Mathematics in Engineering: Identifying, Enhancing and Linking the Implicit Mathematics Curriculum

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Abstract
A study is undertaken to lay out in a structured manner the mathematics skills required of undergraduate students in the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology. The key objective of the research is to identify barriers to deep mathematical understanding among engineering undergraduates. Data from engineering course syllabi and interviews with engineering and mathematics faculty are combined to form an implicit mathematics curriculum, which lists the mathematical skills relevant to core engineering classes along with the flow of learning and utilization. Several problematic areas are identified, including the concept of a function, linearization, and vector calculus. Interview results show that many engineering faculty have an inadequate knowledge of mathematics class syllabi, and often do not know where or how the skills they require are taught, while mathematics instructors often have a limited understanding of how mathematical concepts are applied in downstream engineering classes. A number of recommendations are made, including increased communication between mathematics and engineering faculty, development of joint resources for problematic areas, and dissemination of a formal catalogue of mathematical skills and resources to engineering students and faculty.

Background
Inadequate mathematical skills present a widespread problem throughout engineering undergraduate programs; however, specific, well-documented examples of student difficulties are often lacking, and the exact nature of the difficulty is frequently uncertain. Moreover, there is often little communication between engineering and mathematics faculty dedicated to or addressing mathematics skills related issues. Engineering faculty assume that certain concepts are taught in the mathematics courses, but they are often not familiar with the specifics of the mathematics curriculum, or the methods utilized (for example: terminology and context of use).

The level of mathematics skills of sophomores and juniors at MIT has been identified as a problem by a number of the faculty that teach core subjects in the Department of Aeronautics and Astronautics. This issue manifests itself in a number of ways and, in particular, has a negative impact on students' ability to grasp engineering subject material. Specific problems are observed during lectures, where questions often arise regarding basic mathematic manipulations. These questions are also posed in the form of "muddy cards" – cards on which students anonymously write down the muddiest part of the lecture.[6] Some examples of such muddy cards taken from a junior-level controls class are shown in Table 1. In all cases shown, the question relates to material that a typical junior is expected to know when entering the class. The questions on these cards strongly suggest that lack of mathematical understanding presents a barrier to deep understanding of the control systems concepts, which are the focus of the lectures. Other evidence of mathematics problems has been observed on class quiz results and homework problems.

Table 1: Example muddy card comments from Principles of Automatic Control (junior-level class), fall 2002 and fall 2003.

Lecture subject	**Muddy card comment**
Control system analysis	"Laplace is muddy"
Steady-state errors	"How did you go from $C = \frac{K_v}{s} E$ to $\frac{dc}{dt} = K_v e$?"
State-space analysis	"What is a non-singular transformation" "What does singular mean"

Diagnostics have been performed by several faculty members to document this problem. Figure 1 shows the results from a diagnostic quiz given to students entering the junior class Principles of Automatic Control in 2001. Although the questions were graded very leniently, the results show that many of the students are unable to perform an integration by parts or calculate the eigenmodes of a second-order system. This issue is of great concern, since these mathematical skills are fundamental to much of the material covered in the course. If the students are stumbling on the mechanics of the problem, it is unlikely that they are grasping the true underlying physical principles and core material of the course.

A similar diagnostic was performed in another class in the Department of Aeronautics and Astronautics, Computational Methods in Aerospace Engineering, which is taken primarily by seniors and second-semester juniors. The mathematical concepts tested were Taylor series, first-order ordinary differential equations (ODEs), eigenvalues, integration by parts, minimum finding, mean/standard deviation, root finding, and numerical ODE integration.[3] The results showed that, with the exception of eigenvalues, many students lacked the ability to correctly approach these basic problems. For example, only 20% of students were able to calculate the mean and standard deviation of a linear function. Of particular interest is the result for the eigenvalue question. This was the highest scoring question - over 80% of the students were able to correctly calculate the eigenmodes of a second-order system. This result is in direct contrast to that shown in Figure 1; however, it is interesting to note that all students in the computational methods class had previously completed Principles of Automatic Control, which not only revisits the concept of eigenvalues, but also ties this mathematical concept to application for aerospace systems.

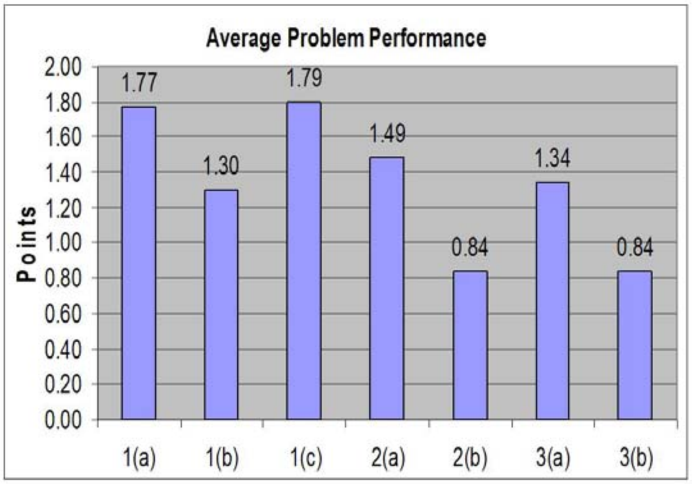

Figure 1: Results from mathematical diagnostic quiz taken by 65 juniors. Questions were each worth two points, and are as follows. 1a: plotting complex numbers; 1b: conversion from Cartesian to polar coordinates; 1c: multiplication and addition of complex numbers; 2a: integration of a function; 2b: integration by parts; 3a: matrix-vector multiplication; 3b: calculate eigenvalues and eigenvectors of a second-order system.

This problem is not unique to students at MIT. The question of how to best teach mathematics in an engineering program has been considered by a number of researchers (for example, [1], [4], [5]). Recently, at the University of Hartford, faculty teaching the freshman engineering design, physics, and calculus courses worked closely together and developed shared outcomes for the three courses.[7] The evaluation showed that this unified approach enabled students to gain better understanding of the linkages between engineering, physics and calculus. In a study to assess mathematics proficiency of students at Grand Valley State University, it was determined that student problems in this area are widespread and originate from many sources.[1] Some resources exist that attempt to address these problems. Examples include the dAimp project, [2] which is currently developing online resources for engineering mathematics. The goal is to put together a series of manipulatives that lend greater understanding of mathematical concepts to engineering undergraduates. Project Links aims to link the concepts of higher mathematics to real-world applications through interactive web-based modules. [8] One of the major challenges associated with developing such resources is the creation of an effective bridge between mathematics and engineering.

The first step to bridging the gap between mathematics and engineering is to comprehend the barriers to deep mathematical understanding among engineering undergraduates. In order to gain such understanding, it is critical to identify specifically what mathematical skills are expected and where in the engineering curriculum these skills are gained. While there were many suppositions regarding this issue in the Department of Aeronautics and Astronautics at MIT, such identification had not been formally carried out or documented. This paper describes an effort to formally identify and document the implicit mathematics curriculum in the undergraduate degree program.

Approach

The implicit mathematics curriculum is a comprehensive list of topics in mathematics relevant to the core undergraduate engineering curriculum in the Department of Aeronautics and Astronautics. The core engineering classes are Thermodynamics, Fluid Dynamics, Structures, Signals and Systems, Computation, and Dynamics for sophomores, and Thermodynamics and Controls for juniors. At MIT, all sophomore courses except Computation are taught together as one subject called Unified Engineering. Many of the mathematics skills are taught in required freshman and sophomore mathematics courses; a few skills are taught explicitly in engineering courses. An initial list of mathematics topics was collected from the syllabi and measurable outcomes documents of the core engineering classes and then organized by subject. For example, eigenvectors and eigenvalues, extracted from the Unified curriculum,[9] were listed under the heading of "Linear Algebra", together with matrix algebra, and linear systems of equations. As found later, disagreement exists about where certain topics belong between the engineering and mathematics community. Our list of topics was modified continuously to approximate a consensus among faculty, but also to serve our original purpose of focusing on key mathematics topics in the context of engineering education in the department. It should be noted that while forming this list, we often found overlap in different disciplines and decided that our classification / organization is not unique. The disagreement among faculty on terms and their organization was also the first pointer towards problematic areas in the students' understanding.

After assembling the initial version of the implicit mathematics curriculum, interviews were scheduled with engineering faculty with the intention to incorporate their feedback on the curriculum structure and to collect their opinion on mathematics-related problems they might have encountered in core undergraduate teaching. The goal was to formally document and organize the implicit mathematics curriculum and to trace the flow of skills learning and utilization. The first question to each faculty member was whether they would modify the mathematics topics list in any way, reorganize or add topics. This helped to form an exhaustive list as viewed by the engineering faculty. The second part of the interview concentrated on the particular set of mathematics skills relevant to the course taught by the interviewee. In particular, he or she had to specify in detail precisely how a mathematics skill is relevant to the class, and whether it is taught anew, reviewed, and/or utilized. If a skill was reviewed and/or utilized, the faculty member was asked to identify the prior course in which the knowledge was assumed to be gained.

After this preliminary round of data was collected, feedback and input was sought from mathematics faculty. The data was discussed in detail with faculty involved in teaching a freshman calculus course and a freshman/sophomore differential equations course, which are respectively pre-requisite and co-requisite for sophomore students in the Department of Aeronautics and Astronautics. This communication with mathematics faculty not only enabled precise identification of mathematics courses and topics where engineering mathematics knowledge and skills are introduced, but also allowed some major gaps and misconceptions to be identified. These findings will be presented in the following section.

Results and Discussion

The condensed implicit curriculum is listed in Table 2. Using input from mathematics and engineering faculty, this final list has evolved considerably from the initial draft, which can be seen in Appendix A on the example of the original form of the questionnaire with entries from the interview with Signals and Systems faculty. It should be noted in Table 2 that the last two categories, probability and statistics and discrete mathematics, are important components of the engineering curriculum but have not yet been fully scoped. The subheadings for the other categories represent what was determined to be the most effective arrangement of topics; however, this classification is not unique. In particular, it was interesting to note that the preferred grouping of topics often differed between engineering and mathematics faculty. Another interesting disparity occurred for the topics of linearization and state. These two areas were identified by many of the engineering faculty as being extremely important concepts and skills. Conversely, mathematics faculty was not at all accustomed to using the term 'state'. Differences were also noted in the way in which linearization was viewed and presented between the two sets of faculty. The first topic, functions, was not originally included as a category; however, after interviewing the engineering faculty, it became clear that there were a number of important mathematical concepts relating to functions that were not captured by other headings. In addition, many of these function-related concepts were identified as problem areas. This will be discussed in more detail later in the paper.

Table 2: The Implicit Mathematics Curriculum. The key for courses is as follows: Fl=Fluids, Dy=Dynamics, Th=Thermodynamics, SS=Signals and Systems, Co=Controls, S=sophomore class, J=junior class.

Mathematical Knowledge	*Utilized*	*Reviewed*	*Taught*
1 Calculus			
1.1 Functions	Fl-S, Th-S, Dy-S, SS-S, Th-J, Co-J		
1.2 Differentiation	Fl-S, Th-S, Dy-S, SS-S, Th-J, Co-J		
1.3 Integration	Fl-S, Dy-S, SS-S, Th-J, Co-J	Th-S	
1.4 Series and sums: Taylor, Fourier	Fl-S, Dy-S, SS-S, Th-J, Co-J		
1.5 Vector Calculus	Fl-S	Fl-S	
2 Geometry			
2.1 Analytical Geometry	Dy-S		
2.2 Trigonometry	Dy-S, Fl-S		
3 Differential Equations			
3.1 ODEs	Th-S, Th-J	Dy-S, Co-J	SS-S, Co-J
3.2 PDEs	Th-J	Th-J	Fl-S
3.3 Integral Equations			Fl-S
4 Linear Algebra			
4.1 Matrix Algebra	SS-S, Co-J	Dy-S, SS-S	
4.2 Linearization, Linear Systems		Co-J	Dy-S
4.3 State (discrete)			SS-S, Co-J
4.4 Tensors (multidimensional objects)			Fl-S
5 Complex Analysis			
5.1 Complex Variables	Co-J	Fl-S	SS-S
5.2 Frequency domain, variables and plots			SS-S, Co-J
5.3 Transforms: Fourier, Laplace	SS-S	Co-J	SS-S, Co-J
6 Probability and Statistics	*To be completed*		
7 Discrete Mathematics	*To be completed*		

Also shown in Table 2 are the engineering core courses in which each mathematical skill is utilized, reviewed and taught. In some cases, a particular course appears in multiple columns for a given topic. For those skills appearing in the ‘Utilized’ and ‘Reviewed’ columns, the engineering faculty was asked to identify where they assumed the appropriate skills to be learned. In many cases, answers were of the vague form “One of the prerequisite mathematics classes”, but the correct course could not be identified. The interview results showed clearly that the majority of engineering faculty has limited familiarity with the syllabi of the mathematics courses, and inadequate knowledge of the context in which particular mathematical skills are learned by the students. Another important point to note about the results in Table 2 is that the mathematical topics appearing in the ‘Taught’ column fall into two general categories. The first consists of topics, such as integral equations, that are taught in the engineering class because they are not part of the prerequisite mathematics curriculum. The second class of topics is those that are taught in mathematics courses, but are re-taught in engineering because the instructor feels that the students’ skills are not sufficient. This need to re-teach material was noted by a number of faculty for several topics, including ODEs, linearization, and complex variables.

Each of the topics in Table 2 was analyzed in detail using the raw data from the interviews. First, we summarized the *specific skills* needed for each mathematical concept as cited by the faculty. Then we included *examples and applications* from class lectures, homework and exams. The third aspect we looked at was *background* assumed by the teaching faculty in order to look for potential matches and mismatches among subjects offered by the mathematics department. The next step was to identify *issues* arising either from comments by faculty or by gaps of instruction found during the analysis. We finally added *resources and recommendations* by listing courses, textbooks, specific lectures, and other resources that are relevant to the skills needed. An example from the analysis of *Functions* is shown in Figure 2.

This investigation resulted in identification of a number of issues and potential barriers, leading to recommendations for curriculum modification and development of supplementary materials. The example above concerns one of the major findings – conceptual problems with the meaning of *function*, its formulations and applications in engineering problems. As evident from the faculty’s comments, many students have difficulty with the concept of argument of a function, which propagates into misunderstanding nested functions and compositions of functions. These notions on the other hand are essential in understanding derivatives and the chain rule. It was determined that the fundamental concept of a *function* is poorly understood by many students, leading to downstream difficulties with engineering applications, such as performing a convolution integral in signals and systems and interpreting engine parametric dependencies in thermodynamics. It was further determined that this mathematical concept is only explicitly taught in the introductory calculus course, taken by only a small percentage of engineering undergraduates (due to advanced placement credit). Conversations with mathematics faculty indicate that the level of high-school mathematics preparation varies widely and that it is highly likely that the concept of a function is never formally encountered by many of the advanced placement students. Our findings indicate that it would be beneficial to further diagnose the problems in this area. One recommendation is to include a diagnostic quiz addressing the concept of function at the beginning of the second-semester calculus course. Based on these findings, consideration should be given to its explicit inclusion in the curriculum of both introductory and second-semester calculus courses.

1.1 Functions

Specific Skills

- Understand the concept of an argument of a function
- Distinguish between independent and dependent variables
- Understand the concept of $f(g(x))$ and $f \circ g$ and apply the chain rule
- Apply change of variables

Examples and Applications

- The convolution integral (*Signals & Systems*):

$$(f \star g)(t) = \int f(\tau)g(t-\tau)d\tau \qquad (1)$$

- Engine parameters (*Thermo*):

$$p_{T2}/p_{T1} = f(\dot{m}_{corr}, N_{corr}) \qquad (2)$$

Background and Assumptions

- Taught in high school and 18.01

Issues

- What is a function? What does it mean for something to be a function of something? Students have a conceptual problem. (Some data available in form of muddy cards feedback.) (*Thermo*)
- Function: what is that? Students refuse to talk about it. Cannot understand the difference between $g(t)$, $g(\tau)$, $g(t-\tau)$, especially important in understanding convolution integrals (1). Can differentiate $f(g(x))$ using chain rule, but not $f(g(h(x)))$, because concept of a function is not well understood.(*Signals & System*)

Resources and Recommendations

- Chain rule: 18.01 Unit 1 - *Differentiation*, Lecture 4 - *Chain rule and implicit differentiation*, Reading: 3.3, 3.5 (*Multivariable Calculus with Analytic Geometry*, Edwards and Penney)
- Change of variables: 18.01 Unit 3 - *Integration*, Lecture 16 - *Changing variables and the second fundamental theorem*, Reading: 6.7
- *Function* fundamentals: recommended candidate for resource development

Figure 2: An example of the analysis and data organization of *Functions*. Extract from complete document.

An interesting experiment regarding the definition of a function was conducted in a more advanced thermodynamics class. Students were asked to write anonymously on paper cards the definition of a function as they understand it, in a sentence, in the context of an example from the class. The results are all over the map from general miscomprehension to some understanding in the class context. For example, one student wrote:

> *Means that there's a graph somewhere I can use to look up the value of the function given the parameters… or something, I don't know.*

Another student tried to think about the problem given:

> *There is some equation relating Y to X1, X2 that is not given to us. I therefore assume it is proportional.*

The most general statement given was similar to:

To say that "Y=f(X1,X2)" is to say that if and only if you know X1 and X2 you can find Y.

It is further recommended that engineering faculty take a small amount of time in their lectures to revisit the concept of a function, and in particular, to tie the mathematical concepts to the appropriate physical problems in the application at hand. In addition, the mathematics resources identified in Figure 2 should be explicitly recommended to the students for background reading.

Another major difficulty among students is dealing with the concepts of linearity and approximation of nonlinear systems in simple terms. In undergraduate engineering classes, linearization is used most often for superposition and linearization around an operating point in disciplines such as controls and dynamics. Deriving simple linearized models from the physics to set up appropriate differential equations is common in both disciplines. This requires basic physical understanding of linear approximation, derivatives and series expansions. Even though all of these ideas are not new to students, they "cannot piece them together to take a nonlinear physical system and create a set of linear ODEs". Discussions with mathematics faculty revealed that linearization is presented in different ways in mathematics and engineering classes. It was also determined that an opportunity exists here for engineering applications to be brought into mathematics courses. For example, derivation of the equations governing the phugoid motion of an aircraft provides an excellent context in which to teach linearization. Use of a physical, engineering example allows students to better understand the importance and relevance of the mathematical concept at hand. Linearization was identified as one of the candidates for further resource development, with particular emphasis on communication of engineering examples to mathematics faculty for inclusion in their classes.

Other major problems were identified in *vector calculus* skills. Performing vector products in conjunction with integrals seems to be a difficulty, while it is an essential skill for fluids and thermodynamics applications (for example, in deriving equations of motion). The term "control volume" was used by most engineering faculty when describing these mathematical skills; however, this term was completely unfamiliar to mathematics faculty. A straightforward approach to address these issues in a timely manner is to build / find resources for both students and faculty to bridge the existing gaps. Several attempts at this have been made before which resulted in a myriad of web-based resources. Some of them have been developed by engineers and thus tailored to engineering purposes. Others involve works by mathematicians. For example, there is an ongoing effort by the MIT Mathematics department to create synthesized web-based content of all core mathematics subjects intended for engineering students.[10,11,12] This can be useful as review or catalogued resource for students who need to refresh their knowledge on basic concepts. Our vision is that a common resource, a product of a joint effort of mathematics and engineering faculty will be more effective at bridging the missing or mismatched mathematics concepts.

Conclusions and Recommendations

This structured attempt to map the implicit mathematics curriculum in the Department of Aeronautics and Astronautics proved to be very valuable to both engineering and mathematics faculty. The interviews conducted identified not only specific problem skill areas, but also highlighted misconceptions and mismatches between engineering and mathematics departments.

Recommendations arising from this research include both the development of supplementary resources and an increase in the communication and awareness between engineering and mathematics faculty.

One of the major findings of this study was that the engineering faculty is unaware of the details of mathematics class curricula – they do not know specifically where and how mathematical concepts are taught. Likewise, for many concepts, mathematics faculty do not have a clear understanding of precisely how their downstream "customers" will use the skills they teach. There are many opportunities for engineering examples to be incorporated into mathematics courses and for a common mathematical terminology to be incorporated into engineering classes.

The next step in this work is to communicate these findings to mathematics and engineering faculty using the detailed data structure shown in Figure 2. The list of resources and recommendations for each topic will then be sorted by engineering course and given to students. The goal is to have the available resources and linkages with mathematics courses clearly laid out for the engineering students and faculty. This awareness will allow engineering instructors to build upon previous mathematical learning, rather than attempt to re-teach skills in their own way. In addition, an effort will continue to provide relevant engineering examples for incorporation to mathematics courses.

The most problematic concepts for undergraduate students in the Department of Aeronautics and Astronautics were found to be functions, linearization, and vector calculus. It is recommended that resources be identified and/or developed in each of these areas jointly between mathematics and engineering faculty.

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APPENDIX A: Math Curriculum Questionnaire 2.1 – Example form from Unified Signals and Systems Interview.

Math Curriculum Questionnaire 1.0
Unified Signals & Systems

1.0 Mathematical Knowledge	***Notes***	***Review***	***Utilize***	***Teach***
1.1 **Calculus**				
1.1.1 Integration	See Note 3 below.		18.01	meaning of integration (vs. mechanics)
1.1.2 Differentiation			18.01	
1.1.3 Series and sums (Taylor), convergence, limits	Utilize small amount, Fourier series in sampling theorem.		18.02	
1.2 **Probability & Statistics**				
1.2.1 Means, standard deviations, averaging integrals				
1.2.2 Random variables				
1.2.3 Distributions (normal, student etc)				
1.2.4 Uncertainty				
1.3 **Differential Equations**				
1.3.1 ODEs	Teach/use extensively. 15 lectures in fall.	In class	18.03	20% class take 18.03 as co-req. Have to re-teach almost all needed info for ode's.
1.3.1.1 Eigenvalues, eigenvectors	methods to solve, 2x2 and 3x3 systems. no generalized evs. See Note 4, 5 below.	In class, mechanical aspects in recitation.	18.03	
1.3.1.2 Linearization, linear systems				
1.3.2 PDEs				
1.3.3 Integral Equations				
1.3.4 State				3-4 lectures on concept of state.
1.4 **Linear Algebra**				
1.4.1 Matrix algebra	Rely very heavily on solving linear eqns. First reading		18.02/18.03	

	assignment: primer on solving linear eqns.			
1.4.1.1 Orthogonality, vectors				
1.4.1.2 Eigenvalues, Eigenvectors	as above			
1.4.1.3 Linear systems of varying dimensions	3x3 max size		18.03	in class
1.4.1.4 State				as above
1.4.2 Tensors (multidimensional objects)				
1.4.2.1 Introduction and some basic operations				
1.5 **Complex Analysis**				
1.5.1 Complex variables	are weak, have to re-teach. Euler's formula for complex exponentials.		18.01	in class
1.5.2 Frequency domain, variables and plots				frequency domain, not plots
1.5.3 Transforms: Fourier, Laplace	see Note 6 below.		18.03	teach FT utilize LT
1.5.4 Convergence, stability and consistency				teach convergence, stability
1.6 **Computation / Discrete math**				
1.6.1 Proof theory				
1.6.2 Number theory (basic), combinatorics				
1.6.3 Algorithms analysis				
1.6.4 Graph theory				
1.6.5 Logic				
1.7 **Numerical Methods** *(includes different/additional knowledge from the above, but requires integrated skills from calculus, algebra, differential equations, complex analysis…)*				

Additional Notes

1. Missing: functions. What is a function? g(t) vs. g(T) vs. g(t-T) or f[g(x)]. cos(t-T) vs. g(t-T) – issue with abstract functions.
2. Missing: trigonometry. e.g. ~ 50% can't expand cos x in exponentials or could not derive. Should learn in 18.01.
3. Integration: learn emphasis on how to find analytical integrals. Not taught what it means for an integral to converge. Need for Laplace transforms. Have seen this in the context of sums – taught using this analogy.

4. Eigenvalues & eigenvectors: taught in 18.03 but don't understand. "They told us it was important, but not why."
5. Also talk about characteristic values – where does this fit?
6. Fourier/Laplace transforms: see Fourier series in 18.03, not FT. In 18.03, LT are taught as a "tool" to solve d.e.'s. Important to teach bilateral transforms and region of convergence.
7. Two major problems: a) have learned material and forgotten it; manageable. b) lack of mathematical sophistication

Additional Questions

For the checked fields, answer the following questions:

1/ If the knowledge is *expected* from the students and *utilized* in your course, in what course from the AA curriculum do you think they learned it?

2/ If the topic is *reviewed*, to what extent do you review it, how long ago do you think the knowledge was gained, in what AA or math course? How long do you spend on review? Who does the review (instructor, TA graduate/undergraduate)?

3/ If the topic is *taught*, would you qualify the instruction as introduction, basic teaching or detailed presentation?

4/ How do you expect each of the checked math concepts to be used by the students: in what way, to what degree, in what types of problems?

5/ Are there any math concepts or skills presented in your course that are missing from this list?