Governance Through Trading and Intervention: A Theory of Multiple Blockholders[*]

Alex Edmans

Wharton School, University of Pennsylvania

Gustavo Manso

MIT Sloan School of Management

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Abstract

Traditional theories argue that governance is strongest under a single large blockholder, as she has large incentives to undertake value-enhancing interventions. However, most firms are held by multiple small blockholders. This paper shows that, while such a structure generates free-rider problems that hinder intervention, the same co-ordination difficulties strengthen a second governance mechanism: disciplining the manager through trading. Since multiple blockholders cannot co-ordinate to limit their orders and maximize combined trading profits, they trade competitively, impounding more information into prices. This strengthens the threat of disciplinary trading, inducing higher managerial effort. The optimal blockholder structure depends on the relative effectiveness of manager and blockholder effort, the complementarities in their outputs, information asymmetry, liquidity, monitoring costs, and the manager's contract. (*JEL* D82, G14, G32)

Corporate governance can have significant effects on firm value. Through ensuring that managers act in shareholders' interest, it reduces the agency costs arising from the separation of ownership and control. In turn, traditional theories argue that concentrated ownership is critical for effective governance, since only large investors have incentives to monitor the manager and, if necessary, intervene to correct value-destructive actions.

However, many firms in reality have multiple small blockholders (Faccio and Lang 2002; Maury and Pajuste 2005; Laeven and Levine 2007; Holderness 2009). Such a structure appears to be suboptimal for governance, as splitting equity between numerous shareholders leads to a free-rider problem: each investor individually has insufficient incentives to bear the cost of monitoring. Should policymakers encourage more concentrated stakes, as suggested by existing models, or can such a structure in fact be efficient? The evidence also demonstrates heterogeneity in blockholder structures. What causes the number of blockholders to vary across firms?

These questions are the focus of this paper. We demonstrate that a multiple blockholder structure can be efficient, and identify the factors that determine the optimal blockholder structure. While splitting a block reduces the effectiveness of direct intervention, it increases the power of a second governance mechanism: trading. By trading on private information, blockholders move the stock price towards fundamental value, and thus cause it to more closely reflect the effort exerted by the manager to enhance firm value. If the manager shirks or extracts private benefits, blockholders follow the "Wall Street Rule" of "voting with their feet" and selling to liquidity traders. This drives down the stock price, reducing the manager's equity compensation and thus punishing him ex post. However, such a mechanism only elicits effort ex ante if it is dynamically consistent. Once the manager has taken his action, blockholders cannot change it and are only concerned with maximizing their trading profits. A single blockholder will strategically limit her order to reduce the revelation of her private information. By contrast, multiple blockholders trade aggressively to compete for profits, as in a Cournot oligopoly. Total quantities (here, trading volumes) are higher than under monopoly, so more information is impounded in prices and they more closely reflect fundamental value and thus the manager's effort.[1] Multiple blockholders therefore serve as a commitment device to reward or punish the manager ex post for his actions.

We derive an interior solution for the optimal number of blockholders that maximizes firm value. This optimum arises from a trade-off between intervention and trading: fewer blocks maximize intervention, but more blocks increase trading. Therefore, this optimum is increasing in the value created by managerial effort and decreasing in the value created by blockholder intervention. If blockholders are passive, such as mutual funds, they are more effective at governing through trading than intervention, and so a large number is optimal. By contrast, with activists and venture capitalists, concentrated ownership is efficient. We show that the firm value optimum may differ from the social optimum that maximizes total surplus (firm value net of effort costs), and the private optimum that would be endogenously chosen by the blockholders if they retraded their stakes to maximize their combined net payoffs (which include informed trading profits). However, the above comparative statics are the same for all three optima.

In the core model, blockholders are automatically informed about firm value. We ex-

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[1]The 2007 hedge fund crisis is a prominent example of the substantial price changes that result from multiple investors trading in the same direction.

tend the model to costly information acquisition. In equilibrium some blockholders may stay uninformed, because trading profits are insufficient to justify information gathering. Since uninformed blockholders do not trade, and reduce intervention by diluting ownership, they lower firm value. Thus, the optimal number of blockholders is bounded above, to ensure that competition in trading is sufficiently low that trading profits are adequate to motivate all block-holders to acquire information. If net trading profits increase, this bound is loosened and so the number of blockholders rises. This in turn occurs if market liquidity and the blockholders' informational advantage increase, and the cost of information falls.

The core model assumes that blockholder and manager efforts are substitutes, with independent effects on firm value. For example, the firm value impact of managerial effort to launch new products is unaffected by the extent to which blockholders extract private benefits or monitor managerial perks. However, in some cases there may be positive complementarities, where the marginal productivity of one party's effort is increasing in other party's effort – for example, the blockholder formulates a strategy which the manager implements. We model positive complementarities by specifying that firm value depends only on the lower of the manager's and blockholders' output levels (where "output" is effort scaled by productivity). Since managerial effort is only productive if it is accompanied by high blockholder effort (and vice versa), the optimal number of blockholders balances the output levels of both parties. The effect of effort productivity changes direction: the optimum is now decreasing (increasing) in the effectiveness of the manager's (blockholders') effort. If blockholder effort is ineffective, concentrated ownership is necessary to "boost" blockholder output so that it is at a similar level to the manager's output.

The opposite case is negative complementarities, where the marginal productivity of one party's effort is decreasing in the other party's output. This occurs if blockholders correct managerial shirking: blockholders are most effective if the manager exerts low effort or consumes private benefits. We model negative complementarities by specifying that firm value depends only on the higher of the output levels of the two parties. The optimum is determined entirely by the more effective action, and ignores trade-off considerations with the less effective action. The efficient number of blockholders is either very low (if blockholder effort is relatively effective) or very high (if managerial effort is relatively effective).

Finally, the optimal number of blockholders is also increasing in the manager's and block-holders' relative weighting on the stock price rather than long-run fundamental value (e.g. as a result of short vesting periods or liquidity needs), since this augments the importance of stock price informativeness for their effort choices.

We close by discussing empirical implications, which fall under two broad themes. First, the model suggests a different way of thinking about the interaction between multiple blockholders, that can give rise to new avenues for empirical research. Prior models perceive blockholders as competing for private benefits, and so existing empirical studies of multiple blockholders typically focus on rent extraction (e.g. Laeven and Levine (2007)). Our paper suggests that future research may be motivated by conceptualizing them as informed traders, competing for trading profits. This link between blockholders and the microstructure literature generates a new set of predictions relating to informed trading and financial markets. The model predicts that blockholder structure impacts price efficiency and consequently firm value, and their power in exerting governance depends on microstructure factors such as liquidity and the blockholders' information advantage. One recent example of such a research direction is Gal-

lagher, Gardner and Swan (2010), who show that an increase in the number of blockholders reduces trading profits, augments price efficiency, and leads to subsequent improvements in firm performance. Gorton, Huang and Kang (2010) find that price informativeness is increasing in the number of blockholders; Boehmer and Kelley (2009) document that it is rising in ownership dispersion. Bharath, Jayaraman and Nagar (2010) find that liquidity improves firm value particularly in firms with multiple blockholders, and Smith and Swan (2008) show that trading by multiple blockholders disciplines managerial compensation. More generally, these implications contribute to the broader literature linking financial markets to corporate finance and demonstrating the real effects of financial markets.[2]

Second, the theory implies that the number of blockholders is important as both a dependent and independent variable in empirical studies. Existing research often focuses on explaining total institutional ownership or the size of the largest blockholder. This paper suggests that the number of blockholders is another important feature of governance structures. As a dependent variable, the model generates testable predictions for the factors that should cause blockholder structure to vary across firms, potentially explaining the heterogeneity observed empirically. As an independent variable, the number of blockholders is a driver of both market efficiency and the strength of corporate governance. Empirical papers frequently use total institutional ownership as a gauge of price efficiency, since institutions are typically more informed than retail investors. However, market efficiency requires not only that investors be informed, but that they impound their information into prices and so the number of informed shareholders is a relevant additional factor. Similarly, governance is typically proxied for using total institutional ownership, or the holding of the largest shareholder, but the number of blockholders is also important. See Bharath et al. (2010) and Gorton et al. (2010) for recent empirical studies of the effect of blockholder numbers.

This paper is organized as follows. Section 1 reviews related literature. Section 2 presents the model and analyzes the effect of blockholder structure on both intervention and trading. Section 3 derives the optimal number of blockholders that maximizes firm value, total surplus, and the blockholders' payoff. Section 4 considers extensions, Section 5 discusses empirical implications, and Section 6 concludes. The Appendix contains all proofs not in the main text, some extensions, and other peripheral material.[3]

1. Related Literature

The vast majority of blockholder models involve the large shareholder adding value through direct intervention, or "voice" as termed by Hirshman (1970). This can involve implementing profitable projects or correcting managerial inefficiency. In Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Maug (1998, 2002), Kahn and Winton (1998) and Mello and Repullo (2004), a larger block is unambiguously more desirable as it reduces the free-rider problem and maximizes incentives to intervene.

By contrast, Burkart, Gromb and Panunzi (1997) show that the optimal block size is finite if blockholder intervention can deter managerial initiative ex ante. Bolton and von Thadden (1998) and Faure-Grimaud and Gromb (2004) achieve a finite optimum through a different

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[2]See, e.g., Fishman and Hagerty (1989), Holmstrom and Tirole (1993), Dow and Gorton (1997), Subrahmanyam and Titman (1999), Fulghieri and Lukin (2001), Chen, Goldstein and Jiang (2007), Goldstein and Guembel (2008), Calcagno and Heider (2008), Ferreira, Ferreira, and Raposo (2010).

[3]All Appendices are available online at http://www.sfsrfs.org.

channel, as too large a block reduces free float. While these papers only consider a single shareholder, Pagano and Roell (1998) point out that if the finite optimum is lower than the total external financing required, the entrepreneur will need to raise funds from additional shareholders. Although this leads to a multiple blockholder structure, the extra blockholders play an entirely passive role: they are merely a "budget-breaker" to provide the remaining funds. Replacing the additional blockholders by creditors or dispersed shareholders would have the same effect. In this paper, all blockholders play an active role. In Winton (1993), a multiple blockholder structure arises as investors face wealth constraints, rather than from price efficiency considerations.

Two recent papers by Admati and Pfleiderer (2009) and Edmans (2009) analyze an alternative governance mechanism: trading (also commonly referred to as "exit"). Informed trading causes prices to more accurately reflect fundamental value, in turn inducing the manager to undertake actions that enhance value.[4] The survey evidence of McCahery, Sautner and Starks (2010) finds that trading is the primary governance mechanism used by institutions; Parrino, Sias and Starks (2003) and Chen, Harford and Li (2007) document direct evidence of governance through trading. However, Admati and Pfleiderer and Edmans both consider a single blockholder and do not feature intervention.

Attari, Banerjee and Noe (2006), Faure-Grimaud and Gromb (2004), and Aghion, Bolton and Tirole (2004) feature a blockholder who can only intervene and a speculative agent who can only trade. The blockholder does not trade; even though the speculator does, such trading does not exert governance as there is no managerial decision. These theories thus consider intervention only. Noe (2002) features multiple blockholders who both intervene and trade. Since stock price informativeness has no effect on managerial effort, blockholder trading again does not exert governance.[5] In Khanna and Mathews (2010), blockholder trading does improve firm value, but through the different channel of countering manipulation by a short-seller. In our model, all blockholders engage in both intervention and trading; the latter affects the manager's incentives and thus exerts governance. Indeed, McCahery et al. find that institutional blockholders use both governance mechanisms frequently.[6] To our knowledge, this paper is the first theory that analyzes both of these major governance mechanisms, and the tradeoffs between them.

Most existing multiple blockholder theories focus on the formation of coalitions to win voting contests (Dhillon and Rossetto 2009) or extract private benefits (Zwiebel 1995; Bennedsen and Wolfenzon 2000; Mueller and Wärneryd 2001; Bloch and Hege 2003; Maury and Pajuste

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[4]In Holmstrom and Tirole (1993), Calcagno and Heider (2008) and Ferreira, Ferreira and Raposo (2010), price efficiency is also desirable as it helps monitor management. In Fulghieri and Lukin (2001), efficient prices reduce the cost of raising funds for a high-quality firm. In Fishman and Hagerty (1989), efficient prices improve the manager's investment decisions. These papers do not analyze the effect of blockholder structure on price efficiency and there is no blockholder intervention. In Fulghieri and Lukin, price efficiency is enhanced via security design; in Fishman and Hagerty it is enhanced by firms' voluntary disclosures.

[5]Similarly, the single blockholder models of Maug (1998, 2002), Kahn and Winton (1998), Mello and Repullo (2004), Brav and Mathews (2010), and Kalay and Pant (2010) allow the blockholder either to intervene or to sell her stake (in the last two papers, the intervention occurs through voting). However, trading again does not exert governance, and so these papers are theories of intervention only.

[6]While trading is the primary mechanism (undertaken by 80% of institutions), 66% vote against management and 55% engage in discussions with the board. Six other channels of intervention as used by at least 10% of respondents. Institutions can both trade freely on information and engage in intervention because the above intervention mechanisms do not require them to have a board seat and become a firm insider.

	All blockholders		Outside blockholders	
N	Number of firms with N blockholders	% of firms with $\geq N$ blockholders	Number of firms with N blockholders	% of firms with $\geq N$ blockholders
0	152	100%	249	100%
1	217	88%	289	80%
2	287	70%	284	57%
3	264	47%	213	34%
4	170	26%	116	17%
5	88	12%	62	7%
6	40	5%	18	2%
7	17	2%	7	1%
8	4	0%	2	0%
9	1	0%	0	0%

Table 1: Frequency of multiple blockholders for 1,240 U.S. firms. This table reports the frequency of blockholder structures for U.S. firms in 2001 using data from Dlugosz et al. (2006).

2005; Gomes and Novaes 2006).[7] This paper derives a multiple blockholder structure through a quite different channel – its effect on governance through trading, rather than control contests. By studying different blockholder actions, the model generates a new range of empirical predictions, in particular those relating to informed trading and financial markets, and more broadly links together the previously disparate literatures on blockholders and microstructure.

We now turn from related theories to the empirical facts that motivate our model. Table 1 illustrates the prevalence of multiple blockholders using U.S. data for 2001 from Dlugosz et al. (2006). They define a blockholder as a shareholder with at least 5% of a firm's equity. The table illustrates that 70% of firms have multiple blockholders, and 26% of firms have at least four blockholders. Focusing on outside blockholders, these figures remain sizable at 57% and 17%. Hence, not only do most firms have multiple blockholders, but even among such firms, the number of blockholders varies. Therefore, we seek not only to show that a multiple blockholder structure can be optimal, but also explain why blockholder numbers vary across firms. Hand-collected data from Holderness (2009) gives consistent results, showing that 74% of firms having multiple blockholders and 26% have at least four blockholders.[8]

Turning to overseas, Laeven and Levine (2007) find that 34% of European firms have more than one blockholder; Maury and Pajuste (2005) document a figure of 48% for Finnish firms. Using Western European data made available by Faccio and Lang (2002) we find a similar ratio of 39%. All these figures are sizable but somewhat lower than the U.S. data, because the above papers require an investor to have at least 10% of the voting rights to be a blockholder,

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[7]Another explanation is that regulation (e.g. Section 13(d) filing requirements upon acquisition of a 5% stake, or becoming classified as an insider upon acquisition of a 10% stake) prevents investors from building large blocks and thus forces firms to be held by multiple blockholders. Existing theories advocating a single large blockholder would suggest that such institutional constraints lead to inefficient ownership structures; this paper reaches a different conclusion.

[8]The Holderness (2009) paper does not contain the frequency of multiple blockholders. We thank Cliff Holderness for providing us with these figures using his underlying data.

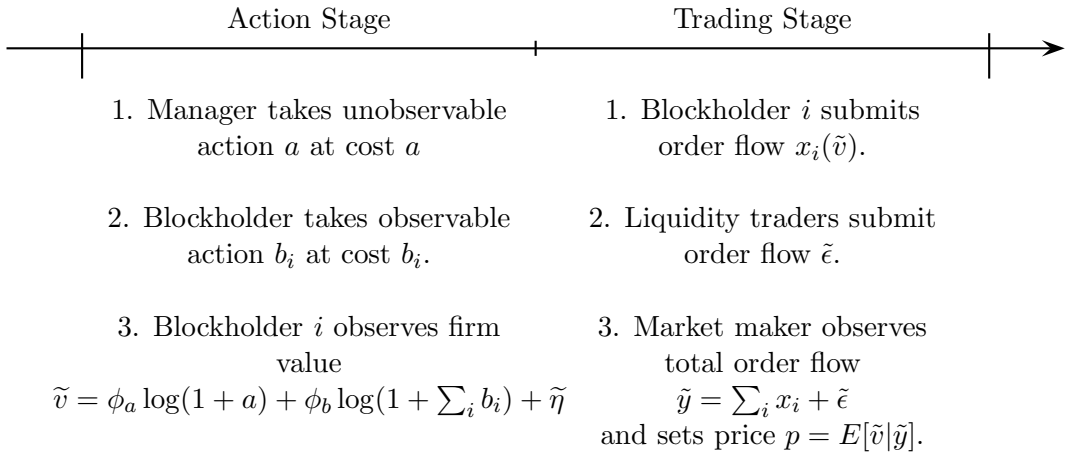| Action Stage	Trading Stage
1. Manager takes unobservable action a at cost a	1. Blockholder i submits order flow $x_i(\tilde{v})$.
2. Blockholder takes observable action b_i at cost b_i.	2. Liquidity traders submit order flow $\tilde{\epsilon}$.
3. Blockholder i observes firm value $\widetilde{v} = \phi_a \log(1+a) + \phi_b \log(1+\sum_i b_i) + \widetilde{\eta}$	3. Market maker observes total order flow $\tilde{y} = \sum_i x_i + \tilde{\epsilon}$ and sets price $p = E[\tilde{v}|\tilde{y}]$.

Figure 1: Timeline of the model

in part motivated by existing theories based on control contests. While a 10% stake may be necessary to exert control, in our model a blockholder is simply a shareholder who has greater information than the market and so the lower threshold of Dlugosz et al. is more appropriate. Even a stake below 5% may be sufficient to gain access to management or give incentives to analyze the firm in detail (for example, mutual funds typically hold under 5%). Under a lower threshold, the prevalence of multiple blockholders and heterogeneity in structures will be even greater. Our model does not assume that blockholders have control rights – a blockholder is simply any party with a sufficient stake to induce intervention, who also has private information and the ability to trade on this information.[9] It thus can apply to shareholders with less than 5% and suggests that empirical studies of blockholders may wish to use data sources other than 13d filings to identify sizable shareholders below the 5% threshold (see, e.g., Gallagher, Gardner and Swan (2010)).

2. Model and Analysis

Our model consists of a game between the manager, a market maker and the I blockholders of the firm. The game has two stages, and the timeline is given in Figure 1.

In the first stage, the manager and blockholders take actions that affect firm value. Firm value is given by

$$\tilde{v} = \phi_a \log\left(1 + a\right) + \phi_b \log\left(1 + \sum_i b_i\right) + \widetilde{\eta}, \tag{1}$$

where $a \in [0, \infty)$ represents the action taken by the manager, $b_i \in [0, \infty)$ represents the action taken by blockholder i, and $\tilde{\eta}$ is normally distributed noise with mean zero and variance σ_η^2. The manager incurs personal cost a when taking action a, while each blockholder i incurs personal cost b_i when taking action b_i.[10] The manager's action is broadly defined to encompass any decision that improves firm value but is personally costly, such as exerting effort or

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[9]For U.S. firms, insiders will typically not meet this definition of a blockholder since they are prohibited from trading on material non-public information by insider trading laws; Table 1 therefore differentiates between inside and outside blockholders. In countries where insider trading laws are weak or not enforced, and insiders do not face other trading restrictions such as wealth constraints or risk aversion, both insiders and outsiders can be considered blockholders in the model.

[10]Firm value depends on the logarithm of the combined blockholder effort level, and the action has a linear cost to each blockholder. This functional form ensures that adding blockholders does not change the available

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forgoing private benefits. We call these actions "initiative" and "managerial rent extraction" respectively. Similarly, the blockholder's action can involve advising the manager ("advising"), inhibiting managerial perks ("perk prevention") or extracting private benefits for themselves ("blockholder rent extraction.")[11] Section 5 discusses which types of action will likely be most important in a given setting. The parameter ϕ_a (ϕ_b) measures the productivity of manager (blockholder) effort. We use the term "effort" to refer to a and b_i and "output" to refer to $\phi_a \log(1 + a)$ and $\phi_b \log(1 + \sum_i b_i)$, i.e. effort scaled by its productivity. To avoid having to deal with the boundary cases where a and/or b_i are zero and explicitly analyze non-negativity constraints, we impose technical restrictions on the parameters to guarantee that both are strictly positive. Sufficient conditions are given in Appendix A.[12]

In the core model, the manager's and blockholders' actions are perfect substitutes, with independent effects on firm value. This benchmark case is appropriate in a number of settings. For example, if blockholders primarily impact the firm through rent extraction, this erodes firm value regardless of the manager's initiative or rent extraction. If the key managerial action is initiative (e.g. designing new products or building client relationships) and blockholders mainly block perks or consume private benefits themselves, these are also independent. However, in some situations, there may be positive or negative complementarities between the manager's and blockholders' actions. These are analyzed in Section 4.2.

Action a is privately observed by the manager, as in any moral hazard problem. In the core model, we assume that b_i is public. This assumption is made only for tractability, since it allows the trading and effort decisions to be solved separately. The key mechanism through which the paper justifies multiple blockholders, that a rise in I generates competition in trading, is unaffected by whether b_i is observable. In Section 4.4 we allow for b_i to be private.

There is one share outstanding. The risk-neutral manager owns α shares, and each risk-neutral blockholder holds β/I shares, where $\alpha + \beta < 1$. Our analysis focuses exclusively on the optimal number of blockholders (I) among which a given level of concentrated ownership is divided, and thus holds the amount of concentrated ownership (β) constant. This separates our paper from previous literature that analyzes the optimal β. For example, Shleifer and Vishny (1986) and Maug (1998, 2002) show that a higher β raises incentives to intervene, but this must be traded off against the potential reduction in managerial initiative (Burkart, Gromb and Panunzi 1997) and free float (Bolton and von Thadden 1998). In this model, free float is fixed at $1 - \alpha - \beta$ and plays no role. Endogenizing β and allowing liquidity (introduced shortly) to depend on free float will lead to the same trade-off as these earlier papers.[13]

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technology (in addition, it leads to substantial tractability). The common assumption of a quadratic cost and a linear effect of b_i on $\tilde{v}$ is inappropriate here: with a convex cost function, the blockholders' technology would improve if there are multiple small blockholders, since each would be operating at the low marginal cost part of the curve. A single blockholder would be able to reduce monitoring costs by dividing herself up into multiple small "units", and increase total effort. Instead, the linear cost means that the monitoring technology is constant, and so there is no mechanical reduction in monitoring costs from splitting a block.

[11]See Barclay and Holderness (1989) for a description of the private benefits that blockholders can extract. Unlike in earlier theories of multiple blockholders, here blockholders do not compete (with either each other or the manager) to consume private benefits.

[12]The analysis of perfect negative complementarities (Proposition 10) does allow for a or $\sum_i b_i$ to be zero, and indeed shows that the optimum involves one of these terms being zero.

[13]We could also extend the model by introducing managerial risk aversion and endogenizing α. Then, the increased price efficiency that results from a greater number of blockholders will lead to the optimal contract involving a greater relative weight on equity compensation versus other performance measures: see Holmstrom

In the second stage of the game, the blockholders, noise traders, and a market maker trade the firm's equity. As in Admati and Pfleiderer (2009), each blockholder observes firm value $\tilde{v}$ perfectly, while noise traders are uninformed. Section 4.1 extends the model to costly information acquisition and Appendix B shows that our results are unchanged if each blockholder obtains an imperfect signal of $\tilde{v}$: we only require that blockholders have superior information to atomistic investors.[14] This superior information can be motivated by a number of underlying assumptions. Blockholders' large stakes may give them greater access to information: given their voting power, management is more willing to meet with them. In reality, managers meet large institutional investors but not households. Even if blockholders have the same access to information as other investors, they have stronger incentives to engage in costly analysis of this information. For example, mutual funds undertake detailed analysis of public information to form their own valuations. Edmans (2009) microfounds this relationship between block size and informedness. If there are short-sales constraints (or nontrivial short-sales costs), blockholders can sell more if information turns out to be negative. Since information is more useful to them, they have a greater incentive to acquire it in the first place. Several empirical studies indeed find that blockholders are better informed than other investors and impound their information into prices through trading. Parrino, Sias and Starks (2003) and Chen, Harford and Li (2007) find that blockholders have superior information about negative firm prospects, which they use to vote with their feet. Bushee and Goodman (2007) show that blockholders trade on private rather than public information. Holthausen, Leftwich and Mayers (1990) and Sias, Starks and Titman (2006) demonstrate that such blockholder trading has a permanent effect on stock prices (suggesting the price moves are due to information rather than liquidity) and Brockman and Yan (2009) find that blockholders impound firm-specific information into prices.[15]

After observing $\tilde{v}$, each blockholder submits a market order $x_i(\tilde{v})$. Noise traders, who trade for exogenous liquidity reasons, submit a market order $\tilde{\epsilon} \sim N(0, \sigma_\varepsilon^2)$, where ε and η are independent. We use the term "liquidity" to refer to σ_ε. After observing total order flow $\tilde{y} = \sum_i \tilde{x}_i + \tilde{\epsilon}$, the competitive market maker sets the price $\tilde{p}$ equal to expected firm value.

The manager's objective is to maximize the market value of his shares less the cost of effort. Each blockholder maximizes her trading profits, plus the fundamental value of her shares, less her cost of effort.[16] In Section 4.3, we allow the objective functions of all players to depend on

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and Tirole (1993) and Calcagno and Heider (2008). As in the present paper, managerial effort unambiguously rises (see Chen and Swan (2010).)

[14]Appendix C allows signal precision to be increasing in the blockholder's individual stake and thus fall with I. This does not change any results as long as signal precision does not decline so rapidly with I that this outweighs the beneficial effect of greater I on competition in trading.

[15]Parrino, Sias and Starks (2003), Sias, Starks and Titman (2006) and Gallagher, Gardner and Swan (2010) document that blockholders typically trade on the market rather than using a negotiated block trade. This is because only the former method allows them to trade on their information by camouflaging with noise traders (as in Kyle (1985).) Blockholders cannot trade on information in a negotiated trade because the counterparty engages in extensive due diligence since she is trading a large stake. Indeed, Barclay and Holderness (1991) find that negotiated block trades are rare and trades lead to stock price increases, inconsistent with the hypothesis that the selling blockholder is exiting on negative information. The event-study returns are independent of whether the block is traded at a premium or discount, rejecting the view that the trading parties have superior information to the market.

[16]Each blockholder thus maximizes her individual objective function. The results are unchanged if blockholders can co-ordinate (either to share the costs of intervention, or limit their trading volumes), but the cost is increasing in the number of co-ordinating parties. An increase in I reduces the co-ordination costs for both

both the stock price and fundamental value.

We solve for the equilibrium of the game by backward induction.

2.1   The Trading Stage

To proceed by backward induction, we take the decisions a of the manager and b_i of the blockholders as given. (In equilibrium, these conjectures will be correct and equal the actions derived subsequently in Proposition 3.) The trading stage of the game is similar to Kyle (1985) and its extensions to multiple informed investors (Kyle 1984; Admati and Pfleiderer 1988; Holden and Subrahmanyam 1992; Foster and Viswanathan 1993.)

Proposition 1. *(Trading Equilibrium)* *The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{v}) = \gamma \left(\tilde{v} - \phi_a \log\left(1 + a\right) - \phi_b \log\left(1 + \sum_i b_i\right) \right) \quad \forall i \tag{2}$$

$$p(\tilde{y}) = \phi_a \log\left(1 + a\right) + \phi_b \log\left(1 + \sum_i b_i\right) + \lambda \tilde{y}, \tag{3}$$

where

$$\lambda = \frac{\sqrt{I}}{I+1} \frac{\sigma_\eta}{\sigma_\epsilon} \tag{4}$$

$$\gamma = \frac{1}{\sqrt{I}} \frac{\sigma_\varepsilon}{\sigma_\eta}, \tag{5}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions. Each blockholder's expected trading profits are given by

$$\frac{1}{\sqrt{I}(I+1)} \sigma_\eta \sigma_\epsilon. \tag{6}$$

Trading profits are increasing in σ_η, the blockholders' informational advantage, and σ_ϵ, their ability to profit from information by trading with liquidity investors. In addition, aggregate blockholder trading profits are decreasing in I, because multiple blockholders compete as in a Cournot oligopoly and trade aggressively. While aggressive trading reduces aggregate profits, it also impounds more information into prices. Our definition of price informativeness is $\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right]$, the expected change in price for a given change in firm value. This definition is particularly relevant for our setting as it measures the incentives to improve fundamental value of an agent compensated according to the stock price. It will thus be used later to derive the manager's optimal action. The common measure used in the microstructure literature is $\left(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})\right) / \mathrm{Var}(\tilde{v})$, the proportion of the variance of $\tilde{v}$ that is captured by prices. Appendix D shows that these measures are equivalent.

The next proposition calculates price informativeness.

Proposition 2. *(Price Informativeness)* *Price informativeness is equal to $I/(I+1)$.*

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intervention and trading, with the same effects as in the core model.

Price informativeness is increasing in I. As I approaches infinity, prices become fully informative. On the other hand, in the monopolistic Kyle model ($I = 1$), the blockholder limits her order, and so prices reveal only one-half of her private information.

The positive link between the number of blockholders and price informativeness does not arise because a greater number of informed agents mechanically increases the amount of information in the market. Indeed, a single blockholder already has a perfect signal of fundamental value; since she faces no trading constraints, she could theoretically impound this entire information into prices. The amount of information in the economy is independent of I; the effect on price informativeness instead arises entirely from competition in trading.

As is standard in Kyle-type models, liquidity σ_ε has no effect on price informativeness. From (5), greater noise trading allows blockholders to trade more aggressively. This increase in informed trading exactly counterbalances the effect of increased noise and leaves price informativeness unchanged. In Section 4.1 we show that liquidity has a positive effect on price informativeness under costly information acquisition.

2.2 The Action Stage

We now solve for the actions of the manager and the blockholders in the first stage. There is a unique symmetric equilibrium.

Proposition 3. (Optimal Actions) *The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I}{I+1} \right) - 1 \tag{7}$$

and combined blockholder actions are

$$\sum_i b_i = \phi_b \beta \left(\frac{1}{I} \right) - 1. \tag{8}$$

In a symmetric equilibrium, the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{9}$$

Proof The manager maximizes the market value of his shares, less the cost of effort:

$$E\left[\alpha \widetilde{p} - a \right]. \tag{10}$$

When setting the price $\widetilde{p}$, the market maker uses his conjecture for the manager's action a. Therefore, the manager's actual action affects the price only through its influence on $\tilde{v}$, and thus blockholders' order flow. The manager's first-order condition is given by:

$$\alpha \left(\mathrm{E}\left[\frac{d\widetilde{p}}{d\tilde{v}} \right] \right) \left(\frac{\phi_a}{1+a} \right) - 1 = 0. \tag{11}$$

From Proposition 2, his optimal action is therefore

$$a = \alpha \left(\frac{I}{I+1} \right) \phi_a - 1. \tag{12}$$

11

Each blockholder maximizes her trading profits, plus the fundamental value of her shares, less her cost of effort. From (6), the blockholder's trading profits do not depend on b_i, because it is public and thus does not affect her informational advantage. Therefore, blockholder i simply chooses b_i to maximize the fundamental value of her shares, less her cost of effort:

$$E\left[\left(\frac{\beta}{I}\right)\widetilde{v} - b_i\right].$$
(13)

Her first-order condition is given by

$$\sum_i b_i = \frac{\beta}{I}\phi_b - 1$$

and so in a symmetric equilibrium, the action of blockholder i is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}.$$
(14)

There also exist asymmetric equilibria, but $\sum_i b_i$ is uniquely defined. Since firm value depends on the sum of blockholder efforts, there is no loss of generality by focusing on symmetric equilibria. ∎

The manager's action a is the product of three variables: the effectiveness of effort ϕ_a, his equity stake α, and price informativeness $\frac{I}{I+1}$. It is increasing in I as a higher I augments price informativeness, and so the stock price more closely reflects the firm's fundamental value and thus the manager's effort. In effect, blockholder trading rewards managerial effort ex post by impounding its effects into the stock price, therefore inducing it ex ante. The dynamic consistency of this reward mechanism depends on the number of blockholders. Critically, trading occurs *after* the manager has taken his action, at which point shareholders are concerned only with maximizing their trading profits. A single blockholder optimizes her profits by limiting her order, at the expense of price informativeness. Therefore, the promise of rewarding effort by bidding up the price to fundamental value is not credible. By contrast, multiple blockholders trade aggressively, augmenting price informativeness, and thus constitute a commitment device to reward the manager ex post for his actions. While such aggressive trading is motivated purely by the private desire to maximize individual profits in the presence of competition, it has a social benefit by eliciting managerial effort.

As is standard, combined blockholder effort $\sum_i b_i$ is decreasing in I, owing to the free-rider problem. Therefore, there is a trade-off between the effect of I on intervention and trading. The co-ordination problems and externalities created by splitting a block play opposing roles in intervention and trading. For intervention, the externalities are positive: intervention improves the value of other shareholders' stakes, but this effect is not internalized by the individual blockholder. Since these externalities are positive, there is "too little" intervention with multiple blockholders, from a firm value standpoint. For trading, the externalities are negative. Higher trading volumes reveal more information to the market maker, leading to a less attractive price for other informed traders. Blockholders trade "too much" from the standpoint of maximizing combined profits. However, firm value does not depend on trading profits as they are a mere transfer from liquidity traders to blockholders. Instead, "too much" trading is beneficial for firm value as it increases price informativeness and induces effort ex ante.

3. The Optimal Number of Blockholders

This section derives the optimal number of blockholders. We start by deriving the optimal number that maximizes firm value, and then analyze the social optimum (that maximizes total surplus) and the private optimum (that maximizes the total payoff to blockholders).

Proposition 4. (Firm Value Optimum) *The number I^* of blockholders that maximizes firm value is*[17]:

$$I^* = \frac{\phi_a - \phi_b}{\phi_b}. \tag{15}$$

Proof From Proposition 3, expected firm value is:

$$E[\widetilde{v}] = \phi_a \log\left[\phi_a \alpha\left(\frac{I}{I+1}\right)\right] + \phi_b \log\left[\phi_b \beta\left(\frac{1}{I}\right)\right]. \tag{16}$$

The first-order condition with respect to I is given by:

$$\frac{\phi_a - \phi_b - \phi_b I}{I + I^2} = 0. \tag{17}$$

$\hat{I} = (\phi_a - \phi_b)/\phi_b$ satisfies the first order condition. Since the left hand side of (17) is positive for $I < \hat{I}$ and negative for $I > \hat{I}$, I^* is indeed a maximum. ■

The number I^* of blockholders that maximizes firm value solves the trade-off between the positive effect of more blockholders on managerial effort, and the negative effect on blockholder intervention. The optimum is therefore increasing in ϕ_a, the productivity of the manager's effort, and declining in ϕ_b, the productivity of blockholder intervention.

While Proposition 4 is concerned with maximizing firm value, the social optimum maximizes total surplus, which also takes into account the effort costs borne by the manager and blockholders. In theory, the social optimum would be chosen by a social planner. If the noise traders are the firm's atomistic shareholders (as in Kahn and Winton (1998) and Bolton and von Thadden (1998)), it will also be chosen by the initial owner when taking the firm public, since IPO proceeds will equal total surplus. The owner will have to compensate the blockholders (in the form of a lower issue price) for their expected intervention costs, and the manager for his effort in the form of a higher wage. Trading profits have no effect on IPO proceeds: while blockholders will pay a premium in expectation of trading gains, small shareholders will demand discounts to offset their future losses.

Proposition 5. (Social Optimum) *The number I^*_{soc} of blockholders that maximizes total surplus is the unique positive solution to*

$$\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} - \frac{\phi_a \alpha}{(I+1)^2} + \frac{\phi_b \beta}{I^2} = 0, \tag{18}$$

which may be higher or lower than I^*. I^*_{soc} is increasing in ϕ_a and β, and decreasing in ϕ_b and α.

---

[17]In reality, the number of blockholders must be a strictly positive integer. To economize on notation, we ignore such technicalities when stating I^*. If $\frac{\phi_a - \phi_b}{\phi_b} < 1$, the optimal number is 1. If $\frac{\phi_a - \phi_b}{\phi_b}$ is a non-integer, the optimal number is found by comparing (16) under the two adjacent integers.

Proof Total surplus is given by:

$$\phi_a \log \left[\phi_a \alpha \left(\frac{I}{I+1} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right] - \phi_a \alpha \left(\frac{I}{I+1} \right) - \phi_b \beta \frac{1}{I} + 2. \qquad (19)$$

Differentiating yields (18). Appendix A proves that there is a unique positive solution and that it maximizes (19). It also addresses the comparative statics. ∎

Compared to (16), (19) contains two additional terms. Increasing I raises the cost of managerial effort, but reduces the combined cost of blockholder effort. The social optimum may thus be higher or lower than the firm value optimum. If β rises, total blockholder costs $\phi_b \beta \frac{1}{I} - 1$ become more important in the social welfare function, and so I^*_{soc} rises to reduce these costs by lowering intervention. Conversely, a rise in α increases the importance of the manager's costs and thus lowers I^*_{soc}. The comparative statics with respect to ϕ_a and ϕ_b are the same as in Proposition 4.

Finally, we analyze the privately optimal division of β that would maximize blockholders' combined payoffs. This optimum would be endogenously chosen by the blockholders themselves and is robust to re-trading.

Proposition 6. (Private Optimum) *The number I^*_{priv} of blockholders that maximizes total blockholders' payoff is the unique positive solution to*

$$\beta \left[\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2} \right] - \frac{(I-1)}{2\sqrt{I}(I+1)^2} \sigma_\eta \sigma_\varepsilon = 0, \qquad (20)$$

which may be higher or lower than I^*, and higher or lower than I^*_{soc}. I^*_{priv} is increasing in ϕ_a and β, and decreasing in ϕ_b and $\sigma_\eta \sigma_\varepsilon$.

Proof Total blockholders' payoff is given by:

$$\beta \left\{ \phi_a \log \left[\phi_a \alpha \left(\frac{I}{I+1} \right) \right] + \phi_b \log \left[\phi_b \beta \frac{1}{I} \right] \right\} - \phi_b \beta \frac{1}{I} + 1 + \frac{\sqrt{I}}{I+1} \sigma_\eta \sigma_\varepsilon. \qquad (21)$$

Differentiating yields (20). Appendix A proves that there is a unique positive solution and that it maximizes (21). It also addresses the comparative statics. ∎

The blockholders' objective function differs from firm value in three ways. They only enjoy β of any increase in firm value; bear the costs of intervention; and are concerned with informed trading profits. Increasing I above I^* has an ambiguous effect: it reduces the combined costs of intervention, but also reduces total trading profits by exacerbating competition. Therefore, as with the social optimum, the private optimum may be higher or lower than the firm value optimum. An increase in β causes blockholders' effort costs to become more important in the objective function and so I^*_{priv} rises. If $\sigma_\eta \sigma_\varepsilon$ increases, trading profits become more important and so I^*_{priv} falls to lower competition.

The blockholders' objective function also differs from the social welfare function in three ways. Blockholders are concerned with trading profits and only β of firm value, but ignore the cost of managerial effort. Again, the sum of these three effects is ambiguous. Increasing I above I^*_{soc} both reduces profits and increases the manager's costs. The comparative statics with respect to ϕ_a and ϕ_b are the same as in Propositions 4 and 5.

4. Extensions

4.1 Costly Information Acquisition

In the core model, blockholders are endowed with private information about firm value $\tilde{v}$. In this subsection, they are initially uninformed but can learn $\tilde{v}$ by paying a cost c in the first stage of the game. Blockholders that do not pay this cost remain uninformed in the second stage. To solve this modified version of the model, we again use backward induction.

Proposition 7. *(Equilibrium With Costly Information) Let J be the number of blockholders that acquire information in the first stage of the game. Then in the unique linear equilibrium of the trading stage, the $I - J$ uninformed blockholders do not trade in aggregate. The J informed blockholders submit demands as in (2) and the market maker sets the price as in (3) with*

$$\lambda = \frac{\sqrt{J}}{J+1} \frac{\sigma_\eta}{\sigma_\epsilon} \tag{22}$$

$$\gamma = \frac{1}{\sqrt{J}} \frac{\sigma_\epsilon}{\sigma_\eta}. \tag{23}$$

In the first stage of the game, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1} \right) - 1 \tag{24}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{25}$$

The number J of blockholders that acquire information is

$$J = \min\{I, n\},$$

where n satisfies

$$\frac{1}{\sqrt{n}(n+1)} \sigma_\eta \sigma_\epsilon = c.$$

Proposition 7 shows that when I is sufficiently large (greater than n), some blockholders choose not to acquire information. If all blockholders become informed, competition in trading is sufficiently fierce that trading profits are insufficient to recoup the cost c. Hence, in equilibrium, some blockholders remain uninformed and do not participate in the trading stage.

Turning to the optimal number of blockholders, it is never efficient to have I greater than n. If $I > n$, then from Proposition 7, some blockholders will not acquire information in equilibrium. Uninformed blockholders do not trade and thus have no effect on governance through trading. Moreover, they dilute ownership and reduce incentives to engage in intervention. Uninformed blockholders are thus unambiguously detrimental to firm value, and so the optimum involves no such blockholders. This leads to the next proposition.

Proposition 8. *(Firm Value Optimum With Costly Information) The number* I^*_{costly} *of blockholders that maximizes firm value with costly information acquisition is equal to*

$$I^*_{costly} = \min\left(\frac{\phi_a - \phi_b}{\phi_b}, n\right). \qquad (26)$$

If $n < \frac{\phi_a - \phi_b}{\phi_b}$, I^*_{costly} *and firm value are increasing in* σ_η *and* σ_ε *and decreasing in* c. *If* $n \geq \frac{\phi_a - \phi_b}{\phi_b}$, I^*_{costly} *and firm value are independent of* σ_η, σ_ε *and* c.

I^*_{costly} is weakly increasing in σ_η and σ_ϵ and weakly decreasing in c. The intuition is as follows. If $n < \frac{\phi_a - \phi_b}{\phi_b}$, the optimum with costless information I^* is so large that competition in trading reduces individual trading profits below the cost of information. Some blockholders would choose to remain uninformed, and their existence would reduce firm value. The optimum is therefore n, the maximum number under which competition is sufficiently low that all blockholders become informed. A fall in the cost of information c, an increase in the informational advantage σ_η, and a rise in liquidity σ_ε all lead to an increase in net trading profits. Higher net profits in turn raise n, as they allow greater competition to be sustained before net profits become negative. This in turn increases I^*_{costly} towards I^*, and thus raises firm value.

By contrast, if $n > \frac{\phi_a - \phi_b}{\phi_b}$, net trading profits are sufficiently high that all blockholders become informed. The analysis is as in the core model of Section 3, where the optimum depends only on ϕ_a and ϕ_b. The constraint that I is sufficiently low to induce information acquisition is not binding. Changes in net trading profits, and thus changes in σ_η, σ_ε and c, have no effect on the optimal number of blockholders or firm value.

4.2   Complementarities

In the core model, the manager's and blockholders' actions are perfect substitutes, with independent effects on firm value. The marginal productivity of the manager's (blockholders') effort is unaffected by the effort level of the other party, i.e. $\frac{\partial^2 v}{\partial a \partial b_i} = 0$. This assumption likely applies to a number of settings: for example, rent extraction by the blockholders reduces firm value regardless of the manager's effort; managerial initiative is unaffected by blockholder perk prevention or rent extraction.

In some cases, there may be complementarities between the manager's and blockholders' efforts. This subsection extends the core model to these cases. If complementarities are positive, the marginal productivity of one party's action is increasing in the effort level of the other party, i.e. $\frac{\partial^2 v}{\partial a \partial b_i} \geq 0$. This arises if manager and blockholder outputs are mutually interdependent – in particular, if the main managerial action is initiative and the main blockholder action is advising. For example, venture capital investors have expertise in devising an effective strategy, which is then executed by the manager. Both strategy formulation and implementation are necessary for firm success.

With positive complementarities, blockholders are "allies" of the manager, providing him with advice. Negative complementarities arise if blockholders are "adversaries" of the manager – for example, if their main value added is perk prevention, and rent extraction is an important managerial action. Blockholders are most productive if managerial effort is low ($\frac{\partial^2 v}{\partial a \partial b_i} \leq 0$), i.e. the manager is pursuing private benefits. Negative complementarities are most likely in mature firms, where the optimal strategy is often clear to the manager. Inefficiencies arise

| | | | Blockholder Action | |
		Advisory	Rent Extraction	Perk Prevention
Manager Action	Initiative	Positive Complements	Substitutes	Substitutes
	Rent Extraction	Substitutes	Substitutes	Negative Complements

Table 2: Classification of blockholders' and manager's actions as substitutes or complements.

not because the manager is unaware of the correct course of action and needs blockholders' advice, but because he has private incentives to depart from the efficient action. For example, managers of "cash cows" know that they should return excess cash to shareholders, but may instead reinvest it inefficiently. Table 2 summaries whether actions are likely to be substitutes or positive or negative complements depending on their type.

We analyze complementarities using the boundary cases of perfect positive (negative) complementarities, where firm value depends only on the minimum (maximum) output level of the manager and blockholders, as these scenarios are most tractable within our framework and thus allow the clearest empirical predictions.[18] Reality will typically lie between these two extremes and the optimum for an interior level of complementarity may be inferred by interpolating between the boundary cases. For example, we will see that the zero complementarities case of the core model lies between the two extremes.

We commence with perfect positive complementarities, which we model with a Leontief production function:

$$\widetilde{v} = \min\left[\phi_a \log\left(1 + a\right), \phi_b \log\left(1 + \sum_i b_i\right)\right] + \widetilde{\eta}. \tag{27}$$

The optimal actions can no longer be derived independently. The manager's optimal action depends on his conjecture $\widehat{b}_i$ for the blockholders' actions. Blockholder i's optimal action depends on her conjecture for the manager's effort $(\widehat{a})$ and for the actions of the other blockholders $(\widehat{b}_j, j \neq i)$.

Proposition 9. (Perfect Positive Complementarities) *The manager's optimal action is*

$$a = \min\left(\phi_a \alpha \left(\frac{I}{I+1}\right) - 1, \exp\left(\frac{\phi_b}{\phi_a} \log\left(1 + \sum_i \widehat{b}_i\right) - 1\right)\right). \tag{28}$$

---

[18]An alternative way to model complementarities is to use a constant elasticity of substitution production function, e.g. $\widetilde{v} = [(\phi_a \log(1 + a))^\rho + (\phi_b \log(1 + \sum_i b_i))^\rho]^{1/\rho} + \widetilde{\eta}$. Such a production function does not yield tractable solutions in our framework owing to the logarithmic functional form, which is necessary for the core model (see footnote 10).

Blockholder i's effort level is:

$$
b_i = \begin{cases}
\phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I} \ \ if \ \phi_a\log\left(1+\widehat{a}\right) \geq \phi_b\log\left[1+\phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I} + \sum\limits_{j\neq i}\widehat{b}_j\right] \\[2ex]
\exp\left(\frac{\phi_a}{\phi_b}\log\left(1+\widehat{a}\right)\right) - \sum\limits_{j\neq i}\widehat{b}_j - 1 \\[1ex]
\qquad if \ \phi_b\log\left(1+\sum\limits_{j\neq i}\widehat{b}_j\right) \leq \ \phi_a\log\left(1+\widehat{a}\right) < \phi_b\log\left[1+\phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I} + \sum\limits_{j\neq i}\widehat{b}_j\right] \\[2ex]
0 \ \ if \ \phi_a\log\left(1+\widehat{a}\right) < \phi_b\log\left(1+\sum\limits_{j\neq i}\widehat{b}_j\right)
\end{cases}
$$

(29)

The number I^* of blockholders that maximizes firm value is the unique positive solution to

$$
\frac{I^2}{I+1} = \frac{\phi_b\beta}{\phi_a\alpha}\exp\left(\phi_b - \phi_a\right).
$$

(30)

I^* is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

As with the core case, I^* is typically an interior solution, i.e. involves multiple, but finite, blockholders. However, the comparative statics with respect to ϕ_a and ϕ_b are *opposite* to the core case. In the core case, I^* is increasing in ϕ_a. If managerial effort becomes more productive, it becomes increasingly important in the trade-off between trading and intervention, and so I^* rises to enhance trading. With complements, I^* must balance the levels of manager and blockholder outputs. If ϕ_a rises, managerial effort is more effective and so it is not necessary to "boost" it via a high I. Instead, I should be used to enhance blockholder effort so that it becomes sufficiently high to complement the manager's output. This involves reducing I.

We now turn to the case of perfect negative complementarities, i.e.

$$
\widetilde{v} = \max\left[\phi_a\log\left(1+a\right), \phi_b\log\left(1+\sum\nolimits_i b_i\right)\right] + \widetilde{\eta}.
$$

(31)

Proposition 10. *(Perfect Negative Complementarities) The manager's optimal action is*

$$
a = \begin{cases}
\phi_a\alpha\frac{I}{I+1} - 1 \ \ if \ \alpha\frac{I}{I+1}\left(\phi_a\log\left[\phi_a\alpha\frac{I}{I+1}\right] - \phi_b\log\left(1+\sum_i b_i\right)\right) \geq \phi_a\alpha\frac{I}{I+1} - 1 \\[1ex]
0 \ \ if \ \alpha\frac{I}{I+1}\left(\phi_a\log\left[\phi_a\alpha\frac{I}{I+1}\right] - \phi_b\log\left(1+\sum_i b_i\right)\right) < \phi_a\alpha\frac{I}{I+1} - 1.
\end{cases}
$$

(32)

Similarly, blockholder i's effort level is:

$$
b_i = \begin{cases}
\phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I} \ \ if \ \frac{\beta}{I}\left(\phi_b\log\left[1+\phi_b\beta\frac{1}{I^2} - \frac{1}{I} + \sum\limits_{j\neq i}\widehat{b}_j\right] - \phi_a\log\left(1+\widehat{a}\right)\right) \geq \phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I} \\[2ex]
0 \ \ if \ \frac{\beta}{I}\left(\phi_b\log\left[1+\phi_b\beta\frac{1}{I^2} - \frac{1}{I} + \sum\limits_{j\neq i}\widehat{b}_j\right] - \phi_a\log\left(1+\widehat{a}\right)\right) < \phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I}.
\end{cases}
$$

(33)

The number of blockholders I^* that maximizes firm value is

$$
I^* = \begin{cases}
\infty \ \ if \ \phi_a\log\left(\phi_a\alpha\right) \geq \phi_b\log\left(\phi_b\beta\right) \\[1ex]
1 \ \ if \ \phi_a\log\left(\phi_a\alpha\right) < \phi_b\log\left(\phi_b\beta\right)
\end{cases}
$$

(34)

In the core model of perfect substitutes, firm value depends on both manager and block-holder efforts. Since the optimal shareholder structure must trade-off both, I^* is typically an interior solution. Here, firm value depends only on the maximum output level and there are no trade-off concerns. If blockholder effort is relatively productive, I^* should be chosen exclusively to maximize the potency of intervention and completely ignores trading; thus I^* is at its minimum value of 1. By contrast, if managerial effort is relatively productive, $I^* = \infty$. This case represents fully dispersed ownership; since empirical studies define a blockholder as a shareholder who owns above a minimum threshold, it will appear in the data as zero blockholders. Therefore, under perfect negative complementarities, there is either zero or one blockholder. Indeed, Table 1 shows that both of these cases are common in the data.

With perfect substitutes, I^* is smoothly increasing in ϕ_a. Here, I^* remains weakly increasing in ϕ_a, but ϕ_a has a discontinuous effect. If $\phi_a \log(\phi_a \alpha) < \phi_b \log(\phi_b \beta)$, I^* is independent of ϕ_a. A small increase in ϕ_a has zero effect on I^*: since blockholder effort is still more productive, I^* continues to be exclusively determined by intervention. However, when ϕ_a rises above the level for which $\phi_a \log(\phi_a \alpha) = \phi_b \log(\phi_b \beta)$, I^* jumps from 1 to ∞. For $\phi_a \log(\phi_a \alpha) \geq \phi_b \log(\phi_b \beta)$, I^* is already exclusively determined by trading, and so further increases in ϕ_a have no effect on I^*. Similarly, changes in ϕ_b have either a zero or infinite effect on I^*. Negative complementarities therefore lead to more extreme results than the core model. The optimal number of blockholders is a corner solution; ϕ_a and ϕ_b have the same directional effect as in the core model, but their impacts are discontinuous.

Combining all of the results, with perfect negative complementarities, I^* is either 1 or ∞ and is driven entirely by the more productive action. As complementarities become less negative, I^* becomes less extreme and is determined by the productivity of both actions; it continues to be increasing in ϕ_a and decreasing in ϕ_b. The core case of perfect substitutes is an example. Once complementarities become sufficiently high, we approach the case of perfect complements, and the effects of ϕ_a and ϕ_b change direction.

4.3   General Objective Functions

In the core model, the manager's payoff stems from the market value of his shares, $\alpha \widetilde{p}$, as in Holmstrom and Tirole (1993). In a more general setting, the manager can be compensated according to the fundamental value $\widetilde{v}$ as well as the market value $\widetilde{p}$, for instance using long-vesting stock. We thus generalize the manager's objective function from (10) to

$$E\left[\alpha\left(\omega\widetilde{p} + (1-\omega)\widetilde{v}\right) - a\right]. \tag{35}$$

The actual level of ω will reflect factors outside the model and introduced in earlier work, such as takeover threat (Stein 1988), concern for managerial reputation (Narayanan 1985; Scharfstein and Stein 1990), or the manager expecting to sell his shares for $\widetilde{p}$ before $\widetilde{v}$ is realized, e.g. to finance consumption (Stein 1989).[19] Even if the manager's sole objective is to maximize long-run shareholder value, he will care about the stock price as it affects the terms at which the firm can raise equity at $t = 2$ (Stein 1996).

Similarly, in the core model, each blockholder maximizes her share of fundamental value less the cost of effort when choosing her action. More generally, the blockholder may place weight

---

[19]Kole (1997) shows that vesting periods are short in practice, perhaps because long vesting periods would subject the manager to excessive risk.

on the short-term stock price, for example if she expects to receive a liquidity shock which will force her to sell her shares in the interim regardless of her private information (Miller and Rock 1985; Faure-Grimaud and Gromb 2004). We thus generalize each blockholder's objective function from (13) to

$$E\left[\left(\frac{\beta}{I}\right)(\zeta\widetilde{p} + (1-\zeta)\widetilde{v}) - b_i\right].$$

The core model has $\omega = 1$ and $\zeta = 0$. The new equilibrium is given below.

Proposition 11. (General Compensation Contract) *The number I^*_{gen} of blockholders that maximizes firm value is the larger root of*

$$\frac{\phi_a\omega}{I+1-\omega} - \frac{\phi_b}{I}\frac{(I+1)^2 - \zeta(2I+1)}{I+1-\zeta} = 0 \tag{36}$$

if equation (36) has solutions for $I \geq 1$. In this case, I^*_{gen} is increasing in ω, ζ and ϕ_a, and decreasing in ϕ_b. If (36) has no solutions for $I \geq 1$, $I^* = 1$.

As in the core model, I^*_{gen} represents a trade-off between price informativeness and intervention. The positive effect of I on stock price efficiency is more important when the manager is more closely aligned with the stock price, and so I^*_{gen} increases in the manager's short-term concerns ω. Similarly, I^*_{gen} is increasing with blockholders' short-term concerns ζ. This is for two reasons. First, when ζ is high, blockholder effort is low: effort affects $\widetilde{p}$ to a lesser extent than $\widetilde{v}$, since the stock price is only partially informative, and so if she places greater weight on $\widetilde{p}$, she is less rewarded for her effort. When intervention is low, the negative effect of increasing blockholders on intervention is less important. Second, when blockholders care about the stock price, their effort depends on price informativeness. Since a rise in I raises price informativeness, this augments their effort.

In addition to generating additional comparative statics for ω and ζ, this extension demonstrates that the results of the core model do not stem from the fact that we modeled the blockholders as having a more long-term objective than the manager (i.e. maximize their share of $\widetilde{v}$ while the manager maximizes his share of $\widetilde{p}$). Even if blockholders have shorter horizons than the manager ($\zeta < \omega$), the results continue to hold; in fact, the case for multiple blockholders is even stronger when blockholders have short-term concerns.

4.4 Unobservable Blockholder Actions

This section extends the model to allowing the blockholders' actions b_i to be unobservable. Now, a blockholder can earn additional trading profits by taking an action different from the market maker's conjecture, so we must compute her trading profits off the equilibrium path. The market maker conjectures an expected firm value of

$$\mu = \phi_a\ln(1+\widehat{a}) + \phi_b\ln\left(1 + \sum_i\widehat{b}_i\right)$$

where $\widehat{a}$ and $\widehat{b}_i$ are his conjectures for the manager's and blockholders' actions. However, blockholder i may choose an action $b_i \neq \widehat{b}_i$, which will yield a different expected value of

$$E\left[\tilde{v}\right] = \phi_a \ln\left(1 + \widehat{a}\right) + \phi_b \ln\left(1 + \sum_{j \neq i} \widehat{b}_j + b_i\right)$$

Her trading profits are given by

$$E\left[x_i(\tilde{v} - \tilde{p})\right] = E\left[\frac{1}{(I+1)\lambda}(\tilde{v} - \mu)\left(\tilde{v} - \mu - \lambda\left(\frac{I}{(I+1)\lambda}(\tilde{v} - \mu) + \varepsilon\right)\right)\right]$$

$$= \frac{1}{\sqrt{I}(I+1)}\sigma_\eta\sigma_\varepsilon + \frac{1}{\sqrt{I}(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\phi_a \log(1+a) + \phi_b \log\left(1 + \sum_i b_i\right) - \mu\right)^2.$$

and so her overall objective function is:

$$\max_{b_i}\left(\frac{\beta}{I}\right)E\left[v\right] - b_i + \frac{1}{\sqrt{I}(I+1)}\sigma_\eta\sigma_\varepsilon + \frac{1}{\sqrt{I}(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\phi_a \log(1+a) + \phi_b \log\left(1 + \sum_i b_i\right) - \mu\right)^2 \tag{37}$$

We wish to show that, if the market maker conjectures $b_i = \frac{\phi_b\beta}{I^2} - \frac{1}{I} \; \forall \; i$, then it is indeed optimal for blockholder i to take action $b_i = \frac{\phi_b\beta}{I^2} - \frac{1}{I}$.

**Proposition 12. *(Unobservable Blockholder Actions)* ** *If either*

$$\frac{\beta}{\phi_b\left(1 + \ln(\phi_b\beta)\right)} > \frac{\sigma_\varepsilon}{\sigma_\eta}, \tag{38}$$

or

$$\frac{\phi_b\beta - 1}{\phi_b^2 \ln(\phi_b\beta)} > \frac{\sigma_\varepsilon}{\sigma_\eta}, \tag{39}$$

then

$$a = \phi_a\alpha\left(\frac{I}{I+1}\right) - 1$$

$$b_i = \phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I},$$

is an equilibrium.

The conjectured action $b_i = \frac{\phi_b\beta}{I^2} - \frac{1}{I}$ maximizes the blockholder's share of firm value less her cost of intervention. By deviating, the blockholder reduces this objective (the "fundamental motive"), but also earns additional trading profits since she how has private information on b_i (the "trading motive".) Either condition (38) or (39) is sufficient to ensure that the trading motive is sufficiently weak to deter such deviations. The parameters in the conditions are intuitive. Recall from equation (5) that the sensitivity of the blockholder's trade to fundamental value is given by $\gamma = \frac{1}{\sqrt{I}}\frac{\sigma_\varepsilon}{\sigma_\eta}$. When ϕ_b is higher, a given deviation in b_i has a larger effect on firm value. When $\frac{\sigma_\varepsilon}{\sigma_\eta}$ is higher, this in turn leads to a greater change in the blockholder's trade, and so the trading motive becomes stronger. Thus, conditions (38) and (39) are more likely to be satisfied if ϕ_b and $\frac{\sigma_\varepsilon}{\sigma_\eta}$ are low. Similarly, if β is high, the blockholder has a high share of fundamental value, and so the fundamental motive is stronger.

If neither condition is satisfied, the actions a and b_i stated in Proposition 12 may not constitute an equilibrium, as trading profits are sufficiently strong that the blockholder will always wish to deviate from the market maker's conjecture. In this case, there is no alternative pure strategy equilibrium.

Proposition 13. *The equilibrium actions stated in Proposition 12 constitute the unique symmetric equilibrium in pure strategies. Moreover, any asymmetric equilibrium in pure strategies satisfies $\sum_i b_i = \phi_b \beta / I - 1$.*

Proposition 13 states that, if actions a and b_i stated in Proposition 12 do not constitute an equilibrium, then there cannot exist a symmetric equilibrium in pure strategies. Moreover, any asymmetric equilibrium in pure strategies must satisfy $\sum_i b_i = \phi_b \beta / I - 1$, and therefore only differs from our equilibrium in terms of the division of rents among blockholders, as in the case of observable actions studied in Proposition 3. The analysis of mixed strategy equilibria is beyond the scope of this paper, which focuses on the trade-off between trading and intervention. (See Maug (1998) and Kahn and Winton (1998) for analysis of mixed strategy equilibria in a single blockholder model.)

5. Empirical Implications

This paper is motivated by the empirical observation that many firms are held by multiple small blockholders, in contrast to theories that advocate highly concentrated ownership. The model generates a number of additional empirical implications, over and above its initial motivation. It suggests new ways of thinking about blockholders that may give rise to novel directions for empirical research. First, the paper views blockholders as competing for trading profits rather than private benefits, thus linking the previously separate blockholder and microstructure literatures. Second, it suggests studying the number of blockholders rather than (or in addition to) total ownership or the stake of the largest shareholder. These two broad themes in turn generate specific predictions for the effects of blockholder structure, and the determinants of blockholder structure. We commence with the former.

The model suggests that the number of blockholders impacts both financial markets and firm value. Starting with the first set of effects, it predicts that a greater number of blockholders reduces total trading profits, but increases price efficiency. Gallagher, Gardner and Swan (2010) find support for both predictions, Gorton, Huang and Kang (2010) show that price informativeness is increasing in the number of blockholders and Boehmer and Kelley (2009) find that it is increasing in the dispersion of ownership among institutional traders (the last two studies do not investigate trading profits). Turning to the second set, multiple blockholders can improve firm value, in contrast to existing models that advocate a single concentrated blockholder.[20] Gallagher et al. find that the threat of disciplinary trading from multiple blockholders leads to superior subsequent firm performance. They use a measure of portfolio churning to specifically test governance through trading rather than control contests. Smith and Swan (2008) show that institutional trading is successful at disciplining executive pay.

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[20]If I is always at the firm value optimum, there should be no relationship between I and firm value, when controlling for the joint determinants of I and firm value. Demsetz and Lehn (1985) made this point in the context of managerial ownership and firm value. However, the empirically observed I is likely to be the private optimum, which differs from the firm value optimum. Moreover, the private optimum may shift for exogenous reasons, such as a blockholder suffering a change in management or a liquidity shock.

Multiple investors with frequent trading have greatest effect; total institutional ownership only matters insofar as it affects trading activity. Kandel, Massa and Simonov (2010) find that multiple shareholders that trade in the same direction are associated with higher firm value and profitability. Bharath et al. (2010) document that U.S. firms with multiple blockholders have higher Tobin's Q than firms with a single blockholder; Laeven and Levine (2007) find a similar result with international data.

The effect of the number of blockholders on prices and firm value suggests that it is an important determinant of both market efficiency and corporate governance. Many empirical papers use total institutional ownership as a measure of market efficiency, since institutions have greater information than retail traders. However, price efficiency depends not only on the amount of information held by investors, but the extent to which this information is impounded into prices. The latter in turn depends on the number of informed shareholders. Similarly, many studies use total institutional ownership or the stake of the largest investor as a proxy for corporate governance, but the model suggests that the number of blockholders is another important factor and thus may be relevant for future empirical work. Bharath et al. (2010) and Gorton et al. (2010) are two recent empirical studies that investigate the effect of blockholder numbers, and Konijn et al. (2009) study the effect of blockholder dispersion which is positively correlated with numbers.

Our model also generates predictions concerning the determinants of blockholder structure. To our knowledge, none of these predictions have been tested formally as empirical studies have largely focused on total institutional ownership or the stake of the largest blockholder rather than the number of blockholders, and so they are potential topics for future research.[21] In the paper, we considered different criteria for the optimal number of blockholders. In practice, sometimes the social optimum may be observed, for instance if the firm has recently undergone an IPO, or lock-ups prevent blockholders from re-trading from the initial structure. For most firms, it is most likely that the private optimum will be observed (see also Maug (1998)). Importantly, both optima share the same predictions for ϕ_a and ϕ_b: the number of blockholders is increasing (decreasing) in the productivity of the manager's (blockholders') effort.

We first consider the core model of perfect substitutes. The magnitude of ϕ_b depends on the nature of blockholders' expertise. Using the terminology of Dow and Gorton (1997), if blockholders have forward-looking ("prospective") information about optimal future investments or strategic choices, intervention is particularly valuable and ϕ_b is high. For example, activist investors (e.g. Kirk Kerkorian or Carl Icahn) are typically expert at preventing perks or empire-building; venture capitalists have skills in advising. On the other hand, passive mutual funds and insurance companies typically lack specialist expertise in managing a firm, but instead are adept at gathering backward-looking ("retrospective") information to evaluate the effect of past decisions on firm value. Their primary benefit is to impound the effects of prior managerial effort into the stock price. In such a case, ϕ_b is low and I^* is high. Another determinant of ϕ_b is blockholders' control rights and thus ability to intervene (holding constant the size of their individual stakes).[22] Black (1990) and Bebchuk (2007) note that U.S. shareholders face substantial legal and institutional hurdles to intervention, compared to their

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[21]Maury and Pajuste (2005) and Laeven and Levine (2007) report the number of blockholders, but do not relate them to cross-sectional determinants.

[22]In reality, control rights will likely be increasing in the size of each blockholder's individual stake β/I. This will reinforce the negative effect of I on intervention currently in this paper.

foreign counterparts. This reduces ϕ_b, thus increasing I^*, and is consistent with smaller and more numerous blockholders in the U.S.

The manager's effectiveness ϕ_a will be higher if he is more talented. Talent can be measured directly using managerial characteristics, such as education, experience or past performance, or proxied by salary (see Gabaix and Landier (2008)). The manager's effectiveness ϕ_a also depends on the manager's scope to use his initiative or extract rents. It is likely lower in regulated firms, and high in firms with free cash flow problems. The latter implication suggests that mature firms should be held by many blockholders, which reinforces the earlier predictions. It is also likely higher in large firms because many managerial actions can be "rolled out" across the entire firm – for example, if the CEO designs a new method to reduce production costs, this can be applied firmwide.

Negative complementarities may arise if the manager has significant scope for rent extraction that can be prevented by intervention, such as in mature firms with high agency costs of free cash flow. If investors are passive, ϕ_a will be significantly higher than ϕ_b, and so the model predicts dispersed ownership. By contrast, if blockholders are activist and skilled in perk prevention, it is efficient to have a single blockholder. Both of these predictions reinforce earlier results.

Positive complementarities typically occur in start-up firms. The main managerial action is initiative, and early-stage investors (such as venture capitalists) are expert at advising the manager (e.g. by devising a strategy for the manager to implement). Typically, ϕ_a will be significantly greater than ϕ_b: the manager is able to add greater value than blockholders, given his close proximity to firm operations. In such a case, Section 4.2 predicts that I^* is lower under positive complementarities than perfect substitutes. Moreover, in start-ups, the manager often has a significant equity stake (high α) which gives him strong incentives to exert effort. From equation (30), I^* should be low to ensure blockholder effort is also high. This may explain the concentrated blockholder structure in early-stage firms, even after such firms go public and the trading governance mechanism becomes available.

Section 4.1 shows that if information is costly, the optimal number of blockholders depends on microstructure features: it is decreasing in the information cost c, increasing in blockholders' private information σ_η and increasing in market liquidity σ_ε. Indeed, Fang, Noe and Tice (2009) find a causal relationship between liquidity and firm value. While many other papers also generate a positive effect of liquidity on firm value (e.g. Holmstrom and Tirole 1993), here the specific mechanism is through changing blockholder structure. Bharath et al. (2010) find that liquidity is particularly beneficial for firm value where there are more blockholders. This is consistent with the model because, if blockholders are numerous, a large volume of noise trading is necessary to induce them all to gather information.

Turning to the predictions regarding c and σ_η, we previously established that institutions skilled at gathering retrospective information have low ϕ_b, increasing I^*. Such institutions also likely have a low cost of monitoring and superior information, further reinforcing the prediction that I^* is high. Indeed, as firms mature, active venture capitalist investors are typically replaced by passive institutional shareholders, and the number of blockholders usually increases. Note that this association could be for reasons outside the model. As firms mature, they typically become larger; if blockholder wealth constraints limit the number of dollars they can invest in a firm (Winton 1993), this will lead to more dispersed ownership. Therefore, the above empirical observation is only tentative support for the model; a formal test will have to control

for factors such as firm size.

The theory also suggests that trading is most important where the manager's short-term concerns ω are highest. Therefore, the number of blockholders should be higher when the manager's stock and options have shorter vesting periods, or takeover defenses are weaker. Again, simple cross-sectional correlations will be insufficient to support this prediction, since blockholders can plausibly affect the compensation contract. In addition, the number of blockholders is increasing in blockholders' short-term concerns ζ, which could be proxied by blockholders' trading frequency. Hence the model predicts a positive correlation between the number of blockholders and trading frequency because of causation in both directions: dispersed blockholders trade aggressively; and if a firm's blockholders are frequent traders who rarely intervene, they should adopt a dispersed structure. Gallagher, Gardner and Swan (2010) find evidence of multiple small blockholders engaging in frequent "churning" trades.

While the theory appears to generate a number of untested predictions through a different conceptualization of blockholders to prior research, we caveat that empirical testing will have to overcome a number of challenges. First, although the model yields clear, closed-form predictions for the optimal number of blockholders in terms of certain variables, a number of these parameters (such as the effectiveness of blockholder and manager effort) are difficult to measure directly. The key challenge for empiricists is to come up with accurate proxies. Second, while the model predicts that these variables have a causal impact on blockholder structure, it may be that additional factors outside the model have an effect on both. Therefore, documenting correlations will be insufficient to support the model; identification of causal effects will require careful instrumentation.

6. Conclusion

Why are so many firms held by multiple blockholders when such a shareholding structure generates free-rider problems in monitoring? This paper offers a potential explanation. The same co-ordination issues that hinder intervention increase blockholders' effectiveness in exerting governance through an alternative mechanism: trading. Multiple blockholders act competitively when trading, impounding more information into prices. This in turn induces higher managerial effort, particularly if the manager has high stock price concerns.

The optimal number of blockholders depends on the relative productivity of managerial and blockholder effort. If outputs are perfect substitutes, the optimum is decreasing in the effectiveness of blockholder intervention and increasing in the potency of managerial effort. It is therefore high if blockholders are mutual funds that gather retrospective rather than prospective information, and low if they are activists. This dependence becomes stronger under negative complementarities. However, if complementarities are positive, the productivity parameters have opposite effects on the optimal shareholder structure. If blockholder effort is unproductive, concentrated ownership is necessary to augment it to a sufficient level to complement the manager's effort.

The paper suggests a number of potential avenues for future research. On the empirical side, the model highlights the importance of the number of blockholders. As an independent variable, it is a relevant determinant of both governance and price efficiency; as a dependent variable, the model identifies a number of underlying factors that affect the optimal blockholder structure. On the theoretical side, the paper assumes symmetric blockholders and focuses the analysis on their optimal number. It would be interesting to extend the analysis to introduce asymmetries

and examine the optimal distribution of shares between a fixed number of blockholders.[23] Another possible asymmetry would be to feature some blockholders specializing in trading and others in intervention, as in Faure-Grimaud and Gromb (2004), Aghion, Bolton and Tirole (2004), and Attari, Banerjee and Noe (2006) (although these models feature only one type of each blockholder). Similarly, while we have focused our study on the efficient number of blockholders, the model can be expanded to consider the simultaneous determination of the manager's stake and total blockholder ownership.

More broadly, the model suggests a new way of thinking about the interactions between multiple blockholders: as competing for trading profits, rather than private benefits. This leads to new empirical predictions linking blockholders to microstructure, and more generally corporate finance to financial markets. In addition, this way of thinking gives rise to new theoretical directions: future corporate finance models of multiple blockholders could incorporate more complex effects currently analyzed in asset pricing models of many informed traders. The present paper assumes a single trading period, but in reality there may be multiple periods in which information may arrive and blockholders may trade. Trading profits, and thus incentives to acquire costly information, then depend not only on the quality of information but its timeliness. A blockholder who receives information late may find that the price has already moved, reducing her trading profits. In addition, in the present paper, blockholders trade on information only. If blockholders are subject to liquidity shocks (see, e.g., Brunnermeier and Pedersen (2005)), the addition of multiple trading rounds may give incentives for other blockholders to "front-run" and sell in advance of an anticipated forced liquidation. This may increase the potency of governance through trading, but reduce incentives to engage in interventions with long-run benefits.

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[23]Studying asymmetric blockholders will likely require a quite different framework. In the current model (as in standard Kyle-type models), block size has no effect on trading behavior as the ability to trade is independent of one's stake. Introducing short-sale constraints will allow block size to be relevant, but will require departures from normal noise distributions to obtain tractability (see, e.g., Edmans (2009)). Moreover, a symmetric equilibrium is necessary to obtain closed-form solutions in the trading stage – see also Kyle (1984), Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), Foster and Viswanathan (1993). The current model does contain an asymmetry in the case of costly information acquisition, as some blockholders may remain uninformed.

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Online Appendix for "Governance Through Trading and Intervention: A Theory of Multiple Blockholders"

Alex Edmans and Gustavo Manso

A Proofs

Proof of Proposition 1. If the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$, blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \,|\, \tilde{v} = v] = (v - \mu - \lambda \sum_{j \neq i} x_j)x_i - \lambda x_i^2.$$

This maximization problem yields

$$x_i(v) = \frac{1}{\lambda}[v - \mu - \lambda \sum_j x_j(v)] \quad \forall i. \tag{40}$$

Summing both sides across i yields

$$\sum_j x_j(v) = \frac{I}{\lambda}[v - \mu - \lambda \sum_j x_j(v)]$$

$$\sum_j x_j(v) = \frac{I}{(I+1)\lambda}[v - \mu]$$

Substituting into (40) yields

$$x_i(v) = \frac{1}{(I+1)\lambda}[v - \mu] \quad \forall i,$$

which means that, in a linear equilibrium, blockholders' strategies are symmetric. Total order flow is thus given by

$$y = \frac{I}{(I+1)\lambda}(v - \mu) + \varepsilon. \tag{41}$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{42}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon},$$

$$\mu = \phi_a \log(1 + a) + \phi_b \log(1 + \sum_i b_i).$$

From this we obtain:

$$x_i(v) = \frac{1}{\sqrt{I}}\frac{\sigma_\epsilon}{\sigma_\eta}(v - \phi_a \log(1 + a) - \phi_b \log(1 + \sum_i b_i)) \quad \forall i,$$

$$p(y) = \phi_a \log\left(1 + a\right) + \phi_b \log\left(1 + \sum_i b_i\right) + \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon}y,$$

as required. Blockholder i's trading profits equal $x_i\left(v - p\right)$ and can be computed immediately using the above expressions.

Proof of Proposition 2. The result follows from $p\left(y\right) = \mu + \lambda y$ and equation (41).

Proof of Proposition 5. Putting equation (18) under a common denominator yields

$$\frac{\phi_a I\left(I+1\right) - \phi_b I\left(I+1\right)^2 - \phi_a \alpha I^2 + \phi_b \beta\left(I+1\right)^2}{I^2\left(I+1\right)^2} = 0. \tag{43}$$

Equation (18) is a cubic, and has at most three roots. The function is discontinuous at $I = -1$ and approaches $-\infty$ either side of $I = -1$ (since the $-\frac{\phi_a \alpha}{(I+1)^2}$ term dominates). It is also discontinuous at $I = 0$ and approaches $+\infty$ either side of $I = 0$ (since the $\frac{\phi_b \beta}{I^2}$ term dominates). It is continuous everywhere else.

As $I \to -\infty$, the $-\frac{\phi_b}{I}$ term in (18) dominates, and so the function asymptotes the x-axis from above. Since it approaches $-\infty$ as I rises to -1, and is continuous between $I = -\infty$ and $I = -1$, there must be one root between these two points. Similarly, since the function tends to $+\infty$ as I rises from just above -1 to just below 0, and is continuous between these two points, there must be a second root within this interval. As $I \to +\infty$, the $-\frac{\phi_b}{I}$ term in (18) again dominates, and so the function asymptotes the x-axis from below. Since the function tends to $+\infty$ as I approaches 0 from above, and is continuous between $I = 0$ and $I = +\infty$, there must be a third root $(\widetilde{I})$ between these two points. There can be no other positive roots, since there are two negative roots and three roots in total. The positive root is a local maximum, since the gradient is positive for $I < \widetilde{I}$ and negative for $I > \widetilde{I}$.

Let $F\left(I, \theta\right)$ denote the left-hand side of (43), where θ is a vector of parameters ϕ_a, ϕ_b, α, β. I_{soc}^* is defined by $F = 0$. Differentiating with respect to θ gives

$$\frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial I}\frac{\partial I}{\partial \theta} = 0.$$

Since the gradient F is positive just below I_{soc}^* and negative just above I_{soc}^*, $\frac{\partial F}{\partial I}\big|_{I=I_{soc}^*} < 0$. Therefore, the sign of $\frac{\partial I}{\partial \theta}$ equals the sign of $\frac{\partial F}{\partial \theta}$, which in turn is the cross-partial derivative of total surplus (19) with respect to I and θ. This generates the comparative statics with respect to α, β, ϕ_a and ϕ_b.

Proof of Proposition 6. Equation (20) can be rewritten

$$2\beta\left(-\frac{\phi_b\left(I+1\right)}{\sqrt{I}} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b\left(I+1\right)}{I^{3/2}}\right) - \frac{I-1}{I+1}\sigma_\eta \sigma_\varepsilon = 0.$$

Let

$$F(I) = 2\beta\left(-\frac{\phi_b\left(I+1\right)}{\sqrt{I}} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b\left(I+1\right)}{I^{3/2}}\right) - \frac{I-1}{I+1}\sigma_\eta \sigma_\varepsilon.$$

We need only consider $I \geq 1$. Since $2\beta\left(-\frac{\phi_b(I+1)}{\sqrt{I}} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b(I+1)}{I^{3/2}}\right)$ is decreasing in $I \in [1, \infty)$ and $\frac{I-1}{I+1}\sigma_\eta \sigma_\varepsilon$ is increasing in $I \in [1, \infty)$, $F(I)$ is decreasing in $[1, \infty)$. Then since $F(\infty) < 0$ and $F(1) > 0$, there exists a unique root of $F(I) = 0$ in $[1, \infty)$.

The comparative statics results follow from taking the cross-partial derivatives of the objective function. The cross-partial with respect to I and β is $\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}$, which is positive from equation (20). The other cross-partial derivatives can be immediately signed.

Proof of Proposition 7. The only difference from the previous analysis is that in the action stage of the game, blockholder i now simultaneously chooses her action b_i and whether to become informed.

We proceed by backwards induction. Let J be the number of blockholders that acquire information. In the trading stage, uninformed blockholders cannot expect to make profits and thus do not trade in aggregate. Therefore, only the J informed blockholders trade and the equilibrium is similar to the one derived in Proposition 1.

Now in the action stage, the manager must choose an action a. Using the same arguments as in Proposition 3, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1} \right) - 1. \tag{44}$$

Blockholders must choose actions b_i and whether to become informed. These decisions can be taken independently since informed trading profits are independent of b_i (which is public), and the choice of b_i depends only on blockholder i's stake β/I. The optimal action of each blockholder is thus

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{45}$$

From equation (6), if there are I informed blockholders, then each blockholder's trading profits are given by:

$$\frac{1}{\sqrt{I}(I+1)} \sigma_\eta \sigma_\epsilon.$$

A blockholder will acquire information if and only if her trading profits are higher than c. This gives the number J of blockholders that decide to become informed in equilibrium.

Proof of Proposition 8. Let n and $J(I)$ be as given in Proposition 7. Using the results of Proposition 3, expected firm value is

$$E[\widetilde{v}] = \phi_a \log \left[\phi_a \alpha \left(\frac{J(I)}{J(I)+1} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{46}$$

We wish to maximize the above expression with respect to I. Since $J(I) = n$ for $I \geq n$, it is never optimal to increase I beyond n since it reduces the second term in the firm value while keeping the first term constant. Therefore, $I^*_{costly} \leq n$. When $I \leq n$, $J(I) = I$ and the problem is the same as in Proposition 4. From (15) we obtain the desired result.

Proof of Proposition 9. The manager will not exert effort above the level for which

$$\phi_a \log (1 + a) = \phi_b \log \left(1 + \sum_i \widehat{b}_i \right),$$

i.e.

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$$a = \exp\left(\frac{\phi_b}{\phi_a} \log\left(1 + \sum_i \widehat{b}_i\right)\right) - 1.$$

This derives the optimal a as given in equation (28). Similarly, blockholder i will not exert effort above the level for which

$$\phi_b \log\left(1 + b_i + \sum_{j \neq i} \widehat{b}_j\right) = \phi_a \log\left(1 + \widehat{a}\right),$$

i.e.

$$b_i = \exp\left(\frac{\phi_a}{\phi_b} \log\left(1 + \widehat{a}\right)\right) - \sum_{j \neq i} \widehat{b}_j - 1.$$

A Nash equilibrium requires the following three conditions to hold:

$$\phi_b \log\left(1 + I b_i\right) = \phi_a \log\left(1 + a\right).$$

$$a \leq \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$$

$$b_i \leq \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}.$$

If the first condition was violated, then the party producing the higher output would gain by reducing effort. The two inequality conditions represent the maximum levels of effort that the manager and blockholders will exert, given the marginal cost of effort.

Out of the continuum of potential Nash equilibria, we seek the one that maximizes firm value. Since firm value is increasing in both a and b_i, it is clear that at least one incentive compatibility constraint will bind. If neither constraint binds, then all parties are exerting suboptimal effort. We could raise the effort levels of all parties while maintaining the equality condition and violating neither constraint.

We now show that, in fact, *both* constraints will bind. Consider the case where $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}$. (Starting with $a = \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$ leads to the same result). Then we have

$$\phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right] = \phi_a \log\left(1 + a\right)$$

$$a = \exp\left(\frac{\phi_b}{\phi_a} \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) - 1.$$

Recall that we also require $a \leq \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$. Hence firm value is optimized by solving:

$$\max_I \exp\left(\frac{\phi_b}{\phi_a} \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) \quad \text{s.t.} \quad \exp\left(\frac{\phi_b}{\phi_a} \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) \leq \phi_a \alpha \left(\frac{I}{I+1}\right).$$

The constraint will bind, and so we obtain

$$\phi_a \log\left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] = \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right]. \tag{47}$$

The firm value optimum setting I to ensure all parties exert their "full" effort levels. The intuition is as follows. Consider a Nash equilibrium where the blockholders are exerting their full effort (i.e. $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}$), and the manager is not (i.e. $a < \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$). b_i is thus constrained by I via the equation $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}$, and so firm value rises if I is reduced to relax this constraint and allow b_i to rise. Unlike in the core model, we do not have the side-effect that reducing I decreases a. I only determines the upper bound to a, not its level. Since $a < \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$, the upper bound is not a constraint anyway. Rather than declining, a will rise to accompany the increase in b_i and ensure that $\phi_b \log(1 + Ib_i) = \phi_a \log(1 + a)$ still holds.

From equation (47), the optimal number of blockholders is determined implicitly by:

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp(\phi_b - \phi_a) = Z.$$

Using the quadratic formula, the unique positive solution is

$$\underline{I} = \frac{Z + \sqrt{Z^2 + 4Z}}{2},$$

which is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

Proof of Proposition 10. We now allow the non-negativity constraints to bind. Deriving $\widetilde{p}$ as in the main model and solving the manager's objective function, he will choose either $a = \phi_a \alpha \frac{I}{I+1} - 1$ or $a = 0$. If $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] < \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)$, exerting $a = \phi_a \alpha \frac{I}{I+1} - 1$ will have no effect on $\widetilde{p}$ and so the manager will choose $a = 0$. Even if $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] \geq \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)$, it is not automatic that the manager will exert effort. Exerting effort increases $\widetilde{p}$ not by $\frac{I}{I+1} \phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right]$, as in the core model, but by only

$$\frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)\right)$$

because blockholder effort "supports" firm value even if $a = 0$. Hence the manager chooses $a = \phi_a \alpha \frac{I}{I+1} - 1$ if and only if

$$\alpha \frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)\right) \geq a.$$

and so the optimal a is as given by (32). Blockholder i's effort level is derived similarly.

There are two candidates for a Nash equilibrium:

$$\begin{cases} a = 0, b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I} \\ a = \phi_a \alpha \frac{I}{I+1} - 1, b_i = 0 \end{cases}.$$

Firm value is thus either $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right]$ or $\phi_b \log\left[\phi_b \beta \frac{1}{I}\right]$. The former is monotonically increasing in I, and maximized at $\phi_a \log(\phi_a \alpha)$ for $I = \infty$. The latter is monotonically decreasing in I, and maximized at $\phi_b \log(\phi_b \beta)$ for $I = 1$. Thus I^* is as given in (34).

Proof of Proposition 11. Proceeding as in the main model, the actions are given by

$$a = \phi_a \alpha \left[1 - \frac{\omega}{I+1} \right] - 1 \tag{48}$$

and

$$b_i = \phi_b \beta \left[\frac{\zeta}{I(I+1)} + \frac{1-\zeta}{I^2} \right] - \frac{1}{I}. \tag{49}$$

Firm value is given by:

$$E[\widehat{v}] = \phi_a \ln \left[\phi_a \alpha \left[1 - \frac{\omega}{I+1} \right] \right] + \phi_b \ln \left[\phi_b \beta \left[\frac{\zeta}{I+1} + \frac{1-\zeta}{I} \right] \right]. \tag{50}$$

The first-order condition is given by (36). Putting this under a common denominator yields

$$F(I, \omega, \zeta) = \frac{I(I+1-\zeta)\phi_a \omega - \phi_b \left[(I+1)^2 - \zeta(2I+1) \right](I+1-\omega)}{I(I+1-\omega)(I+1-\zeta)}.$$

It is a cubic, and has at most three roots. If $I \to \pm\infty$, the numerator becomes dominated by the term containing $(I+1)^2$ and so F tends to $\frac{-\phi_b(I+1)^2}{I(I+1-\zeta)}$. It thus asymptotes the x-axis from below. If $I \to 0$ or $I \to -(1-\zeta)$, then F tends to $\frac{-\phi_b \left[(I+1)^2 - \zeta(2I+1) \right]}{I(I+1-\zeta)}$. For I close to 0, we have $\frac{(I+1)^2 - \zeta(2I+1)}{I(I+1-\zeta)} > 0$ and so the sign depends on $-\frac{\phi_b}{I}$. It is positive (negative) as I approaches 0 from below (above). For I close to $-(1-\zeta)$, we have $\frac{(I+1)^2 - \zeta(2I+1)}{I} < 0$ and so the sign depends on $\frac{\phi_b}{I+1-\zeta}$. It is negative (positive) as I approaches $-(1-\zeta)$ from below (above). If $I \to -(1-\omega)$, then F tends to $\frac{\phi_a \omega}{I+1-\omega}$ and is negative (positive) as I approaches $-(1-\omega)$ from below (above).

To identify the roots, consider $-(1-\zeta) < -(1-\omega)$. (The same arguments apply for $-(1-\omega) < -(1-\zeta)$.) At $I = -\infty$, F asymptotes the x-axis from below, and declines until it reaches $-\infty$ when I is just below $-(1-\zeta)$, so there are no roots for $I < -(1-\zeta)$. When I is just above $-(1-\zeta)$, $F \to \infty$. It then decreases, crosses through zero and becomes $-\infty$ just below $-(1-\omega)$. There is one root for $-(1-\zeta) < I < -(1-\omega)$. $F \to \infty$ just above $I = -(1-\omega)$ and just below $I = 0$, so there are either 0 or 2 roots for $-(1-\omega) < I < 0$. Thus, there can be at most 2 roots for $I > 0$. $F \to -\infty$ when I is just above 0, and asymptotes the x-axis from below as $I \to \infty$. Therefore, F crosses the x-axis either 0 or 2 times for $I > 0$. If F has no roots, it is negative for all $I > 0$ and so the optimal number of blockholders is its minimum value of 1. If it has two roots greater than 1, the upper root I_u is the maximum since the derivative is positive below I_u and negative above I_u. As in the proof of Proposition 5, the cross-partials are sufficient to determine the sign of the comparative statics. The cross-partials with respect to ϕ_a and ϕ_b are immediate. For ω and ζ, we have:

$$\frac{\partial^2 E[v]}{\partial I \partial \omega} = \frac{\phi_a}{(I+1-\omega)^2} > 0$$

$$\frac{\partial^2 E[v]}{\partial I \partial \zeta} = \frac{\phi_b}{(I+1-\zeta)^2} > 0.$$

Proof of Proposition 14. Suppose the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$ and blockholders use a linear demand of the form $x_i(\nu) = \gamma(\tilde{\nu} - \mu)$. Then blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \mid \tilde{\nu}_i = \nu] = \left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) - \lambda(I - 1)\gamma\left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu)\right)\right)x_i - \lambda x_i^2.$$

This maximization problem yields

$$x_i(\nu) = \frac{1}{2\lambda}\left[\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) - \lambda(I - 1)\gamma\left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu)\right)\right] \quad \forall i.$$

The strategies of the blockholders are symmetric and we thus have

$$x_i(\nu) = \frac{1}{2}\left(\frac{1}{\lambda} - (I - 1)\gamma\right)\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) \quad \forall i.$$

which implies that

$$\gamma = \frac{\sigma_\eta^2}{((I + 1)\sigma_\eta^2 + 2\sigma_\delta^2)\lambda}$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{51}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}\sigma_\eta^2}{\sigma_\epsilon((I + 1)\sigma_\eta^2 + 2\sigma_\delta^2)}$$

$$\mu = \phi_a \log(1 + a) + \phi_b \log\left(1 + \sum\nolimits_i b_i\right).$$

From this we obtain:

$$x_i(\nu_i) = \frac{\sigma_\epsilon}{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}}\left(\nu_i - \phi_a \log(1 + a) - \phi_b \log\left(1 + \sum\nolimits_i b_i\right)\right) \quad \forall i,$$

$$p(y) = \phi_a \log(1 + a) + \phi_b \log\left(1 + \sum\nolimits_i b_i\right) + \frac{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}\sigma_\eta^2}{\sigma_\epsilon((I + 1)\sigma_\eta^2 + 2\sigma_\delta^2)}y,$$

as required.

Proof of Proposition 12. Dropping terms that do not contain b_i, blockholder i's objective function (37) becomes

$$\max_{b_i}\left(\frac{\beta}{I}\right)\phi_b \log\left(1 + \sum\nolimits_i b_i\right) - b_i + \frac{1}{\sqrt{I}(I + 1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\phi_a \log(1 + a) + \phi_b \log\left(1 + \sum\nolimits_i b_i\right) - \mu\right)^2$$

Given the conjecture $b_i = \frac{\phi_b\beta}{I^2} - \frac{1}{I}$, we have

$$\mu = \phi_a \ln(1 + a) + \phi_b \ln\left(\frac{\phi_b\beta}{I}\right)$$

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and so the objective becomes

$$\max_{b_i} \left(\frac{\beta}{I}\right) \phi_b \log\left(1 + \sum_i b_i\right) - b_i + \frac{1}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta} E\left[\left(\phi_b \log\left(1 + \sum_i b_i\right) - \phi_b \log\left(\frac{\phi_b\beta}{I}\right) + \eta\right)^2\right]$$

with first-order condition

$$\frac{\phi_b\beta}{I\left(1 + \sum_i b_i\right)} - 1 + \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\phi_b \log\left(1 + \sum_i b_i\right) - \phi_b \log\left(\frac{\phi_b\beta}{I}\right)\right)\frac{\phi_b}{\left(1 + \sum_i b_i\right)} = 0. \quad (52)$$

where

$$1 + \sum_i b_i = \frac{1}{I} + \frac{I-1}{I^2}\phi_b\beta + b_i.$$

One solution is $b_i = \frac{\phi_b\beta}{I^2} - \frac{1}{I}$. The second-order condition is:

$$-\frac{\phi_b\beta}{I\left(1 + \sum_i b_i\right)^2} + \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\begin{array}{c} -\frac{\phi_b}{\left(1 + \sum_i b_i\right)^2}\left(\phi_b \ln\left(1 + \sum_i b_i\right) - \phi_b \ln\left(\frac{\phi_b\beta}{I}\right)\right) \\ + \frac{\phi_b}{\left(1 + \sum_i b_i\right)}\frac{\phi_b}{\left(1 + \sum_i b_i\right)} \end{array}\right)$$

which has the same sign as

$$-\frac{\beta}{I} + \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\phi_b\left(1 - \left(\ln\left(\frac{1}{I} + \frac{I-1}{I^2}\phi_b\beta + b_i\right) - \ln\left(\frac{\phi_b\beta}{I}\right)\right)\right).$$

To show that $b_i = \frac{\phi_b\beta}{I^2} - \frac{1}{I}$ is a global maximum, it is sufficient to show that the function is globally concave, i.e. is negative for all b_i. Since the second-order condition is decreasing in b_i, it is sufficient to show that it is negative when b_i is at its lowest possible value of 0. Then, it becomes

$$-\frac{\beta}{I} + \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\phi_b\left(1 - \left(\ln\left(\frac{I-1}{I^2}\phi_b\beta + \frac{1}{I}\right) - \ln\left(\frac{1}{I}\phi_b\beta\right)\right)\right) < 0 \quad (53)$$

which is satisfied if

$$-\frac{\beta}{I} + \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\phi_b\left(1 + \ln\left(\frac{1}{I}\phi_b\beta\right)\right) < 0.$$

Since $I \geq 1$, this is in turn satisfied if

$$-\beta + \frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\phi_b\left(1 + \ln\left(\phi_b\beta\right)\right) < 0.$$

Since $\frac{\sqrt{I}}{(I+1)}$ is decreasing in I, a sufficient condition is

$$\frac{\beta}{\phi_b\left(1 + \ln\left(\phi_b\beta\right)\right)} > \frac{\sigma_\varepsilon}{\sigma_\eta}, \quad (54)$$

i.e. (38).

The alternative sufficient condition is obtained without studying second-order conditions. First, observe that plugging $b_i = \infty$ into the objective function yields a value of $-\infty$, so the global maximum is either $b_i = 0$ or involves b_i satisfying the first-order condition (52). Defining

$$A = \frac{1}{I} + \frac{I-1}{I^2}\phi_b\beta,$$

$$B = \frac{\phi_b\beta}{I},$$

$$C = \frac{\phi_b\beta}{I^2} - \frac{1}{I} = B - A,$$

$$K = \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\phi_b^2,$$

the first-order condition (52) can be rewritten:

$$\frac{B}{A+b_i} - 1 + \frac{K}{A+b_i}\ln\left(\frac{A+b_i}{B}\right) = 0$$

$$C - b_i + K\ln\left(1 + \frac{b_i - C}{B}\right) = 0. \tag{55}$$

As considered above, $b_i = C$ is a solution to the first-order condition. If $b_i \neq C$, then the first-order condition can be rewritten as

$$-1 + \frac{K}{b_i - C}\ln\left(1 + \frac{b_i - C}{B}\right) = 0. \tag{56}$$

Note that the function $\ln(1+x)/x$ is decreasing in x, and so $-1 + \frac{K}{b_i-C}\ln\left(1 + \frac{b_i-C}{B}\right)$ is decreasing in b_i. If $-1 - \frac{K}{C}\ln\left(1 - \frac{C}{B}\right) < 0$, then (56) has no solution. Then $b_i = C$ is the unique solution for (55). Also note that $-1 - \frac{K}{C}\ln\left(1 - \frac{C}{B}\right) < 0$ implies that $\frac{\partial f(b_i)}{\partial b_i}\big|_{b_i=0} > 0$, and so $b_i = 0$ cannot be the global maximum. Hence, if $-1 - \frac{K}{C}\ln\left(1 - \frac{C}{B}\right) < 0$, the global maximum must be $b_i = C$. This sufficient condition implies $-C < K\ln\left(1 - \frac{C}{B}\right)$, which eventually yields:

$$1 > \frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}\ln\left(1 - \frac{1}{I} + \frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta} - \frac{1}{I}} \tag{57}$$

Since $\frac{\ln(1+x)}{x}$ is decreasing in x, the function

$$\ln\left(1 - \frac{1}{I} + \frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta} - \frac{1}{I}}$$

is decreasing in I. Also note that the function

$$\frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}$$

is decreasing in I for $I \geq 1$. Thus

$$\frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}\ln\left(1 - \frac{1}{I} + \frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta} - \frac{1}{I}}$$

is decreasing in I for $I \geq 1$. Hence a sufficient condition for (57) to hold is that

$$1 > \frac{\sigma_\varepsilon}{\sigma_\eta} \frac{\phi_b}{\beta} \ln \left(\frac{1}{\phi_b \beta} \right) \frac{1}{\frac{1}{\phi_b \beta} - 1}$$

$$\frac{\sigma_\varepsilon}{\sigma_\eta} < \frac{\phi_b \beta - 1}{\phi_b^2 \ln (\phi_b \beta)}. \tag{58}$$

Note that sufficient condition (54) or (58) may be weaker, depending on parameter values, so we provide them both in the Proposition.

Proof of Proposition 13. Suppose the conjectured equilibrium actions are $\hat{b}_i$ such that $\sum_i \hat{b}_i \neq \phi_b \beta / I - 1$. Can $b_i = \hat{b}_i$ be an optimal response of blockholder i?

We first analyze the case $\sum_i \hat{b}_i > 0$. In this case, there exists i such that $\hat{b}_i > 0$. Blockholder i's objective function (37) becomes

$$\max_{b_i} \left(\frac{\beta}{I} \right) \phi_b \log \left(1 + b_i + \sum_{j \neq i} \hat{b}_j \right) - b_i$$

$$+ \frac{1}{\sqrt{I}(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} E \left[\left(\phi_a \log (1 + a) + \phi_b \log \left(1 + b_i + \sum_{j \neq i} \hat{b}_j \right) + \eta - \mu \right)^2 \right] \tag{59}$$

with

$$\mu = \phi_a \ln (1 + a) + \phi_b \ln \left(1 + \sum_i \hat{b}_i \right).$$

The first-order condition is

$$0 = \left(\frac{\phi_b \beta}{I \left(1 + b_i + \sum_{j \neq i} \hat{b}_j \right)} - 1 \right)$$

$$+ \frac{2}{\sqrt{I}(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \left(\phi_a \log (1 + a) + \phi_b \log \left(1 + b_i + \sum_{j \neq i} \hat{b}_j \right) - \mu \right) \frac{\phi_b}{1 + b_i + \sum_{j \neq i} \hat{b}_j} \tag{60}$$

When $b_i = \hat{b}_i$, the second term on the right-hand side of (60) is equal to zero. However, the first term on the right-hand side of (60) is different from zero since $\sum_i \hat{b}_i \neq \phi_b \beta / I - 1$. The first-order condition cannot be satisfied and thus we cannot have $\sum_i \hat{b}_i \neq \phi_b \beta / I - 1$ and $\sum_i \hat{b}_i > 0$ at the same time.

The only other possible symmetric equilibrium in pure strategies involves $\sum_i \hat{b}_i = 0$, which implies $\hat{b}_i = 0$ for all i. For this to be an equilibrium, we would need the right-hand side of (60) to be negative at $b_i = \hat{b}_i = 0$. Since we have $I b_i = \frac{\phi_b \beta}{I} - 1 > 0$, we have $\phi_b \beta > I$ and so this cannot be the case.

Sufficient Conditions for $a > 0$ and $b_i > 0$. From (7), we have

$$a = \phi_a \alpha \left(\frac{I}{I+1} \right) - 1.$$

Since $I \geq 1$, $\frac{I}{I+1} \geq \frac{1}{2}$ and so a sufficient condition for $a \geq 0$ is

$$\phi_a \alpha \geq 2.$$

The sufficient conditions for $b_i \geq 0$ depend on the variant of the model we are considering. We start with the analysis of the firm value optimum in the core model, Proposition 4, which yielded $I^* = \frac{\phi_a - \phi_b}{\phi_b}$. From (9), we have

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I},$$

and so $b_i = 0$ at $I = \phi_b \beta$. Thus, $\frac{\phi_a - \phi_b}{\phi_b} < \phi_b \beta$ is sufficient to guarantee that $b_i > 0$ at $I = \frac{\phi_a - \phi_b}{\phi_b}$. However, in the presence of non-negativity constraints, firm value (16) is no longer a concave function of I and so an additional condition is necessary to guarantee that $I = \frac{\phi_a - \phi_b}{\phi_b}$ is a global, rather than only local, optimum. While increasing I above $\frac{\phi_a - \phi_b}{\phi_b}$ initially reduces firm value (because the detrimental effect on intervention outweighs the beneficial effect on trading), once I hits $\phi_b \beta$, intervention is already at its minimum level of zero. Thus, further increases in I have no negative effect on intervention, but continue to improve trading, and thus unambiguously boost firm value. The global optimum may be either $I = \frac{\phi_a - \phi_b}{\phi_b}$ or $I = \infty$. For $I = \frac{\phi_a - \phi_b}{\phi_b}$, we have

$$E[v] = \phi_a \log\left(\phi_a \alpha \frac{I}{I+1}\right) + \phi_b \log\left(\phi_b \beta \frac{1}{I}\right)$$
$$= (\phi_a - \phi_b) \log(\phi_a - \phi_b) + \phi_a \log \alpha + \phi_b \log\left(\phi_b^2 \beta\right),$$

and for $I = \infty$, we have
$$E[v] = \phi_a \log(\phi_a \alpha).$$

Thus,
$$(\phi_a - \phi_b) \log(\phi_a - \phi_b) + \phi_b \log\left(\phi_b^2 \beta\right) > \phi_a \log \phi_a$$

is sufficient to guarantee that $I^* = \frac{\phi_a - \phi_b}{\phi_b}$ in the presence of non-negativity constraints.

Similar analysis yields sufficient conditions for the analysis of the social optimum, Proposition 5, as

$$I_{soc}^* < \phi_b \beta,$$

$$\phi_a \log\left[\phi_a \alpha \left(\frac{I_{soc}^*}{I_{soc}^* + 1}\right)\right] + \phi_b \log\left[\phi_b \beta \left(\frac{1}{I_{soc}^*}\right)\right] - \phi_a \alpha \left(\frac{I_{soc}^*}{I_{soc}^* + 1}\right) - \phi_b \beta \frac{1}{I_{soc}^*} > \phi_a \log(\phi_a \alpha) - \phi_a \alpha,$$

where I_{soc}^* is defined by (18). The sufficient conditions for the analysis of the private optimum, Proposition 6, are

$$I_{priv}^* < \phi_b \beta,$$

$$\beta \left\{\phi_a \log\left[\phi_a \alpha \left(\frac{I_{priv}^*}{I_{priv}^* + 1}\right)\right] + \phi_b \log\left[\phi_b \beta \frac{1}{I_{priv}^*}\right]\right\} - \phi_b \beta \frac{1}{I_{priv}^*} + \frac{\sqrt{I_{priv}^*}}{I_{priv}^* + 1} \sigma_\eta \sigma_\varepsilon > \beta \phi_a \log(\phi_a \alpha),$$

where I^*_{priv} is defined by (20). The sufficient conditions for the model with the general objective function, Proposition 11, are

$$I^*_{gen} < \phi_b \beta$$

$$\phi_a \ln \left[\phi_a \alpha \left[1 - \frac{\omega}{I^*_{gen} + 1} \right] \right] + \phi_b \ln \left[\phi_b \beta \left[\frac{\zeta}{I^*_{gen} + 1} + \frac{1 - \zeta}{I^*_{gen}} \right] \right] > \phi_a \ln (\phi_a \alpha),$$

where I^*_{gen} is defined by (36). The sufficient conditions for the model with imperfect signals, Proposition 17, are

$$\frac{(\phi_a - \phi_b)(2\sigma_\delta^2 + \sigma_\eta^2)}{\phi_b \sigma_\eta^2} > \phi_b \beta$$

$$(\phi_a - \phi_b) \log (\phi_a - \phi_b) + \phi_b \log \left[\frac{\phi_b{}^2 \beta \sigma_\eta^2}{(2\sigma_\delta^2 + \sigma_\eta^2)} \right] > \phi_a \log \phi_a.$$

For the analysis of perfect positive complementarities (Proposition 9), it is automatic that the optimum cannot involve a non-negativity constraint binding, since firm value is zero if a or $\sum_i b_i$ is zero. For perfect negative complementarities (Proposition 10), we do allow for a or $\sum_i b_i$ to be zero, and indeed the optimum involves one of these terms being zero.

B   Imperfect Signals

The key mechanism through which we achieve the optimality of a multiple blockholder structure is the positive effect of blockholder numbers on price informativeness. It is therefore important to verify the robustness of this result to other specifications of the information structure. In the core model, blockholders have perfect information about firm value $\tilde{v}$; Appendix C shows that the results hold with imperfect signals when blockholders receive the same signal. Here, we consider the case in which blockholders observe imperfect and uncorrelated signals.

Each blockholder observes a signal $\tilde{\nu}_i = \tilde{v} + \tilde{\delta}_i$ where $\tilde{\delta}_i$, $i \in I$ are independent and $\tilde{\delta}_i \sim N(0, \sigma_\delta^2)$. Propositions 14-17 are the analogs of Propositions 1-4 in the core model.

Proposition 14. *(Trading Equilibrium)* *The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{\nu}_i) = \gamma (\tilde{\nu}_i - \phi_a \log (1 + a) - \phi_b \log (1 + \textstyle\sum_i b_i)) \quad \forall i \tag{61}$$

$$p(\tilde{y}) = \phi_a \log (1 + a) + \phi_b \log (1 + \textstyle\sum_i b_i) + \lambda \tilde{y}, \tag{62}$$

where

$$\lambda = \frac{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}\sigma_\eta^2}{\sigma_\epsilon((I + 1)\sigma_\eta^2 + 2\sigma_\delta^2)} \tag{63}$$

$$\gamma = \frac{\sigma_\epsilon}{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}}, \tag{64}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions.

Proposition 15. *(Price Informativeness) Price informativeness is equal to*

$$\frac{I\sigma_\eta^2}{(I+1)\sigma_\eta^2 + 2\sigma_\delta^2}.$$

Proposition 16. *(Optimal Actions) The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I\sigma_\eta^2}{(I+1)\sigma_\eta^2 + 2\sigma_\delta^2} \right) - 1 \tag{65}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{66}$$

Proposition 17. *(Firm Value Optimum) The optimal number I^* of blockholders maximizes:*

$$E[\widetilde{v}] = \phi_a \log \left[\phi_a \alpha \left(\frac{I\sigma_\eta^2}{(I+1)\sigma_\eta^2 + 2\sigma_\delta^2} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{67}$$

Solving the maximization problem, we obtain:

$$I^* = \frac{(\phi_a - \phi_b)(2\sigma_\delta^2 + \sigma_\eta^2)}{\phi_b \sigma_\eta^2}. \tag{68}$$

The number of blockholders has exactly the same effects as in the core model. An increase in I raises price informativeness (Proposition 15) and thus managerial effort (Proposition 16), but reduces blockholder effort. Therefore, I^* remains increasing in ϕ_a and decreasing in ϕ_b (Proposition 17). An additional result in the case of imperfect signals is that I^* is also increasing in the noise in the blockholders' signals σ_δ^2 and decreasing in the variance of firm value σ_η^2. Proposition 15 shows that, if σ_η^2 is high, price informativeness is already high under a single blockholder, and so there is less scope to increase it further through augmenting I. The opposite intuition applies to the effect of σ_δ.

The model can also be extended to multiple trading rounds and long-lived private information. Since these extensions have been undertaken in the microstructure literature (albeit without linking price informativeness to manager actions), we can use these prior studies to establish the robustness of our results. Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) consider the effect of competition among identically informed investors with long-lived private information. As in our model, they find that price discovery is accelerated when compared to Kyle's monopolistic case. Foster and Viswanathan (1996) extend the analysis to the case of heterogeneously informed investors and show that the degree of competition depends on the correlation structure of investors' signals. In particular, competition is more intense when the correlation between initial signals is high.

Back, Cao, and Willard (2000) extend the Kyle model to continuous time and a general correlation structure of investors' signals. They show that price informativeness is again higher under multiple informed traders for some fixed initial period, after which the relationship

reverses. This fixed initial period is typically a very long time, and only ends close to the public announcement date. Thus, price informativeness is higher under multiple informed traders for all but the very end of the trading period. It is the initial period that is relevant for our setting: the microfoundations for the manager's stock price concerns discussed in Section 4.3 show that the stock price the manager cares about is a long time before the date when fundamental value is publicly released. For example, the manager can be fired (for a low stock price), headhunted (for a high stock price), sell his own shares or raise equity within a few months. By contrast, the recent corporate scandals and financial crisis highlight that it may take several years for fundamental value to become known.

As discussed in more detail in Section 5, empirical evidence also supports the robustness of our model. In the real world, blockholders have heterogenous signals and there are multiple trading periods. Boehmer and Kelley (2009) and Gallagher, Gardner and Swan (2010) find that competition among blockholders increases price efficiency.

C   Precision of Information Varies with I

In the core model, all blockholders observe the value of the firm perfectly. We now allow for blockholders to receive the same noisy signal, the precision of which is increasing in each blockholder's stake (β/I) and thus decreasing in the number of blockholders I. Blockholders now observe a signal $\tilde{\nu} = \tilde{v} + \tilde{\delta}$ where $\tilde{\delta} \sim N(0, \sigma_\delta^2(I))$. We show that the results of the core model are unchanged as long as signal precision does not decline too rapidly with I.[24]

Proposition 18. *(**Trading Equilibrium**) The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{\nu}) = \gamma\left(\tilde{\nu} - \phi_a \log(1+a) - \phi_b \log(1+\textstyle\sum_i b_i)\right) \quad \forall i \tag{69}$$

$$p(\tilde{y}) = \phi_a \log(1+a) + \phi_b \log(1+\textstyle\sum_i b_i) + \lambda \tilde{y}, \tag{70}$$

where

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta^2}{\sigma_\epsilon(\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2})} \tag{71}$$

$$\gamma = \frac{1}{\sqrt{I}}\frac{\sigma_\varepsilon}{\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2}}, \tag{72}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions.

Proof If the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$, blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \mid \tilde{\nu} = \nu] = \left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}\nu - \mu - \lambda\sum_{j\neq i}x_j\right)x_i - \lambda x_i^2.$$

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[24] Appendix B considers noisy and uncorrelated signals. Here, blockholders receive the same signal. This represents the toughest case for our model, since it means that the amount of information in the economy declines as I rises – there is a single signal which becomes less precise.

This maximization problem yields

$$x_i(\nu) = \frac{1}{\lambda}\left[\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}\nu - \mu - \lambda\sum_j x_j(\nu)\right] \quad \forall i.$$

The strategies of the blockholders are symmetric and we thus have

$$x_i(\nu) = \frac{\sigma_\eta^2}{(I+1)\lambda(\sigma_\eta^2 + \sigma_\delta(I)^2)}(\nu - \mu) \quad \forall i.$$

Total order flow is thus given by

$$y = \frac{I}{(I+1)\lambda}\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}(v - \mu) + \varepsilon. \tag{73}$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{74}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta^2}{\sigma_\epsilon(\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2})}$$

$$\mu = \phi_a \log(1+a) + \phi_b \log\left(1 + \sum_i b_i\right).$$

From this we obtain:

$$x_i(\nu) = \frac{\sigma_\epsilon}{\sqrt{I}}\frac{1}{\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2}}(\nu - \phi_a \log(1+a) - \phi_b(1 + \sum_i b_i)) \quad \forall i,$$

$$p(y) = \phi_a \log(1+a) + \phi_b(1 + \sum_i b_i) + \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta^2}{\sigma_\epsilon(\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2})}y,$$

as required. ∎

The next proposition calculates price informativeness.

Proposition 19. *(Price Informativeness)* *Price informativeness is equal to*

$$\frac{I}{I+1}\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}$$

Proof The result follows from $p(y) = \mu + \lambda y$ and equation (73). ∎

It is easy to see that if $\sigma_\delta(I)$ does not increase too quickly, then price informativeness is increasing in I. As in the core model, when I increases, blockholders trade more competitively and impound more information into prices. This outweighs the fact that there is less information in the economy and each blockholder has less precise information. Also as in the core model, liquidity σ_ε has no effect on price informativeness.

We now solve for the actions of the manager and the blockholders in the first stage.

Proposition 20. *(Optimal Actions)* *The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I}{I+1} \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2} \right) - 1 \tag{75}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{76}$$

Proof The manager maximizes the market value of his shares, less the cost of effort:

$$E\left[\alpha \tilde{p} - a \right]. \tag{77}$$

When setting the price $\tilde{p}$, the market maker uses his conjecture for the manager's action a. Therefore, the manager's actual action affects the price only through its influence on $\tilde{v}$, and thus blockholders' order flow. The manager's first-order condition is given by:

$$\alpha \left(E\left[\frac{d\tilde{p}}{d\tilde{v}} \right] \right) \left(\frac{\phi_a}{1+a} \right) - 1 = 0. \tag{78}$$

From Proposition 19, we obtain (75). The action of each blockholder is the same as in the paper. ∎

If $\sigma_\delta(I)$ does not increase too quickly, the number of blockholders has a positive impact on managerial effort a. The mechanism is the same as in the core model. An increase in the number of blockholders makes prices more informative, increasing the reward to the manager for exerting effort. As in the core model, increasing the number of blockholders always has a negative impact on blockholders effort b_i.

The optimal number I of blockholders maximizes:

$$E[\tilde{v}] = \phi_a \log\left[\phi_a \alpha \left(\frac{I}{I+1} \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2} \right) \right] + \phi_b \log\left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{79}$$

It is easy to see that the optimal number of blockholders is strictly higher than 1 if $\sigma_\delta(I)$ does not increase too quickly. The intuition is similar to the core model. On one hand, an increase in I exacerbates the free-rider problem and hinders intervention. On the other hand, an increase in I can raise price informativeness and thus managerial effort. In this extension, there is an additional negative effect of raising I, which is that each blockholder becomes less informed. The optimal number of blockholders is thus lower than in the core model.

D   Measures of Price Informativeness

This section proves that our measure or price informativeness, $E\left[\frac{d\tilde{p}}{d\tilde{v}} \right]$, is equivalent to the measure commonly used in the microstructure literature, $(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) / \mathrm{Var}(\tilde{v})$.

Using the formula for the conditional variance of a bivariate normal distribution

$$\mathrm{Var}(\tilde{v}|\tilde{p}) = \left(1 - \mathrm{Corr}(\tilde{v}, \tilde{p})^2 \right) \mathrm{Var}(\tilde{v}),$$

we have

$$(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) \, / \, \mathrm{Var}(\tilde{v}) = \mathrm{Corr}(\tilde{v}, \tilde{p})^2. \qquad (80)$$

Since, in equilibrium, the price is a linear function of $\tilde{v}$ and $\tilde{\epsilon}$,

$$\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right] = \frac{\mathrm{Cov}(\tilde{v}, \tilde{p})}{\mathrm{Var}(\tilde{v})}.$$

From the law of iterated expectations and (42),

$$\mathrm{Var}(\tilde{p}) = \mathrm{Cov}(\tilde{v}, \tilde{p}).$$

Therefore,

$$\mathrm{Corr}(\tilde{v}, \tilde{p})^2 = \mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right]. \qquad (81)$$

Combining (80) and (81) shows that

$$\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right] = (\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) \, / \, \mathrm{Var}(\tilde{v}).$$

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