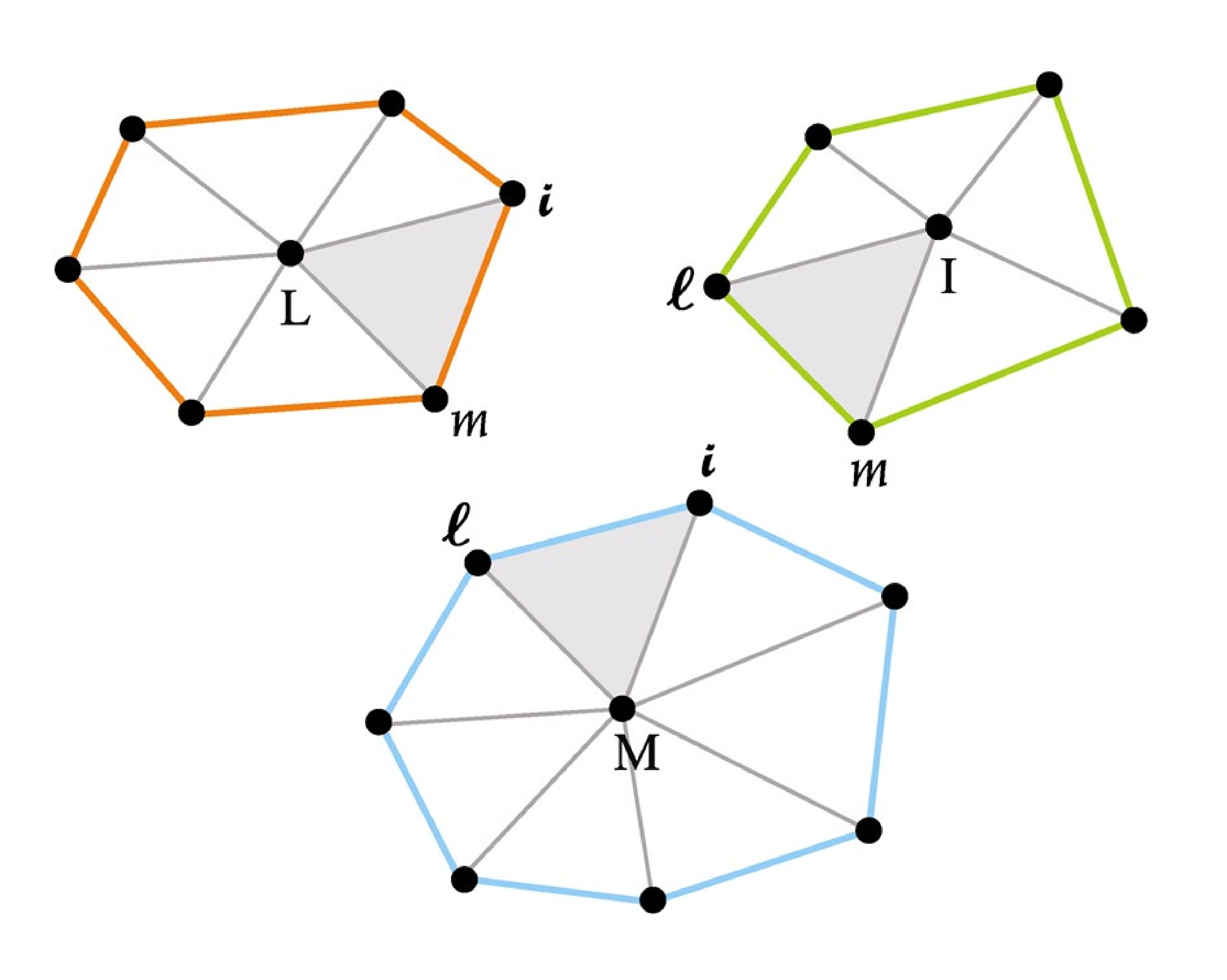


(a) Traditional mesh (b) New scheme mesh

Example analysis of bracket



Overlapping elements, with overlapped region shown in grey



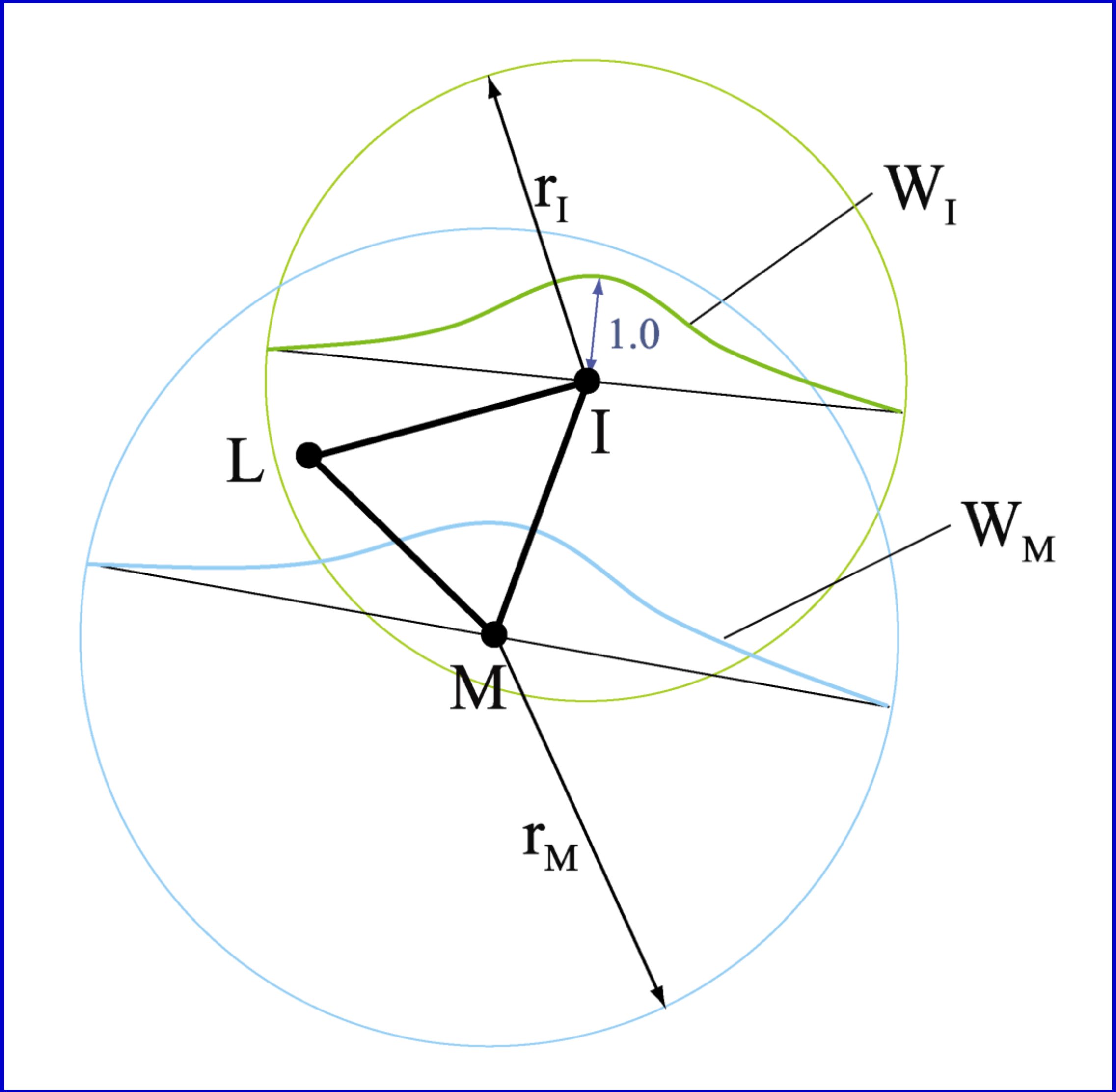
The 3 overlapping polygonal elements

We use the concept of the Method of Finite Spheres (MFS)

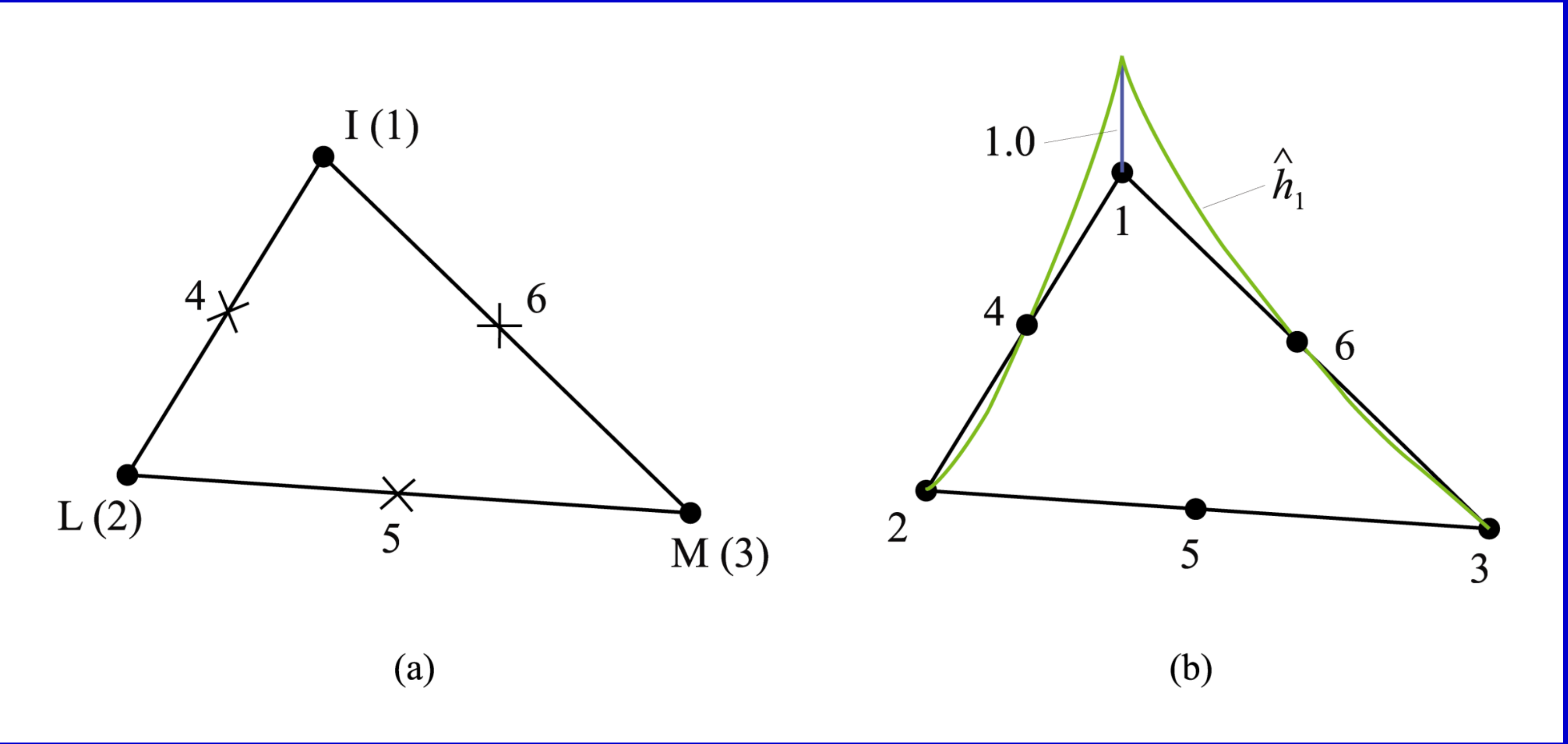
We place a sphere with its interpolation functions at each node *I, L, M* ; hence at each node the DOF of the MFS are used (polynomials, or ...)

The displacements of the spheres on the overlapped region *I-L-M* are "weighted" by the linear interpolations of the triangular element *I-L-M*

We interpolate the Shepard functions used in the MFS, to only have polynomials in the interpolations and solution efficiency



Shepard weight functions used



Interpolation of Shepard functions

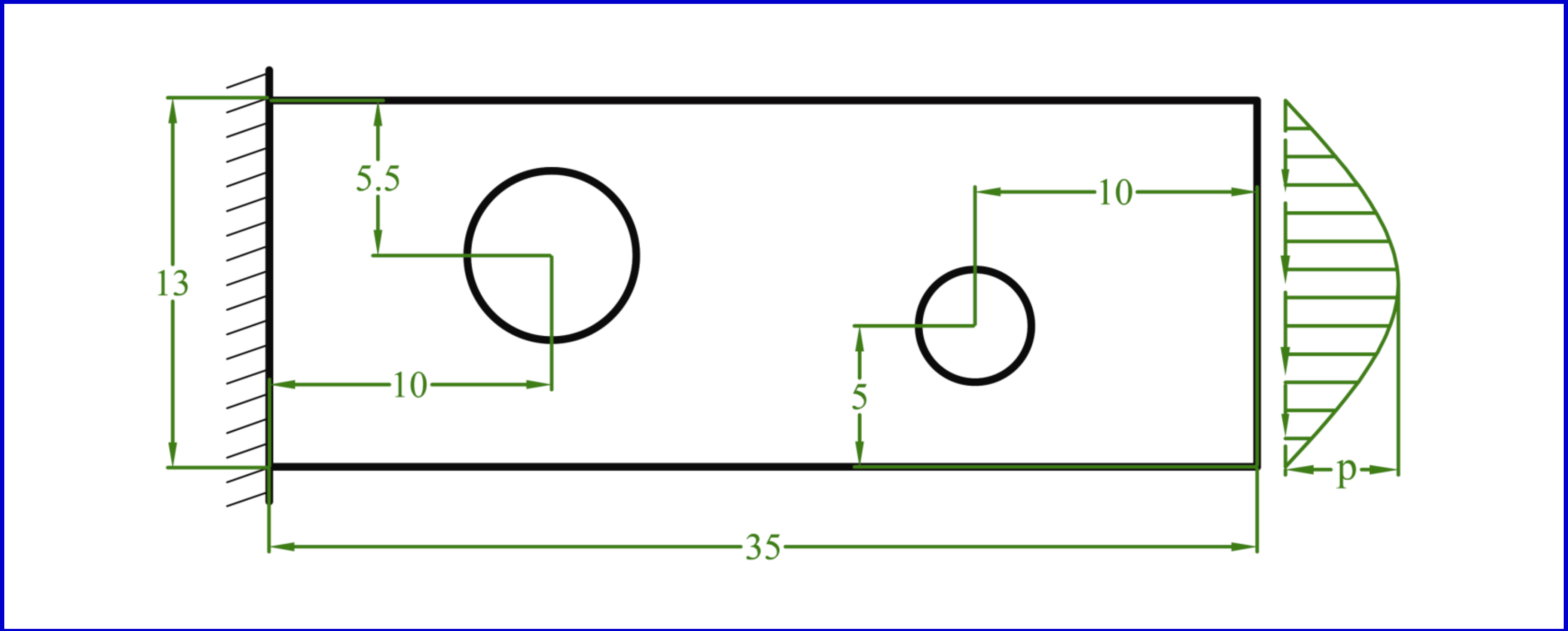
$$\mathbf{u}(\mathbf{x}) = \rho_I\, p_n \mathbf{a}_{In} + \rho_L\, p_n \mathbf{a}_{Ln} + \rho_M\, p_n \mathbf{a}_{Mn}$$

$$\rho_I = h_I\, \hat{h}_i \hat{\phi}_{Ii}^{I} + h_L\, \hat{h}_i \hat{\phi}_{Ii}^{L} + h_M\, \hat{h}_i \hat{\phi}_{Ii}^{M}$$

$$\rho_L = h_I\, \hat{h}_i \hat{\phi}_{Li}^{I} + h_L\, \hat{h}_i \hat{\phi}_{Li}^{L} + h_M\, \hat{h}_i \hat{\phi}_{Li}^{M}$$

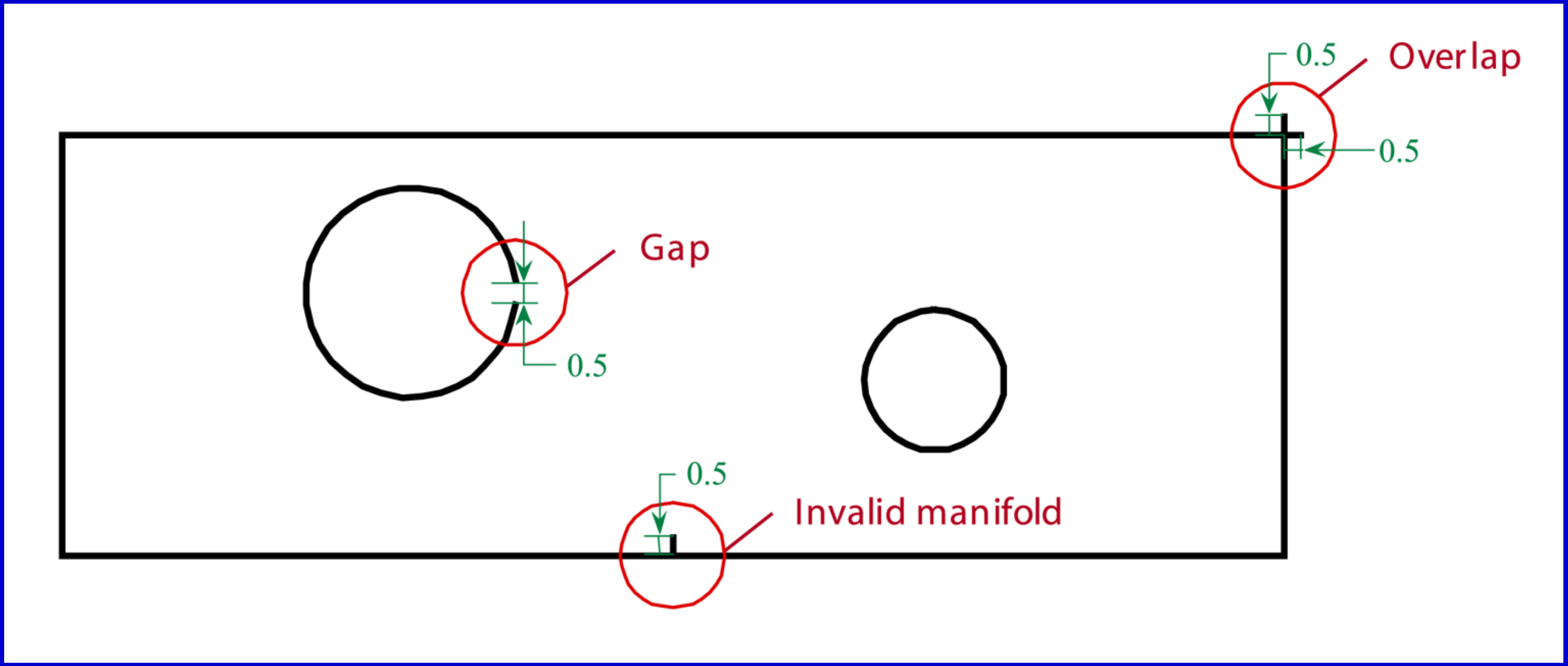
$$\rho_M = h_I\, \hat{h}_i \hat{\phi}_{Mi}^{I} + h_L\, \hat{h}_i \hat{\phi}_{Mi}^{L} + h_M\, \hat{h}_i \hat{\phi}_{Mi}^{M}$$

Interpolation functions for triangle *I-L-M*

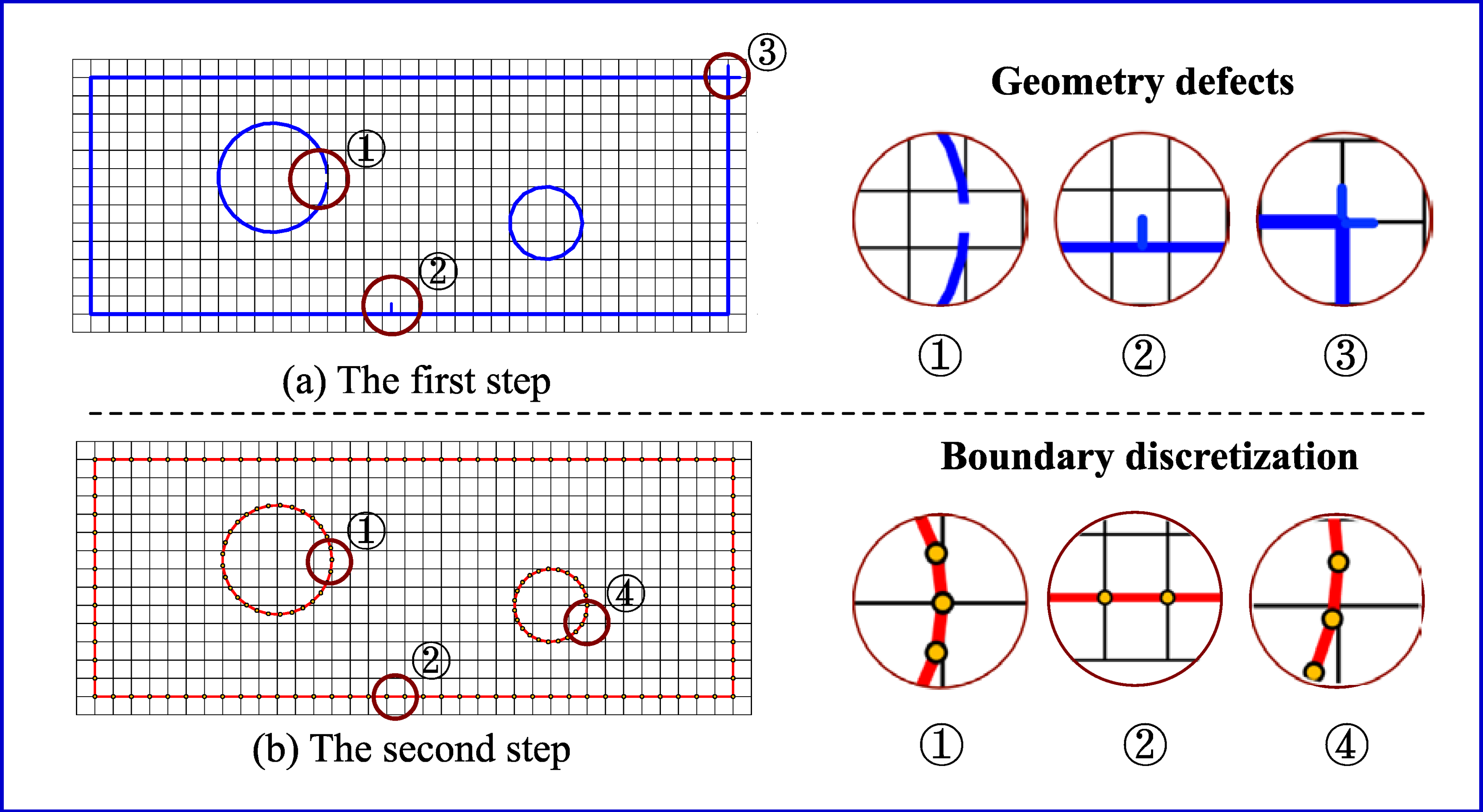


Example: Analysis of a cantilever plate

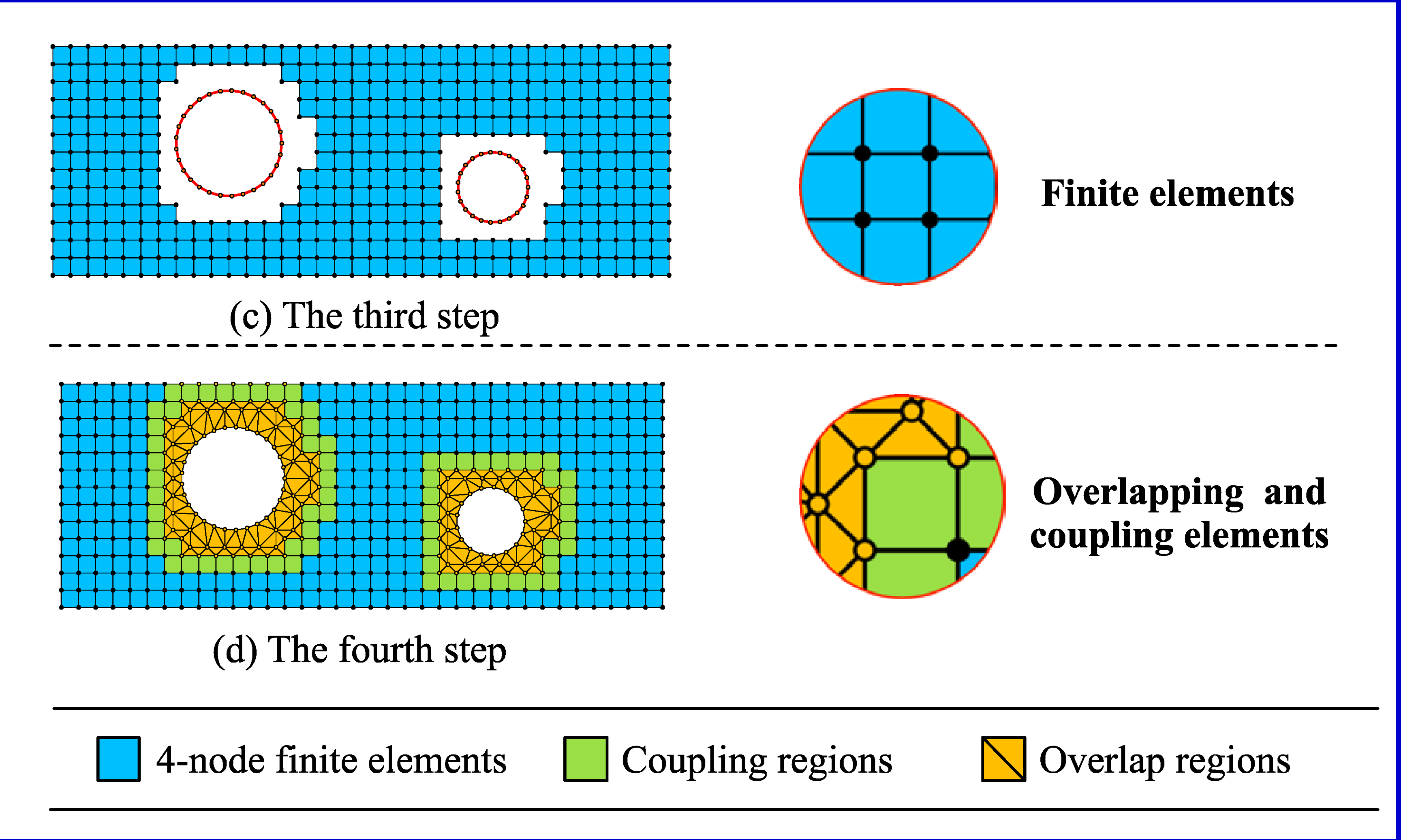
L Zhang, KJ Bathe. Overlapping finite elements … . C & S 2017
L Zhang, KT Kim, KJ Bathe. The new paradigm … . C & S 2018



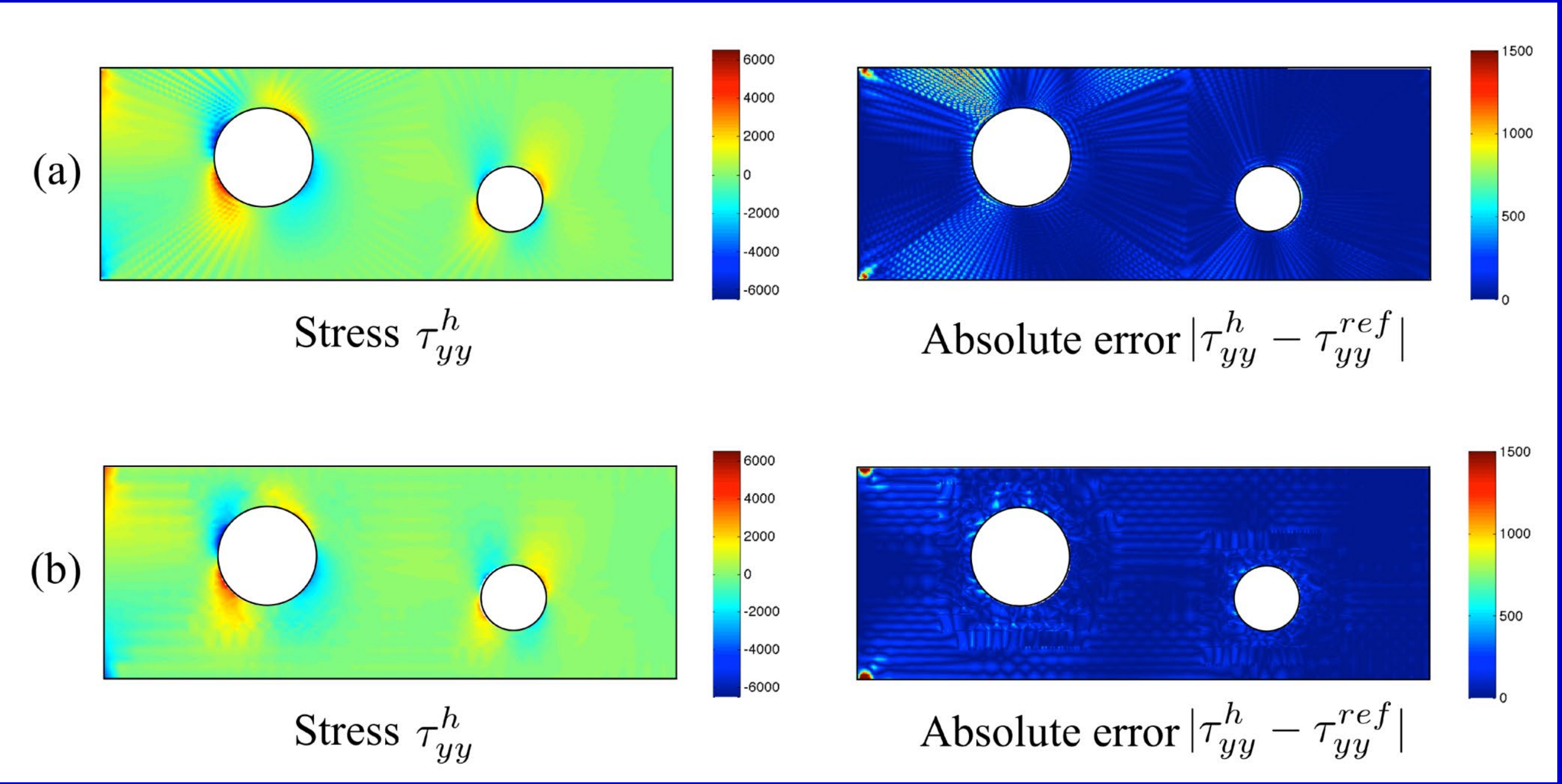
CAD geometry –
Analysis of a cantilever plate



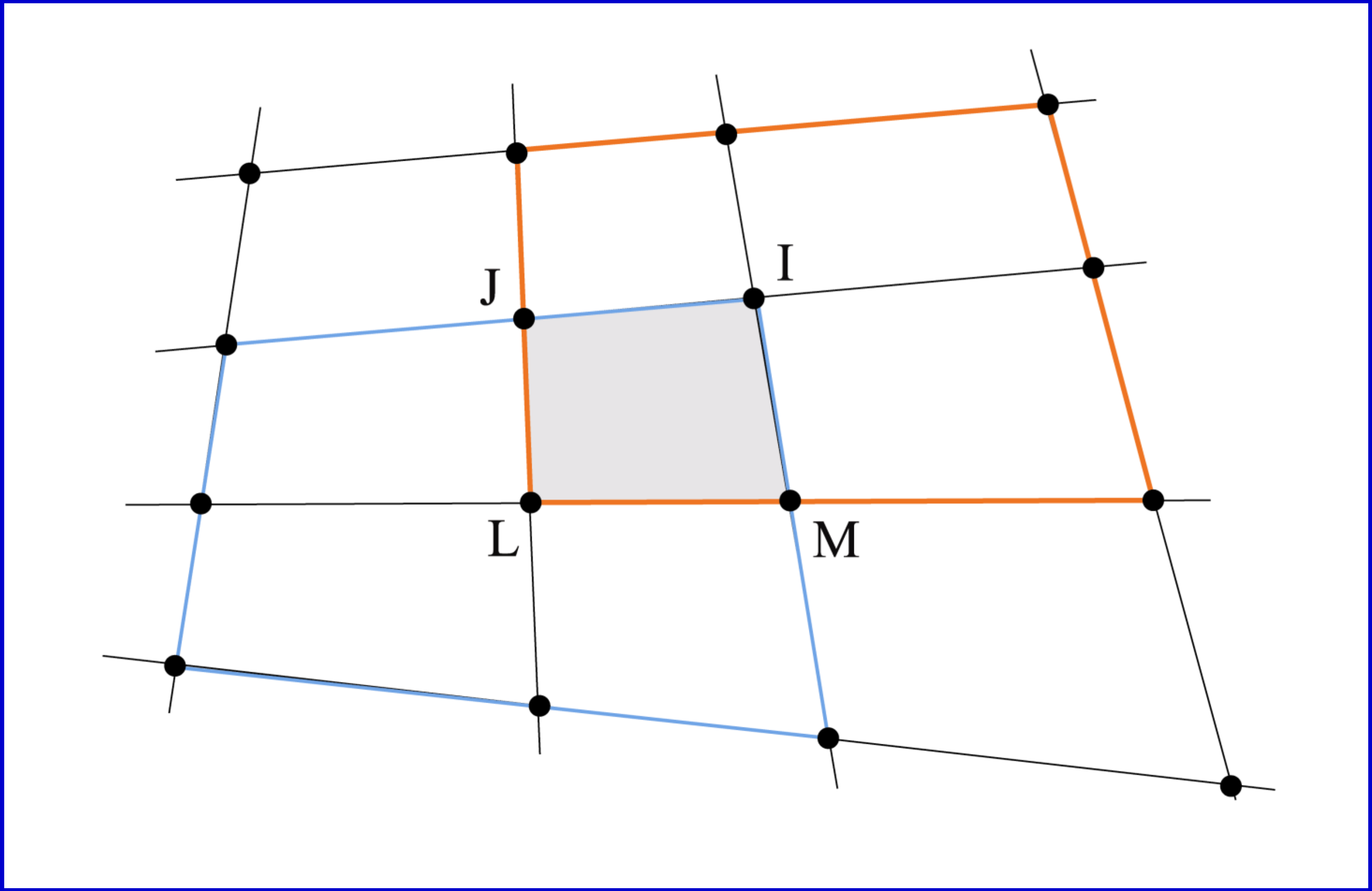
Analysis of a cantilever plate



Analysis of a cantilever plate

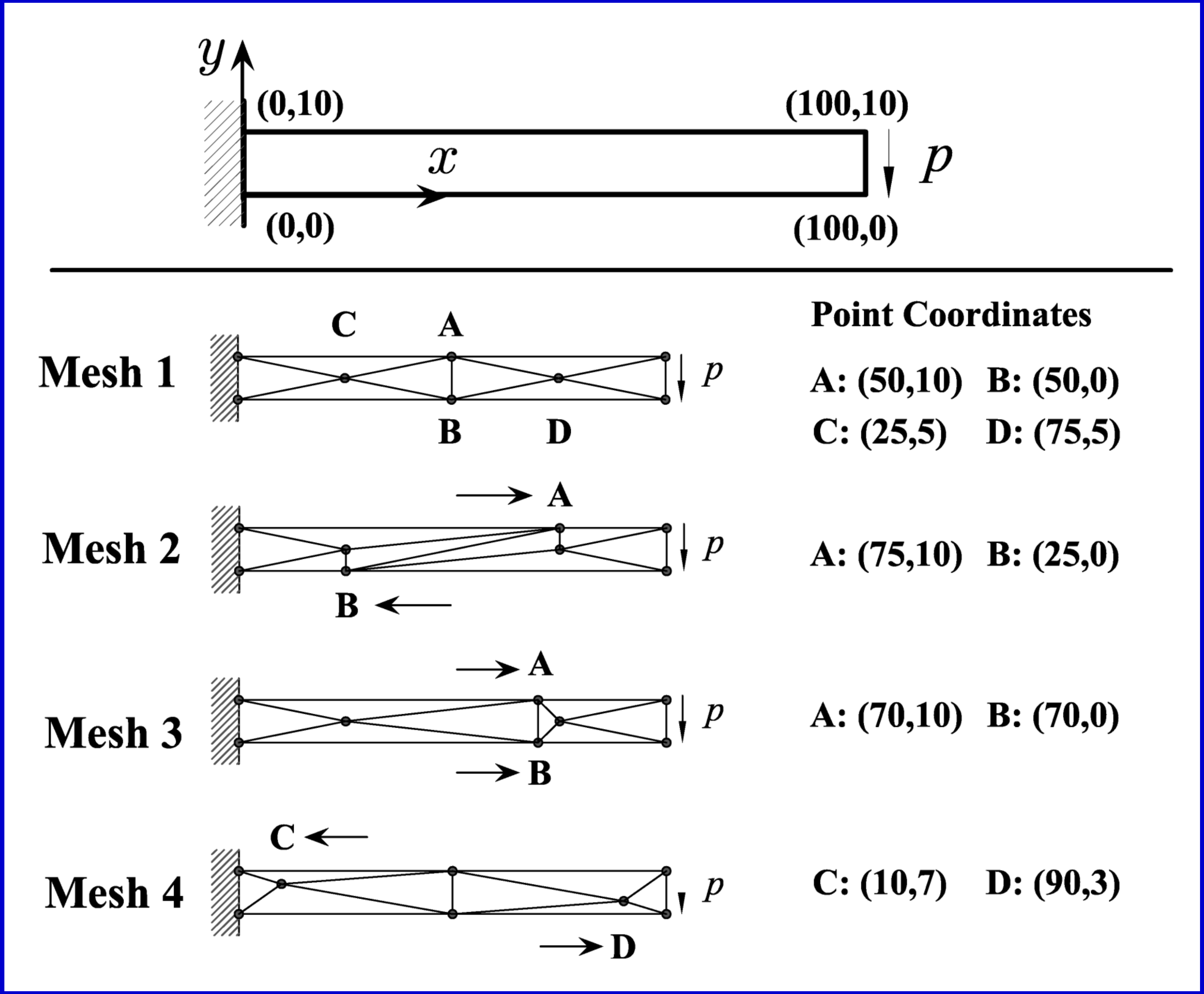


Results: Analysis of a cantilever plate
(a) traditional analysis, (b) AMORE scheme

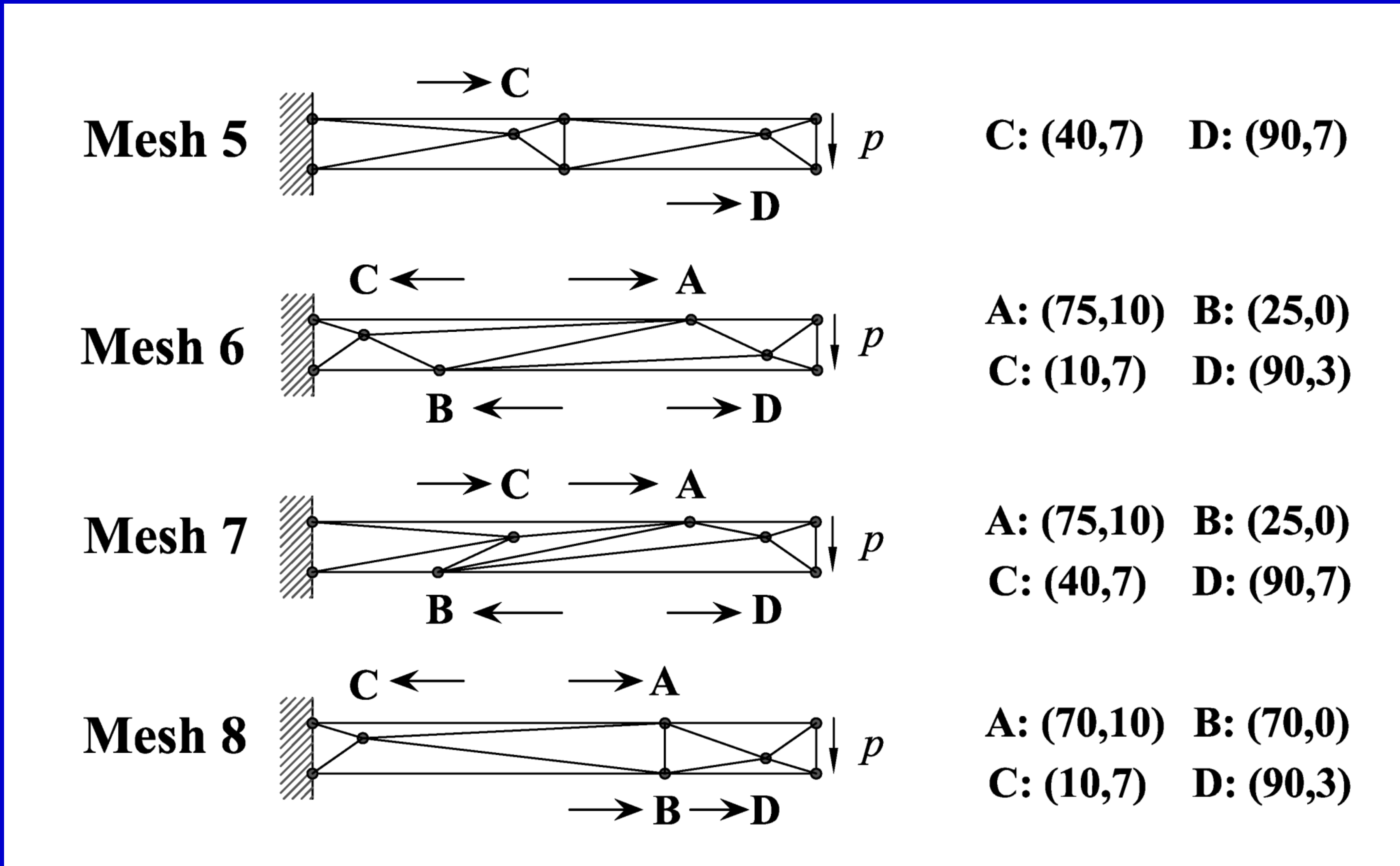


Quadrilateral overlapping elements

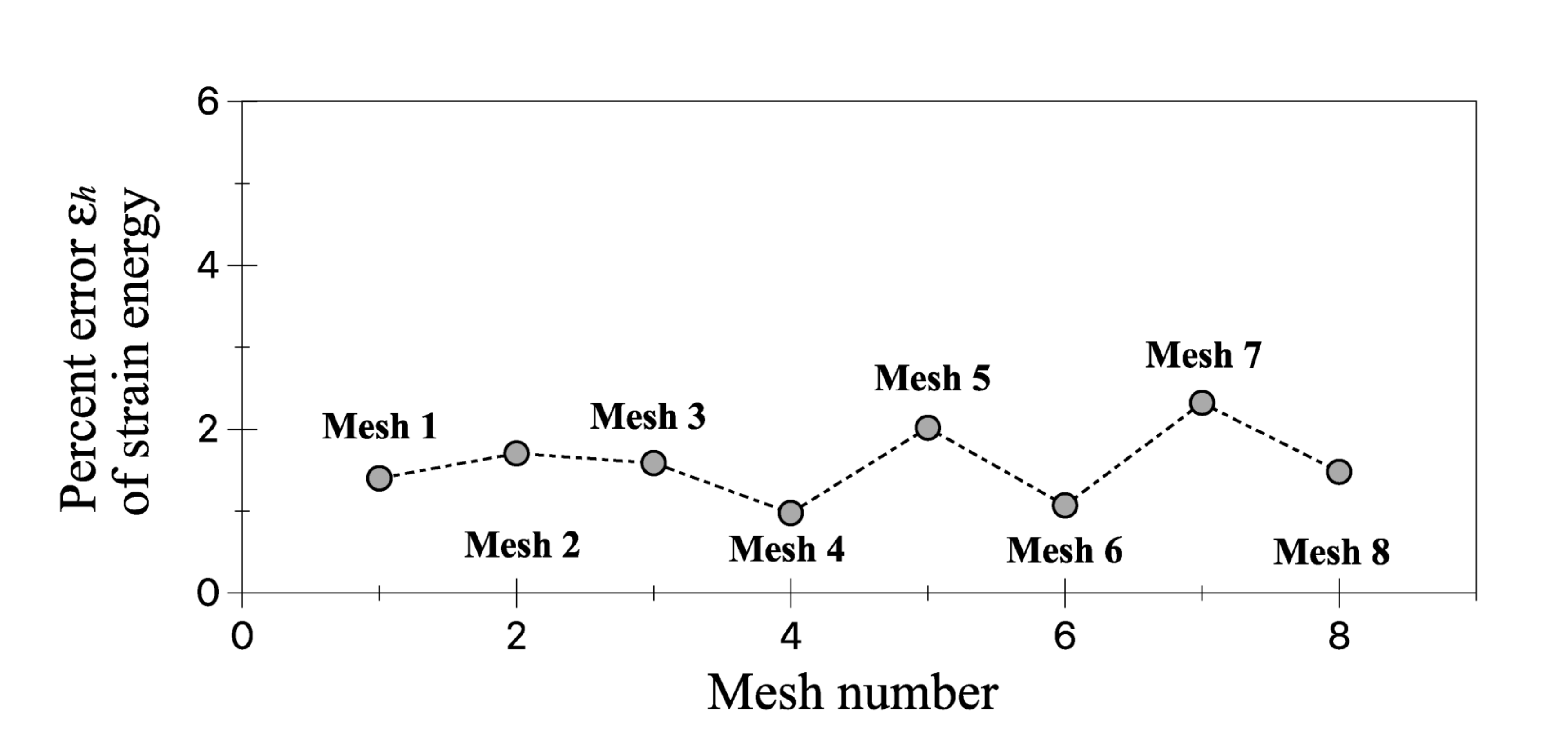
J. Huang, KJ Bathe. Quadrilateral … . In prep.



Analysis of cantilever

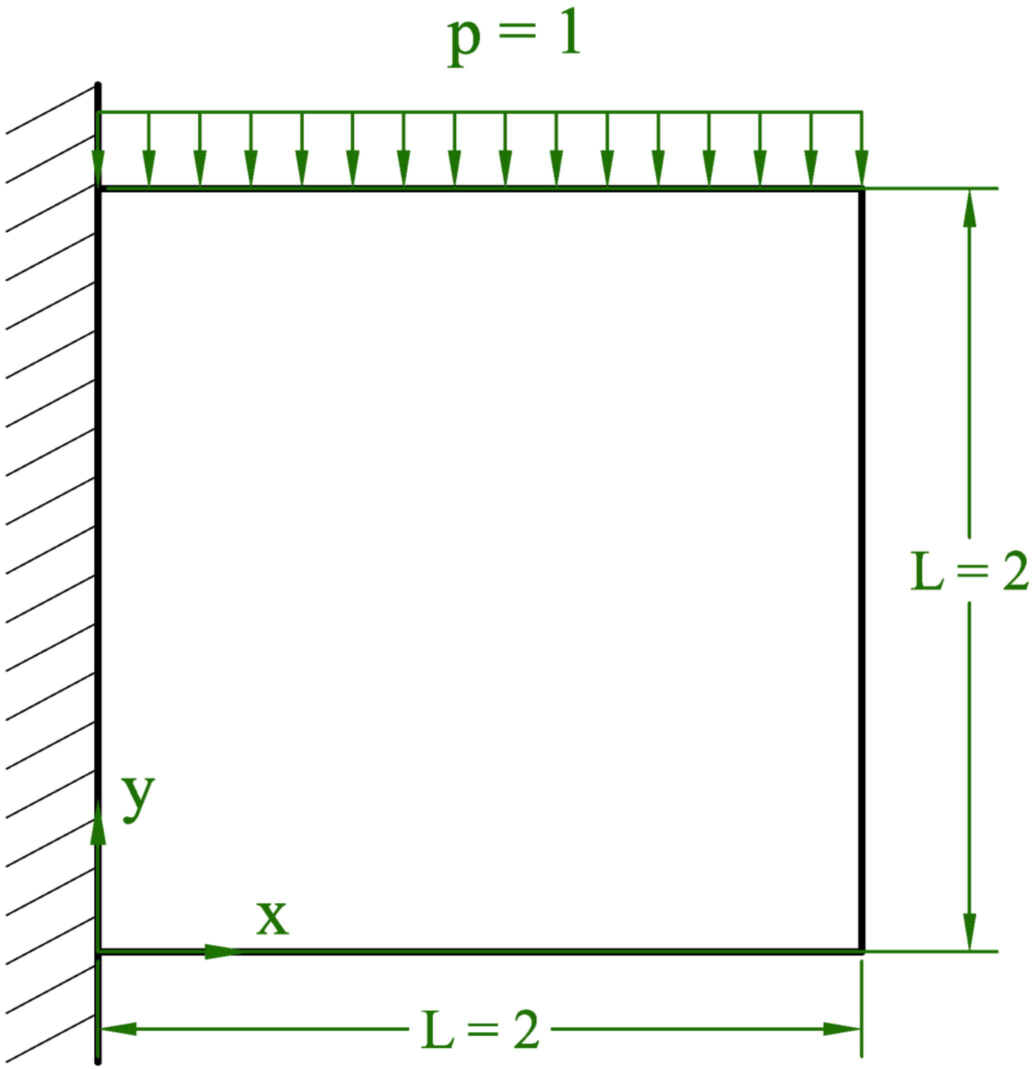


Analysis of cantilever, continued

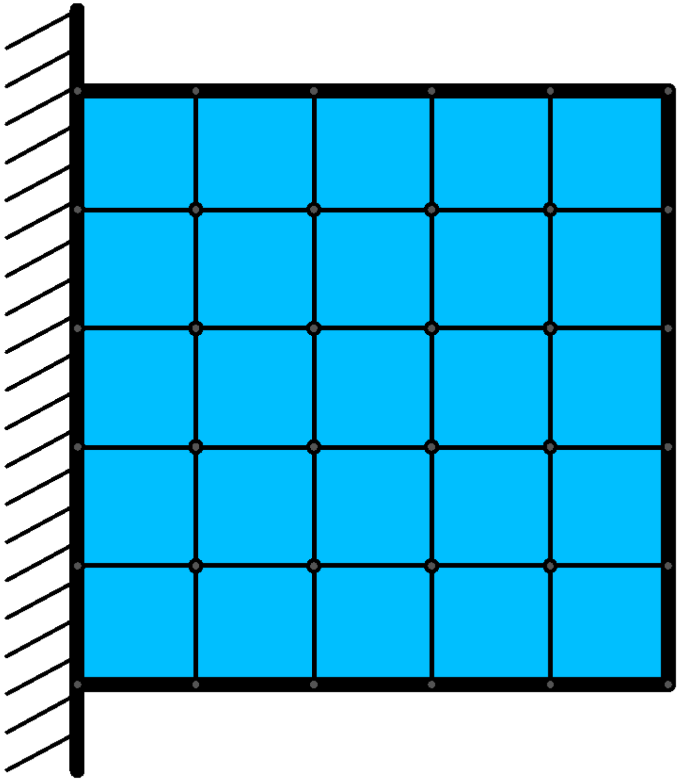


% strain energy error

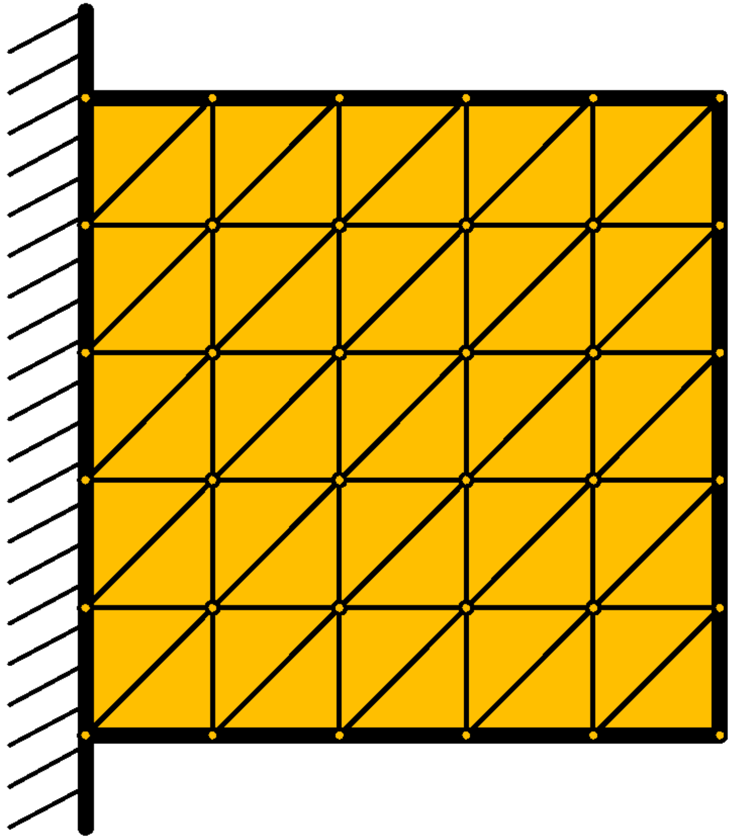
L Zhang, KT Kim, KJ Bathe. The new paradigm … . C & S 2018



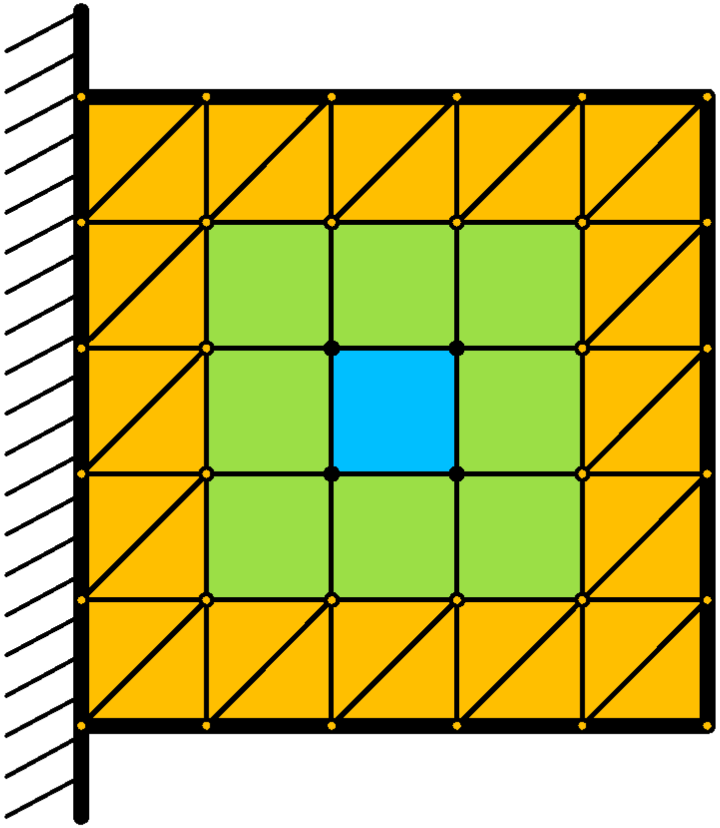
Analysis of clamped plate



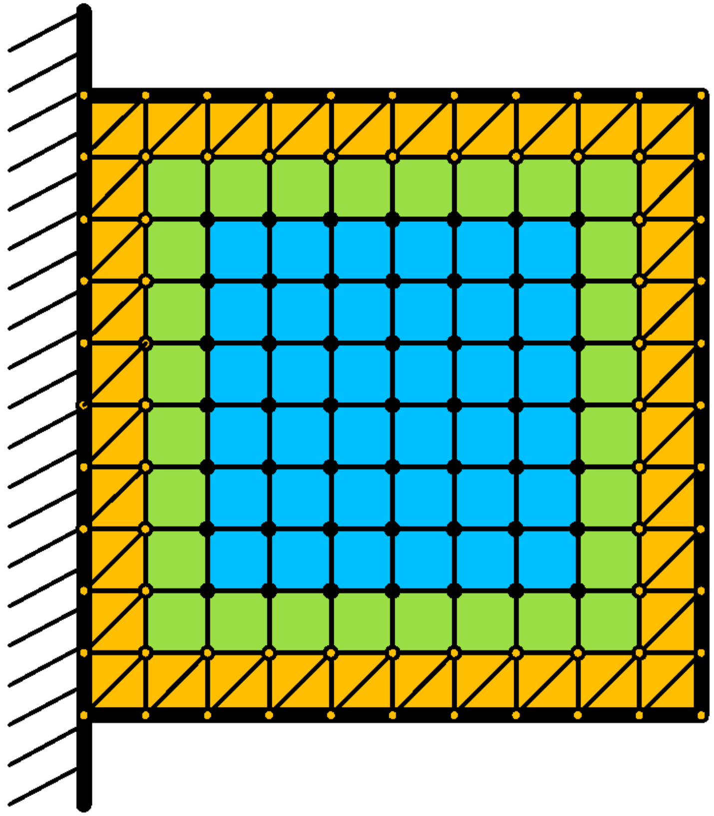
Meshes used for clamped plate, traditional mesh



OFE only used



OFE, coupling and 1 traditional element



OFE, coupling and traditional elements

Solution times for the cantilever plate problem

Scheme		$\log\left(\lvert E_{ref} - E_h \rvert / E_{ref}\right)$	Actual CPU time (s)	Predicted CPU time *SolE* (s)
4-node finite elements		-3.09	1.3	—
Overlapping finite elements	Bilinear basis	-3.25	0.16	0.16
	Quadra-tic basis	-3.04	0.007	0.004
AMORE	Bilinear basis	-3.35	0.4	0.4
	Quadra-tic basis	-3.44	1.0	1.0

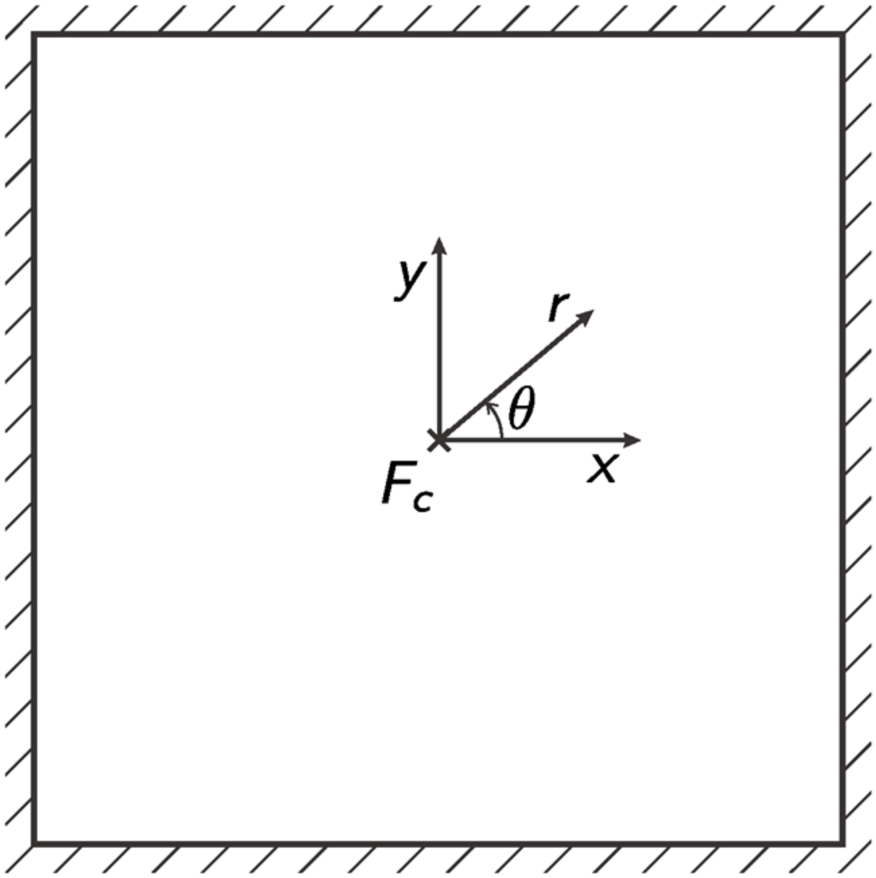
Analysis of wave propagations

Need to have ‘effective time integration’ and ‘effective spatial interpolations’

We explored the use of the OFE with the Bathe time integration scheme and obtained good results

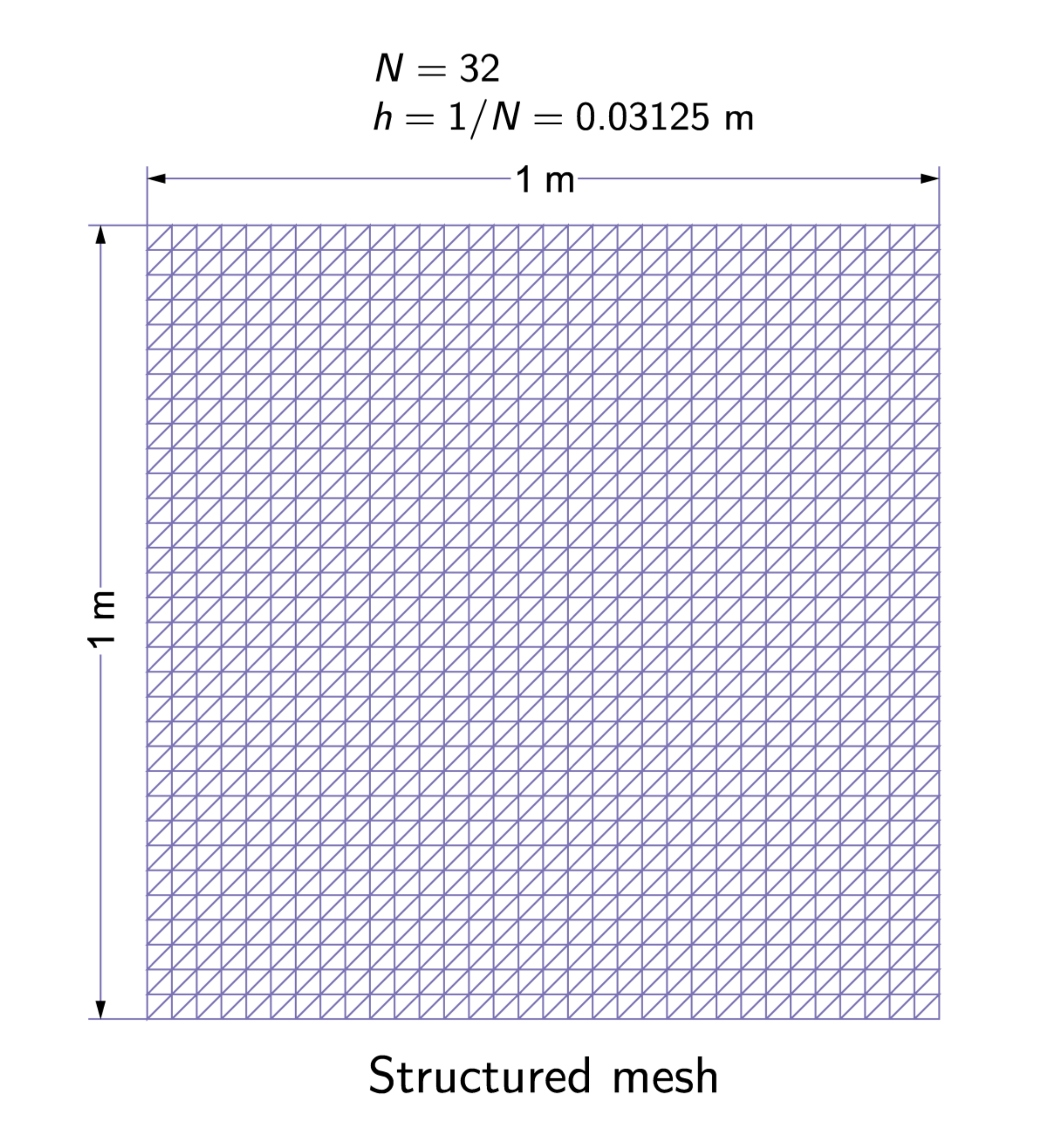
In the overlapping finite elements we use the bilinear and harmonic functions …

KT Kim, L Zhang, KJ Bathe. Transient … . C & S 2018

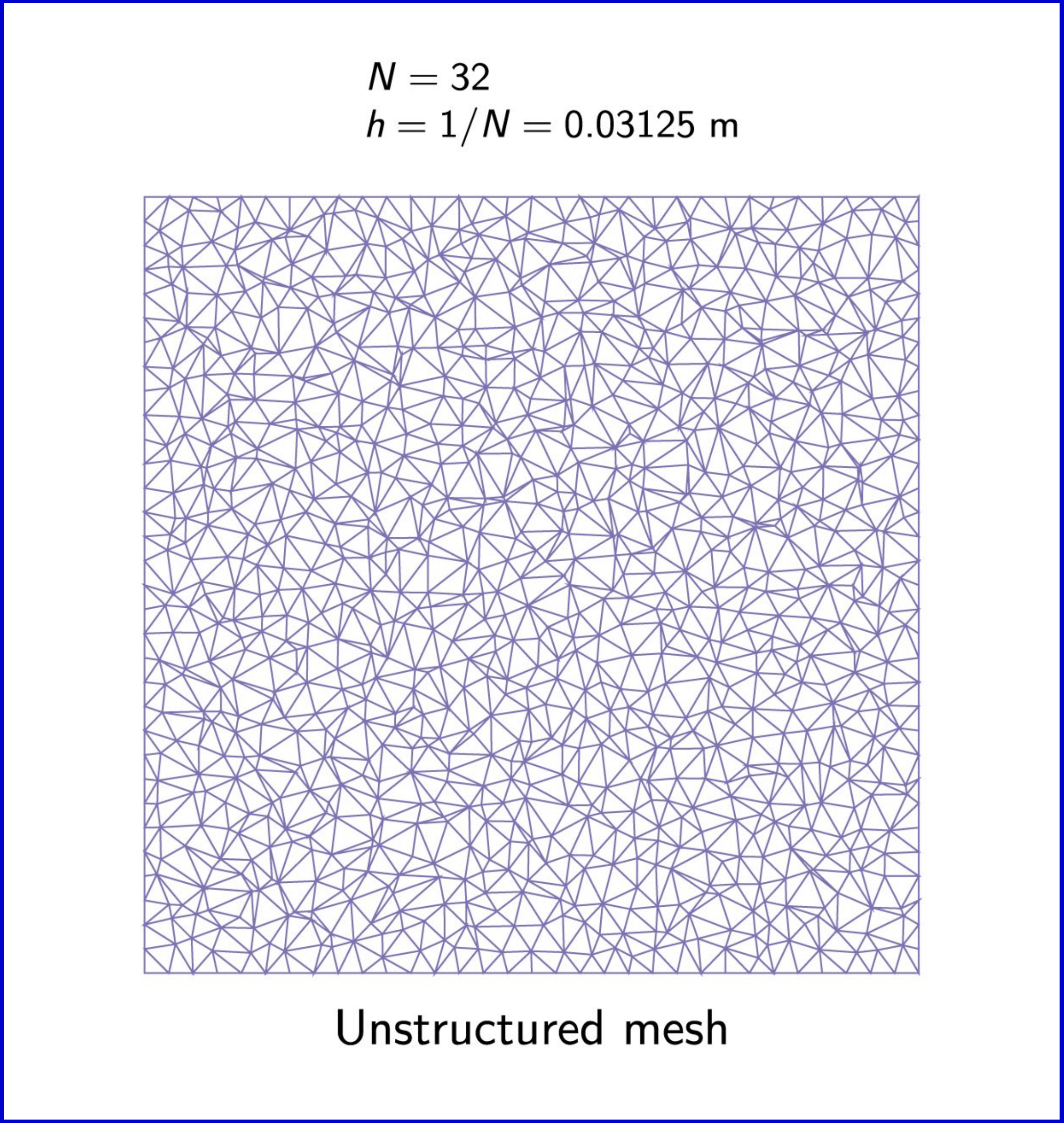


$$c = 1 \text{ m/s}$$
$$u(\mathbf{x}, t = 0) = 0 \text{ m}$$
$$\dot{u}(\mathbf{x}, t = 0) = 0 \text{ m/s}$$

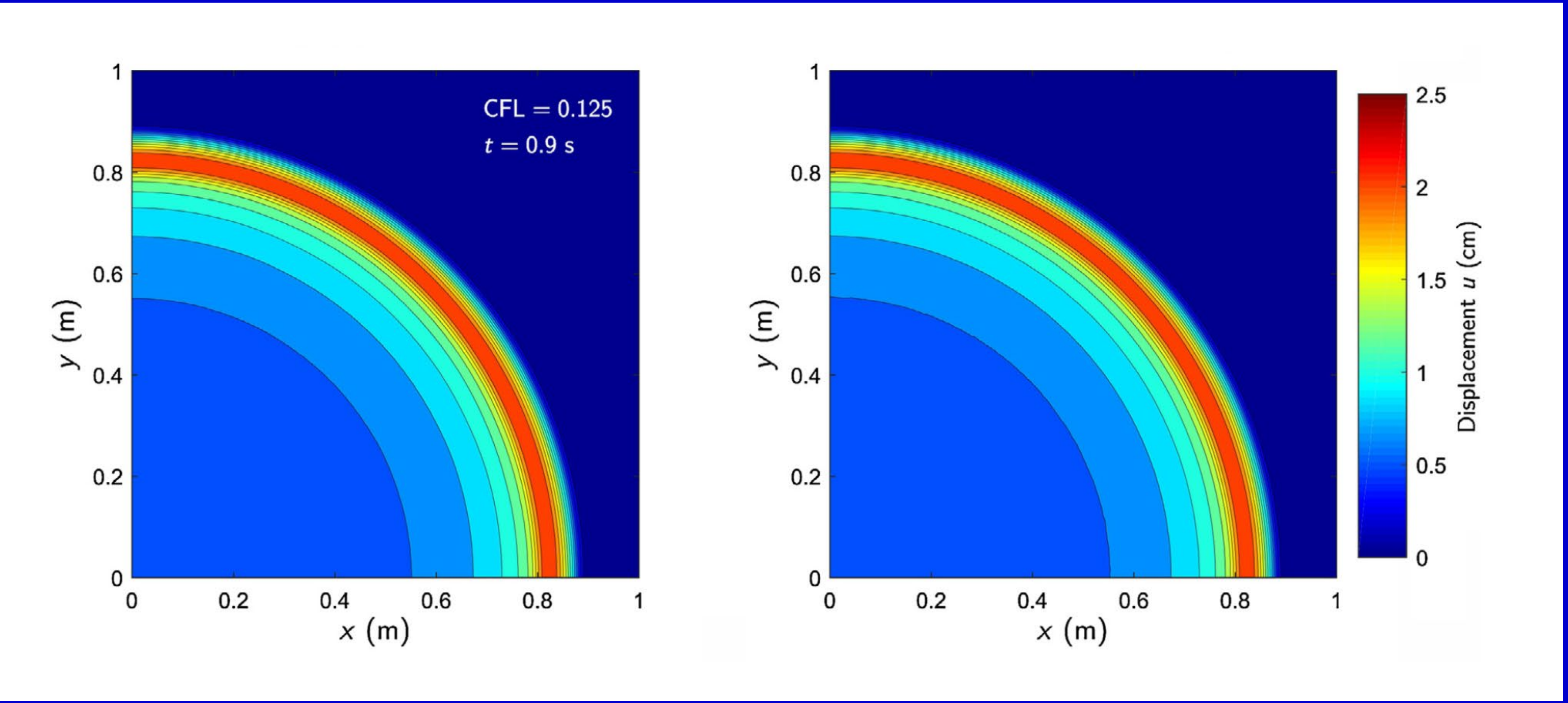
Analysis of wave propagation problem



Structured mesh used

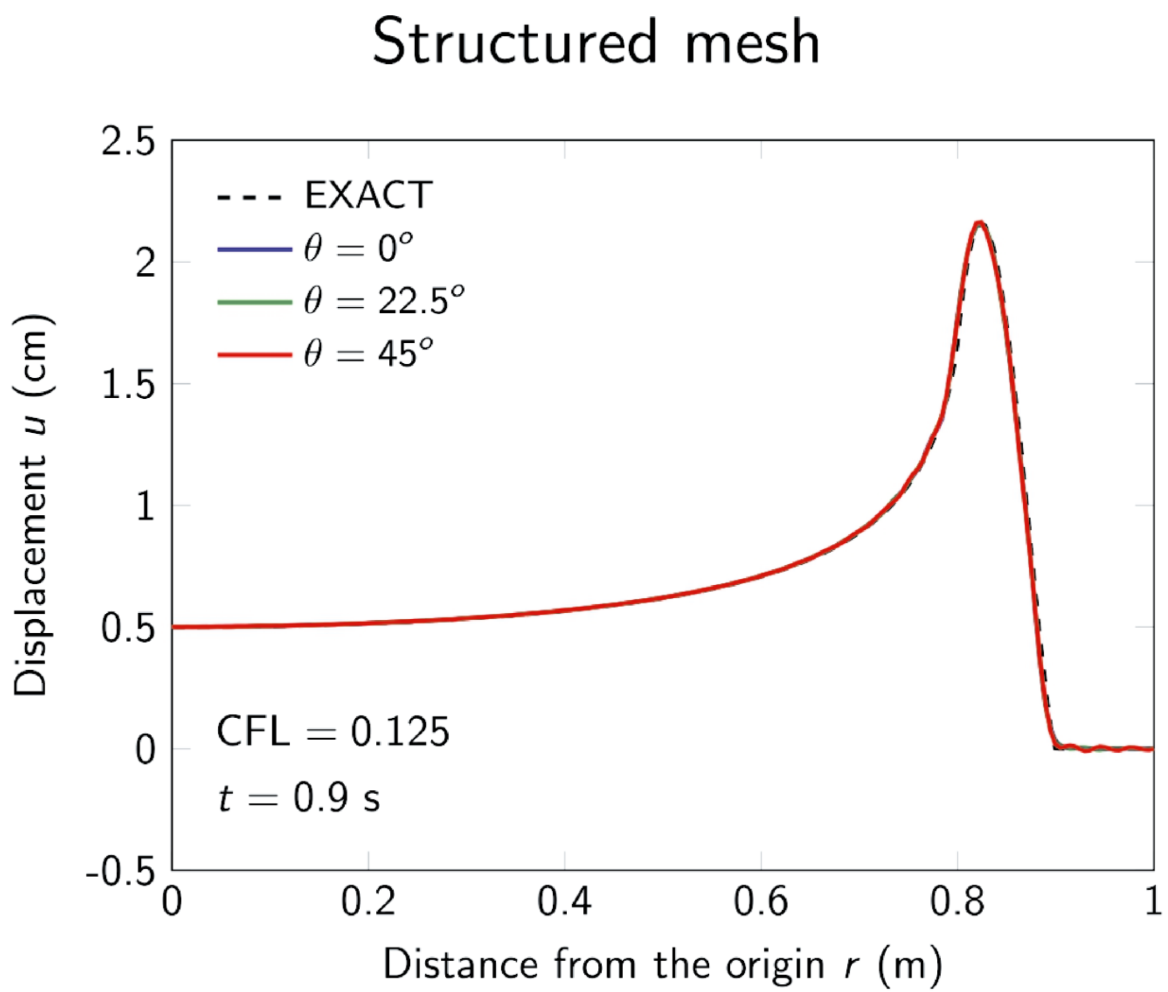


Unstructured mesh used

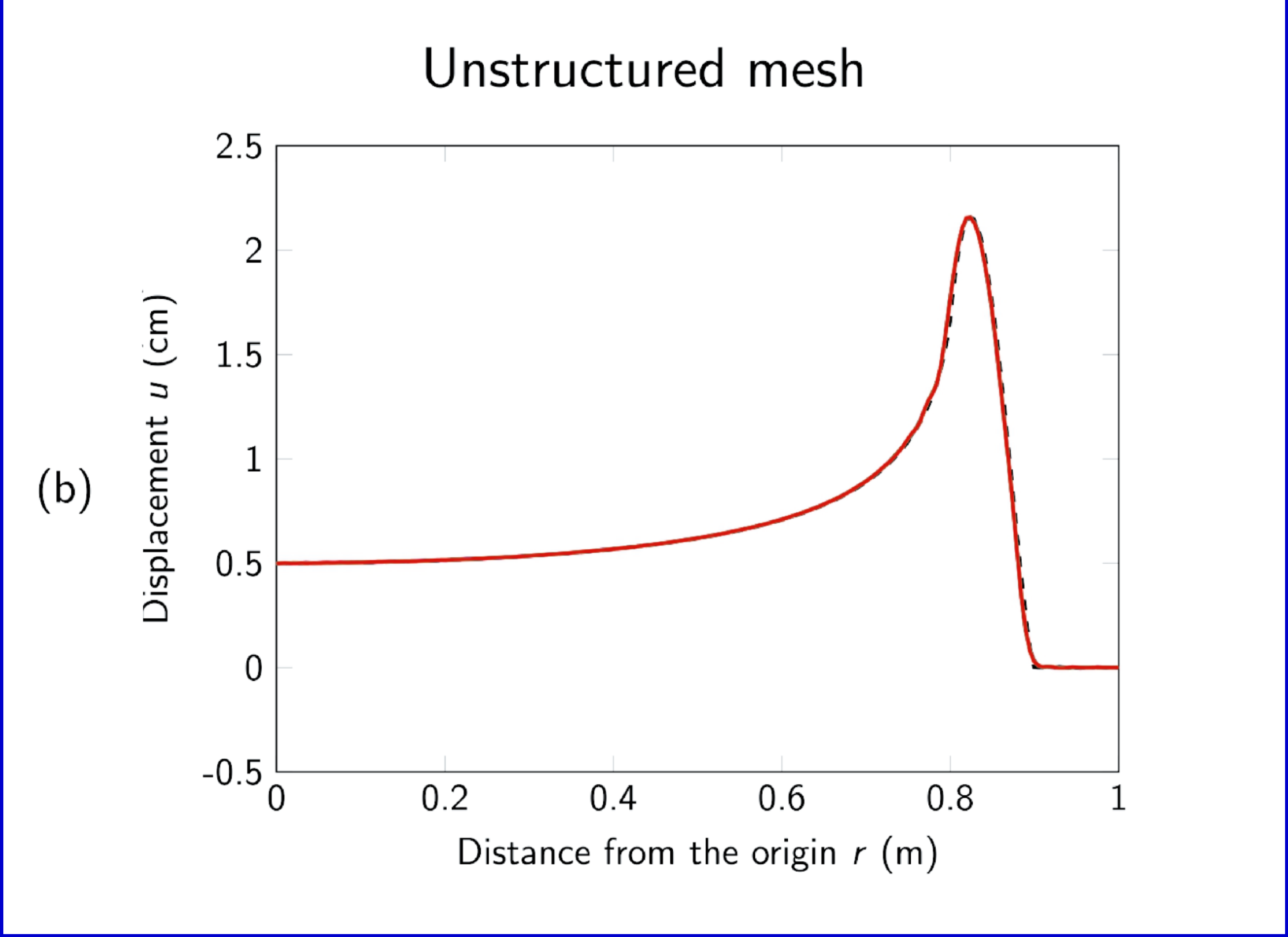


Results, contours of wave,
structured mesh left, unstructured mesh right

(b)



Details of results



Details of results

Conclusions
and a look into the future

- Very powerful capabilities are now available, but there are also still many exciting research challenges – a message to our young (in age and heart !) researchers

Considering analyses and capabilities:

“ We are really only at the beginning of the use of simulations on the computer and the extent to which these will greatly

ENRICH OUR LIFE ! ”