Some Tools for Event Frequency Decomposition and Heterogeneous Transfer Line Analysis

by

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Chapter 1

Introduction

Modeling and analyzing manufacturing systems is a difficult task. Accounting for the variety of events that take place in such systems requires either very detailed and sophisticated methods, or a great amount of simplification, considering only a few, fundamental characteristics of the behavior of the system.

Analytical models that represent relevant characteristics in manufacturing system, and particularly for transfer lines, are available (Gershwin 1994). However, they are usually based on assumptions about the system's behavior that limit their applicability to the modeling of some phenomena. Extending these models to account for other characteristics of the line, or building new models, is often very difficult.

In this thesis we develop a set modeling and analysis tools for two classes of transfer lines. The methods we describe in this thesis allow the treatment of a larger set of transfer lines with the analytical tools already available to model continuous-material transfer lines with exponentially distributed failures and repairs.

First, we consider multiple-event-frequency transfer lines. That is, lines whose machines differ in the frequencies at which failures and repairs occur. We show how a hierarchical decomposition based on frequency can be defined and provide a method for the representation of the line at the different levels of deterministically or exponentially distributed. Considering this class of transfer lines allows the analysis of phenomena like set-ups or scheduled maintenance.

We describe the two classes of transfer lines we consider in this thesis in Sections 1.1 and 1.2.

1.1 Hierarchical Decomposition of Transfer Lines

Manufacturing systems are often large and complex. The manufacturing of a product can involve many operations, each of a different nature and, consequently, each performed by a different machine or work center. Each of these different machines processes the raw material at a particular speed (processing rate) and, according to the nature of the operation, its failure-repair behavior can differ from that of other machines in the line. Moreover, a single machine can experience several types of failures and other processing interruptions, each occurring at a different rate and requiring different times to be repaired.

This issue of complexity must not be ignored in the development of modeling, analysis and control tools for manufacturing systems. Events of different nature have a different effect in the behavior of the system, and must be dealt with in a different manner.

One natural way to deal with the complexity issue is to divide the original problem in more levels, each taking into account only a *class* of events, that is, a set of events grouped by some criterion. Control mechanisms can then be built at each level, which deal only with a class of events and that share information with controllers at other levels.

One possible criterion to characterize events is the frequency at which they take place. More specifically, we focus on the frequency at which machine *failures* and *repairs* occur.

This frequency characterization induces a hierarchical representation of the line. At each level of the hierarchy, the dynamics of only one class of events is considered in detail. More frequent events are treated at other levels.

Although this hierarchical representation is based on failure and repair behavior, it can be extended to include other events, like preventive maintenance and others of deterministic nature, each inducing new hierarchy levels according to its frequency.

The objective of this thesis is to develop tools for the representation of transfer lines at the different hierarchical levels induced by the frequency of individual machine events. We explore how events of higher frequency can be integrated in a lower level of detail at each level of the hierarchy, while maintaining enough information to provide accurate estimates of the performance of the line.

1.2 Heterogeneous Transfer Lines

One of the elements that contributes to the complexity and diversity of manufacturing systems and, in particular, of transfer lines, is the presence of several classes of events. Events may differ in their nature (operations, machine failures and repairs, set-ups, scheduled maintenance), in the way they occur over time (with some random distribution, at some fixed times and dates, after some amount of material has been processed, etc.), and in their duration (deterministic, random).

Such complex systems can be analyzed at different levels of detail. In this process one may choose to include only a set of the events in the line, or to ignore some of the characteristics of such events. In Chapters 2 through 4 we present one of these simplifications: we develop tools for a decomposition of the line based on the frequency at which events occur. The continuous-material transfer line model described in Section 2.1.1 considers only a subset of all possible machines events, operations, failures and repairs, and assumes failure and repair times to be exponentially distributed.

A model that only takes into account failures and repairs is a useful tool to predict measures of the performance of the line *during normal operation*, that is, ignoring interruptions of processing caused by set-ups, maintenance, weekends or holidays.

However, scheduled events do occur in a manufacturing line and do affect its performance. For instance, buffers should be sized to keep the line processing after a failure, but also during the maintenance of a particular machine, during the change of a tool or while a machine is switched to serve a different line. This buffer sizing would require a model that accounts for both unpredictable failures and scheduled interruptions.

The need for more complete models presents us with a trade-off between exact representations and workable models; that is, between a model that includes all relevant information and one for which a solution is attainable (mathematically or numerically) in a finite time.

A mathematical model with deterministic and exponentially distributed processing interruptions is difficult to obtain and solve. The approach in this thesis is to investigate how to represent deterministic events using the models currently available for the analysis of transfer lines with exponentially distributed failure and repair times.

1.3 Outline of the Thesis

In Chapter 2, we provide a definition of a continuous-material transfer line, and characterize the subset of multiple-frequency transfer lines. We also define frequency-induced hierarchical decomposition for such multiple-frequency lines.

In Chapter 3 we develop a method for the representation of a two-frequency two-machine at the higher hierarchical level, where only low frequency events are considered. We examine the behavior of the original system and of its simplified representation and suggest a mechanism for improving the estimates provided by the simplified system.

In Chapter 4 we extend the representation method to F-frequency, k-machine transfer lines. We provide empirical evidence of the performance of the representation at various level of the hierarchy.

Chapter 5 applies the hierarchical decomposition method of Chapters 3 and 4, to the modeling of two particular situations. First, we present the case of transfer lines with machines with several classes of failures. We describe how multilevel event machines can be represented as regular transfer lines and then we show how to use the decomposition method to analyze performance measures for these systems. Then, we apply the method for the case in which machines in a transfer line are grouped in cells, according to the nature of the operations that they perform.

Chapter 6 considers the case of transfer lines with mixed event distributions, specifically, transfer lines where one or more of the machines have deterministic failures and repairs, while the rest have exponentially distributed failures and repairs. We first formally describe these heterogeneous lines. Then we show that an equivalent exponentially-failing machine exists such that when it replaces the deterministic machine, it preserves the performance measures.

Chapter 7 presents a practical application of the methods described in Chapter 6. We describe and present an example of the use of the method for modeling deterministic machine set-ups.

Finally, Chapter 8 summarizes the results of this thesis and provides some directions for further research and applications of the methods here described.

Chapter 2

Multiple-Event-Frequency Transfer Lines - A Method for Hierarchical Decomposition

In this chapter we present a method for hierarchically decomposing continuous material transfer lines according to the frequencies at which failures and repairs take place at individual machines.

In Section 2.1 we describe the dynamics and introduce the basic notation for continuous material transfer lines, then we formalize the concept of a transfer line with *different machine event frequencies* and finally we provide a formal definition of the event frequency hierarchy.

2.1 Definitions

This section introduces the definitions and notation for the transfer line models we analyze in this chapter. First, we describe the continuous material transfer line model, then we characterize the subset of transfer lines with *multiple machine event frequen-*

cies and finally we present a formal definition of a *hierarchy* induced by the machine event frequencies.

2.1.1 Continuous Material-Continuous Time Transfer Lines

The analysis and results presented in this chapter are based on a continuous material transfer line, with unreliable machines and finite buffers. In this section we present a description of the basic characteristics and assumptions of the model. Gershwin (1994) provides a detailed description of the dynamics of these systems.

In a transfer line, raw material enters the first machine or working station where processing start. Then the material visits consecutive machines or working stations, at each of which some processing is performed, adding value to the part. The final product exits the last machine. Each machine is visited only once, and all machines are always visited in the same order.

Machines are unreliable, with operation dependent failures. This means that machines can fail only when they are processing material. The failure and repair times are exponentially distributed and, since failures are operation dependent, the failure rate is proportional to the rate at which material is processed. Each machine suffers only one kind of failure; more general machines, with multiple failures modes can be modeled using single failure machines. (See Section 5.2.)

Finite *buffers* or storage areas exist between consecutive machines. Whenever the downstream machine stops processing and the buffer is not full, material is stored in it. The model is based on the assumption that buffers are perfectly reliable and that there is no delay associated with the passing of material from one machine to the next through the buffer.

Material is considered to be continuous. That is, it behaves as a fluid. Material is continuously processed by each machine and no piece of material resides at a machine but rather, flows through it. This is the case of chemical processing operations, where

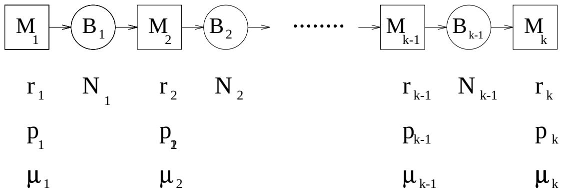

Figure 2-1: Schematic representation of a continuous-material transfer-line

some liquid is subjected to successive transformations. Although we do not study this issue in this thesis, the continuous material model can be of use in modeling high speed discrete material processing (Suri and Fu, 1992).

Line Parameters Now we introduce the parameters that characterize the behavior of machines and buffers in a continuous material transfer line. Each machine M_i is characterized by three parameters:

- Processing Rate μ_i. This is the rate at which material flows and gets processed through machine M_i in the absence of blockage or starvation. In the presence of starvation or blockage, machine M_i will process material at a fraction of μ_i. The actual processing rate at each individual instant t is called $\mu_i(t)$.

- Failure Rate p_i. This is the rate at which machine M_i fails when working at its maximum processing rate μ_i. In the presence of blockage and starvation it will fail at a rate proportional to the fraction of the maximum processing speed at which it is operating. The actual failure rate at instant t is

$$p_i(t) = \frac{\mu_i(t)}{\mu_i} p_i$$

- Repair Rate r_i. This is the rate at which machine i gets repaired if it is down for a failure. This rate never varies. At any instant t:

$$r_i(t) = r_i$$

Using parameters p_i, r_i and μ_i, we can define the two following measures of the performance of each machine M_i:

- Isolated Efficiency e_i. This is the fraction of the time that machine M_i is processing material. e_i is defined in terms of the parameters p_i and r_i as follows:

$$e_i = \frac{r_i}{r_i + p_i}$$

- Mean isolated processing rate ρ_i. This is the mean processing rate of machine M_i when working in isolation. Unlimited supply of raw material is assumed. It is defined as follows:

$$\rho_i = e_i \mu_i$$

Buffer i stores material between machines i and $i+1$. (Fig. 2-1). It is characterized by a single parameter:

- Buffer N_i. N_i is the maximum amount of material that can be stored in buffer i.

Model Assumptions. The model is based on the following set of assumptions:

1. There is an infinite amount of raw material to be processed. The first machine in the line is never starved.
2. There is an infinite demand for the finished material. The last machine is never blocked.
3. Machines work at the maximum possible rate whenever material is available.

Performance Measures. Two measures of performance of line are taken into account in this chapter:

- Production Rate P. P is the average amount of material that exits the line per time unit.

- Mean Buffer Levels $\overline{x}_i$. $\overline{x}_i$ is the average amount of material in buffer i.

A method for the analysis of continuous two-machine material transfer line and for obtaining the values of the performance measures is provided in Gershwin (1994). Solution techniques are also available, such as the accelerated DDX algorithm, or ADDX (Burman, 1995).

2.1.2 Multiple Event Frequency Continuous Material Continuous Time Transfer Lines

In this section we deal with a particular subset of continuous material continuous time transfer lines where events occur at different frequencies at different machines.

In our model, each machine M_i has only one class of failures and repairs, i.e., failures and repairs occur at rates p_i and r_i, and these quantities may vary from one machine to another in magnitude.

To help define multiple event frequency lines, we first define Mean Time to Fail (MTTF), Mean Time to Repair (MTTR) and Mean Time Between Failures (MTBF) as follows:

$$
\begin{aligned}
MTTF_i &= \frac{1}{p_i} \\
MTTR_i &= \frac{1}{r_i} \\
MTBF_i &= \frac{1}{p_i} + \frac{1}{r_i} = MTTF_i + MTTR_i
\end{aligned}
$$

The multiple frequency continuous machine transfer line we focus on in this chapter is characterized by the following condition:

There is at least one machine M_i, $i = 1, ..., N$; such that:

$$MTBF_i \ll min(MTTF_j, MTTR_j), \quad j = 1, ...N, \quad j \neq i \tag{2.1}$$

Condition (2.1) only says that there exists at least one machine whose event frequency is much higher than that of other machines in the line. It requires the time elapsed between any two consecutive events of the lower event-frequency to be, on average, much longer than the time needed for a full failure-repair cycle of the high frequency machine. When this condition is satisfied, on average between any consecutive events of the low frequency machine M_j, machine M_i is operating at a rate equal to its isolated production rate ρ_i. This fact will be used in the frequency decomposition method.

Condition (2.1) does not by itself define a hierarchy, it just identifies the fact that events are occurring at significantly different rates in the line. In Section 2.1.3 we define a hierarchical decomposition of the line based on the existence of multiple event frequencies.

2.1.3 Frequency-Induced Hierarchy

As mentioned in Section 1.1, one possible criterion for a hierarchical decomposition of a transfer line is the frequency at which events occur at individual machines. If we can identify machines that operate with similar event rates, we can group them in a single *level* of the hierarchy, while machines with higher event frequencies belong to lower levels of the hierarchy. (See Fig. 2-2.)

We will define each level of the hierarchy as a set of machines with similar event frequencies as well as all machines with lower frequency. Each level of the hierarchy is called $\mathcal{M}_k$ where k goes from 0, the lowest frequency level, to L, the highest frequency level.

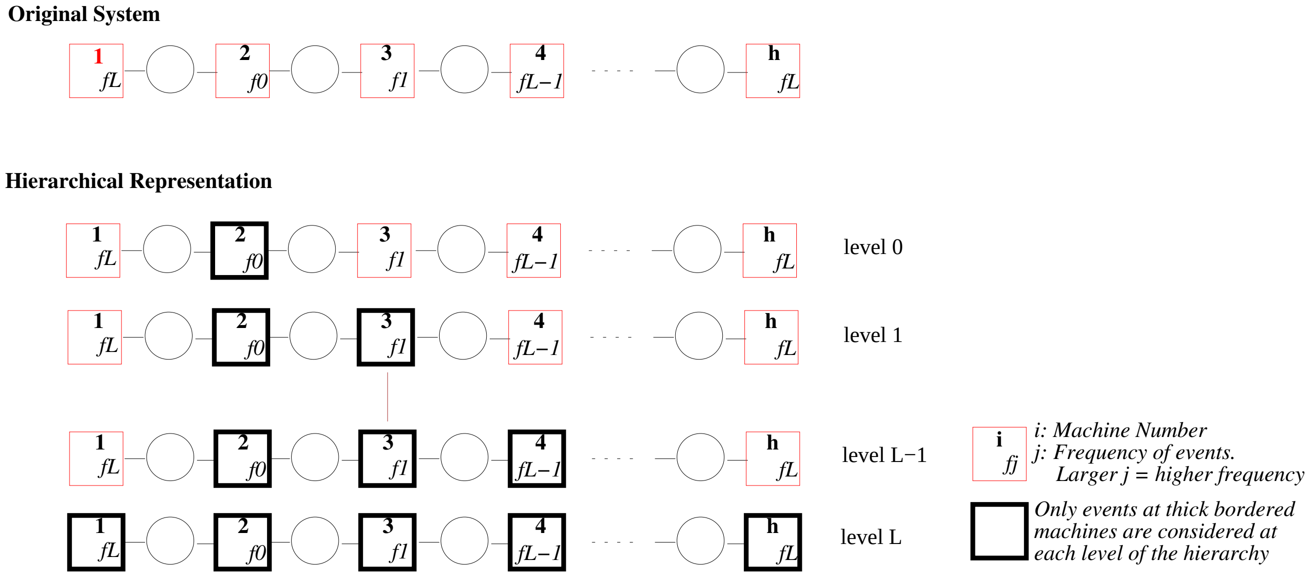

Figure 2-2: Event frequency hierarchical decomposition of a transfer line. At each level of the hierarchy only events of a given frequency an lower are considered in detail. Machine whose events are considered are represented by thick bordered squares. The behavior of machines with higher level events (thin bordered squares) is considered at some higher level of aggregation.

Each level $\mathcal{M}_i$ is such that:

$$\mathcal{M}_k = \{M_i | MTBF_i \ll min(MTTF_j, MTTR_j), M_j \in \mathcal{M}_{k-1}\} \tag{2.2}$$

where each machine's $MTBF_i$, $MTTF_i$ and $MTTR_i$ are defined as in Section 2.1.1.

According to Condition (2.2), level $\mathcal{M}_i$ of the hierarchy contains all machines in levels $\mathcal{M}_{i+1}$ and lower (higher value of index i), of the hierarchy.

This definition is not a precise one, in the sense that no specific size of the frequency difference between levels is mentioned and that no range for the event frequencies of machines within a given level is imposed. However, rather than specify the minimum frequency difference a priori, we analyze how the minimum frequency difference affects the performance of the representation methods we develop.

Chapter 3

Two-Frequency, Two-Machine Lines. Frequency Decomposition Method

In Chapter 2 we define a multiple-event-frequency transfer line and the corresponding frequency-induced hierarchy. In this chapter we develop a method for the representation of a two-frequency two-machine line at the higher level of such a hierarchy. That is, we show how to obtain a single-frequency two-machine transfer line that approximately models the behavior of the original two-frequency line.

We start by defining a two-frequency two-machine line and the two-level hierarchy induced by the two different frequencies in the line in Section 3.1. In Section 3.2 we present a method for the representation at the higher hierarchical level (where higher frequency events are ignored). In Sections 3.3 and 3.4 we evaluate the accuracy of this representation. Finally, in Section 3.5, we characterize the behavior of the buffer level and, on the basis of this characterization, we propose a method for improving the estimates of mean buffer levels obtained from the higher level representation of the line.

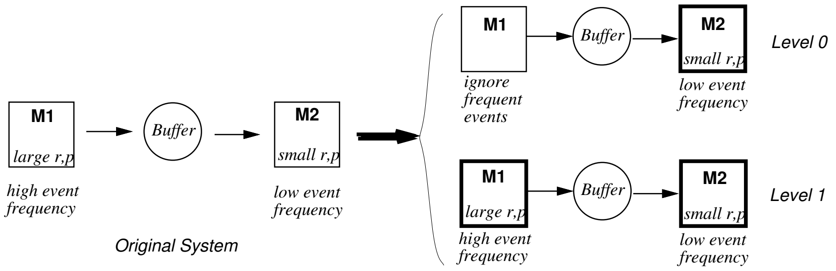

Figure 3-1: Schematic representation of the two-level hierarchy for the two-machine, two frequency transfer line.

3.1 The Problem: Two-Frequency Two-Machine Lines

This section analyzes a two-machine transfer line, where events occur with different frequencies in each of the machines. More specifically, machine M_1 events occur at a higher rate than events at machine M_2; we call M_1 the *high frequency* machine and M_2 the *low frequency* machine.

Event Frequency Separation. Using the definition of multiple frequency transfer line of Section 2.1.2, the original system satisfies:

$$MTBF_1 \ll min(MTTF_2, MTTR_2) \tag{3.1}$$

Condition (3.1) is an application of condition (2.1) to the two-machine case.

Two-Level Hierarchy The event frequency difference between M_1 and M_2 induces a two-level hierarchy. At the higher (less detailed) level only lower frequency events are taken into account; at the lower (more detailed) level both machines events are considered, that is, at this level the original line is considered.

According to the event frequency hierarchy definition in Section 2.1.3, we have two hierarchical levels $\mathcal{M}_0$ and $\mathcal{M}_1$ (Fig.3-1):

$$\begin{aligned}\mathcal{M}_0 &= \{M_2\} \\ \mathcal{M}_1 &= \{M_1, M_2\}\end{aligned}$$

At the higher level of the hierarchy (level $\mathcal{M}_0$) we need consider only lower frequency failures, that is, repairs and failures of machine M_2, while ignoring the detailed dynamics of M_1. This means that we need to build a *higher level representation* or *simplification* of the original line. We will refer to the alternate representation of the original line as the *simplified line.* Figure 3-2 illustrates such a transformation.

The objective of the representation method is to find a single frequency two-machine line which approximately models the behavior of the original two-frequency two-machine line.

Behavior of the Two-Frequency, Two-Machine Line. We define a two-frequency, two-machine transfer line as a transfer line such that, on average, failure-repair cycles of one machine occur much faster than a failure-repair cycle of the second machine. (Condition (3.1). Consequently, on average, from the point of view of the second machine the rate at which M_1 is operating between any consecutive two M_2 events is approximately equal to the average production rate of M_1, ρ_1.

Given the variability of the failure and repair processes at both machines, this is not necessarily true for each individual failure-repair cycle of M_1 and M_2. In some cases the actual time between two M_1 failures will exceed time between two M_2 events while in other cases many M_1 failure-repair cycles will occur during the interval between two machine M_2 events.

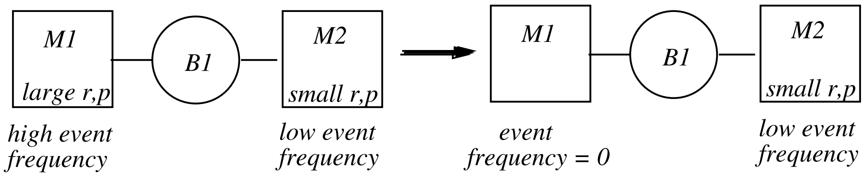

Figure 3-2: Schematic representation of the line as a *simplified* line at level $\mathcal{M}_0$ of the hierarchy.

3.2 Representation of Two Frequency-Two Machine Transfer Lines: Simplified System

At higher levels of the hierarchy we need consider only lower frequency events. At each level we want to build an approximately equivalent system that reflects the operating characteristics of the original system, either by directly preserving the performance measures or by providing enough information to allow inferring those measures.

In this section we present a method by which the original two-machine line can be represented at the higher level of the hierarchy, at which only low frequency events are considered, by a simplified two machine line, while preserving enough information to provide accurate approximations to the original line's performance measures.

3.2.1 Equivalent Two-Machine Line

The purpose of the transformation is to replace the original two-machine line (which includes a low level, high frequency, machine), by a *simplified* line, in which both machines operate at an event frequency equal or lower than that of the second original machine.

In order to obtain the simplified system, we replace the first machine by a machine with events less frequent than or equally frequent as those of machine M_2. Machine M_1 will be replaced by a lower frequency machine $\hat{M}_1$.

We impose the following conditions on machine $\hat{M}_1$:

1. **Event Frequency**: $\hat{M}_1$ events must occur at an equal to or lower rate than those of M_2.

2. **Production Rate**: The isolated production rate must remain unchanged:

$$\hat{\rho}_1 = \rho_1 \tag{3.2}$$

Considering the above requirements for the equivalent first machine, and using the fact that on average between any two events of M_2, M_1 process at a rate equal to its mean production rate ρ_1, we can replace the original line by a simplified two machine line, with machines $\hat{M}_i$ and buffer $\hat{B}$ as follows:

1. **Buffer**: The intermediate buffer remains unchanged:

$$\hat{N} = N$$

2. **Machine** M_2: The second machine is kept identical to the original:

$$\hat{p}_2 = p_2\ ,\quad \hat{r}_2 = r_2\ \ and\ \ \hat{\mu}_2 = \mu_2$$

3. **Machine** M_1: Machine M_1 is replaced by a perfectly reliable machine with reduced processing rate as follows:

$$\begin{aligned} \hat{p}_1 &= 0 \\ \hat{r}_1 &= c \quad c > 0 \\ \hat{\mu}_1 &= \rho_1 \end{aligned}$$

The repair rate $\hat{r}_1$ can be set to any positive number, to allow the efficiency of the first machine, $\hat{e}$, to be 1. We choose $c = 1$ in the remaining discussion and examples.

Since the simplified first machine $\hat{M}_1$ experiences no events, the simplified system satisfies the condition regarding event frequency: both machines operate with an event frequency less than or equal to that of the second (lower frequency) machine.

Condition (3.2), which requires the mean production rate of the individual machines to remain unchanged, is also satisfied, since the mean processing rate of the first machine remains unchanged: $\hat{\rho}_1 = \hat{\mu}_1 \hat{e}_1 = \hat{\mu}_1 = \rho_1$.

Example 3.1. Now, we provide an example of the application of the method to a two-frequency two-machine line. Table 2.1 indicates the parameters of the original line. Table 2.2 contains the parameters of the simplified line:

Table 2.1. Example: Original System						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	0.558	0.594	1.284	0.484	0.622	80
2	0.021	0.015	1.087	0.583	0.634	n/a

Table 2.2. Example: Simplified Line						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	1.000	0.000	0.622	1.000	0.622	80
2	0.021	0.015	1.087	0.583	0.634	n/a

One natural way of examining the accuracy of the representation is to compare the performance measures, production rate P and mean buffer level $\overline{x}_i$, for both the original and the modified system.We use the analytical solution for a two-machine continuous-material line developed by Gershwin (1994) to obtain these performance measures for each of the lines. Table 2.3 shows the performance measures values and the relative errors.

Table 2.3. Example: Performance Measures.		
	Line Performance Measures	
Model	Mean Prod. Rate	Mean Buffer Level
Original	0.5577	30.931
Simplified	0.5597	30.535
Error	0.004	-0.012

We observe that the information provided by the simplified line about the original system is very accurate. Both mean production rate P and mean buffer level $\overline{x}$ obtained using the simplified line are about one percent from the actual values, slightly overestimating the line production rate and underestimating the mean buffer level.

The accuracy of the method varies from case to case. The simplified model provides very good approximations for the mean production rate of the line for all the cases we examined. However, errors in the predicted mean buffer level can sometimes be large. Section 3.3 explores in more detail the accuracy of the simplified model in approximating the values of the performance measures.

3.3 Performance of the Transformation

Measures of the Accuracy of the Method Since the objective of the representation method is to build a simplified system as close to the original in terms of its performance, one way to evaluate its accuracy is to analyze the effect of the transformation in the values estimated for production rate and mean buffer level.

For the two-machine line case, we select three measures for the accuracy of the simplification method.

Let

- P: Production rate of the original two-machine line.
- $\hat{P}$: Production rate of the simplified two-machine line.
- $\overline{x}$: Mean buffer level of the original two-machine line.
- $\hat{\overline{x}}$: Mean buffer level of the simplified two-machine line.
- N: Buffer size.

Define

1. **Production Rate Error**:

$$\mathcal{E}_P = \frac{\hat{P} - P}{P}$$

2. **Mean Buffer Level Error**:

$$\mathcal{E}_{\overline{x}} = \frac{\hat{\overline{x}} - \overline{x}}{\overline{x}}$$

3. **Mean Buffer Level Error, with respect to buffer size**:

$$\mathcal{E}_N = \frac{\hat{\overline{x}} - \overline{x}}{N}$$

$\mathcal{E}_{\overline{x}}$ and $\mathcal{E}_N$ are closely related measures of the error in the mean buffer level. They differ in the way the magnitude of the error is measured. $\mathcal{E}_{\overline{x}}$ measures the error relative to the actual mean buffer level $\overline{x}$ while $\mathcal{E}_N$ measures them relative to the buffer size. We use both measures because $\mathcal{E}_{\overline{x}}$ magnifies error magnitudes when the actual buffer level, $\overline{x}$, is much less than one half the buffer size, and especially when it is very small.

For an almost empty buffer, a very small difference between $\hat{\overline{x}}$ and $\overline{x}$ can lead to a high percent error if calculated relative to $\overline{x}$, while qualitatively in both cases the buffer is *almost empty*. In this case, the percent error calculated relative to N will be much smaller.

Generation of Random Two-Machine Lines. To evaluate the accuracy of the representation method, we randomly generated a large number of two-machine lines, obtained the simplified line and compared the values of the performance measures obtained with the original and the simplified lines.

The two-machine lines were generated following these rules:

- The first machine has more frequent events than the second.
- Failure and repair rates p_1 and r_1 are uniformly distributed from 0.001 to 1.0.
- Failure and repair rates p_2 and r_2 are uniformly distributed from 0.001 to 1.0.
- Machine processing rates μ_1 and μ_2 are uniformly distributed from 0.1 to 10.0
- Individual machine production rates ρ_1 and ρ_2 are within 20% of each other.
- The buffer size N is chosen to be larger than half and smaller than five times the amount of material produced during by M_1 during the average duration of

a M_2 failure. That is,

$$N \in (0.5\frac{\rho_1}{r_2}, 5\frac{\rho_1}{r_2})$$

- Condition (3.1) is implemented as

$$MTBF_1 < \frac{1}{5}\, min(MTTF_2, MTTR_2) \tag{3.3}$$

Evaluation of the Method 5000 cases were randomly generated as described above and the error measures calculated. Here we analyze the accuracy of the method in terms of each of the error measures.

Production Rate Error $\mathcal{E}_P$ Figure 3-3 shows the production rate error $\mathcal{E}_P$ for the randomly generated cases, ranked by the size of the error. We observe from the figure that the estimate of production rate obtained from the simplified system is always very close to the actual value. This error is always less than 2%, and in less than 10% of the cases exceeds 1%. The average production rate error was 0.0005 (or less than 0.05%).

We observe that the production rate error is always positive. That is, the simplified system always overestimates the line production rate, even though the first machine in the simplified system operates at the same mean production rate as the first machine in the original system. This can be attributed to the fact that the first machine is a *perfectly reliable* machine which does not allow the second machine to stop due to complete starvation. Partial starvation is still possible if $\rho_1 = \widehat{\mu_1} < \mu_2$.

From the results of these experiments we conclude that the method does accurately estimate the production rate of the two-machine line.

Mean Buffer Level Error $\mathcal{E}_{\overline{x}}$. Figure 3-4 shows the production rate error $\mathcal{E}_{\overline{x}}$ for the randomly generated cases, ranked by the size of the error.

The range of the error is significantly larger for mean buffer level than the one

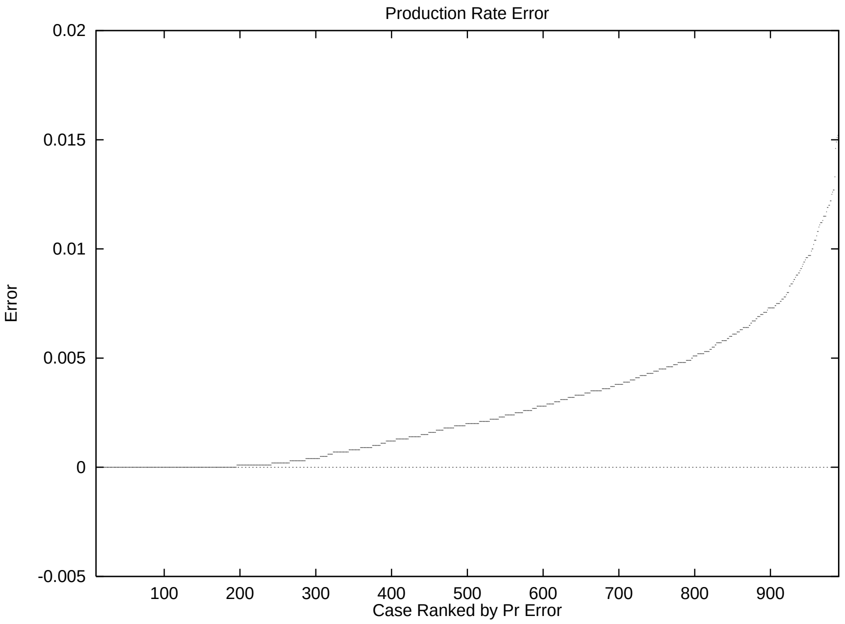

Figure 3-3: Percentage error for production rate P for the randomly generated two-machine lines.

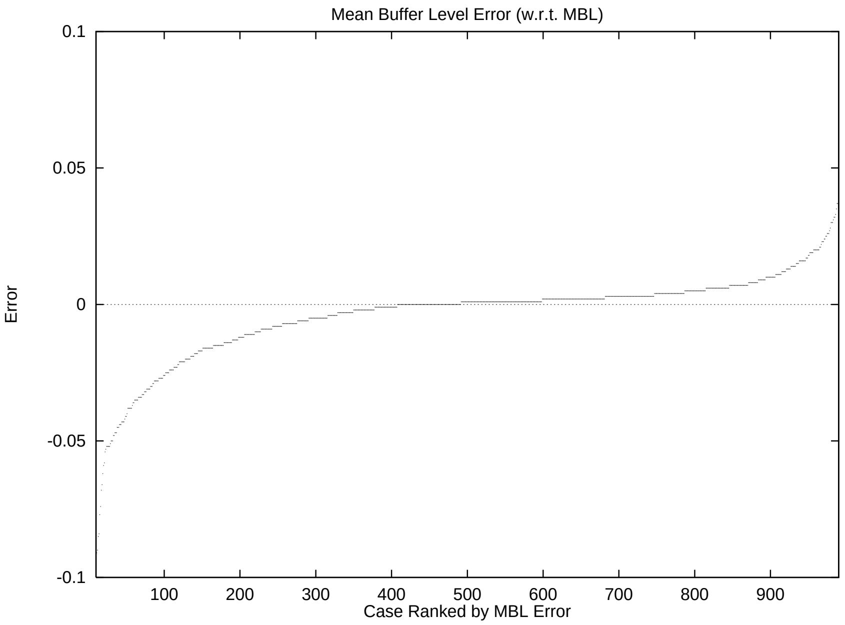

Figure 3-4: Percentage $\overline{x}$ error (with respect to actual mean buffer level). Original vs. Simplified.

we found for production rate. It goes from -9% to 5%. Positive error correspond to cases in which $\rho_1 < \rho_2$, that is, when the first machine was on average slower than the second.

The observed differences between the actual and the estimated mean buffer levels are to be expected. This is because $\overline{x}$ is strongly dependent on the precise failure-repair behavior of the line. The number of events that occur in the original line is larger than the number of events that occur at the simplified line, whose first machine never fails. We examine more closely the issue of the buffer level dynamics in Section 3.5.

We also observe in Fig 3-4 that error magnitudes are not symmetrical, that is, negative errors are larger in magnitude than the positive ones. This is not related to the transformation method, but rather to the way the error measure is defined. This measure of mean buffer level error tends to magnify the error whenever the actual mean buffer level is very small (compared to the size of the buffer), as it is whenever the isolated production rate of the first machine is less than the isolated production rate of the second. That is, it magnifies small errors when the buffer is usually empty or almost empty. In these cases, the simplified method could provide a mean buffer level that is also almost empty, but the relative error $\mathcal{E}_{\overline{x}}$ would show a large error. This magnifying phenomenon does not occur when the buffer is almost full.

For the 5000 random cases generated, the magnitude of the buffer error was on average 0.011. The average for negative errors was -0.016 and and the average was 0.006 for positive errors. That is, on average, mean buffer level error are small, and we can say that the representation method provides a very close approximations. However, we must be careful in using estimates of mean buffers levels obtained from the method since they can be, according to this experiment, as large as 10% in error.

If work in progress (WIP) and location of inventory in the line are important in the task where this method is applied, as they usually are, it is necessary to provide a mechanism to obtain a better approximation of mean buffer levels. Section 3.5

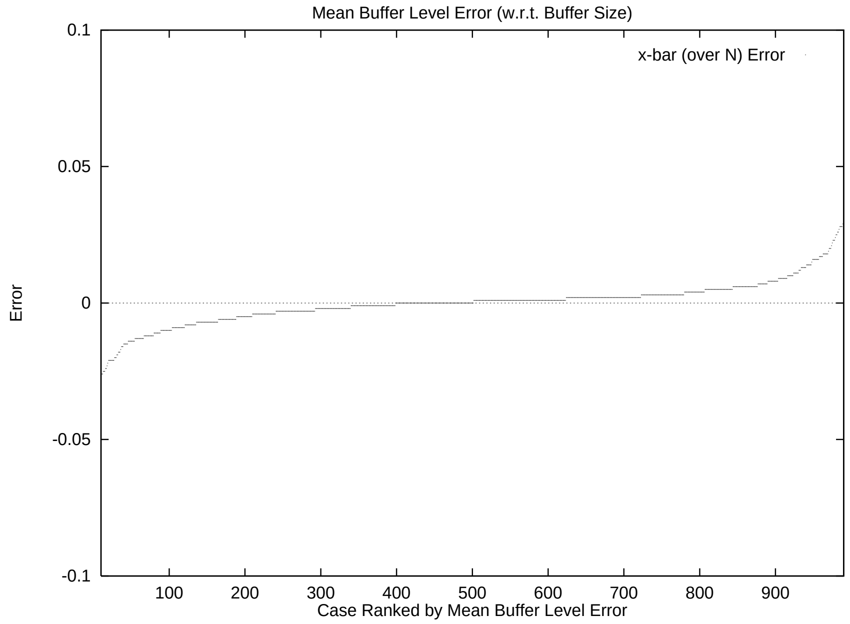

Figure 3-5: Percentage mean buffer level error (with respect to buffer size). Original vs. Simplified.

examines the behavior of the buffer levels and proposes a method for improving the mean buffer level values estimated by the simplified system.

Mean Buffer Level Error Relative to Buffer Size $\mathcal{E}_N$. Figure 3-5 shows the production rate error $\mathcal{E}_N$ for the randomly generated cases, ranked by the size of the error.

The results for this measure of error confirms the fact that the mean buffer level estimate obtained using the simplified system while very good on average can be poor in some cases. Specifically, the average magnitude of the error was 0.006, less than 1%. However, it reached 3% in some cases.

Since the magnitude of the error is the same for both measures of mean buffer level error, we have in this case the same change of sign depending on what machine has the larger mean processing rate. As a consequence of measuring the error relative to the buffer size this measure of error shows similar magnitudes for both positive and negative errors.

3.4 Method Performance as a Function of Event Frequency Separation

In Section 3.2 we present a method for representing a two-frequency, two-machine line by a simplified single-frequency two-machine line. This representation method replaces the machine with higher frequency events by a reliable machine with a reduced processing rate. The reduce rate accounts for the time that the machine is down due to failures. This representation approach is based on the fact that the event frequency separation between M_1 and M_2 is so large that material arrives at M_2 at a rate similar to the isolated production rate of M_1 without being strongly influenced by its specific failure-repair dynamics.

Condition (3.1) in Section 3.1 only requires the event frequency difference between the two machines to be very large. In the evaluation of the performance of the method in Section 3.3 we chose that difference to be of at least a factor of 5. In this section we examine how the performance of the simplification method introduced in the previous section changes as the difference between machine event frequencies increases.

In order to compare the effects of different event frequency separations, we generated eight different sets of random cases. Failure, repair and processing rates as well as buffer sizes were obtained used the same procedure as described in Section 3.3.

Cases satisfy the following event frequency separation condition:

$$\begin{aligned} MTBF_1 &\in (\frac{1}{k+5}min(MTTF_2, MTTR_2), \frac{1}{k}min(MTTF_2, MTTR_2)) \quad (3.4) \\ & k = 1, 6, 11, 16, 21, 26, 31, 36 \end{aligned}$$

That is, each set includes an non-overlapping event frequency separation interval.

We evaluate the performance of the transformation for each random case in each set in the same way as in Section 3.3. That is, we obtained measures of the errors that the simplification incurs in estimating mean production rate and mean buffer level. We use $\mathcal{E}_P$, $\mathcal{E}_{\overline{x}}$ and $\mathcal{E}_N$ as defined in Section 3.3. Then, we compare how these error measures vary for the different event frequency separation sets.

We obtained error measures for 2000 randomly generated cases for each event frequency separation set. Table 2.3 summarizes the results of these experiments. It reports the mean and extreme errors for the measures of the method performance.

Table 2.3. Two-machine lines. Performance of the Transformation Method

	Errors Due to the Transformation					
separation	Max $\mathcal{E}_P$	Mean $\mathcal{E}_P$	Range $\mathcal{E}_{\overline{x}}$	Mean $\mathcal{E}_{\overline{x}}$	Range $\mathcal{E}_N$	Mean $\mathcal{E}_N$
1 to 6	0.0942	0.0119	(-0.3839,0.2132)	0.0304	(-0.1850,0.1670)	0.0213
6 to 11	0.0198	0.0029	(-0.1273,0.1220)	0.0103	(-0.0788,0.1160)	0.0089
11 to 16	0.0129	0.0016	(-0.0646,0.0713)	0.0074	(-0.0231,0.0622)	0.0066
16 to 21	0.0095	0.0012	(-0.0383,0.0578)	0.0054	(-0.0131,0.0518)	0.0043
21 to 26	0.0070	0.0010	(-0.0344,0.0392)	0.0040	(-0.0120,0.0357)	0.0032
26 to 31	0.0063	0.0009	(-0.0273,0.0324)	0.0039	(-0.0103,0.0343)	0.0031
31 to 36	0.0059	0.0009	(-0.0245,0.0294)	0.0038	(-0.0093,0.0303)	0.0029
36 to 41	0.0056	0.0008	(-0.0221,0.0242)	0.0038	(-0.0987,0.0283)	0.0028

Results in Table 2.3 show that the method performs worst for very small event frequency separation factors (less than 5),with errors as large as 9.5% for production rate and 38% for mean buffer levels. For larger event frequencies it performs significantly better and the errors decrease as the separation increases. We now examine these results more closely.

Production Rate Error $\mathcal{E}_P$. Production rate errors are shown in Figures 3-6 and 3-7. Figure 3-6 shows the error range as well as the average error. We observe that error is always positive and it decreases with frequency separation, particularly from the first to the second separation range. On average, the error is less than 0.3% for any event frequency separation larger than 5.

Figure 3-7 shows the error for all the examined cases (ranked by error magnitude); we observe that, for any separation larger than 5, in less than 5% of the cases the error exceeds 1%; and for separations larger than 15, the error is always below 1%.

These results confirm our previous conclusion that the method provides a very

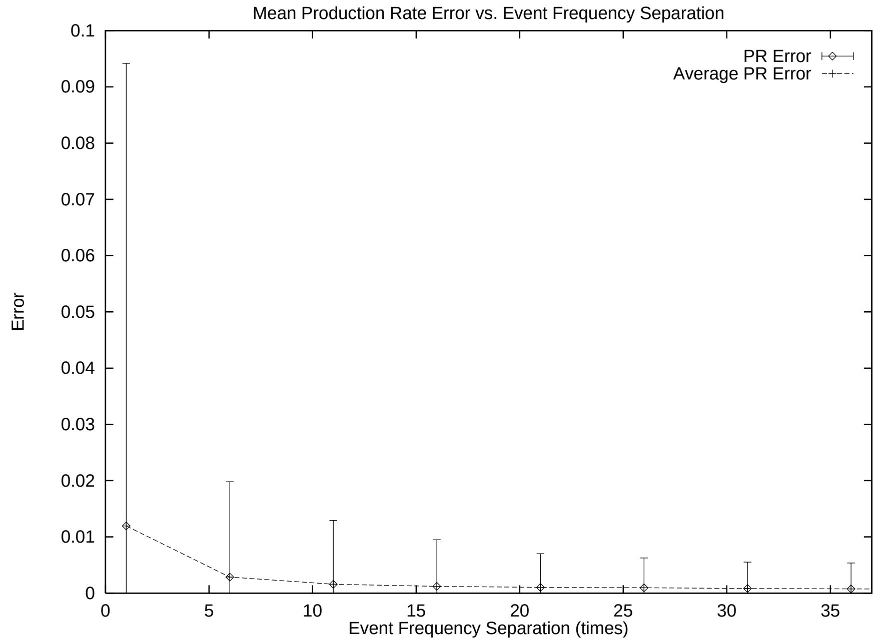

Figure 3-6: Evaluation of the performance of the simplification method for a two frequency-two machine line. Ranges and mean of production rate error $\mathcal{E}_P$ for different event frequency separations.

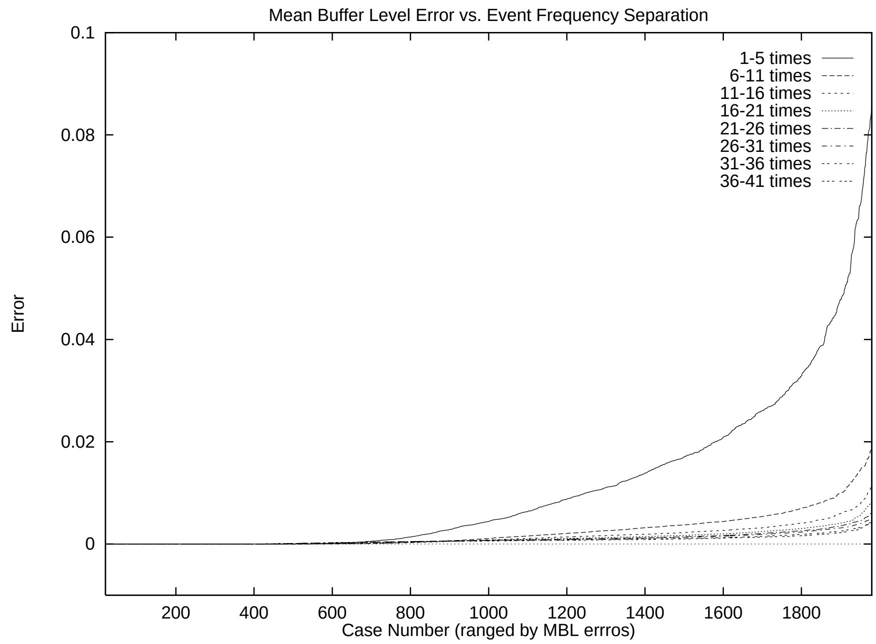

Figure 3-7: Evaluation of the performance of the simplification method for a two frequency-two machine line. Production rate error $\mathcal{E}_{\overline{x}}$ for different event frequency separations.

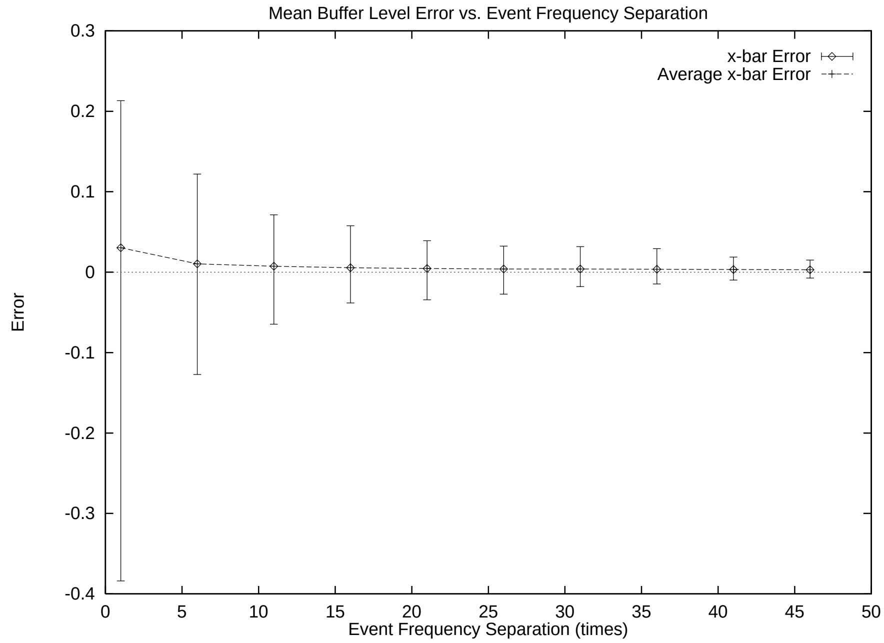

Figure 3-8: Evaluation of the performance of the simplification method for a two frequency-two machine line. Mean and range of buffer level error $\mathcal{E}_{\overline{x}}$ for different event frequency separations.

good approximation for the production rate.

Mean Buffer Level Errors $\mathcal{E}_{\overline{x}}$ and $\mathcal{E}_N$. Results for mean buffer errors are shown in Figures 3-8, 3-9 and 3-10. Figure 3-8 and 3-10 shows the range of the error as well as the its average magnitude for the various frequency separations. Figure 3-9 shows the errors for each of the cases ranked by the error size. We observe that the errors are smaller in magnitude for larger frequency differences.

For very small event frequency separations the mean buffer level error reaches 40% when measured relative to the actual mean buffer level $\overline{x}$ and 20% when measured relative to the buffer size N. The error decreases significantly as the frequency separation factor increases to be at least 5, with maximum $\mathcal{E}_{\overline{x}}$ and $\mathcal{E}_N$ of approximately 10%. A separation factor of at least 20 is needed for obtaining errors of less than 2% in magnitude. The average mean buffer level errors are close to or below 1% for all separations larger than 5.

Figure 3-9 shows the mean buffer errors $\mathcal{E}_{\overline{x}}$ for all the cases random cases sorted

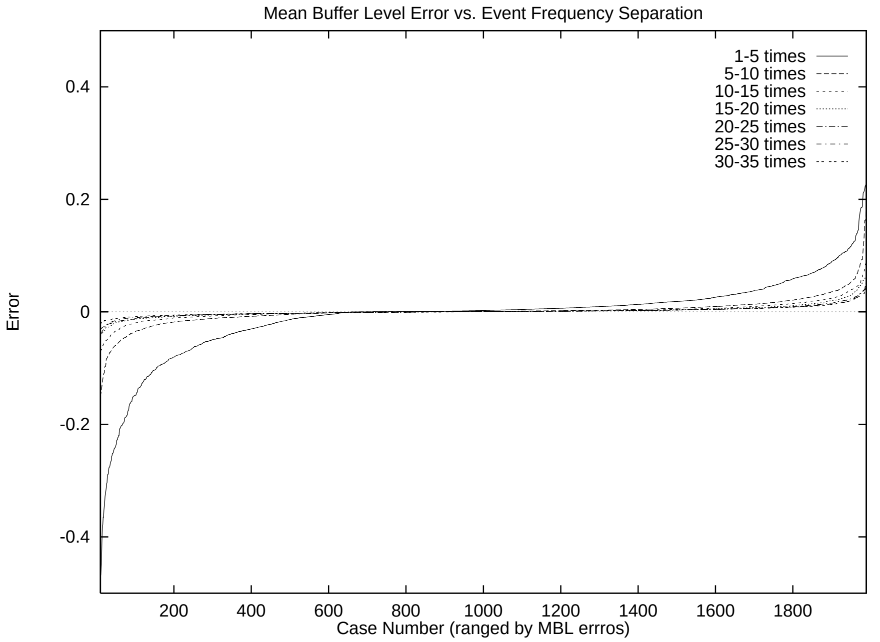

Figure 3-9: Evaluation of the performance of the simplification method for a two frequency-two machine line. Mean buffer level error $\mathcal{E}_{\overline{x}}$ for different event frequency separations.

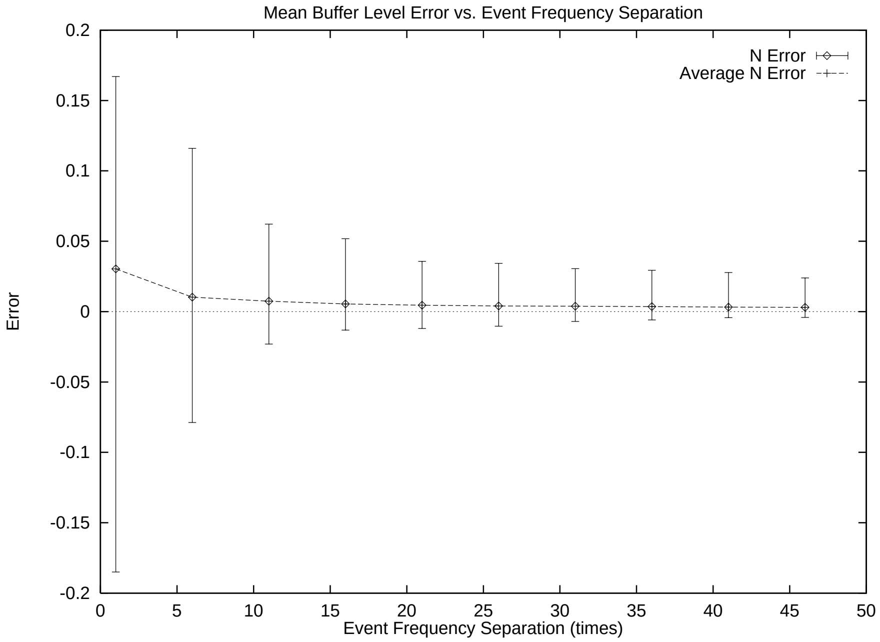

Figure 3-10: Evaluation of the performance of the simplification method for a two frequency-two machine line. Mean and range of buffer level error $\mathcal{E}_N$ for different time scale separations

by the size of the error. We observe that approximately 75% of the cases show errors below 2%.

These results show that the simplification method provides better approximations of the mean buffer levels as the event frequency gets larger and that on average the error is small. However, an accurate approximation can be guaranteed only for large frequency separations.

Remarks on the Accuracy of Representation Method We have tested how close the long term behavior of the original two frequency-two machine system is imitated by the simplified system in terms of how some their performance measures differ. We have calculated these differences for a large number of cases and examine how large they are.

However, choosing what measures of error have to be used and determining what error sizes are acceptable depend on how the method is applied. While the results of the previous sections allow us to conclude that the method is adequate if only a measure of the long term throughput of the line is required, its adequacy in providing estimates for mean buffer level does depend on how the method will be used.

In applications where a very close estimate of $\overline{x}$ is needed or where repeated applications of the method are required and errors may build up, a better approximation may be needed, especially if the frequency separation is small. On the other hand, a 10% or 5% error will not change qualitative information such as the fact that the buffer is nearly empty or full. Moreover, the measure of performance to be used may change according to what information is required; for instance, measuring the mean buffer level error relative to the buffer size ($\mathcal{E}_N$) or relative to the actual value ($\mathcal{E}_{\overline{x}}$).

Since we are interested in providing a tool for a hierarchical decomposition of a transfer line and this decomposition may require several applications of a simplification method, we need to obtain a better estimate of the mean buffer levels. That is, we need to develop a mechanism to reduce the errors of the method in estimating

$\overline{x}$. In Section 3.5 we compare the buffer level trajectories of the original and the simplified system and in Section 3.5.1 we use the differences between the trajectories to explain the mean buffer level errors and to develop a correction method.

3.5 Qualitative Differences in Buffer Level Trajectory.

This section examines how the buffer level trajectory of the original system and that of the simplified system differ.

In the model we are analyzing in this section, machine M_1 events are much more frequent that M_2 events. We have argued that the time scale at which events occur at M_2 is so large with respect to that of M_1 that M_2 can be regarded as receiving material at a steady rate equal to the isolated mean production rate of M_1, ρ_1. That is, between any two M_2 events, machine M_1's production rate approaches its long term average production rate ρ_1. Results in Section 3.4 sustain this argument.

However, we have seen that mean buffer levels differ between the original and the simplified system even though the throughput of both systems is very similar. This phenomenon can be explained by the fact that the mean buffer level depends on the buffer level dynamics and that this depends strongly on the failure-repair behavior of both the upstream and the downstream machines. Figure 3-11 compares the trajectories of two lines working at the same mean rate ρ, but with different failure and repair rates. The figure shows the buffer level trajectory when the second machine is not operating. The flow through the system is the same and the mean buffer level is the same, but the buffer trajectories and the mean buffer level differ.

In the case of the simplification method we have two different trajectories. The first corresponds to the original system where the buffer level increases at the maximum rate μ_1 when M_1 is up and stays unchanged during M_1 failures. The trajectory for the simplified system is a constant increase in the buffer level at the maximum

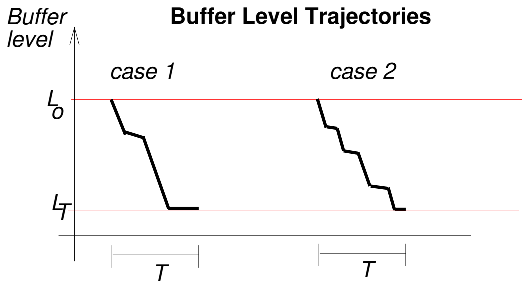

Figure 3-11: Differences in Buffer Level Trajectory. Each curve represents the buffer level trajectory when the buffer is starts full and the second machine is down. L_0 denotes the buffer level at the beginning of the interval and L_T the level at the end of the interval. The total incoming flow is the same in both cases and so is the mean incoming flow during a period of length T. However, since M_1failures and repair behavior differ, the buffer trajectories differ.

rate of the simplified first machine $\hat{\mu}_1$, which is the same as the mean processing rate of the original machine, ρ_1. We observe that, even though the simplified first machine has been selected to have the same mean processing rate, we can expect differences in the buffer trajectories and consequently in the mean buffer level.

Figure 3-12 shows the buffer level trajectories for an original two-frequency, two-machine line and its corresponding simplified two-machine line. This trajectories were obtained using computer simulation[1] and show the the buffer level for 2000 units of time. We observe that the buffer level trajectory for the original system presents intervals of fast depletion, corresponding to failures M_1 followed by periods of slower depletion when M_1 is up. For the simplified system we see that while M_2 is up, the depletion rate is constant and less than the faster depletion rate of the original system; this is due to the fact that the first machine processes at a reduced maximum processing speed $\hat{\mu}_1$ and never fails.

Figure 3-11 also shows how that the difference between the actual buffer trajectory and the constant rate trajectory corresponding to a reliable machine is smaller when the unreliable machine has more frequent events (case 2). This coincides with the results in Section 3.4 where we found that the method better estimates the buffer

[1]We used a continuous time continuous material simulator. See Appendix A.

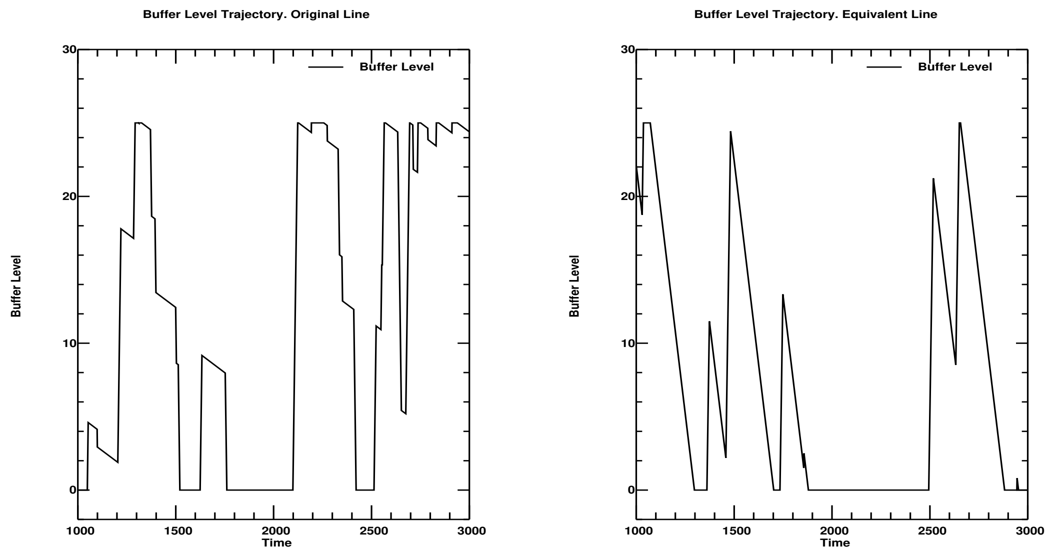

Figure 3-12: Buffer Level Trajectory. The plots show the buffer level trajectory for a two-machine line and a corresponding simplified two-machine line. The parameters of the line are: $r_1 = 0.1, p_1 = 0.01, \mu_1 = 1.0, r_2 = 0.01, p_2 = 0.001, \mu_2 = 1.01, N = 25$.

level as the event frequency difference increases, that is, when the events of M_1 are more frequent.

In the following section we use the observations on buffer level trajectories of this section to find an approximate characterization of the trajectory differences.

3.5.1 Characterization of Buffer Trajectory Differences

In this section we construct an approximation of the difference in buffer trajectories between the original and the simplified system based in the observations in the previous section.

Before we present this approximate characterization of buffer differences, we point out that it is not the purpose of this research to obtain the exact description of the dynamics of a continuous material-two machine line. This detailed description exists in the literature (Gershwin 1994) and solution methods to obtain the performance measures are available (Dallery 1988, Burman 1995).

The purpose of this section is rather to estimate the difference in the buffer trajectories between the original and the simplified system. This will enable us to develop estimates of mean buffer level from the analysis of the simplified system and the parameters of the original system, without having to analyze the original system.

To characterize the difference in buffer level trajectory, we analyze a *representative* buffer level trajectory for both the original and the modified system and estimate the *expected* difference in mean buffer level.

The difference in the mean buffer level corresponds to the difference in the areas below the corresponding buffer level trajectories. Therefore, one way of estimating the error in mean buffer level due to the simplification method is to obtain an analytical characterization of the buffer trajectories, obtain the corresponding areas and observe the differences.

If we let:

$$\begin{aligned} \mathcal{A} &= \text{Area below the buffer trajectory of the original system,} \\ \widehat{\mathcal{A}} &= \text{Area below the buffer trajectory of the simplified system,} \end{aligned}$$

the difference in mean buffer level can be expressed in terms of areas below buffer trajectories as:

$$\mathcal{E}_{\overline{x}} = \mathcal{E}_{\mathcal{A}} = \frac{\widehat{\mathcal{A}} - \mathcal{A}}{\mathcal{A}} \tag{3.5}$$

We restrict the areas $\widehat{\mathcal{A}}$ and $\mathcal{A}$ to be the average areas below the buffer trajectory between two failures of the lower event frequency machine M_2. We define the mean time to empty the buffer when it starts full as

$$MTEB = \frac{N}{(\mu_2 - \hat{\mu}_1)}$$

We can then obtain an approximate expression for the area below the mean buffer level trajectory for the simplified system $\widehat{\mathcal{A}}$. This expression uses the mean buffer size

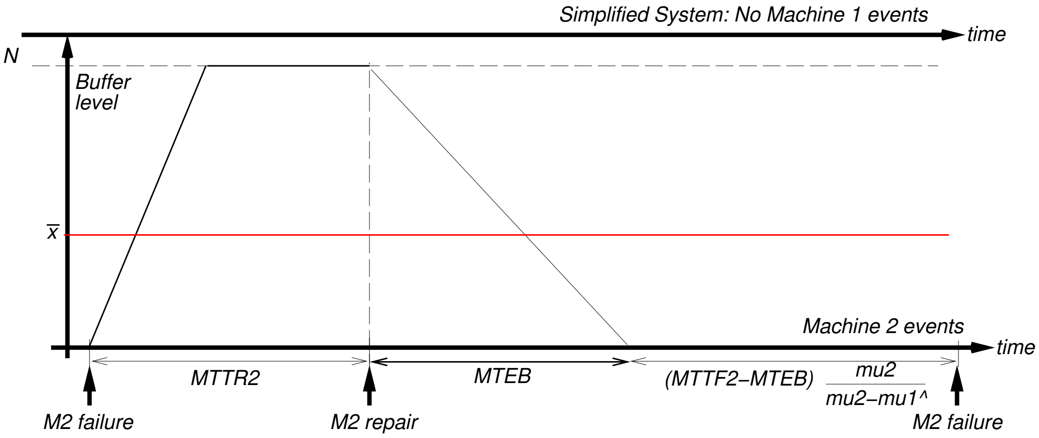

Figure 3-13: Buffer Level Trajectory for the Simplified System. We observe the change in buffer level due to events of machine M_2.

$\hat{\overline{x}}$ and the mean times to fail and repair of the second machine:

$$\hat{\mathcal{A}} = \begin{cases} \hat{\overline{x}} \left(MTTR_2 + MTEB + (MTTF_2 - MTEB)(\frac{\mu_2}{\mu_2 - \hat{\mu}_1}) \right) & \text{if } \hat{\mu}_1 > \mu_2 \\ \hat{\overline{x}} \left(MTTR_2 + MTTF_2 \right) & \text{otherwise} \end{cases} \tag{3.6}$$

Expression (3.6) takes into account the fact that when the buffer is empty and ρ_1 is less than μ_2, the actual failure rate of machine M_2 is a fraction of p_2 equal to the fraction of the maximum speed at which M_2 is operating. Equation (3.6) is an approximation of the actual area, since we assume that the buffer always starts full and is always depleted before M_2 fails.

We want to obtain an approximation of the difference between the areas below the buffer trajectory. Figure 3-13 shows a representative buffer trajectory, based on mean times to fail and to repair. The trajectory in the figure corresponds to the case in which M_2 starts to operate when the buffer is full and does not fail before it is emptied.

Having an approximation for the buffer level trajectory for the simplified system we can now approximately quantify the difference between this this trajectory and the one for the original system. Figure 3-14 shows a *representative* buffer level trajectory for a two-frequency two-machine transfer line as well as the trajectory of its simplified

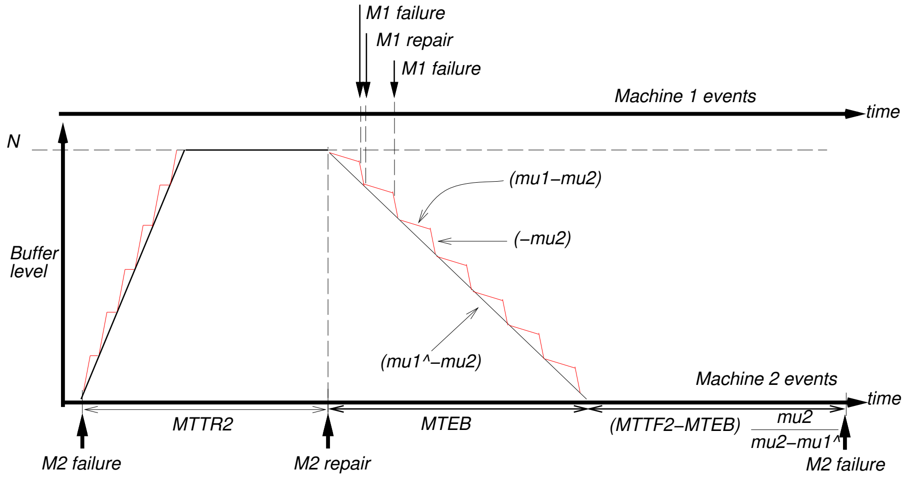

Figure 3-14: Buffer Level Trajectories. We observe the change in buffer level due to events of M_1 between any two consecutive events of M_1. The rate at which the buffer fills and empties is the same for both system but the level path differs.

system. The trajectory shown considers only mean times to fail and to repair for both machines. We observe a stepwise depletion of the buffer for the original case, as the one described in the previous section and shown in Figure 3-11. The actual size and form of the steps depends on the actual line parameters; the trajectory shown corresponds to those cases where $\mu_1 < \mu_2$ and $\rho_1 < \rho_2$.

We can obtain an approximation of the difference in buffer trajectories based in the mean times to fail and to repair of the machines. We will construct such an approximation of the buffer trajectory differences for the case in which $\mu_1 < \mu_2$. Then, we calculate the difference in the areas below the buffer trajectories and use this difference as an estimate of the error in mean buffer level incurred by the simplified system, $\mathcal{E}_{\overline{x}}$.

Characterization of the Buffer Trajectory when $\mu_1 < \mu_2$ and $\rho_1 < \rho_2$. We focus on the subset of two-frequency, two machine systems where the maximum processing rate of the first machine is less than that of the second, and where the mean isolated processing speed of the first machine is less than that of the second machine, that is:

$$\mu_1 < \mu_2, \quad \rho_1 < \rho_2 \tag{3.7}$$

In the simplified system, when M_2 is up, the buffer level decreases at a rate equal to $(\mu_2 - \hat{\mu}_1)$ until the buffer completely empties. When M_2 is down, it increases at $\hat{M}_1$'s processing rate $\hat{\mu}_1$.

For the original line, the buffer fills and empties at the same average rates, but the actual filling and depletion rates change according to the state of M_1. While M_2 is down, machine M_1 fails and gets repaired several times: the buffer fills at the processing rate of M_1 when M_1 is operating and remains unchanged while M_1 is waiting to be repaired. After M_2 gets repaired and before it fails again, M_1 fails and gets repaired several times: the buffer empties at a rate equal to $\mu_2 - \mu_1$, when M_1 is operating and fills at the processing rate of M_2, μ_2 when M_1 is down. This behavior is illustrated in Figure 3-14.

We can express the mean buffer level error $\mathcal{E}_{\overline{x}}$ in terms of the differences in the areas below the buffer level trajectories, as in equation (3.5) and we can approximate the differences between the areas taking into account the representative behavior described above.

If the buffer fills during an M_2 failure and empties when M_2 is repaired, as is the case in Figure 3-14, the difference between the areas corresponds to the area between the steps of the original machine trajectory and the straight line of the simplified trajectory. We denote this difference as $\Delta_{\mathcal{A}}$ and, using mean values, it can be expressed as:

$$\Delta_{\mathcal{A}} = \underbrace{N(MTTF_1)}_{\Delta \text{ filling buffer}} + \underbrace{MTTF_1(1 - \frac{\mu_2 - \mu_1}{\mu_2 - \hat{\mu}_1})\frac{N}{2} + \frac{N}{2r_1}}_{\Delta \text{ emptying buffer}} \tag{3.8}$$

$\Delta_{\mathcal{A}}$ approximates the difference between the areas when the buffer completely fills after an M_2 failure and empties after M_2 gets repaired. Even though, on average, the buffer completely fills, the buffer will not be completely filled or emptied during every failure-repair cycle of machine M_2. In these cases, the actual difference between the areas will be smaller.

To avoid overestimation of the actual difference in the area below the buffer trajectories, we calculate the probability that buffer completely fills and empties between two consecutive M_2 failures, P_L. If we define TTR_2 as the time to repair machine M_2, TTF_2 as the time for a failure of machine M_2, TFB as the time required to fill the buffer when it is empty and TEB as the time required to empty the buffer when it is full, we obtain the following expression for P_L:

$$\begin{aligned} P_L &= Prob(\text{Buffer Fills and Empties in a cycle}) \\ &= Prob(TTR_2 > TFB \ \text{ and } \ TTF_2 > TEB) \\ &= Prob(TTR_2 > \frac{N}{\hat{\mu}_1})Prob(TTF_2 > \frac{N}{\mu_2 - \hat{\mu}_1}) \\ &= e^{-r_2 \frac{N}{\hat{\mu}_1}} e^{-p_2 \frac{N}{\mu 2 - \hat{\mu}_1}} \end{aligned} \tag{3.9}$$

Weighting $\Delta_{\mathcal{A}}$ by the probability P_L, we obtain the following approximation for the error in the area below the buffer level trajectory $\mathcal{E}_{\mathcal{A}}$:

$$\mathcal{E}_{\mathcal{A}} = P_L(\Delta_{\mathcal{A}}) + (1 - P_L)\frac{(\Delta_{\mathcal{A}})}{4} \tag{3.10}$$

In order to maintain the simplicity of the approximation we decided to use a fixed fraction of the differences for the set of cases where the buffer does not completely fill or empty. The fraction used corresponds to the one that allowed the best prediction of the actual difference in area, and consequently in mean buffer level. Other expressions can be constructed that more closely represent the different sample buffer trajectories but with added complexity.

Since we claim that the area error $\mathcal{E}_{\mathcal{A}}$ equals the mean buffer level error $\mathcal{E}_{\overline{x}}$ we can use this approximation to correct the estimates of mean buffer level provided by the simplified method, $\overline{x}$. We obtain *corrected* values of $\overline{x}$ that we can denote as $\overline{x}_c$ and are defined as follows:

$$\overline{x}_c \approx \overline{x}(1 + \mathcal{E}_{\mathcal{A}}) \tag{3.11}$$

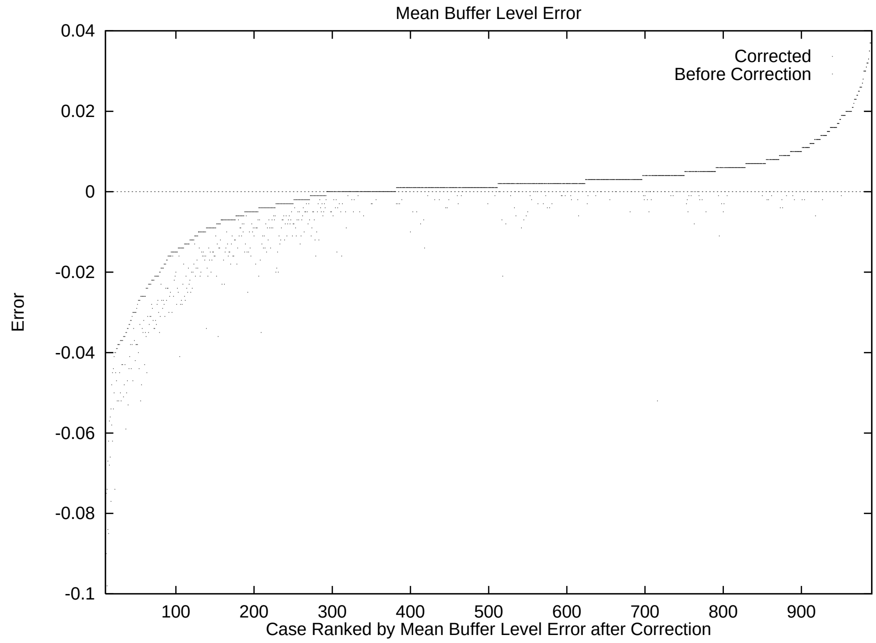

Figure 3-15: $\overline{x}$ errors, $\mathcal{E}_{\overline{x}}$, before and after applying the buffer level correction.

The use of the area error to correct the estimates of the method leads to the two-step method that we present in Section 3.6.

Numerical evaluation of the correction To examine how well expression (3.10) for $\mathcal{E}_{\mathcal{A}}$ estimates the mean buffer error, we generated a large number of random two-frequency, two-machine cases, applied the simplification method, calculated the error $\mathcal{E}_{\overline{x}}$ and compared it to $\mathcal{E}_{\mathcal{A}}$. We first generated 2000 cases following the same rules as in Section 3.3 when we examined the performance of the simplification method.

Figure 3-15 shows the mean buffer level errors due to the simplification, $\mathcal{E}_{\overline{x}}$, for each of the cases and its corresponding error after correction. We observe that the correction in general reduces the size of the errors obtained after the simplification method. In approximately 9% of the cases, the error estimated by $\mathcal{E}_{\mathcal{A}}$ exceeds the actual error, leading to the positive errors in the figure. However, only in 1% of the cases did the magnitude of the error increases after correction.

There is much variability in the performance of the correction: in some cases it predicts 100% of the error while other times it only accounts for 1% of the error.

On average, the correction seems to account for approximately 60% of the error and. more importantly, reduces the range of the magnitude of the error by nearly 30%.

We also examined the use of $\mathcal{E}_{\mathcal{A}}$ to predict the mean buffer level error for different event frequency separations. We used the same event frequency separation ranges as in Section 3.4 and applied the correction to the same cases generated in that section.

Table 2.4 summarizes the results for each of the ranges, comparing maximum and average errors before and after the correction.

Table 2.4. Performance of the Correction

	Errors Due to the Transformation Before and After Correction			
separation	Mean $\mathcal{E}_{\overline{x}}$ before	Mean $\mathcal{E}_{\overline{x}}$ after	Max $\lvert\mathcal{E}_{\overline{x}}\rvert$ before	Max $\lvert\mathcal{E}_{\overline{x}}\rvert$ after
> 5	0.0917	0.0609	0.1301	0.0583
1 to 6	0.1162	0.0432	0.3839	0.1798
6 to 11	0.0303	0.0124	0.1273	0.0533
11 to 16	0.0175	0.0072	0.0646	0.0345
16 to 21	0.0122	0.0059	0.0383	0.0211
21 to 26	0.0089	0.0043	0.0344	0.0141
26 to 31	0.0069	0.0031	0.0273	0.0134
31 to 36	0.0057	0.0023	0.0245	0.0132
36 to 41	0.0047	0.0021	0.0221	0.0102

Results in Table 2.4 show the performance of the correction for the different event frequency separations. We observe that in all cases the mean buffer error $\mathcal{E}_{\overline{x}}$ decreased in absolute value both for average value as well for the maximum error.

Is is important to notice that for event frequency separation factors larger than 5 corrected errors are bounded by 5% in absolute value, and they are always less than 1% for any separation factor larger than 10.

Although the correction presented here accounts for only a fraction than the actual error, we consider the effect of the correction to be significant, especially due to the fact that the magnitude of the error for the worst case has a smaller bound.

3.6 Simplification and Correction: Two-Step Representation Method

In Section 3.2 we introduce a method for the simplification of a two-frequency, two-machine transfer line into a single-frequency, two machine transfer line. We analyze the accuracy of the method in terms of its power to predict the mean production rate and the mean buffer level of the system and found the method to be very accurate in predicting the mean production rate but less so in predicting the mean buffer level.

The difference in predicted values for the mean buffer level led us to more closely examine how the simplification affects the buffer level dynamics. This analysis allowed us to develop an approximation for the expected difference between the areas below the buffer trajectories of the original and the modified lines.

We can combine the method described in Section 3.2 and the correction of mean buffer level errors based in area differences in a single two-step representation method. A two-frequency, two-machine transfer line can be represented at the higher level of the hierarchy as follows:

1. Obtain the equivalent single frequency-two machine line as in Section 3.2. Use the mean production rate of the simplified system, $\hat{P}$ to estimate the production rate of the original line, P.

2. Obtain the estimate of the error in buffer trajectories $\mathcal{E}_{\mathcal{A}}$, as an approximation to the mean buffer level error $\mathcal{E}_{\overline{x}}$. Use the corrected buffer level $\hat{x}_c$ (expression (3.11)) as the estimate for the mean buffer level of the original system, $\hat{x}$.

Example We provide an example of the application of the two-step method to a two frequency-two machine line. Table 2.5 indicates the parameters of the original line.

Table 2.5. Example: Original System						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	0.900	0.300	1.000	0.750	0.750	80
2	0.040	0.015	1.080	0.785	0.785	n/a

Once the simplification method is applied, we obtain the parameters in Table 2.6 for the simplified line.

Table 2.6. Example: Modified Line						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	1.000	0.000	0.750	1.000	0.750	80
2	0.040	0.015	1.080	0.727	0.785	n/a

Following step 1 in the two-step simplification method, we use the mean production rate of the simplified system, $\hat{P}$, as an estimate for P. For this line we have $P \approx \hat{P} = 0.7220$.

In step 2 of the method we calculate the approximate area error $\mathcal{E}_{\mathcal{A}}$. Applying formulae (3.8) to (3.10) we obtain $\Delta_{\mathcal{A}} = 234.34$ and $\mathcal{E}_{\mathcal{A}} = 0.0084$. We correct the buffer level obtained from the simplified system and correct it according to $\mathcal{E}_{\mathcal{A}}$:

$$\overline{x}_c = \overline{x}(1 + \mathcal{E}_{\mathcal{A}}) = 28.07(1 + 0.0084) = 28.32$$

Table 2.7 shows the performance measure corresponding to the original line, the simplification method and the two-step simplification method.

Table 2.7. Example: Performance Measures.			
	Line Performance Measures		
Model	Mean Prod. Rate	Mean Buffer Level	MBL After Correction
Original	0.7220	28.42	n/a
Simplified	0.7220	28.08	28.32
Error	0.0013	-0.0117	-0.0033

.

In the example we see how the correction method improved the estimate of the mean buffer level $\overline{x}$, decreasing the error $\mathcal{E}_{\overline{x}}$ from -1.2% to -0.33%.

Chapter 4

Hierarchical Decomposition Method for Long Continuous Material Transfer Lines

In this chapter we extend the frequency decomposition method developed in Chapter 3 for two-frequency, two-machine transfer lines to multiple frequency-k machine transfer lines.

A k-machine transfer line can be hierarchically decomposed in as many as k different event frequency levels. Each level corresponds to a set of machines $\mathcal{M}_l$ as defined in Section 2.1.3. At each level of the hierarchy only machines whose events occur at a frequency equal to or less than a given value are considered. In this section we show how the line can be represented at each of the hierarchical levels $\mathcal{M}_l$. We follow the same approach as the one used for representing a two frequency-two machine line as a single frequency line at the higher level of the hierarchy.

The simplification method is introduced in Section 4.1. The performance of the method is evaluated in terms of the errors in the estimation of the line performance measures. These measures of the accuracy of the method are introduced in Section 4.2. We provide examples of the application of the method and evaluate its performance for three-machine lines in Section 4.3. In Section 4.4 we present the results of the application of the method to longer multiple-frequency lines.

4.1 Multiple frequency transfer lines and the frequency decomposition method.

In this section we present a decomposition method for multiple-frequency, k-machine lines. We call a transfer line a *multiple-frequency* line when it satisfies the condition that failure and repair event occur at significantly different frequencies at different machines. As we explain in Section 2.1.3, these event frequency differences induce a hierarchical decomposition of the line. Each hierarchical level consists of a set of machines, as defined by expression (2.2):

$$\mathcal{M}_l = \{M_i | MTBF_i \ll min(MTTF_j, MTTR_j), M_j \in \mathcal{M}_{l-1}\}.$$

At level $\mathcal{M}_l$ of the hierarchy, only events of machines $M_i \in \mathcal{M}_l$ must be considered.

At each level $\mathcal{M}_l$ of the hierarchy, the line can be represented by a *simplified* line. This simplified line must model the actual line preserving the general operational characteristics of the line but ignoring higher frequency events.

The method we propose aggregates the higher frequency event rates with the processing rate of their corresponding machines into a single, reduced processing rate.

The conditions to be satisfied by the simplified line and the representation method can be formalized as follows:

Conditions for the simplified line. At each level l of the hierarchy a simplified k-machine line is constructed that satisfies the following conditions:

1. **Event Frequency**: all machine events occur at an equal or lesser rate that those of machines in level l of the hierarchy:

$$\hat{M}_i \text{ satisfies } \widehat{MTBF}_i \geq min(MTTF_j, MTTR_j), M_j \in \mathcal{M}_{l-1} \tag{4.1}$$

2. **Production Rate**: The processing rate of all machines must remain unchanged:

$$\hat{\rho}_i = \rho_i \quad i = 1, ..., k \tag{4.2}$$

Simplified transfer line for hierarchical level l. The simplified line for level l of the hierarchy is obtained as follows:

1. **Buffers**: Buffer sizes remain unchanged:

$$\hat{N}_i = N_i$$

2. **Machines** $M_i \in \mathcal{M}_l$: Those machines whose events occur at the frequency of level l or at a smaller rate, remain unchanged:

$$\hat{p}_i = p_i \ , \ \ \hat{r}_i = r_i \ \text{ and } \ \hat{\mu}_i = \mu_i$$

3. **Machine** $M_i \notin \mathcal{M}_l$: Machines whose events occur with a higher frequency are replaced by perfectly reliable machines with reduced processing rates:

$$\begin{aligned} \hat{p}_i &= 0 \\ \hat{r}_i &= c, \qquad c > 0 \\ \hat{\mu}_i &= \rho_1 \end{aligned}$$

The repair rate $\hat{r}_1$ can be set to any positive number, to allow the efficiency of the first machine, $\hat{e}$, to be 1. We choose $c = 1$ in the remaining discussion and examples.

Conditions (4.1) and (4.2) are satisfied at each hierarchical level by this simplification. Condition (4.1) is automatically satisfied by machines in level l (i.e., $M_i \in \mathcal{M}_l$) since they already belong to level l of the hierarchy. The simplified machines that replace those that originally were not in the set $\mathcal{M}_l$ satisfy this condition because they experience no events. As a consequence all machines in the simplified line operate with an event frequency less than or equal to the machines originally in level l.

Condition (4.2) is satisfied since the mean processing rate of the machine remains unchanged ($\hat{\rho}_i = \rho_i$).

4.2 Measures of the performance of the method

To evaluate the power of the simplified system in estimating the mean production rate and mean buffer levels of the original line, we define five measures of error. Three of them are global measures for the production rate and mean total inventory, and the last two are measures for the individual buffers levels. With P, $\hat{P}$, $\overline{x}_i$ and $\hat{\overline{x}}_i$ defined as in Section 3.3 and $\overline{X}$ and N_{total} defined as follows:

$$\begin{aligned} \overline{X} &= \sum_{i=1}^{k} \overline{x}_i \\ N_{total} &= \sum_{i=1}^{k} N_i \end{aligned}$$

we define the following error measures:

- Global error measures
  - Production Rate Error:
$$\mathcal{E}_P = \frac{\hat{P} - P}{P}$$
  - Total Mean Buffer Level Error, with respect to actual total buffer level $\overline{X}$:
$$\mathcal{E}_{\overline{X}} = \frac{\sum_{i=1}^{h} (\widehat{\overline{x}}_i - \overline{x}_i)}{\overline{X}}$$

– Total Mean Buffer Level Error, with respect to total buffer space N_{total}:

$$\mathcal{E}_{N_{total}} = \frac{\sum_1^h (\widehat{\overline{x}} - \overline{x})}{N_{total}}$$

- Individual buffer level measures.

  – Mean Buffer Level Error with respect to actual buffer level $\overline{x}_i$:

$$\mathcal{E}_{\overline{x}_i} = \frac{\widehat{\overline{x}}_i - \overline{x}_i}{\overline{x}_i}$$

  – Mean Buffer Level Error, with respect to buffer size N_i:

$$\mathcal{E}_{N_i} = \frac{\widehat{\overline{x}}_i - \overline{x}_i}{N_i}$$

4.3 Three-Machine Continuous Material Transfer Lines

In this section we apply the simplification method to multiple frequency-three machine lines. Three-machine lines can be decomposed in two or three hierarchical levels, according to the frequencies of their machines. In Section 4.3.1 we examine the case in which one machine has much higher frequency events that the other two machines, build the corresponding two-level hierarchy, apply the simplification method at the higher level of the hierarchy and evaluate the accuracy of the performance measure estimates. In Section 4.3.2 we focus on three frequency-three machine lines, build the three-level hierarchy, apply the simplification method at each of the higher levels and evaluate the accuracy of the performance measures.

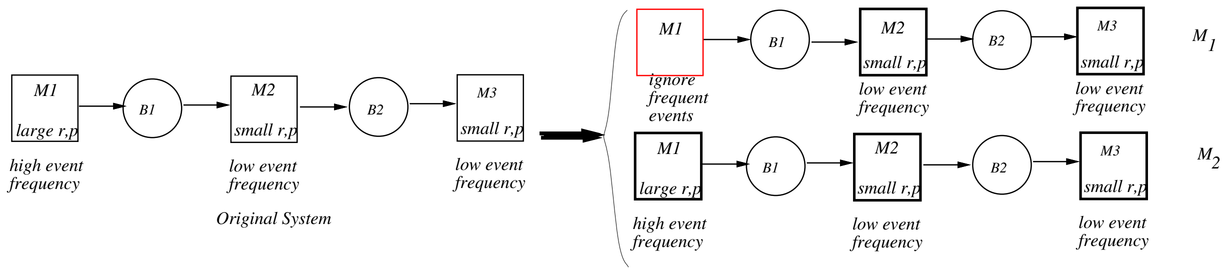

Figure 4-1: Schematic representation of the two level hierarchy for the two frequency-three machine line.

4.3.1 Two frequency-three machine lines.

In this section we consider three-machine lines where one of the machines operates with failure-repair cycles much more frequent that the events of the two other machines. More specifically, we focus on lines where the first machine, M_1, has much frequent events than the second and third machine M_2 and M_3.

Two-Level Hierarchy. The event frequency separation between the first machine, M_1, and its downstream machines M_2 and M_3, induces a *two-level hierarchy.* At the higher level (lower detail) only lower frequency events are taken into account; at the lower level (higher detail) all machines events are considered, that is, the original line is considered.

According to the event frequency hierarchy definition in Section 2.1.3, we have two hierarchical sets of machines $\mathcal{M}_1$ and $\mathcal{M}_2$ (see Figure 4-1):

$$\begin{aligned}\mathcal{M}_1 &= \{M_2, M_3\}\\ \mathcal{M}_2 &= \{M_1, M_2, M_3\}\end{aligned}$$

Level 2 line representation. At level 2 no simplification of the line is required, since events of all machines are considered.

Level 1 line representation. At level 1 we ignore the events of machine M_1. The representation method constructs a simplified line replacing M_1 by a perfectly reliable machine with reduced rate. The parameters of the resulting simplified line are obtained as follows:

- **Buffer Sizes:** $\hat{N}_1 = N_1, \ \ \hat{N}_2 = N_2.$
- **Machines** M_2 **and** M_3: $\hat{p}_i = p_i, \ \ \hat{r}_i = r_i$ and $\hat{\mu}_i = \mu_i; \quad i = 2, 3.$
- **Machine** M_1: $\hat{p}_1 = 0 \ , \ \ \hat{r}_1 = 1 \ \ and \ \ \hat{\mu}_1 = \rho_1.$

We illustrate the representation method in Example 4.1.

Example 4.1. The parameters of a two-frequency, three-machine are shown in Table 4.1.

Table 4.1. Example 4.1: Original System

	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	0.1000	0.01000	1.0000	0.9091	0.9091	1100
2	0.0012	0.00012	1.0200	0.9091	0.9272	2200
3	0.0011	0.00011	1.0100	0.9091	0.9191	n/a

Replacing M_1 by a reliable machine as indicated by the representation method, we obtain a simplified line for hierarchy level 0. The parameters for the simplified line are shown in Table 4.2.

Table 4.2. Example 4.1: Level 0 Representation

	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	1.0000	0.00000	0.9091	1.0000	0.9091	1100
2	0.0012	0.00012	1.0200	0.9091	0.9272	2200
3	0.0011	0.00011	1.0100	0.9091	0.9191	n/a

In order to test the accuracy of the representation, we calculate the values of the performance measures P, $\overline{x}_i$ and $\overline{X}$, and calculate the corresponding errors $\mathcal{E}_P$, $E_{\overline{X}}$, E_{Ntotal}, $\mathcal{E}_{\overline{x}i}$ and $\mathcal{E}_{Ni}$. Table 4.3 summarizes the results:

Table 4.3. Example 4.1: Performance Measures.				
	Line Performance Measures			
Model	P	$\overline{x}_1$	$\overline{x}_2$	$\overline{X}$
Original	0.86405	405.48	883.14	1288.62
Simplified	0.86326	401.84	889.85	1290.85
Errors	$\mathcal{E}_P = -0.0009$	$\mathcal{E}_{\overline{x}1} = -0.0121$	$\mathcal{E}_{\overline{x}2} = -0.0076$	$\mathcal{E}_{\overline{X}} = 0.0016$
		$\mathcal{E}_{N1} = -0.0033$	$\mathcal{E}_{N2} = 0.0030$	$E_N = 0.0007$

The results in Table 4.3 show that the simplified line provides very close estimates of the performance measures of the original line. Only one error exceeds 1% in magnitude ($\mathcal{E}_{\overline{x}1}$).

In general, the accuracy of the performance measures estimates will differ for each particular system. To examine the general performance of the representation method we follow the same approach as for the two frequency-two machine case in Section 3.3. We generate random two-frequency, two-machine lines, obtain their simplified representations for level 0 of the hierarchy and calculate the errors.

Performance of the Representation Method for Two-Frequency, Three-machine lines. We randomly generated 100 two-frequency, three-machine cases, characterized as follows:

- Failure and repair rates p_i and r_i are uniformly distributed from 0.001 to 1.0, for $i = 1, 2, 3$.
- Machine processing rates μ_i are uniformly distributed from 0.1 to 10.0, for $i = 1, 2, 3$.
- Individual mean processing rates ρ_i are within 20% of each other, for $i = 1, 2, 3$.
- The buffer size N_i is chosen to be larger than one half and smaller than five times the amount of material produced during by M_i during the average duration of a M_{i+1} failure.

$$N_i \in (0.5\frac{\rho_i}{r_{i+1}}, 5\frac{\rho_i}{r_{i+1}})$$

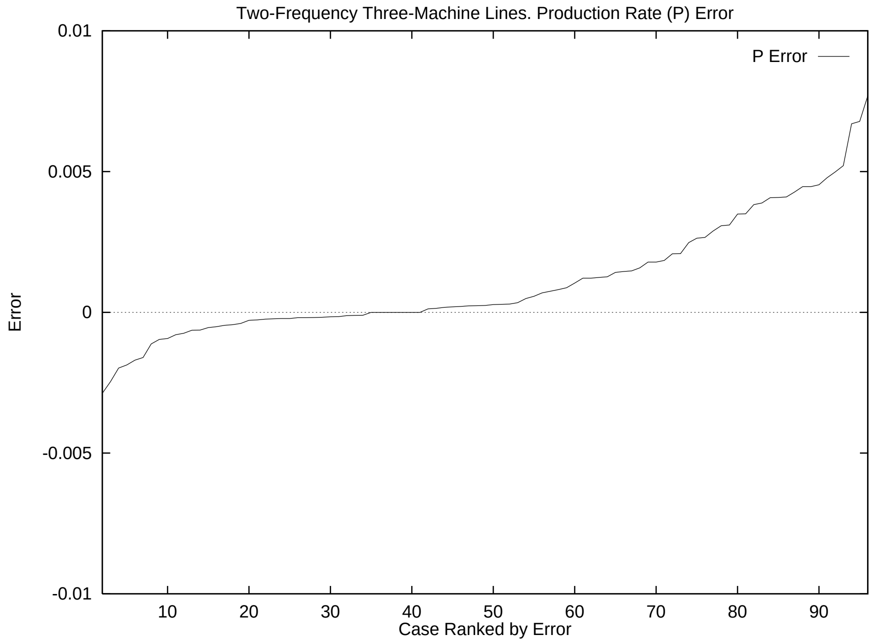

Figure 4-2: Evaluation of the performance of the simplification method for a two frequency-three machine line. Production rate error $\mathcal{E}_P$ for 100 randomly generated lines.

- The event frequency separation condition (3.1) is implemented as

$$MTBF_1 < \frac{1}{5}\, min(MTTF_i, MTTR_i) \;\; i = 2, 3 \tag{4.3}$$

Using simulations[1] we obtained the performance measures for each system and its corresponding simplified line, and calculated the error measures.

Figures 4-2, 4-3 and 4-4 show the errors in the prediction of the mean production rate P, the individual buffer levels $\overline{x}_i$ and the total in process inventory $\overline{X}$.

We observe that the production rate error $\mathcal{E}_P$ is always very small, below 1%. This coincides with the findings in Chapter 3 on the accuracy of the representation method for estimating P for two-machine lines to two frequency-three machine lines.

The individual mean buffer levels and the mean total buffer level estimates show maximum errors of approximately 6% when measured with respect to the actual mean

[1]See appendix A for details.

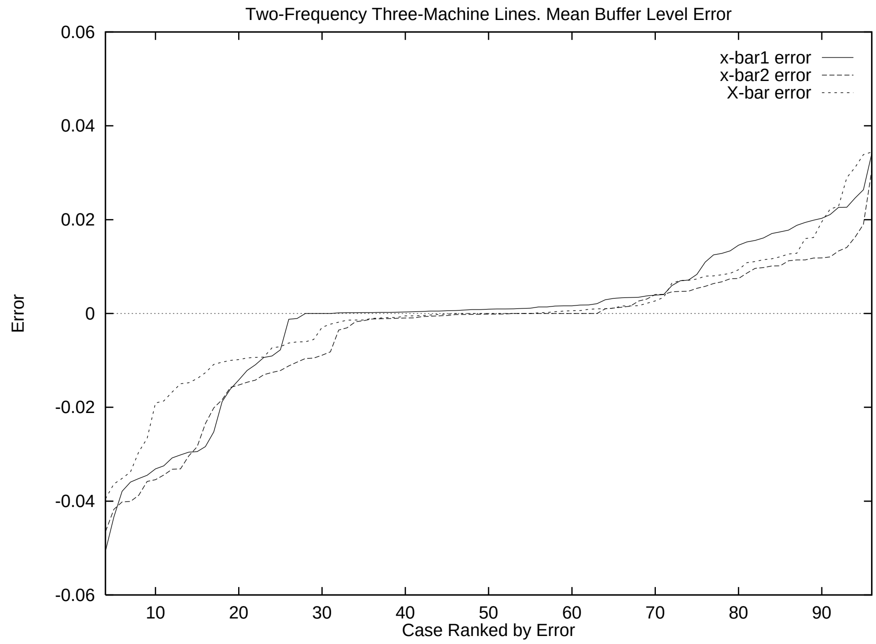

Figure 4-3: Evaluation of the performance of the simplification method for a two frequency-three machine line. Mean buffer errors $\mathcal{E}_{\overline{x}1}$ and $\mathcal{E}_{\overline{x}2}$ and mean total buffer level error $\mathcal{E}_{\overline{X}}$ for for 100 randomly generated lines.

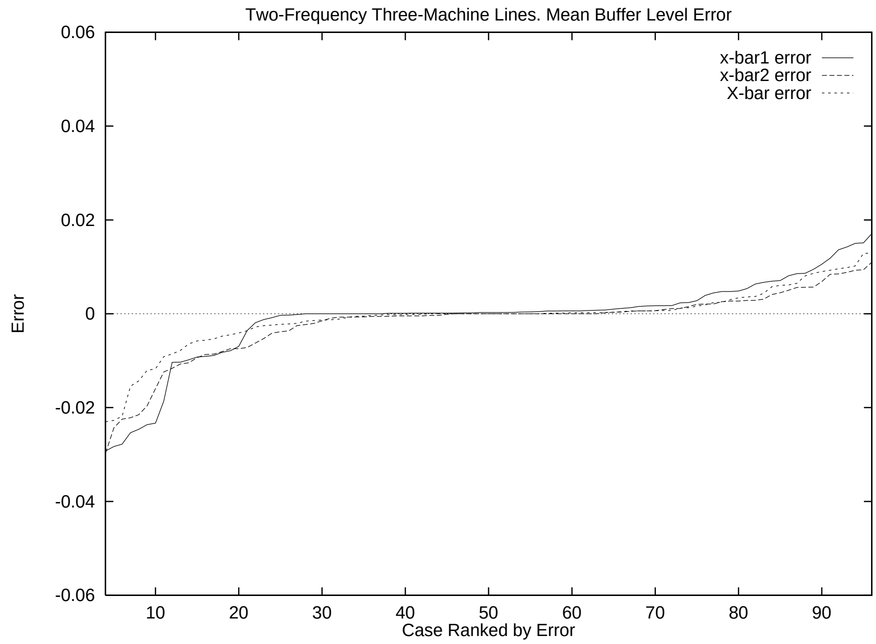

Figure 4-4: Evaluation of the performance of the simplification method for a two frequency-three machine line. Mean buffer errors $\mathcal{E}_{N1}$ and $\mathcal{E}_{N2}$ and mean total buffer level error $\mathcal{E}_N$ for for 100 randomly generated lines.

buffer levels $\overline{x}_i$ and $\overline{X}$, and of approximately 3% when measured with respect to buffer sizes (N_i and N_{total}). These results also coincide with our findings in the two-machine line case. We expect the mean buffer level errors to decrease as larger event frequency separations are imposed between M_1 and the downstream machines.

4.3.2 Three-frequency, three-machine lines.

In this section we apply and evaluate the simplification method for three-machine lines where each of the machines operates with a different failure-repair frequency. More specifically, we focus on lines where the first machine, M_1, has much more frequent events than the second, M_2, and M_2 has much more frequent event than the third, M_3.

Three-Level Hierarchy. The event frequency separation between the machines induces a *three-level hierarchy.* At the higher level (lower detail) only lower frequency events are taken into account; at the middle level only medium and low frequency event are taken into account and, finally, at the lower level (higher detail) all three machines events are considered, that is, at this level the original line is considered.

According to the event frequency hierarchy definition in Section 2.1.3, we have three hierarchical levels $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$ (see Figure 4-5):

$$\begin{aligned}
\mathcal{M}_1 &= \{M_3\} \\
\mathcal{M}_2 &= \{M_2, M_3\} \\
\mathcal{M}_3 &= \{M_1, M_2, M_3\}
\end{aligned}$$

Level 3 line representation. At level 3 no simplification of the line is required, since events of all machines are considered.

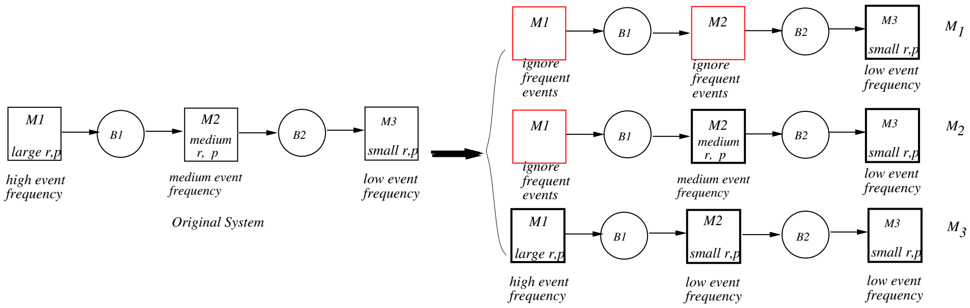

Figure 4-5: Schematic representation of the three level hierarchy for the three frequency-three machine line.

Level 2 **line representation.** At level 2 the representation method ignores the events of machine M_1, replacing it by a reliable machine. The simplified line for this case is of the form:

- **Buffers Sizes:** $\hat{N}_1 = N_1, \quad \hat{N}_2 = N_2$.
- **Machines** M_2 **and** M_3: $\hat{p}_i = p_i, \quad \hat{r}_i = r_i$ and $\hat{\mu}_i = \mu_i; \quad i = 2, 3$.
- **Machine** M_1: $\hat{p}_1 = 0\ , \quad \hat{r}_1 = 1 \ \ \mathit{and} \ \ \hat{\mu}_1 = \rho_1$.

Level 1 **line representation.** At level 1 the representation method ignores the events of machines M_1 and M_2. M_1 and M_2 are replaced by reliable machines. The simplified line for this level is of the form:

- **Buffers** Buffer sizes: $\hat{N}_1 = N_1, \quad \hat{N}_2 = N_2$.
- **Machine** M_3: $\hat{p}_3 = p_3, \quad \hat{r}_3 = r_3 \ \ \mathit{and} \ \ \hat{\mu}_3 = \mu_3$;
- **Machines** M_1 **and** M_2: $\hat{p}_i = 0\ , \quad \hat{r}_i = 1 \ \ \mathit{and} \ \ \hat{\mu}_i = \rho_i\ ; \quad i = 1, 2$.

We illustrate the representation method in Example 4.2.

Example 4.2. The parameters of a three-frequency three-machine are shown in Table 4.4.

Table 4.4. Example 4.2: Original System						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	0.100000	0.010000	1.000000	0.9091	0.9091	1100
2	0.001200	0.000120	1.020000	0.9091	0.9273	13000
3	0.000040	0.000004	1.010000	0.9091	0.9192	n/a

Level 2 **representation** We replace M_2 by a reliable machine as indicated by the representation method. We obtain a simplified line for hierarchy level 1. The parameter for the simplified line are shown in Table 4.5

Table 4.5. Example 4.2: Level 1 Representation						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	1.00000	0.000000	0.9091	1.0000	0.9091	1100
2	0.00120	0.000120	1.0200	0.9091	0.9272	13000
3	0.00004	0.000004	1.0100	0.9091	0.9191	n/a

We calculate the values of the performance measures P, $\overline{x}_i$ and $\overline{X}$, and calculate the corresponding errors $\mathcal{E}_P$, $E_{\overline{X}}$, E_{Ntotal}, $\mathcal{E}_{\overline{x}i}$ and $\mathcal{E}_{Ni}$. Table 4.6 summarizes the results:

Table 4.6. Example 4.2: Level 1 Performance Measures.				
	Line Performance Measures			
Model	P	$\overline{x}_1$	$\overline{x}_2$	$\overline{X}$
Original	0.85756	654.93	5072.74	5727.67
Simplified	0.85679	648.82	5103.35	5752.18
Errors	$\mathcal{E}_P = -0.0009$	$\mathcal{E}_{\overline{x}1} = -0.0093$	$\mathcal{E}_{\overline{x}2} = -0.0060$	$\mathcal{E}_{\overline{X}} = 0.0042$
		$\mathcal{E}_{N1} = -0.0054$	$\mathcal{E}_{N2} = 0.0024$	$E_N = 0.0017$

The results in Table 4.6 are very similar to those for the two-frequency, three machine line in Example 4.1. Again, the simplified line provides very close estimates for the performance measures of the line. All errors are below 1%.

Level 1 **representation** We replace M_1 and M_2 by reliable machines. We obtain a simplified line for hierarchy level 1. The parameters for the simplified line are shown in Table 4.7.

Table 4.7. Example 4.2: Level 1 Representation						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	1.00000	0.000000	0.9091	1.0000	0.9091	1100
2	1.00000	0.000000	0.9072	1.0000	0.9272	2200
3	0.00004	0.000004	1.0100	0.9091	0.9192	n/a

The performance measures for the original and the simplified systems as well as the errors are summarized in Table 4.8:

Table 4.8. Example 4.2: Level 0 Performance Measures.				
	Line Performance Measures			
Model	P	$\overline{x}_1$	$\overline{x}_2$	$\overline{X}$
Original	0.85756	654.93	5072.74	5727.67
Simplified	0.86204	632.82	5180.35	5813.17
Errors	$\mathcal{E}_P = -0.0009$	$\mathcal{E}_{\overline{x}1} = -0.0149$	$\mathcal{E}_{\overline{x}2} = -0.0213$	$\mathcal{E}_{\overline{X}} = 0.0016$
		$\mathcal{E}_{N1} = -0.0020$	$\mathcal{E}_{N2} = 0.0083$	$E_N = 0.0061$

We observe in Table 4.8 that the simplified line for this level is a very good representation of the original line in terms of the mean production rate, which differs by less than 0.1%. The mean buffer level errors are clearly larger for level 0 representation than for level 1 representation (Table 4.6). This is to be expected due to the fact that a larger number of events are being ignored and the two first machines in the simplified line are reliable machines. Since only M_3 fails, the buffer between M_1 and M_2, B_1, accumulates material only when the effect of an M_3 failure propagates upstream. In general, we would expect the first mean buffer level $\overline{x}_1$ to be underestimated and the second, $\overline{x}_2$, to be overestimated.

Now we examine the performance of the representation at levels 1 and 0 of the hierarchy from a set of randomly generated three-frequency, three-machine lines.

Performance of the Representation Method for Three-Frequency, Three-Machine Lines. We randomly generated 100 two-frequency, three-machine cases. The cases are generated as those generated in the evaluation of the two-frequency, three machine case. The event frequency separation condition (3.1) is implemented

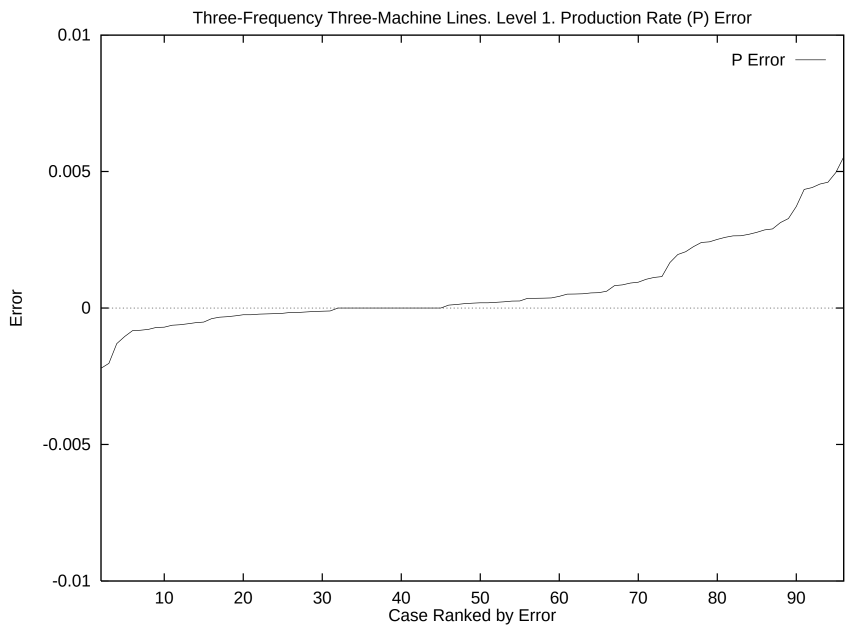

Figure 4-6: Evaluation of the performance of the simplification method for a two-frequency, three-machine line. Production rate error $\mathcal{E}_P$ for 100 randomly generated lines.

as

$$\begin{aligned} MTBF_1 &< \frac{1}{5}\, min(MTTF_2, MTTR_2) \\ MTBF_2 &< \frac{1}{5}\, min(MTTF_3, MTTR_3) \end{aligned} \tag{4.4}$$

Using simulations[2] we obtained the performance measures for each system and its corresponding simplified line and calculated the error measures.

Performance of the Level 1 Representation Figures 4-6, 4-8 and 4-7 show the errors in the prediction of the mean production rate P, the individual buffer levels $\overline{x}_i$ and the total in process inventory $\overline{X}$.

As in previous cases, the magnitude of the production rate error is always below 1%.

[2]See appendix A for details.

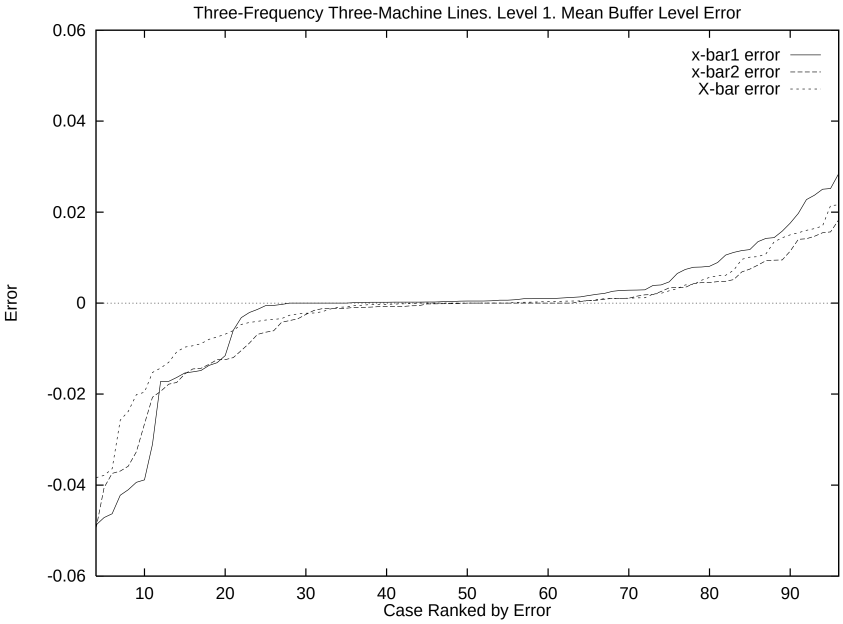

Figure 4-7: Evaluation of the performance of the simplification method for a two frequency-three machine line. Mean buffer errors $\mathcal{E}_{\overline{x}1}$ and $\mathcal{E}_{\overline{x}2}$ and mean total buffer level error $\mathcal{E}_{\overline{X}}$ for for 100 randomly generated lines.

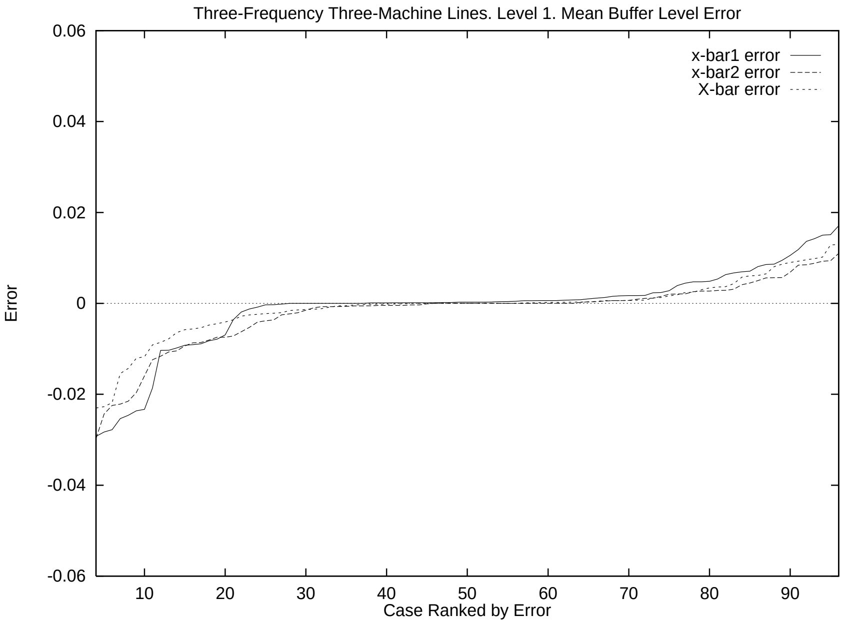

Figure 4-8: Evaluation of the performance of the simplification method for a two frequency-three machine line. Mean buffer errors $\mathcal{E}_{\overline{x}1}$ and $\mathcal{E}_{\overline{x}2}$ and mean total buffer level error $\mathcal{E}_{\overline{X}}$ for for 100 randomly generated lines.

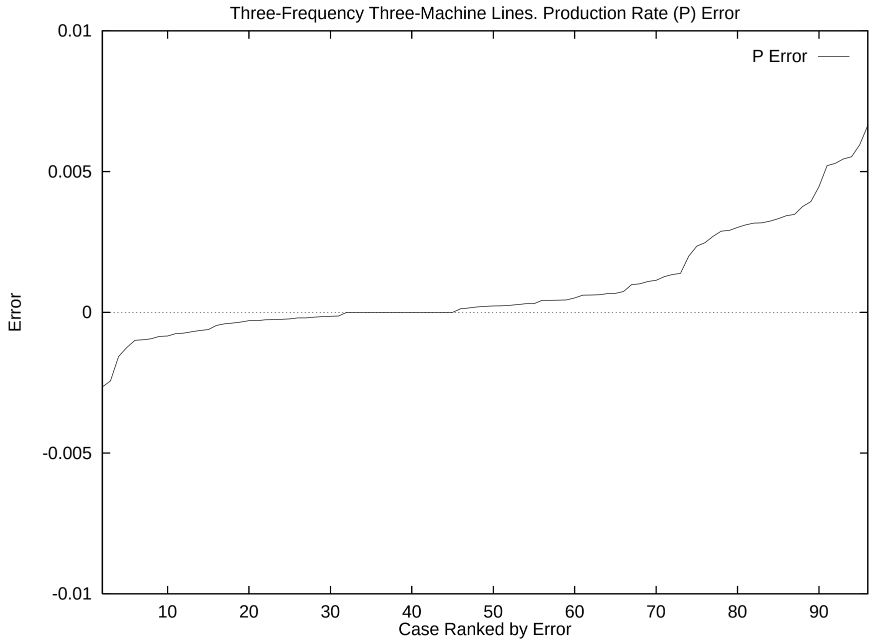

Figure 4-9: Evaluation of the performance of the simplification method for a three frequency-three machine line. Production rate error $\mathcal{E}_P$ for 100 randomly generated lines.

Errors for buffer level are very similar to those we found for the two frequency-two machine case. The maximum magnitude of the mean buffer level errors (individual and total) is approximately 5%, when the error is measured relative to actual buffer levels and below 3% when measured with respect to buffer sizes.

The similarity between these results and those obtained for the two frequency-three machine line is not surprising. In both cases the original line is replaced by a line where the first machine, M_1, is replaced by a perfectly reliable machine. The level of simplification of the line in both cases in essentially the same.

Performance of the Level 0 **Representation** Figures 4-9, 4-10 and 4-11 show the errors in the prediction of the mean production rate P, the individual buffer levels $\overline{x}_i$ and the total in process inventory $\overline{X}$.

In Figure 4-9 we observe that the mean production rate estimate provided by the simplified line is very close to the actual value P. The error is for all the examined cases below 1%. We also observe that the error is seems to be larger than for the

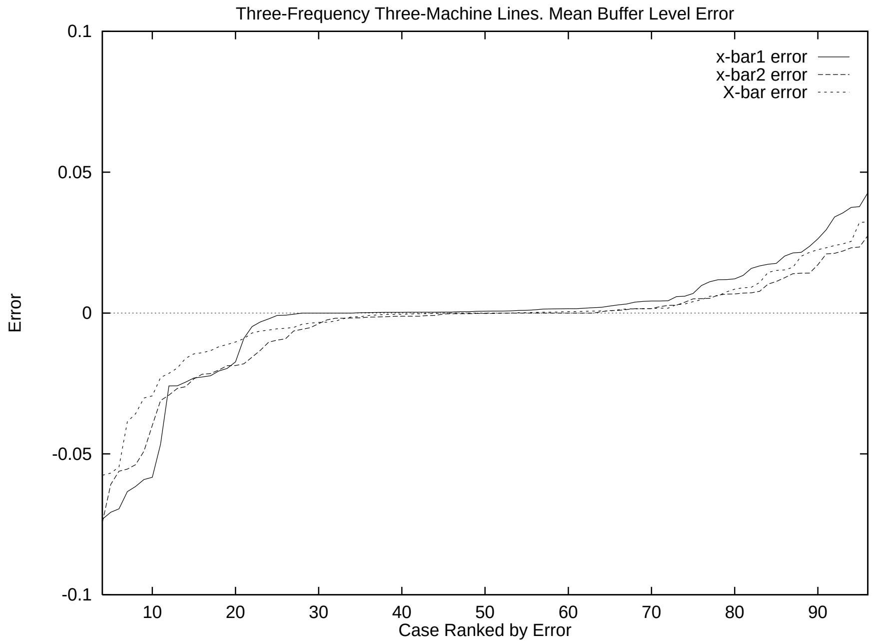

Figure 4-10: Evaluation of the performance of the simplification method for a three frequency-three machine line. Mean buffer errors $\mathcal{E}_{\overline{x}1}$ and $\mathcal{E}_{\overline{x}2}$ and mean total buffer level error $\mathcal{E}_{\overline{X}}$ for for 100 randomly generated lines.

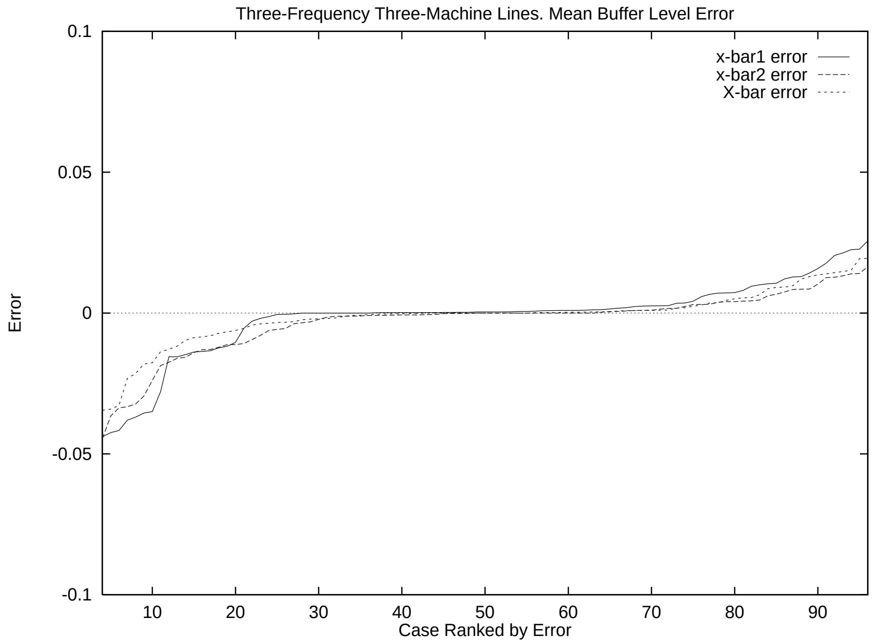

Figure 4-11: Evaluation of the performance of the simplification method for a three frequency-three machine line. Mean buffer errors $\mathcal{E}_{\overline{x}1}$ and $\mathcal{E}_{\overline{x}2}$ and mean total buffer level error $\mathcal{E}_{\overline{X}}$ for for 100 randomly generated lines.

level 1 simplified line.

As we have already observed in Example 4.2 the estimates of the mean buffer level are less accurate for this level of simplification. When measured relative to actual mean buffer levels, the magnitude of the maximum error reaches approximately 7%. For approximately 90% of the cases the error magnitude was below 5%. Errors relative to buffer sizes (see Figure 4-11) reach a maximum magnitude of approximately 5%.

We also observe in Figures 4-10 and 4-11 that the estimate of $\overline{x}_1$ has a larger error than the the estimates for $\overline{x}_2$ and the total mean buffer level. Larger negative errors can be explained by the fact that the buffer will fill only when a machine M_3 failure propagates upstream. If the processing rate of the modified second machine $\hat{M}_2$ is larger, buffer B_1 will be empty unless machine M_3 fails and buffer B_2 completely fills. We observed this phenomenon in Example 4.2. We can expect larger positive errors when the processing rate of simplified machine $\hat{M}_1$, $\hat{\mu}_1$, is larger than that of simplified machine $\hat{M}_2$, $\hat{\mu}_2$. In this case, the buffer will tend to fill and stay full, while in the original system failures of the first machine will allow the buffer to partially deplete in some occasions, which in turn causes a smaller mean buffer level $\overline{x}_i$.

In Section 4.4 we summarize the results obtained for longer multiple-frequency transfer lines.

4.4 Long Multiple-Frequency Transfer Lines

The decomposition described in Section 4.1 can be applied to any f-frequency, k-machine transfer line. The original line is represented by a simplified k-machine line at each level $\mathcal{M}_l$ of the hierarchy. In this section we analyze the accuracy of the representation method at different levels of the hierarchy for k-machine transfer lines. The analysis is based on the results of simulations of the original and simplified lines.

Section 4.4.1 describes how the cases used in this analysis were generated. Section 4.4.2 summarizes and examines the results obtained for the different line lengths and number of frequency levels.

4.4.1 Random Generation of f-frequency, k-machine lines

To examine the performance of the method for longer lines, we generated a large number of sample f-frequency, k machine transfer lines, with k varying from 4 to 9 and f varying from 2 to k.

For each pair (f, k), we randomly generate 25 f-frequency, k-machine transfer lines. Once the transfer line is obtained, we generate its corresponding level 0 simplified line (the level of maximum aggregation). Using simulation we obtain the performance measures for the original and the simplified system.

The f-frequency, k machine lines are generated as follows:

- Randomly obtain parameters p_j, r_j and μ_j $(i = 1, ..k)$ such that:
  - p_j and r_j are uniformly distributed from 0.0000001 to 1.0.
  - Machine processing rates μ_j are uniformly distributed from 0.1 to 10.0.
  - Individual mean processing rates ρ_j are within 20% of each other.
  - Event frequency separation condition (3.1) is implemented as

$$MTBF_j < \frac{1}{5}\, min(MTTF_{j+1}, MTTR_{j+1}) \;\; j = 1, ..., k \tag{4.5}$$

- The machines M_j are randomly reordered so that machines with more frequent events are not necessarily upstream of those with lower event frequencies. We call the machines resulting from the new ordering M_i, $i = 1, ..., k$.
- The buffer size N_i is chosen to be larger than half and smaller than five times the amount of material produced during by M_i during the average duration of a M_{i+1} failure.

$$N_i \in (0.5\frac{\rho_i}{r_{i+1}}, 5\frac{\rho_i}{r_{i+1}})$$

To avoid the effect of randomness on the values of the performance measures obtained using simulation, each of the lines is simulated 10 times for very long periods. 95% confidence interval are obtained for the mean of the performance measures obtained in the 10 simulations and we require the extremes of the interval to be less than 2% from the mean.

The errors reported in section 4.4.2 are obtained by comparing the mean for the 10 simulations for the simplified system to that of the original system.

4.4.2 Random Case Results

In this section we examine the performance of the transformation method at the maximum aggregation level (level 1) observing the differences between the actual mean production rate P and total mean buffer level $\overline{X}$ and the corresponding values obtained by the simplified representation of the line. Since level 1 is the maximum aggregation level, we expect the errors to be the larger among the different aggregation levels. Consequently, the results shown in Table 4.12 can be regarded as upper bounds to the errors for lower levels of aggregation.

Production Rate Error Table 4.12 presents the maximum errors obtained for the mean production rate estimates for the different f-frequency, k-machine line. The values correspond to the error of larger magnitude for the 25 random cases generated for each (f, k) combination. Table 4.13 shows the average errors for the same set of experiments.

Table 4.12. Long Lines. Maximum $\mathcal{E}_P$ for level 1 representation.

	Number of hierarchical levels f							
Number of machines	2	3	4	5	6	7	8	9
4	0.00712	0.00603	0.00584	n/a	n/a	n/a	n/a	n/a
5	0.00698	0.00693	0.00545	0.00564	n/a	n/a	n/a	n/a
6	0.00792	0.00598	0.00654	0.00642	0.00589	n/a	n/a	n/a
7	0.00602	0.00652	0.00543	0.00623	0.00629	0.00691	n/a	n/a
8	0.00667	0.00603	0.00614	0.00599	0.00573	0.00675	0.00702	n/a
9	0.00627	0.00631	0.00645	0.00547	0.00631	0.00692	0.00699	0.00712

Table 4.13. Long Lines. Average magnitude of $\mathcal{E}_P$ for level 0 representation.								
	Number of hierarchical levels f							
Number of machines	2	3	4	5	6	7	8	9
4	0.00184	0.00117	0.00105	n/a	n/a	n/a	n/a	n/a
5	0.00182	0.00140	0.00141	0.00135	n/a	n/a	n/a	n/a
6	0.00194	0.00149	0.00142	0.00111	0.00133	n/a	n/a	n/a
7	0.00209	0.00143	0.00151	0.00158	0.00147	0.00144	n/a	n/a
8	0.00196	0.00148	0.00148	0.00146	0.00153	0.00157	0.00146	n/a
9	0.00185	0.00176	0.00156	0.00148	0.00148	0.00122	0.00161	0.00133

We observe that the mean production rate of the original line is very closely matched by the simplified line. The maximum error is always below 1% while the average error is approximately 0.2%.

This result is to be expected as a consequence of condition (4.2), which requires all machines in the simplified line to preserve the mean isolated processing rate of the corresponding machine in the original line. The fact that higher frequency events are not included in the simplified line does not significantly affect the estimates of mean production rate.

Mean Buffer Level Errors Table 4.14 presents the maximum errors obtained for the mean total buffer level estimates for the different f-frequency, k-machine line. The values correspond to the errors of largest magnitude for the 25 random cases generated for each (f, k) combination. Table 4.15 shows the average errors for the same set of experiments. These errors are calculated relative to the actual total mean buffer level $\overline{X}$.

Table 4.14. Long Lines. Maximum $\mathcal{E}_{\overline{X}}$ for the level 0 representation.								
	Hierarchical level l							
Number of machines	0	1	2	3	4	5	6	7
4	0.05436	0.05362	0.05651	n/a	n/a	n/a	n/a	n/a
5	0.05343	0.04927	0.05754	0.2824	n/a	n/a	n/a	n/a
6	0.05979	0.06039	0.05876	0.0543	0.0714	n/a	n/a	n/a
7	0.04324	0.05453	0.05578	0.0572	0.0534	0.0614	n/a	n/a
8	0.05466	0.05796	0.06489	0.0568	0.0645	0.0532	0.2345	n/a
9	0.04456	0.05123	0.04156	0.0616	0.0576	0.0615	0.0554	0.0501

Table 4.15. Long Lines. Average magnitude of $\mathcal{E}_{\overline{X}}$ for the level 0 representation.								
	Hierarchical level l							
Number of machines	0	1	2	3	4	5	6	7
4	0.01364	0.01396	0.01414	n/a	n/a	n/a	n/a	n/a
5	0.01082	0.01340	0.01531	0.01852	n/a	n/a	n/a	n/a
6	0.01112	0.01496	0.01235	0.01516	0.01809	n/a	n/a	n/a
7	0.01209	0.01443	0.01351	0.01482	0.01912	0.02182	n/a	n/a
8	0.01112	0.01268	0.01562	0.01495	0.01853	0.02101	0.02165	n/a
9	0.01082	0.01176	0.01310	0.01383	0.01734	0.02008	0.02095	0.02514

Results in Tables 4.14 and 4.15 show a slight increase in errors as the number of levels approaches the number of machines.

In two cases the error exceeds 20%, (8-frequency, 8-machine line, 29% and 5 frequency-5 machine line, 23%). The corresponding maximum errors measured with respect to total buffer space are much smaller, 3.5% and 2.9% respectively. This is due to the fact that the actual mean total buffer level was very small causing some magnification of the error.

Excluding the two values mentioned above the errors reach as much as 8.5% (9-frequency, 9-machine lines). The maximum errors range from 5% to 7% for the rest of the cases. The average error range from approximately 1% to 2.5%.

Since the results reported in Tables 4.14 and 4.15 are for the maximum level of aggregation we can expect the errors to increase as the number of different event frequencies approaches the number of machines of the lines. When $f = k$, the failures and repairs of only one of the machines are preserved in the level 1 simplified model, and all buffers not contiguous to this machine fill and deplete only when its failures propagate through the line.

Chapter 5

Multiple-Event-Frequency Transfer Lines - Applications of the Representation Method

In Chapters 3 and 4 we describe a method for event-frequency decomposition of continuous-material transfer lines. The method was developed for transfer lines whose machines differ significantly in the frequencies at which they experience failure and repair events.

In this chapter we examine two different situations in which the decomposition tools of the previous chapters can be applied. The transfer lines that correspond to these cases do not directly satisfy the definition of a multiple-frequency transfer line presented in Chapter 2. However, they can be modeled in such a way that the decomposition method can be applied.

The two situations we consider in this chapter are:

1. **Machine Grouping.** Machines in a transfer line are grouped to form *cells*. Each cell is considered to perform a single *logical* operation. We suggest how to apply the decomposition method to represent the line after this grouping has

taken place.

2. **Multiple-Failure-Class Transfer Lines.** In this case, each machine can experience several classes of failures. Failures in different classes occur and are repaired at different frequencies. We model this line as a regular single-failure-class transfer line and apply the event-frequency decomposition tools.

In Section 5.1 we examine the machine grouping case and in Section 5.2 we discuss the multiple-failure-class transfer line case.

5.1 Machine Grouping

Large manufacturing systems are usually characterized by their complexity and the diversity of the operations that are performed. Such a complex manufacturing process may involve subprocesses. Each subprocess consists of a series of consecutive individual operations, each performed by a different machine, but that can be logically enclosed into a single *higher level* operation.

The subprocesses we consider in this section have two major characteristics:

(1) The set of individual process are logically, operationally or spatially related. That is, the set of operations performed can be regarded as portions of a higher level operation or they are performed in a common space, perhaps physically separated from other portions of the line.

(2) The subprocess performance must be, to a degree, independent of the performance of the upstream and downstream portions of the line. That is, failures inside the subprocess should not propagate very rapidly through the rest of the line and failures in the rest of the line should not cause major disruptions to the operation inside the subprocess.

When this kind of subprocess are present one can represent the line as a set of processing *cells* each performing a single *logical* operation (although not necessarily a single *physical* operation). This *grouping* process can be carried out further, grouping more machines and cells into a more complex, higher level cell.

According to the level of detail at which the line has to be analyzed, we can choose a particular level of aggregation. Aggregation ranges from the level where each single machine and its events are taken into account to the maximum aggregation level at which all the line can be represented as a single cell and individual buffer levels and events can be ignored. Figure 5-1 illustrates the machine grouping concept.

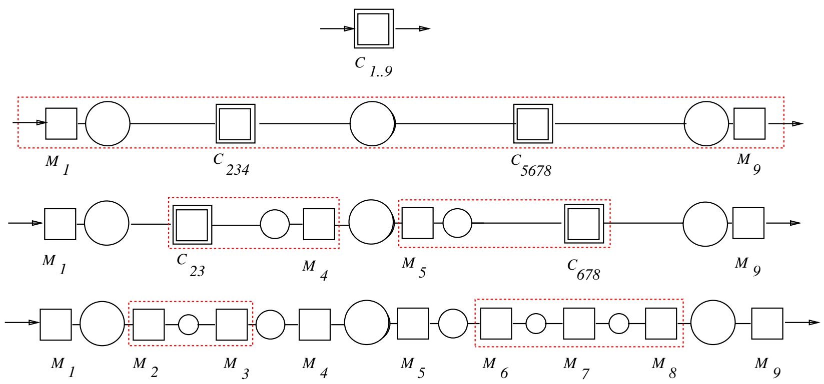

Figure 5-1: Pyramid Shaped Hierarchy. Grouping of consecutive machines and buffers into a single higher level *cell*. M_i denotes an individual machine and C_I denotes cell of machines grouped together. At the highest level of aggregation all the line can be represented by a single cell.

A hierarchical machine grouping similar to the one proposed above is described by Gershwin (1994) as a first step for the construction of a hierarchically-structured controller. We do not pursue the development of such a controller in this thesis but suggest how a the simplification method presented in previous chapters can be used to represent the line at different levels of the hierarchy. A hierarchical controller like the proposed by Gershwin would look at the simplified representation of the line at each level of the hierarchy.

In Section 5.1.1 we propose a method for representing the different levels of aggregation using the simplification tools presented in Chapters 3 and 4. Finally, in Section 5.1.2 we show examples of the use of the representation method.

5.1.1 Representation of the Line at Different Levels of Aggregation

The criteria for grouping a particular sequence of machines of a transfer line into a single logical group do not necessarily include the frequency at which failure and repair events occur in those machines. Therefore, machine grouping does not directly correspond to the situation we described in Chapters 3 and 4, where the main criterion for simplification is the relative frequency of events.

However, as a set of machines is grouped into a single cell, the cell, as seen by its upstream and downstream machines, experience more frequent events than the machines originally at the ends of the sub-line. This is true because the first and last machines of the cell stop processing not only as a consequence of their own failures but also due to blockage (for the first machine) or starvation (in the case of the last machine).

Based in the above observation, we assume that the cell resulting from the grouping of several machines can be treated as a machine with a higher frequency of events than other machines in the line. This assumption allows us to apply a frequency-based simplification to represent the line at different levels of grouping or aggregation.

For this application of the method, the characteristics (1) and (2) required for a group of machines and buffers to be grouped into a single cell, translate to:

(1) Machines and buffers must constitute a *sub-line*. That is, a sequence of machines and buffers originally contained in the line. Letting M_i be the first machine in the cell, and M_j the last, we denote such a sub-line $C_{i,j} = (M_i, B_i, ..., B_{j-1}, M_j)$.

(2) The upstream and downstream buffers of the cell, B_{i-1} and B_{j+1} must be large enough to isolate the cell from the rest of the line. That is, B_{i-1} must rarely fill as a consequence of failures within the cell or empty due to failures of the upstream portion of the line; B_{j+1} must rarely empty due to failures in the cell, or fill due to failures of the downstream portion of the line.

When condition (2) is closely met, the sub-line is very rarely starved or blocked at its extremes. In this case, we can consider the sub-line as an isolated transfer line and from the point of view of the rest of the line, only its mean production rate is significant.

Given the above conditions and observations, we propose the following method for representing lines with cells consisting of several machines and decoupled from the rest of the line by large buffers.

Representation Method Given a transfer line, where machines are grouped in cells $C_{i,j}$, a simplified representation of the line can be obtained as follows:

1. For each cell $C_{i,j}$: obtain the mean production rate as if the machines and buffers in the line formed an independent transfer line $(M_i, B_i, .., B_{j-1}, M_j)$

2. Represent the original transfer line as follows:

   - All machines and buffers not grouped into a cell remain unchanged.

   - Machines and buffers grouped in a cell are replaced by a single reliable machine, with processing rate $\hat{\mu}$ equal to the processing rate of the machines in the cell obtained in step 1.

3. Obtain the production rate for the simplified system and the mean levels of the higher level (not grouped) buffers using the simplified representation of the line.

5.1.2 Examples

In this section we present two examples where grouping can be used to model and hierarchically analyze a transfer line. We use the representation method of Section 5.1.1 at each aggregation level.

Example 5.1. One Level Aggregation The first example correspond to a transfer line where a set of machines is grouped into a cell. At this level of aggregation, the original 7-machine line is represented by a 4-machine line. Figure 5-2 illustrates this situation.

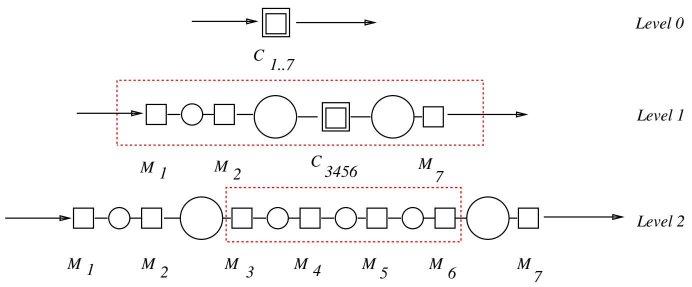

Figure 5-2: Machine grouping. Schematic representation of the situation of Example 5.1. Machines M_3, M_4, M_5 and M_6 are grouped in cell $C_{3,6}$. Buffers B_2 and B_6 are large enough to partially isolate the grouped machines from the rest of the line.

The parameters of the original 7-machine line are as shown in Table 5.1.

Table 5.1. Example 5.1: Original System

	Parameters				
Machine	r_i	p_i	μ_i	e_i	N_i
M_1	0.12	0.012	1.02	0.92727	10
M_2	0.09	0.010	1.03	0.92700	20
M_3	0.12	0.014	1.02	0.91343	5
M_4	0.10	0.010	1.00	0.90909	7
M_5	0.13	0.016	1.01	0.89932	6
M_6	0.10	0.010	1.02	0.92727	20
M_7	0.09	0.009	1.03	0.93636	n/a

Machines M_3, M_4, M_5 and M_6 are grouped to form cell $C_{3,6}$.

Following step 2 of the method presented in Section 5.1.1, the cells are replaced by reliable machines with a processing rate equal to the production rate of the two-machine line consisting of the machines in each cluster.

Using simulation, we obtain the following production rate for the sub-line $C_{3,6}$, considered as an independent line,

$$P_{C_{3,6}} = 0.78434$$

and the resulting simplified 4-machine line is of the form:

Table 5.2. Example 5.1: Simplified System						
	Parameters					
Machine	r_i	p_i	μ_i	e_i	ρ_i	N_i
M_1	0.12	0.012	1.02	0.9091	0.9273	10
M_2	0.09	0.010	1.03	0.9000	0.9270	20
$C_{3,6}$	1.00	0.000	0.78434	1.0000	0.78434	20
M_7	0.09	0.009	1.03	0.9091	0.93636	n/a

We observe in Table 5.2 that the machines that constitute cell $C_{3,6}$ are replaced by a single reliable machine with processing rate equal to the mean production rate of the transfer line consisting of the sequence of machines and buffers in the cell.

We can test the accuracy of this representation comparing the performance measures obtained with the original line to those obtained with its simplified representation. We use the error measures defined in Chapter 4. Table 5.3 summarizes these results.

Table 5.3. Example 5.1: Performance Measures.					
	Line Performance Measures				
Model	P	$\overline{x}_1$	$\overline{x}_2$	$\overline{x}_6$	$\overline{X}$
Original	0.7599	7.3587	14.6734	3.3555	25.3876
Simplified	0.7605	7.5123	15.3564	2.9495	25.8182
Errors	$\mathcal{E}_P = -0.0009$	$\mathcal{E}_{\overline{x}1} = 0.0208$	$\mathcal{E}_{\overline{x}2} = 0.0465$	$\mathcal{E}_{\overline{x}5} = -0.1212$	$\mathcal{E}_{\overline{X}} = 0.01696$
		$\mathcal{E}_{N1} = 0.0153$	$\mathcal{E}_{N2} = 0.0341$	$\mathcal{E}_{N5} = -0.0203$	$E_N = 0.0086$

Quantities shown are averages of values for 10 simulation runs.

Results in Table 5.3 suggest that the representation method provides a good approximation to the actual performance characteristics of the original system. The mean production rate obtained with the simplified line coincides almost perfectly with that of the original system. The mean buffer levels for B_1, B_2 and B_5, the only buffers considered at this level of aggregation, are estimated with errors ranging approximately from 2 to 12% when measured relative to actual mean buffer levels, $\overline{x}_i$, and from 1 to 3% when measured relative to buffer sizes, N_i.

We observe that mean buffer level errors are larger than those observed when we evaluated the representation method for multiple-frequency transfer lines in Chapter 4. This is to be expected since the frequency separation condition required in previous chapters is not strictly satisfied here. As we mentioned, we assume that after grouping machines in a cell, the resulting cell can be viewed as having higher frequency events by its upstream and downstream machines.

We show in Chapter 3 that the performance of the representation method improves as the frequency separation increases. Therefore, we expect the performance of the application of the method to the representation of aggregated transfer lines to be affected by how closely the frequency separation assumption is met.

The following example applies the representation method to a long line at two different levels of aggregation.

Example 5.2. Two Level Aggregation. We show the application of the representation method to a 25-machine transfer line. The parameters of the 25-machine line are indicated in Table 5.4.

At the first level of aggregation, three machine groups are identified. Each group is replaced by a single cell. At a second level of aggregation two larger groups of machines are identified and each replaced by a cell. Figure 5-3 illustrates this situation.

We perform the representation method described in Section 5.1.1 at each of the two aggregation levels.

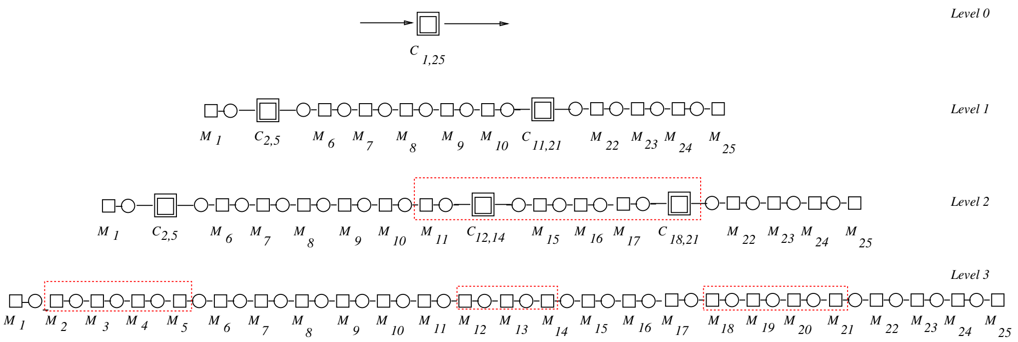

Figure 5-3: Machine grouping. Schematic representation of the situation of Example 5.1. Machines M_2 and M_3 are grouped in cell C_{23} and Machines M_5 and M_6 are grouped in cell C_{56}. Buffers B_1, B_3, B_4 and B_6 are large enough to isolate the grouped machines from the rest of the line.

Table 5.4. Example 5.2: Original 25-machine line

	Parameters					
Machine	r_i	p_i	μ_i	e_i	ρ_i	N_i
M_1	0.081642	0.006701	1.124977	0.924148	1.03965	20
M_2	0.195633	0.014536	0.851581	0.930837	0.79268	7
M_3	0.075617	0.009373	0.940581	0.889716	0.83685	9
M_4	0.091165	0.011229	0.870384	0.890335	0.77493	8
M_5	0.125230	0.014741	1.197967	0.894685	1.07180	20
M_6	0.122331	0.012254	1.453302	0.90895	1.32098	14
M_7	0.095787	0.011645	0.817289	0.891606	0.72870	9
M_8	0.122711	0.014014	0.911592	0.897502	0.81815	10
M_9	0.061875	0.004455	1.163847	0.932836	1.08568	16
M_{10}	0.083344	0.005744	1.170096	0.935524	1.09465	23
M_{11}	0.053100	0.004397	1.146019	0.923526	1.05838	20
M_{12}	0.086811	0.005995	0.870216	0.935403	0.81400	5
M_{13}	0.187416	0.014358	1.007357	0.928841	0.93567	4
M_{14}	0.161726	0.014571	1.014169	0.91735	0.93034	16
M_{15}	0.101440	0.010187	0.993424	0.908741	0.90276	11
M_{16}	0.109036	0.012119	1.009345	0.899971	0.90838	4
M_{17}	0.178095	0.016371	1.139120	0.915816	1.04322	20
M_{18}	0.120733	0.011204	0.987732	0.915081	0.90385	7
M_{19}	0.158710	0.013676	1.114927	0.920666	1.02648	8
M_{20}	0.106307	0.013058	1.086835	0.890604	0.96794	6
M_{21}	0.069997	0.005628	1.075921	0.92558	0.99585	25
M_{22}	0.145569	0.011617	1.171297	0.926094	1.08473	6
M_{23}	0.170581	0.015470	1.123093	0.916851	1.02971	10
M_{24}	0.076152	0.006548	1.068077	0.920822	0.98351	6
M_{25}	0.164611	0.012827	0.841827	0.92771	0.78097	n/a

First Level of Aggregation. At the first level of aggregation, each of three sets of machines will be grouped into a single logical unit. Machines M_2, M_3, M_4 and M_5 and their intermediate buffers are grouped to form cell $C_{2,5}$; machines M_{12}, M_{13} and M_{14} and their intermediate buffers are grouped to form cell $C_{12,14}$ and finally, machines M_{18}, M_{19}, M_{20} and M_{21} are grouped to form cell $C_{18,21}$.

Following step 2 of the method presented in Section 5.1.1, the cells are replaced by reliable machines with processing rate equal to the mean production of the transfer line consisting of the machines in each cluster. We construct these transfer lines and, using simulation, for each transfer line we obtain the following mean production rates:

$$P_{C_{2,5}} = 0.69747, \quad P_{C_{12,14}} = 0.77790 \text{ and } P_{C_{18,21}} = 0.82285.$$

The parameters of the resulting line are shown in Table 5.5.

Table 5.5. Example 5.2: First level of aggregation line

	Parameters					
Machine	r	p	μ	e	ρ	N
M_1	0.081642	0.006701	1.124977	0.92414	1.03965	20
$C_{2,5}$	1.00000	0.00000	0.697474	1.0000	0.69747	20
M_6	0.122331	0.012254	1.453302	0.90895	1.32098	14
M_7	0.095787	0.011645	0.817289	0.891606	0.72870	9
M_8	0.122711	0.014014	0.911592	0.897502	0.81815	10
M_9	0.061875	0.004455	1.163847	0.932836	1.08568	16
M_{10}	0.083344	0.005744	1.170096	0.935524	1.09465	23
M_{11}	0.053100	0.004397	1.146019	0.923526	1.05838	20
$C_{12,14}$	1.00000	0.000000	0.77790	1.00000	0.77790	16
M_{15}	0.101440	0.010187	0.993424	0.908741	0.90276	11
M_{16}	0.109036	0.012119	1.009345	0.899971	0.90838	4
M_{17}	0.178095	0.016371	1.139120	0.915816	1.04322	20
$C_{18,21}$	1.000000	0.000000	0.822850	1.00000	0.82285	25
M_{22}	0.145569	0.011617	1.171297	0.926094	1.08473	6
M_{23}	0.170581	0.015470	1.123093	0.916851	1.02971	10
M_{24}	0.076152	0.006548	1.068077	0.920822	0.98351	6
M_{25}	0.164611	0.012827	0.841827	0.92771	0.78097	n/a

The original 25-machine line has been reduced to a 17-machine transfer line by means of grouping several machines in cells $C_{2,5}$, $C_{12,14}$ and $C_{18,21}$.

We compare the performance measures obtained using the simplified model to those obtained from the original line to evaluate the accuracy of the representation. Table 5.6 shows the results.

Table 5.6. Example 5.2: First level of aggregation.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.66826	0.67142	0.00473	
Measure			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_1$	18.99	19.14	0.0079	0.0075
$\overline{x}_5$	8.14	7.67	-0.0577	-0.0235
$\overline{x}_6$	10.51	9.27	-0.0839	-0.0630
$\overline{x}_7$	2.19	2.16	-0.0137	-0.0033
$\overline{x}_8$	0.87	0.71	-0.1839	-0.0160
$\overline{x}_9$	1.97	1.67	-0.1522	-0.0188
$\overline{x}_{10}$	5.12	4.58	-0.0859	-0.0235
$\overline{x}_{11}$	8.93	8.82	-0.0123	-0.0055
$\overline{x}_{14}$	3.92	3.89	-0.0100	-0.0019
$\overline{x}_{15}$	3.07	2.98	-0.0293	-0.0082
$\overline{x}_{16}$	0.68	0.57	-0.1617	-0.0275
$\overline{x}_{17}$	5.01	4.85	-0.0320	-0.0080
$\overline{x}_{21}$	5.29	4.78	-0.0964	-0.0204
$\overline{x}_{22}$	2.23	2.12	-0.0493	-0.0183
$\overline{x}_{23}$	4.23	4.11	-0.0282	-0.0120
$\overline{x}_{24}$	3.67	3.49	-0.0490	-0.0300
$\overline{X}$	80.90	80.81	-0.0012	-0.0003

From Table 5.5 we observe that, as it has been the case in previous examples, the simplified system provides a very accurate estimate of the mean production rate of the original system.

Estimates for mean buffer levels $\overline{x}_i$ show errors ranging from 0.1% to approximately 16% when measured relative to the actual mean buffer level. This error exceeds 10% only when the actual mean buffer level is very small. When measured relative to buffer sizes, errors range from 0.1 to 6%. The estimate for mean total buffer level $\widehat{\overline{X}}$ is very accurate, with an error of only 0.1%.

We now perform the step for the representation of the line at a higher level of aggregation.

Second Level of Aggregation. At the second level of aggregation, a new set of machines is grouped into a single operation. Machines M_{11} through M_{21} and their intermediate buffers are grouped to form cell $C_{11,21}$. In fact, this cell includes two of the cells defined at the first level of aggregation, namely $C_{12,14}$ and $C_{18,21}$.

Following step 2 of the method presented in Section 5.1.1, the cell is represented by a reliable machine with processing rate equal to the mean production of the transfer line consisting of machines M_{11} through M_{21} and their intermediate buffers. We construct such a line and obtain the following mean production rate

$$P_{C_{11,21}} = 0.73260$$

The parameters of the resulting line are shown in Table 5.7.

Table 5.7. Example 5.2: Second level of aggregation.

	Parameters					
Machine	r	p	μ	e	ρ	N
M_1	0.081642	0.006701	1.124977	0.924148	1.03965	20
$C_{2,5}$	1.00000	0.00000	0.697474	1.0000	0.69747	20
M_6	0.122331	0.012254	1.453302	0.90895	1.32098	14
M_7	0.095787	0.011645	0.817289	0.891606	0.72870	9
M_8	0.122711	0.014014	0.911592	0.897502	0.81815	10
M_9	0.061875	0.004455	1.163847	0.932836	1.08568	16
M_{10}	0.083344	0.005744	1.170096	0.935524	1.09465	23
$C_{11,21}$	1.00000	0.000000	0.77790	1.00000	0.73260	25
M_{22}	0.145569	0.011617	1.171297	0.926094	1.08473	6
M_{23}	0.170581	0.015470	1.123093	0.916851	1.02971	10
M_{24}	0.076152	0.006548	1.068077	0.920822	0.98351	6
M_{25}	0.164611	0.012827	0.841827	0.92771	0.78097	n/a

The original 25-machine line has now been reduced to a 11-machine transfer line by means of grouping several machines in cells $C_{2,5}$ and $C_{11,21}$.

We obtain and compare the performance measures for both the original and the simplified system. Table 5.7 summarizes these results.

Table 5.6. Example 5.2: First level of aggregation.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.66826	0.67172	0.00518	
			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_1$	18.99	19.12	0.0070	0.0066
$\overline{x}_5$	9.34	7.12	-0.2376	-0.1110
$\overline{x}_6$	10.51	9.41	-0.1046	-0.0786
$\overline{x}_7$	2.19	2.21	0.0091	0.0022
$\overline{x}_8$	0.87	0.76	-0.1264	-0.012
$\overline{x}_9$	1.97	1.90	-0.0355	-0.0044
$\overline{x}_{10}$	5.12	6.58	-0.2851	0.0634
$\overline{x}_{21}$	5.29	4.71	-0.1096	-0.0363
$\overline{x}_{22}$	2.23	2.04	-0.0852	-0.0317
$\overline{x}_{23}$	4.23	4.03	-0.0473	-0.0200
$\overline{x}_{24}$	3.67	3.29	-0.1935	-0.0633
$\overline{X}$	64.41	61.27	-0.0488	-0.0209

From Table 5.7 we observe that the simplified system provides a very accurate estimate of the mean production rate of the original system.

Estimates for mean buffer levels $\overline{x}_i$ show errors ranging from 0.1% to approximately 23% when measured relative to the actual mean buffer level. When measured relative to buffer sizes, errors range from 0.1% to 11%. The estimate for mean total buffer level $\widehat{\overline{X}}$ shows an error of approximately 5% when measured with respect to the actual mean total buffer level $\overline{X}$ and slightly exceeds 2% when measure relative to the total buffer size N.

This example shows how the method can be used to represent a transfer line at different levels of aggregation. We observe that the simplified system provides accurate estimates of mean production rates at the two aggregation levels. Mean buffer level errors increase as the level of aggregation increases, that is, as the line is represented is studied at a coarser level of detail.

5.1.3 Remarks on the use of the simplification method

The method used here to represent machine cells is based on the representation method developed in the previous sections.

However, we do not start from a transfer line with event frequency separation. We start with a general transfer line and we group machines in cells. We assume that, as a consequence of the grouping, these cells have more frequent events than their adjacent machines in the line. The accuracy of the estimates provided by this method depend on how much more frequent are cell events with respect to those of the regular machines or of those of the cells at lower levels in the hierarchy.

It is also important to remember that the grouping procedure is based on the assumption that this machine cells are indeed separated from the rest of the line by large buffers. This assumption is important for two reasons. First, it improves the accuracy of the simplification method since it makes the rest of the line less dependent of the actual behavior of the machine included in the cell. Second, it isolates the cell from the rest of the line, justifying the use of the mean buffer levels obtained from examination of the cell as an independent transfer line to estimate the level of those buffers in the actual longer line.

From the above remarks we observe that the quality of the estimates of the line performance measures obtained by this method are strongly dependent on how well those assumptions are satisfied.

5.2 Multiple-Failure-Class Machine Transfer Lines

In this section we examine the case of transfer lines where machines can have more than one class of failures. Each class occurs at a different rate and requires different times to be repaired. We can build a hierarchical representation of the transfer line based on the different failure classes. At high levels of the hierarchy, only major failures will be considered, while at lower levels a larger number of classes of failures are considered.

This situation is common in industry. A given machine can fail for different reasons, and different failures are more or less frequent and the times required for repair also differ. For instance, we can identify at least three different machine failures classes:

- Major Failures. These failures occur rather infrequently and take long time to repair. This can be the case of failures that require replacing or fixing a part of a machine, possibly with the additional delay of procuring the failing part.
- Regular Failures. This class includes failures that require fixing a part on site, or replacing it by a spare part readily available on site.
- Minor Failures. These are interruptions that occur very frequently and can be corrected rapidly. They may not imply breakage of the machine, but rather a minor misadjustment.

When different classes of failures occur with different frequencies, there is a close correspondence between a failure-class induced hierarchy and the frequency-induced hierarchy described in Chapter 2. This suggests the use of a simplification method as the one in described Chapter 2, to represent the line at each the levels of such a hierarchy.

However, the representation model was developed for transfer lines where each machine M_i experiences only one class of failures, which occur at a rate p_i and are repaired at a rate r_i. Therefore, before we can use the method, we need to model the multiple failure class machine line as a regular transfer line.

In Section 5.2.1 we define a multiple failure class transfer line. In Section 5.2.2 we show a method for the representation of these lines as single-failure-class transfer lines. Finally, in Section 5.2.4 we show examples of the use of the simplification method of Chapter 2 to represent the transfer line at different levels of the event class induced hierarchy.

5.2.1 Definition of the Multiple-Failure-Class Transfer Line

In this section we define a multiple-failure-class transfer line and introduce the notation we use in later sections.

The behavior of a *multiple-failure-class transfer line* can be described as in Section 2.1.1 with the exception that at each machine more than one class of failures can occur. As in the single-failure-class transfer line described in Section 2.1.1, continuous raw material is processed at consecutive machines, always in the same order. Finite, reliable buffers or storage areas exists between consecutive machines. Whenever the downstream machine stops processing and the buffer is not full, material is stored in it.

Machines are unreliable, with operation-dependent failures. Each machine can suffer more than one class of failures. Failures and repair times are exponentially distributed for each of the failure classes. Failures are independent among failure classes and among machines.

A k machine F-failure-class transfer line is characterized by the following parameters.

Line Parameters. Machines are multiple-failure-class machines and are denoted as M_i^*. Each machine is characterized by the following parameters:

- Processing Rate μ_i, $i = 1, ..., k$. This is the rate at which material flows and is processed at machine M_i^* in the absence of blockage or starvation. In the presence of starvation or blockage, machine M_i^* will process material at a fraction of μ_i. The actual processing rate at each individual instant t is called $\mu_i(t)$.

- Failure Rate p_i^f, $f = 1, ...F$, $i = 1, ..., k$. This is the rate at which class f failures occur at machine M_i^* when it is working at its maximum processing rate μ_i. In the presence of blockage and starvation, class f failures will occur at a rate proportional to the fraction of the maximum processing speed at which the machine is operating. The actual failure rate at instant t is

$$p_i^f(t) = \frac{\mu_i(t)}{\mu_i} p_i^f$$

- Repair Rate r_i^f, $f = 1, ...F$, $i = 1, ..., k$. This is the rate at which machine i gets repaired if it is down because of an class f failure. This rate never varies. At any instant t:

$$r_i^f(t) = r_i^f$$

As in the single-frequency transfer line, buffer B_i stores material between machines i and $i + 1$. (Fig. 2-1). Buffer B_i characterized by a single parameter:

- Buffer Size N_i. N_i is the maximum amount of material that can be stored in buffer i.

Multiple-failure-class machines are then characterized by a *set* of failure and repair rates. In Section 5.2.2 we show how to model these machines as a set single-failure-class machines.

5.2.2 Modeling as an F-failure-class transfer line as a single failure class transfer line

We can model a multiple-failure-class transfer line as a single-failure-class transfer line by replacing each multiple-failure-class machine of the original line by a series of connected single-failure-class machines, with no intermediate buffers. Each multiple-failure-class machine M_j^* is replaced by a F-machine zero-buffer transfer line.

The absence of buffer space between the machines allows us to model the behavior of the original multiple-failure-class machine. Once one failure has occurred, the $F-1$ remaining machines are prevented from processing and, consequently, of failing. Blockage and starvation behavior of the original machine is also preserved, since as soon the Fth machine of the line is blocked, all the machines in the zero-buffer line stop processing. Analogously, as the first machine is starved, the entire zero-buffer line stops processing.

We formalize the representation of a multiple-failure-class transfer line as a single-failure class line, as follows:

1. Each multiple-failure-class machine M_i^*, $i = 1, ..., k$ is replaced by a set of machines $\{M_{i,1}, ..., M_{i,F}\}$ with parameters:
   - Processing rate:
   $$\mu_{i,f} = \mu_i$$
   - Failure rate:
   $$p_{i,f} = p_i^f$$
   - Repair rate:
   $$r_{i,f} = r_j^f$$
2. A zero-buffer transfer line with machines $M_{i,1}, ..., M_{i,F}$ from step 1, is constructed.
3. Multiple-failure-class machine M_i^* is replaced by the zero-buffer transfer line constructed in step 2.

Figure 5-4 illustrates the modeling method for an F-failure-mode two-machine transfer line.

Once the equivalent single-failure-mode transfer line has been obtained, a hierarchy based on the frequency of the different failure classes can be obtained. At each

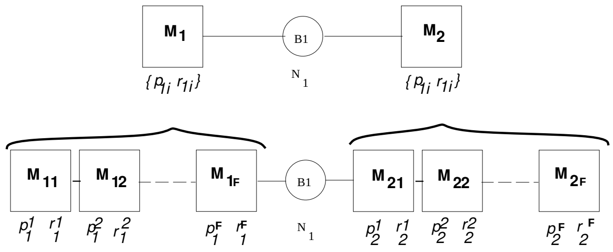

Figure 5-4: Failure Hierarchy. Representation of a multilevel failure machine as a regular transfer line: each original multilevel failure machine is represented by a zero-buffer transfer line, where each machine experiences only one type of failure.

level of such a hierarchy the representation method introduced in Section 3.2 can be used. The next section provides some examples of the use of the representation method for a failure hierarchy.

5.2.3 Correspondence between Multiple-Failure-Class Transfer Lines and Multiple-Frequency Transfer Lines

If failures of the different classes differ significantly in their frequency, we can build a frequency-based hierarchy as the one described in Section 2.1.3. Once the multiple failure mode transfer line has be modeled as a regular transfer line, Condition (2.2) can be rewritten in term of the failure classes f, $f = 1, .., F$.

If all machines $M_{i,f}$, $i = 1, 2, ..k$, $f = 1, ..., F$ satisfy:

$$MTBF_{i,f} \ll min(MTTF_{j,g}, MTTR_{j,g}) \quad j = 1, ..k, \quad g > f; \tag{5.1}$$

we can construct a frequency-based F level hierarchy, where each hierarchical level M_f consists of all the single-failure-mode machines with failures of mode f:

$$\mathcal{M}_f = \{M_{i,f}\ ;\ i = 1, 2, .., k\} \tag{5.2}$$

Once such a hierarchy has been constructed, we can represent the line at different levels of the hierarchy using the tools introduced in Chapters 2 and 3.

In Section 5.2.4 we present an example of the use of the hierarchical decomposition of multiple-failure-class transfer lines.

5.2.4 Examples of Application of the Representation Method

In this section we present examples of the hierarchical decomposition of multiple-failure-class transfer lines. In each example, we start by representing the multiple-failure-class transfer line as a single-failure-class transfer line. Once the single-failure-class has been obtained, we construct the corresponding frequency-based hierarchy and use the tools introduced in Chapter 2 and 3 to represent the line at the different hierarchical levels.

Now we show one example of the application of the representation method to the failure mode hierarchy.

Example 5.3. Three-Failure-Class Two-Machine Transfer Line We consider a three-failure-class transfer two-machine line: a two-machine transfer line whose machines can experience three different classes of failures. The frequency at which failures occur differ significantly among failure classes.

The parameters of the multiple-failure-class line are indicated in Table 5.8.

Table 5.8. Example 5.3: Multiple-failure-class line

	Parameters							
Machine	r_i^1	p_i^1	r_i^2	p_i^2	r_i^3	p_i^3	μ_i	N_i
M_1^*	0.100000	0.010000	0.001800	0.000180	0.000030	0.000003	1.00	1000
M_2^*	0.100000	0.010000	0.001800	0.000180	0.000030	0.000003	1.00	n/a

This line is replaced by a single-failure-mode six-machine line with parameters as indicated in Table 5.9:

Table 5.9. Example 5.3: Equivalent line						
	Parameters					
Machine	$r_{i,f}$	$p_{i,f}$	$\mu_{i,f}$	e_i	ρ_i	N_i
M_{11}	0.100000	0.010000	1.00	0.9091	0.9091	0
M_{12}	0.001800	0.000180	1.00	0.9091	0.9091	0
M_{13}	0.000030	0.000003	1.00	0.9091	0.9091	10000
M_{21}	0.100000	0.010000	1.00	0.9091	0.9091	0
M_{22}	0.001800	0.000180	1.00	0.9091	0.9091	0
M_{23}	0.000030	0.000003	1.00	0.9091	0.9091	n/a

According to the hierarchy definition in Section 5.2.3 we obtain the following three hierarchical levels:

$$
\begin{aligned}
\mathcal{M}_0 &= \{M_{13}, M_{23}\} \\
\mathcal{M}_1 &= \{M_{13}, M_{12}, M_{23}, M_{22}\} \\
\mathcal{M}_2 &= \{M_{13}, M_{12}, M_{11}, M_{23}, M_{22}, M_{21}\}
\end{aligned}
$$

We show the representation of the line at each of the hierarchical levels.

Representation for Level 2. No simplification needed, since all events are considered.

Representation for Level 1. Only events of machines in $\mathcal{M}_1$ are considered. The parameters of the simplified transfer line for this level of the hierarchy are shown in Table 5.10.

Table 5.10. Example 5.3: Level 1 representation						
	Parameters					
Machine	$r_{i,f}$	$p_{i,f}$	$\mu_{i,f}$	e_i	ρ_i	N_i
M_{11}	1.000000	0.000000	0.9091	1.0000	0.9091	0
M_{12}	0.001800	0.000180	1.0000	0.9091	0.9091	0
M_{13}	0.000030	0.000003	1.0000	0.9091	0.9091	10000
M_{21}	1.000000	0.000000	0.9091	1.0000	0.9091	0
M_{22}	0.001800	0.000180	1.0000	0.9091	0.9091	0
M_{23}	0.000030	0.000003	1.0000	0.9091	0.9091	n/a

We compare the performance measures obtained from this simplified representation of the line to those obtained from the original line. These quantities are summarized in Table 5.11.

Table 5.11. Example 5.3: Performance Measures		
	Line Performance Measures	
Model	P	$\overline{x}_1$
Original	0.72452	5000
Simplified	0.72935	5029
Errors	$\mathcal{E}_P = -0.0066$	$\mathcal{E}_{\overline{x}1} = 0.0058$
		$\mathcal{E}_{N1} = 0.0029$

Representation for Level 0. Only events of machines in $\mathcal{M}_0$ are considered. The parameters of the simplified transfer line for this level of the hierarchy are shown in Table 5.12.

Table 5.12. Example 5.3: Level 0 representation						
	Parameters					
Machine	$r_{i,f}$	$p_{i,f}$	$\mu_{i,f}$	e_i	ρ_i	N_i
M_{11}	1.000000	0.000000	0.8333	1.0000	0.8333	0
M_{12}	1.000000	0.000000	0.8333	1.0000	0.8333	0
M_{13}	0.000030	0.000003	1.0000	0.9091	0.9091	10000
M_{21}	1.000000	0.000000	0.8333	1.0000	0.8333	0
M_{22}	1.000000	0.000000	0.8333	0.9091	0.8333	0
M_{23}	0.000030	0.000003	1.0000	0.9091	0.9091	n/a

Parameters μ_{i1} and μ_{i2}, for $i = 1, 2$, correspond to the mean production rate of the two-machine zero-buffer line consisting of machines M_{i1} and M_{i2}. This is done to guarantee that the flow through the first two machines of each of the zero-buffer transfer lines used to represent the original multiple-failure-class machines is the same in both the simplified and the original representation of the line.

We compare the performance measures obtained from this simplified representation of the line to those obtained from the original line. These quantities are summarized in Table 5.13.

Table 5.13. Example 5.3: Performance Measures		
	Line Performance Measures	
Model	P	$\overline{x}_1$
Original	0.72452	5000
Simplified	0.73054	5017
Errors	$\mathcal{E}_P = 0.0083$	$\mathcal{E}_{\overline{x}1} = 0.0034$
		$\mathcal{E}_{N1} = 0.0017$

Tables 5.11 and 5.13, show errors below 1% for production rate and mean buffer level estimates. These results show that the simplified representation of the line accurately estimates the performance measures P and $\overline{x}$.

Example 5.4. Three-Failure-Class, Six-Machine Transfer Line The parameters of the multiple-failure-class line are as indicated in Table 5.14.

Table 5.14. Example 5.4: Multiple-failure-class line

	Parameters							
Machine	r_i^1	p_i^1	r_i^2	p_i^2	r_i^3	p_i^3	μ_i	N_i
M_1^*	0.605202	0.0734504	0.0107024	0.00145312	0.00296958	0.00037494	1.04942	12355
M_2^*	0.675859	0.0662592	0.0103513	0.00140538	0.00310244	0.000346737	1.17386	9732
M_3^*	0.609012	0.074068	0.0100002	0.00135764	0.00261027	0.000318531	1.12701	13342
M_4^*	0.67967	0.0668767	0.0121492	0.00130989	0.00274314	0.000365329	1.0408	19342
M_5^*	0.67967	0.0668767	0.0121492	0.00130989	0.00274314	0.000365329	1.0408	13242
M_6^*	0.612827	0.0746855	0.0117982	0.00126215	0.00287599	0.000337126	1.01747	n/a

This line is replaced by a single-failure-mode six-machine line with parameters as indicated in Table 5.15.

Table 5.15. Example 5.3: Equivalent line

	Parameters					
Machine	$r_{i,f}$	$p_{i,f}$	$\mu_{i,f}$	e_i	ρ_i	N_i
M_{11}	0.60520	0.07345	1.049	0.89177	0.935	0
M_{12}	0.01070	0.00145	1.049	0.88045	0.923	0
M_{13}	0.00296	0.00037	1.049	0.88789	0.931	12355
M_{21}	0.67585	0.06625	1.173	0.91072	1.069	0
M_{22}	0.01030	0.00140	1.173	0.88046	1.033	0
M_{23}	0.00310	0.00034	1.173	0.89947	1.055	9732
M_{31}	0.60901	0.07406	1.127	0.89157	1.004	0
M_{32}	0.01000	0.00135	1.127	0.88046	0.992	0
M_{33}	0.00261	0.00032	1.127	0.89124	1.004	13342
M_{41}	0.67967	0.06688	1.041	0.91042	0.947	0
M_{42}	0.01214	0.00131	1.041	0.90268	0.939	0
M_{43}	0.00274	0.00037	1.041	0.88247	0.918	19342
M_{51}	0.67967	0.06688	1.041	0.91042	0.947	0
M_{52}	0.01215	0.00131	1.041	0.90268	0.939	0
M_{53}	0.00274	0.00037	1.041	0.88247	0.918	13242
M_{61}	0.61283	0.07469	1.017	0.89137	0.906	0
M_{62}	0.01180	0.00126	1.017	0.90336	0.919	0
M_{63}	0.00288	0.00034	1.017	0.89508	0.910	n/a

According to the hierarchy definition in section 5.2.3 we obtain the following three hierarchical levels:

$$\begin{aligned}\mathcal{M}_0 &= \{M_{i3};\} \\ \mathcal{M}_1 &= \{M_{i3}, M_{i2}; i = 1, ..., 6\} \\ \mathcal{M}_2 &= \{M_{i3}, M_{i2}, M_{i1}; i = 1, ..., 6\}\end{aligned}$$

Representation for Level 2. No simplification is needed, since all events are considered.

Representation for Level 1. Only events of machines in $\mathcal{M}_1$ are considered. The parameters of the simplified transfer line for this level of the hierarchy are shown in Table 5.16.

Table 5.16. Example 5.4: Level 1 representation

	Parameters					
Machine	$r_{i,f}$	$p_{i,f}$	$\mu_{i,f}$	e_i	ρ_i	N_i
M_{11}	1.00000	0.00000	0.935	1.0000	0.935	0
M_{12}	0.01070	0.00145	1.049	0.8805	0.923	0
M_{13}	0.00296	0.00037	1.049	0.8879	0.931	12355
M_{21}	1.00000	0.00000	1.069	1.0000	1.069	0
M_{22}	0.01030	0.00140	1.173	0.8805	1.033	0
M_{23}	0.00310	0.00034	1.173	0.8995	1.055	9732
M_{31}	1.00000	0.00000	1.004	1.0000	1.004	0
M_{32}	0.01000	0.00135	1.127	0.8805	0.992	0
M_{33}	0.00261	0.00032	1.127	0.8912	1.004	13342
M_{41}	1.00000	0.00000	0.947	1.0000	0.947	0
M_{42}	0.01214	0.00131	1.041	0.9027	0.939	0
M_{43}	0.00274	0.00037	1.041	0.8825	0.918	19342
M_{51}	1.00000	0.00000	0.947	1.0000	0.947	0
M_{52}	0.01215	0.00131	1.041	0.9027	0.939	0
M_{53}	0.00274	0.00037	1.041	0.8825	0.918	13242
M_{61}	1.00000	0.00000	0.906	1.0000	0.906	0
M_{62}	0.01180	0.00126	1.017	0.9034	0.919	0
M_{63}	0.00288	0.00034	1.017	0.8951	0.910	n/a

We compare the performance measures obtained from this simplified representation of the line to those obtained from the original line. These quantities are summarized in Table 5.17.

Table 5.17. Example 5.4: Performance Measures.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.77439	0.77589	0.00194	
			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_3$	316	299	-0.0461	-0.0013
$\overline{x}_6$	886	867	-0.0211	-0.0021
$\overline{x}_9$	2570	2597	0.0105	0.0024
$\overline{x}_{12}$	6400	6356	-0.0069	-0.0022
$\overline{x}_{15}$	8800	8694	-0.0121	-0.0080
$\overline{X}$	18972	18823	-0.0083	-0.0022

Representation for Level 0. Only events of machines in $\mathcal{M}_0$ are considered. The parameters of the simplified transfer line for this level of the hierarchy are shown in Table 5.18.

Table 5.18. Example 5.4: Level 0 representation						
	Parameters					
Machine	$r_{i,f}$	$p_{i,f}$	$\mu_{i,f}$	e_i	ρ_i	N_i
M_{11}	1.00000	0.00000	0.835	1.0000	0.835	0
M_{12}	1.00000	0.00000	0.835	1.0000	0.835	0
M_{13}	0.00296	0.00037	1.049	0.8879	0.931	12355
M_{21}	1.00000	0.00000	0.951	1.0000	0.951	0
M_{22}	1.00000	0.00000	0.951	1.0000	0.951	0
M_{23}	0.00310	0.00034	1.173	0.8995	1.055	9732
M_{31}	1.00000	0.00000	0.897	1.0000	0.897	0
M_{32}	1.00000	0.00000	0.897	1.0000	0.897	0
M_{33}	0.00261	0.00032	1.127	0.8912	1.004	13342
M_{41}	1.00000	0.00000	0.863	1.0000	0.863	0
M_{42}	1.00000	0.00000	0.863	1.0000	0.863	0
M_{43}	0.00274	0.00037	1.041	0.8825	0.918	19342
M_{51}	1.00000	0.00000	0.863	1.0000	0.863	0
M_{52}	1.00000	0.00000	0.863	1.0000	0.863	0
M_{53}	0.00274	0.00037	1.041	0.8825	0.918	13242
M_{61}	1.00000	0.00000	0.828	1.0000	0.828	0
M_{62}	1.00000	0.00000	0.828	1.0000	0.828	0
M_{63}	0.00288	0.00034	1.017	0.8951	0.910	n/a

Parameters μ_{i1} and μ_{i2}, for $i = 1, ..., 6$ correspond to the mean production rate of the two-machine zero-buffer line consisting of machines M_{i1} and M_{i2}.

We compare the performance measures obtained from this simplified representation of the line to those obtained from the original line. These quantities are summarized in Table 5.19.

Table 5.19. Example 5.4: Performance Measures.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.77439	0.77689	0.00322	
			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_3$	316	287	-0.0912	-0.0023
$\overline{x}_6$	886	803	-0.0936	-0.0085
$\overline{x}_9$	2570	2628	0.0225	-0.0044
$\overline{x}_{12}$	6400	6209	-0.0298	-0.0099
$\overline{x}_{15}$	8800	8654	-0.0166	-0.0110
$\overline{X}$	18972	18581	-0.0206	-0.0057

Tables 5.17 and 5.19 show errors of the estimates obtained from the simplified lines at levels 1 and 0 of the hierarchy. We observe that the estimate of the line mean production rate, $\hat{P}$ is very accurate, with an error of approximately 0.3%.

Mean buffer level errors relative to actual mean buffer size, $\mathcal{E}_{\overline{x}i}$ range approximately from 1 to 10%. However, $\mathcal{E}_{\overline{x}i}$ exceedes 3% only for buffers B_3 and B_6, which actual mean buffer level is less than 10% the size of the buffer. Thus, we can attribute these large errors to the magnification to which are prone the measures of error relative to $\overline{x}_i$.

Measures of error relative to buffer sizes, $\mathcal{E}_{Ni}$ are below or very close to 1%, even when the corresponding $\mathcal{E}_{\overline{x}i}$ is large.

Chapter 6

Representation of Heterogeneous Transfer Lines

6.1 Introduction

In this chapter we develop tools for representing heterogeneous continuous-material transfer lines as continuous-material transfer lines with exponentially distributed times to fail and repair.

For the purposes of this thesis, we define an *heterogeneous* transfer line as a transfer line whose times to repair and fail differ in their distributions at different machines. In particular, we focus on transfer lines whose times to fail and repair can be deterministic or exponentially distributed.

A mathematical model with both deterministic and exponentially distributed processing interruptions is difficult to obtain and solve. The objective of this chapter is to investigate how to represent deterministic events using the models currently available for the analysis of transfer lines with exponentially distributed failure and repair times.

Here, we investigate the possibility of representing such heterogeneous lines as continuous-material transfer lines like the one defined in Section 2.1.1, where all machines obey the same distribution of failure and repair time. If such a representation is obtained, the tools available for continuous-material transfer line could be applied in the study of the more general heterogeneous lines.

In Section 6.2 we describe the behavior and introduce the notation for continuous-material heterogeneous transfer line. In Section 6.4 we focus on the heterogeneous two-machine line case, we show empirical evidence that there exist a two-exponential-machine transfer line equivalent to the original heterogeneous line in terms of mean production rate and mean buffer levels. We show how to obtain the equivalent two-exponential-machine line and discuss the characteristics of this equivalent line. In Section 6.5 we present a method for representing a long, heterogeneous transfer line by an approximately equivalent transfer line with exponentially distributed failure and repair times.

6.2 Definition of Heterogeneous Transfer Lines

In this chapter we consider transfer lines whose machines can be of two classes. For one class of machines failures and repair occur at *exponentially distributed times*. For the other class, processing stops after a *fixed* amount of material has been processed and resumes after a *fixed* amount of time. We call this kind of transfer line a *heterogeneous* transfer line. This section introduces the definitions and notation for the transfer line models we analyze in this chapter.

6.2.1 Heterogeneous Continuous-Material Continuous-Time Transfer Lines

The analysis and results presented in this chapter are based on a continuous material transfer line, with two classes of machines and finite buffers. In this section we present a description of the characteristics and assumptions of the model.

The processing behavior of a heterogeneous continuous-material transfer line is the same as the one described in Section 2.1.1. Raw material enters the first machine or working station where the process starts and then visits consecutive machines at which processing is performed. Perfectly reliable, finite buffers exist between consecutive machines. Whenever the downstream machine stops processing and the buffer is not full, material is stored in it.

According to their failure and repair behavior, machines are grouped into two classes:

- *Deterministically Failing* Machines. These machines fail (and processing stops) after a predetermined amount of material has been processed. It gets repaired (resumes processing) after a predetermined amount of time. We denote deterministically failing machines as M_i^D.

- *Exponentially Failing* Machines. These machines have exponentially distributed failure and repair times. We denote exponentially failing machines as M_i^E.

Line Parameters Now we introduce the parameters that characterize the behavior a heterogeneous continuous-material transfer line.

Exponentially Failing Machines Each exponentially failing machine, M_i^E, is characterized by the following parameters:

- Processing Rate μ_i. This is the rate at which material flows and get processed through machine M_i in the absence of blockage or starvation. In the presence of starvation or blockage, machine M_i will process material at a fraction of μ_i. The actual processing rate at each individual instant t is called $\mu_i(t)$.

- Failure Rate p_i. This is the rate at which machine M_i fails when working at its maximum processing rate μ_i. In the presence of blockage and starvation it will fail at a rate proportional to the fraction of the maximum processing speed at which it is operating. The actual failure rate at instant t is

$$p_i(t) = \frac{\mu_i(t)}{\mu_i} p_i$$

- Repair Rate r_i. This is the rate at which machine i gets repaired if it is down for a failure. This rate never varies. At any instant t:

$$r_i(t) = r_i$$

Deterministically Failing Machines Each deterministically failing machine, M_i^D, is characterized by the following parameters:

- Processing Rate μ_i. This is the rate at which material flows and get processed through machine M_i in the absence of blockage or starvation. In the presence of starvation or blockage, machine M_i will process material at a fraction of μ_i. The actual processing rate at each individual instant t is called $\mu_i(t)$.

- Material Processed Between Failures $\mathcal{P}_i$. This is the *amount of material* that has to be processed for machine M_i^D to fail. This amount of material is predetermined and remains fixed during the operation of the line.

- Repair Time $\mathcal{R}_i$. This is the amount of time after which machine M_i^D is repaired. This time is predetermined and remains unchanged during the operation of the line.

Buffers Buffer i stores material between machines i and $i + 1$. It is characterized by a single parameter:

- Buffer Size N_i. N_i is the maximum amount of material that can be stored in buffer i.

Performance Measures The measures of performance of line are production rate P, and mean buffer levels $\overline{x}_i$, as defined in Section 2.1.1.

6.3 Deterministically and Exponentially Failing Machines

Before we describe how to obtain an equivalent two-exponential-machine transfer line to represent the original heterogeneous transfer line, we analyze the behavior of two machines, one in each class, with *similar* average production behavior.

We study the difference in behavior between a deterministically and an exponentially failing machine, when operating in isolation. We consider a pair of machines with the same average isolated production rate.

We consider two machines, one failing exponentially, which we denote M^E, and one failing deterministically, denoted as M^D. Letting mean time to repair, $MTTR^E$ and mean time to fail, $MTTF^E$ be defined as in Section 3.3, and $MTTR^D$ and $MTTF^D$ be defined as follows:

$$\begin{aligned} MTTR^D &= \mathcal{R} \\ MTTF^D &= \frac{\mathcal{P}}{\mu^D} \end{aligned}$$

we consider a pair of machines M^E and M^D such that:

$$MTTR^E = MTTR^D \tag{6.1}$$

$$MTTF^E = MTTF^D \tag{6.2}$$

Conditions (6.1) and (6.2) require failures of both machines to have the same average duration, and repairs to take the same mean time to be completed. When these conditions are satisfied, we observe that the efficiency e of the isolated machines are the same:

$$e^D = e^E \tag{6.3}$$

We also impose the condition that both machines can process at the same maximum rate:

$$\mu^D = \mu^E \tag{6.4}$$

Expression (6.3) and Condition (6.4) lead to the following relationship between the two machines:

$$\rho^D = \rho^E. \tag{6.5}$$

That is, the mean production rates of the machines are the same.

A machine that satisfies conditions (6.4) and (6.5) is a good candidate for providing a good approximation of the long term average production rate of the original machine. We use this fact in the construction of a two-exponential-machine transfer line approximately equivalent to the heterogeneous system in Section 6.4.

Although the mean production rates for both isolated machines are the same, the failure and repair behavior differs significantly. Figure 6-1 shows how different are the distributions of failures for two machines, one with deterministic and the other with exponentially distributed failure times. Machines in Figure 6-1 satisfy Condition (6.2). The same difference can be observed between the repair distributions of the two machines.

This difference in the failure time distributions strongly influence the transient behavior of the machines and of their outputs. Figure 6-2 shows sample paths of the actual processing rate for each of the classes of machines. We observe that the sequence of failure and repair events over time differs significantly between the two machines.

The difference in the failure-repair behavior of the machines and, consequently, of the processing rate behavior, influences the performance of a line containing the

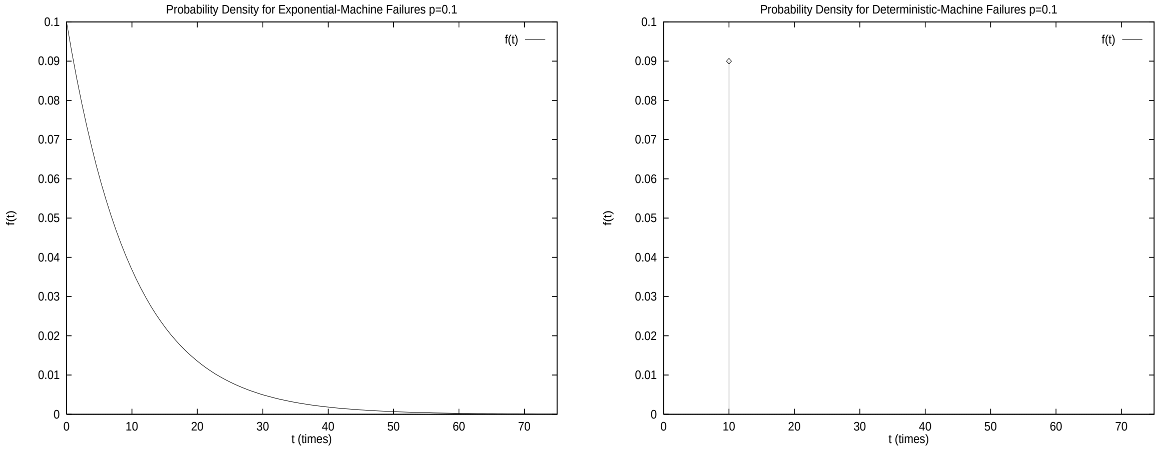

Figure 6-1: Distribution for the failure times for an exponentially-failing machine with $p = 0.1, \mu = 1.0$ and a deterministically-failing machine with $\mathcal{P} = 10, \mu = 1.0$ for the deterministic machine. Both machines have the same mean times to fail, but the distribution of the failure times differ significantly.

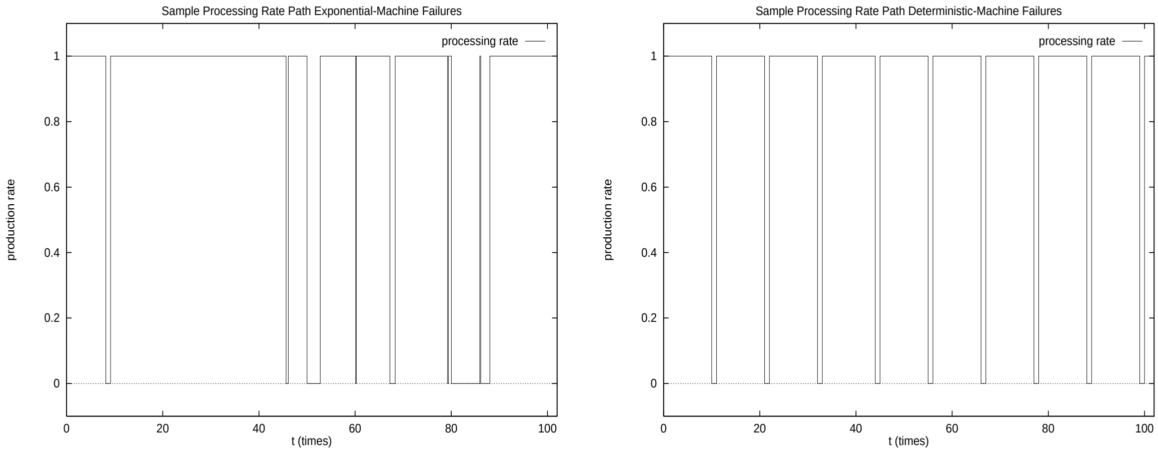

Figure 6-2: Sample Isolated Production Rate Paths. The plot in the left corresponds to an exponentially-failing machine, the one in the right to a deterministically- failing machine. Although the amount of material processed at the end of the period is approximately the same, the succession of failure and repair events differs significantly

machine. Even though the failure and repair rates are kept equal for both the deterministically-failing and the exponentially-failing machines, we may expect the trajectories of the buffers adjacent to the machines to differ.

To illustrate the difference in the trajectories of the levels of the buffers adjacent to the machines, we construct two two-machine lines. The parameters of the lines are shown in Table 6.1 and 6.2.

Table 6.1. Heterogeneous Transfer Line

	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
1	1.0	0.1	1.0	0.9091	0.9091	10
2	1.0	0.0	0.5	1.0000	0.5000	n/a

Table 6.2. Exponential Transfer Line

	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	1.0	0.1	1.0	0.9091	0.9091	10
2	1.0	0.0	0.5	1.0000	0.5000	n/a

For both lines, the second machine is a perfectly reliable machine. The processing rate is chosen to be lower than the one for the first machine to allow the buffer to fill and deplete. Figure 6-3 shows the buffer level trajectory for the two machines. The failure and repair events correspond to those shown in Figure 6-2.

We observe in Figure 6-3 that the buffer trajectories differ and, consequently, the mean buffer level differ from one line to the other.

In order to construct an exponentially-failing transfer line that represents the behavior of a heterogeneous line, we need to find exponentially-failing machines that satisfy conditions (6.4) and (6.5) and which transient failure-repair behavior allows for good estimates of mean buffer level.

In Section 6.4 we develop a method that numerically finds a two-exponential-machine line that closely represents the behavior of a heterogeneous transfer line in terms of mean production rate and mean buffer levels.

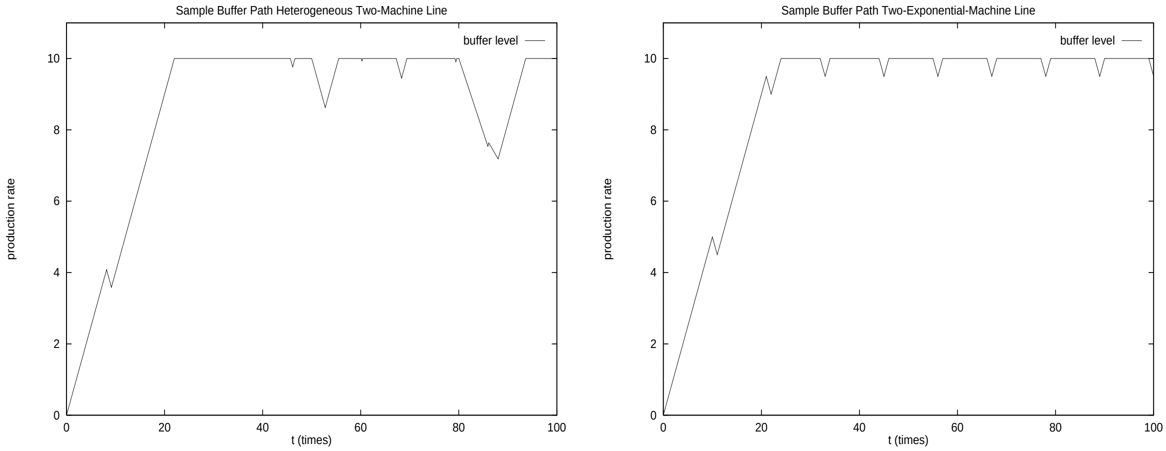

Figure 6-3: Sample Buffer Level Paths. The plot in the left corresponds to an exponentially-failing machine, the one in the right to a deterministically- failing machine. Although the amount of material processed at the end of the period is approximately the same, buffer level trajectory is different

6.4 Heterogeneous Two-Machine Transfer Lines

In this section we study heterogeneous continuous-material two-machine transfer lines. We focus on two-machines transfer lines where the first machine is a deterministically failing machine and the second is an exponentially failing machine. Accordingly to the definitions in Section 6.2, we denote the machines M_1^D and M_2^E.

We investigate the existence of a representation of this heterogeneous line as a two-machine transfer line with exponentially distributed failures and repair times and analyze how this representation is influenced by the parameters of the line.

6.4.1 Equivalent Two-Exponentially-Failing-Machine Transfer Line

In this section we obtain the approximately equivalent two-exponential-machine transfer line for several heterogeneous two-machine lines.

For the purposes of this research, we consider two transfer lines to be equivalent if their mean production rate P and mean buffer level $\overline{x}$ are the same. That is, we want to find a two-machine line, whose machine failures and repair are exponential and with approximately the same P and $\overline{x}$. We will evaluate the accuracy of the estimates using the error measures defined in Section 3.3, $\mathcal{E}_P$, $\mathcal{E}_x$ and $\mathcal{E}_N$.

The equivalent two-exponential-machine transfer line will consist of machines $\hat{M}_1$ and $\hat{M}_2$, with parameters $\hat{r}_i$, $\hat{p}_i$ and $\hat{\mu}_i$ for $i = 1, 2$; and buffer $\hat{B}_1$ with size $\hat{N}$.

Based on the discussion of the previous section about the behavior of heterogeneous and regular two-machine lines, we impose the following conditions on the equivalent exponential two-machine line:

1. **Maximum Processing Rate:** The maximum processing rate of the machines must remain unchanged, or

$$\hat{\mu}_1 = \mu_1^D \tag{6.6}$$

$$\hat{\mu}_2 = \mu_2^E \tag{6.7}$$

2. **Mean Production Rate:** The isolated production rate of the machines must remain unchanged, or

$$\hat{\rho}_i = \rho_i, \quad i = 1, 2. \tag{6.8}$$

Conditions (6.6) and (6.7) preserve the behavior of the machines while they are processing. Condition (6.8) ensures that thee average production rate remains unchanged.

However, Conditions (6.6) through (6.8) are not sufficient to guarantee that the transient behavior of the line is being preserved. As we discussed in Section 6.3, the deterministic and exponential failure and repair time distributions are very different, even when their means are the same. We also observed that the variance of the exponential distribution with mean λ is larger than the variance of the deterministically failing machine.

Obtaining a full analytical characterization of the equivalent transfer line is beyond the scope of this thesis. In the absence of a characterization of the equivalent two-exponential-machine line beyond what is specified by Conditions (6.6) through (6.8), we propose a search mechanism for obtaining a two-exponential-machine transfer line that approximately represents the behavior of the original heterogeneous line both in terms of production rate P and mean buffer level $\overline{x}$.

Such a mechanism allows us to check the hypothesis that such an approximately equivalent two-exponential-machine line exists. The search mechanism is also used in Section 6.5 to find an k-exponential-machine line approximately equivalent to a heterogeneous k-machine-line.

Search method Conditions (6.6) through (6.8) together with the previous observation about the variances of the two systems, lead to the following mechanism for obtaining the approximately equivalent two-exponential-machine line:

1. **Buffer:** The intermediate buffer remains unchanged:

$$\hat{N} = N$$

2. **Machine M_2:** Machine M_1 remains identical to the original:

$$\hat{p}_2 = p_2 \ , \quad \hat{r}_2 = r_2 \ \ and \ \ \hat{\mu}_2 = \mu_2$$

3. **Machine M_1:** Machine $\hat{M}_1$ is obtained numerically as follows:

   (a) **Maximum Processing Speed:** According to condition (6.6), the maximum processing rate remains identical:

$$\hat{\mu}_1 = \mu_1^D$$

(b) **Initialization of $\hat{p}_1$ and $\hat{r}_i$:** The starting values for $\hat{p}_1$ and $\hat{r}_i$, which we denote $\hat{p}_1^0$ and $\hat{r}_i^0$, which we are selected as follows:

$$\begin{aligned}\hat{p}_1^0 &= \frac{\mu_1^D}{\mathcal{P}}\\ \hat{r}_1^0 &= \frac{1}{\mathcal{R}}\end{aligned}$$

(c) **Search step:** Starting from the initial line parameters $(\hat{p}_1^0, \hat{r}_i^0, \hat{\mu}_1)$ increase the failure and repair rates $\hat{p}_1$ and $\hat{r}_i$, keeping unchanged the efficiency e_1 of the machine. For a given value of $\hat{p}_1$, this condition is satisfied by making $\hat{r}_1$ as follows:

$$\hat{r}_1 = \frac{\hat{e}_1}{1-\hat{e}_1}\hat{p}_1 = \frac{e_1}{1-e_1}\hat{p}_1 \tag{6.9}$$

(d) **Stopping Condition:** Let $\hat{p}_1$ increase until the buffer error $\mathcal{E}_{\overline{x}}$ is less than some small value ϵ.

We provide an example of the application of the search mechanism.

Example 6.1 We illustrate the method for obtaining the equivalent two-exponential-machine line for a heterogeneous two-machine line. The parameters of the original heterogeneous line are shown in Table 6.3.

Table 6.3. Example 6.1: Heterogeneous Line						
	Parameters					
i	$\mathcal{R}_1^{-1}$,r_2	$\mathcal{P}_1^{-1}$,p_2	μ_i	e_i	ρ_i	N_i
1	0.10	0.010	1.000	0.9091	0.9091	10
2	0.10	0.009	1.010	0.9174	0.9266	n/a

Following steps (1), (2), (3.a) and (3.b) of the method, we obtain the an initial equivalent system. The parameters of this system are shown in Table 6.4:

Table 6.4. Example 6.1: Initial Equivalent Line						
	Parameters					
i	r_2	p_2	μ_i	e_i	ρ_i	N_i
1	0.10	0.010	1.000	0.9091	0.9091	10
2	0.10	0.009	1.010	0.9174	0.9266	n/a

Before we show the results obtained after performing the search step (3.c) we check how accurate is the initial line whose parameters are shown in Table 6.4. Table 6.5 summarizes the performance measures of the original and the initial equivalent systems.

Table 6.5. Example 6.1: Performance Measures.		
	Line Performance Measures	
Model	Mean Prod. Rate	Mean Buffer Level
Original	0.874	2.595
Simplified	0.868	4.336
Error	-0.008	0.672

We observe in Table 6.5 that the initial system provides a very good estimate of the production rate P, with an error below 1%. However, this initial setting of the line parameters does not provide a good estimate for the mean buffer level $\overline{x}$. This initial representation overestimates the mean buffer level by approximately 67%.

To obtain a two-exponential-machine transfer line with a very close estimate for the mean buffer level $\overline{x}$, we perform the search step (3.c). We select ϵ, the maximum acceptable mean buffer error to be 0.01. The parameters of the final equivalent line are shown in Table 6.6.

Table 6.6. Example 6.1: Final Equivalent Line					
	Parameters				
i	r_i	p_i	μ_i	e_i	N_i
1	1.26	0.260	1.000	0.9091	10
2	0.10	0.009	1.010	0.9266	n/a

The performance measures obtained from the original and the modified system are summarized in Table 6.7.

Table 6.7. Example 6.1: Performance Measures.		
	Line Performance Measures	
Model	Mean Prod. Rate	Mean Buffer Level
Original	0.874	2.595
Simplified	0.876	2.618
Error	0.002	0.009

We observe in Table 6.6 that the failure and repair rates of the equivalent system, $\hat{p}_1$ and $\hat{r}_1$, are significantly larger than the initial setting for these parameters. That is, a machine with the same efficiency e and production rate μ, but significantly higher event frequency was required to replicate both the mean production rate and the mean buffer level of the original system. This is to be expected because of the lower variability of the output of the deterministically-failing machine compared to that of the exponentially-failing machine with equal mean times to fail and repair. Increasing the event frequency reduces the variability of the output.

Existence of the Approximately Equivalent Two-Exponential-Machine Transfer Line In Example 6.1 we show how a two-exponential-machine transfer line can be obtained such that its performance measure closely match those of the original heterogeneous two-machine line. We find such transfer line searching transfer lines that satisfy conditions (6.4) and (6.5).

Although we do not have theoretical proof of the existence of such an approximately equivalent transfer line for every heterogeneous two-machine line, we do have empirical evidence of it. We generated a large number of heterogeneous two-machine line and performed the search mechanism described earlier in this section. In all cases, an equivalent two-exponential-machine transfer line was found, with mean production rate P and mean buffer level $\overline{x}$ within 1% of the corresponding performance measures for the original heterogeneous system.

In all cases, the failure rate for the first machine of the equivalent two-exponential machine line $\hat{p}_1$ was equal to or larger than the initial setting $\hat{p}_1^0$. The ratio between the initial setting of $\hat{p}_1$ and its value at the equivalent line varies, for the cases we tried, from 1 to approximately 16.

6.5 Representation of Heterogeneous k-Machine Transfer Lines

In this section we describe how the search method presented in the previous chapter can be used to find an approximately equivalent representation of a heterogeneous k-machine line, using a k-exponential-machine transfer line. We introduce an algorithm for obtaining such an approximately equivalent line and then provide examples of its use.

Given a heterogeneous k-machine transfer line, we propose the following method for obtaining an equivalent k-exponential-machine transfer line:

Replace each deterministically-failing machine M_i^D in the line, by an exponentially-failing machine $\hat{M}_i$ obtained as follows:

1. Construct a heterogeneous two-machine line with machines M_i^D and M_{i+1}^E and buffer B_i.

2. Obtain the equivalent two-exponential-machine line, using the search mechanism described in Section 6.4. In the resulting two-exponential-machine transfer line, exponentially-failing machine $\hat{M}_i$ replaces the original deterministically-failing machine M_i^D.

3. In the original k-machine-line, replace machine M_i^D by machine $\hat{M}_i$ obtained in step (2).

The resulting line is a k-exponential-machine transfer line whose performance measures, $\hat{P}$ and $\hat{\bar{x}}_i$, can be used to estimates those of the original heterogeneous transfer line.

We illustrate the use of the representation method with the following example.

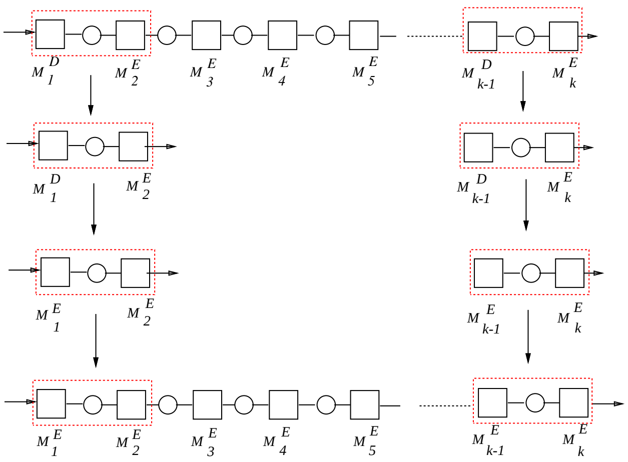

Figure 6-4: Schematic Representation of the Representation Method for a Heterogeneous k-Machine Transfer Line.

Example 6.2 We show the application of the representation method for a heterogeneous five-machine-line. The first machine of the line experiencesdeterministic failures and repairs, while the remaining machines have exponentially distributed failure and repair times. The parameters of the original heterogeneous line are shown in Table 6.8.

Table 6.8. Example 6.2: Original Line						
	Parameters					
i	$\mathcal{R}_1^{-1}, r_i$	$\mathcal{P}_1^{-1}, p_i$	μ_i	e_i	ρ_i	N_i
1	0.03333	0.00640	1.080	0.8852	0.9561	40
2	0.13158	0.01625	1.000	0.8901	0.8901	32
3	0.14158	0.01525	1.100	0.9028	0.9931	32
4	0.14158	0.01525	1.050	0.9028	0.9479	37
5	0.13558	0.01825	0.980	0.8813	0.8637	n/a

In order to obtain a representation of this heterogeneous line, we need to find an five-exponential-machine with similar mean production rate and mean buffer level.

First we construct the heterogeneous two-machine line consisting of machines M_1^D, M_2^E and buffer B_1. Using the method described in Section 6.4 we obtain the equivalent two-exponential-machine line.

The parameters of the heterogeneous two-machine line and the initial setting for the equivalent two-exponential-machine line are shown in Tables 6.9 and 6.10.

Table 6.9. Example 6.2. Heterogeneous Two-Machine Line						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
1	0.03333	0.00640	1.000	0.8852	0.9561	40
2	0.13158	0.01625	1.000	0.8901	0.8901	32

Table 6.10. Example 6.2 Initial Two-Exponential Line						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	0.03333	0.00640	1.000	0.8852	0.9561	40
2	0.13158	0.01625	1.000	0.8901	0.8901	n/a

Once the search step has been performed, we obtain the equivalent two-exponential-machine transfer line. The parameters of this line are shown in Table 6.11.

Table 6.11. Example 6.2: Equivalent Two-Exponential Line						
	Parameters					
i	r_i	p_i	μ_i	e_i	ρ_i	N_i
1	0.38297	0.05745	1.000	0.8852	0.9561	40
2	0.13158	0.01625	1.000	0.8901	0.8901	n/a

We replace the original deterministically-failing machine M_1 by the first machine of the two-exponential-machine line obtained in the previous step and whose parameters are shown in Table 6.11. The resulting equivalent line is a five-exponential-machine transfer line. The parameters of the equivalent line are summarized in Table 6.12.

Table 6.12. Example 6.2: Equivalent Five-Machine Line						
	Parameters					
i	$\mathcal{R}_1^{-1}, r_i$	$\mathcal{P}_1^{-1}, p_i$	μ_i	e_i	ρ_i	N_i
1	0.38297	0.05745	1.000	0.8852	0.9561	40
2	0.13158	0.01625	1.000	0.8901	0.8901	32
3	0.14158	0.01525	1.100	0.9028	0.9931	32
4	0.14158	0.01525	1.050	0.9028	0.9479	37
5	0.13558	0.01825	0.980	0.8813	0.8637	n/a

The performance measures obtained from the original heterogeneous line and from its equivalent exponential-machine line are summarized in Table 6.13.

Table 6.13. Example 6.2: Performance Measures.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.8474	0.8468	0.0008	
Measure			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_1$	18.21	19.01	0.0439	0.0200
$\overline{x}_2$	10.56	10.78	0.0208	0.0069
$\overline{x}_3$	15.76	15.83	0.0044	0.0022
$\overline{x}_4$	24.71	24.79	0.0032	0.0022
$\overline{X}$	69.24	70.41	0.0169	0.0083

The results, shown in Table 6.13, show that the equivalent five-exponential-machine line obtained using the representation method provides very close estimates for the values of the performance measures of the original system.

The estimate of mean production rate $\hat{P}$ is within 0.1% from the actual value P. The estimates for mean buffer level show errors ranging from 0.3 to approximately 4.4%, when measured relative to the actual mean buffer level, and from 0.2 to 2% when measured relative to buffer size. The total inventory in the line is estimated with less than 2% error.

Example 6.3 In this example, we consider a heterogeneous eight-machine transfer line. The second and seventh machines of the line experience deterministic failures and repairs, while the remaining machines have exponentially distributed failure and repair times. The parameters of the original heterogeneous line are shown in Table 6.14.

Table 6.14. Example 6.2: Original Line						
	Parameters					
machine	$\mathcal{R}_2^{-1}, \mathcal{R}_7^{-1}, r_i$	$\mathcal{P}_2^{-1}, \mathcal{P}_7^{-1}, p_i$	μ_i	e_i	ρ_i	N_i
M_1	0.778222	0.029663	1.083662	0.963283	1.043870	30
M_2	0.333333	0.028571	1.000000	0.921053	0.921053	38
M_3	0.252601	0.023618	0.920716	0.914495	0.841991	17
M_4	0.119707	0.016406	1.087048	0.879468	0.956024	35
M_5	0.187270	0.013213	1.197303	0.934094	1.118390	70
M_6	0.167882	0.015926	0.908481	0.913355	0.829766	50
M_7	0.133333	0.013333	1.000000	0.909091	0.909091	69
M_8	0.142255	0.013051	1.084158	0.915966	0.993052	n/a

For each of the deterministically-failing machines, we construct a heterogeneous two-machine line. Each line consists of the deterministically-failing machine, its downstream machine and their intermediate buffer.

The parameters of the heterogeneous two-machine lines corresponding to the two deterministically-failing machines are shown in Tables 6.15 and 6.16.

Table 6.15. Example 6.3. Heterogeneous Two-Machine Line for M_2^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_2	0.333333	0.028571	1.000000	0.921053	0.921053	38
M_3	0.252601	0.023618	0.920716	0.914495	0.841991	n/a

Table 6.16. Example 6.3. Heterogeneous Two-Machine Line for M_7^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_7	0.133333	0.013333	1.000000	0.909091	0.909091	69
M_8	0.142255	0.013051	1.084158	0.915966	0.993052	n/a

Once the heterogeneous two-machine line representation method of Section 6.4, we obtain the following two-exponential-machine lines, corresponding to the heterogeneous ones whose parameters are shown in Tables 6.15 and 6.16. The parameters of the equivalent lines are shown in Tables 6.17 and 6.18.

Table 6.17. Example 6.3: Equivalent Two-Exponential Line for M_2^D						
	Parameters					
machine	r_i	p_i	μ_i	e_i	ρ_i	N_i
M_2	1.373049	0.117690	1.000000	0.921053	0.921053	38
M_3	0.252601	0.023618	0.920716	0.914495	0.841991	n/a

Table 6.18. Example 6.3: Equivalent Two-Exponential Line for M_7^D						
	Parameters					
machine	r_i	p_i	μ_i	e_i	ρ_i	N_i
M_7	1.275748	0.127575	1.000000	0.909091	0.909091	69
M_8	0.142255	0.013051	1.084158	0.915966	0.993052	n/a

We replace the original deterministically-failing machines M_2^D and M_7^D by the first machine of the corresponding two-exponential-machine line obtained in the previous step. The resulting equivalent line is a five-exponential-machine transfer line. The parameters of the equivalent line are summarized in Table 6.19.

Table 6.19. Example 6.3: Equivalent Exponential Line						
	Parameters					
machine	r_i	p_i	μ_i	e_i	ρ_i	N_i
M_1	0.778222	0.029663	1.083662	0.963283	1.043870	30
M_2	1.373049	0.117690	1.000000	0.921053	0.921053	38
M_3	0.252601	0.023618	0.920716	0.914495	0.841991	17
M_4	0.119707	0.016406	1.087048	0.879468	0.956024	35
M_5	0.187270	0.013213	1.197303	0.934094	1.118390	70
M_6	0.167882	0.015926	0.908481	0.913355	0.829766	50
M_7	1.275748	0.127575	1.000000	0.909091	0.909091	69
M_8	0.142255	0.013051	1.084158	0.915966	0.993052	n/a

The performance measures obtained from the original heterogeneous transfer line and from its equivalent exponential-machine transfer line are summarized in Table 6.20.

Table 6.20. Example 6.3: Performance Measures.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.82028	0.81934	0.0012	
Measure			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_1$	29.75	29.76	0.0003	0.0003
$\overline{x}_2$	35.90	36.80	0.0251	0.0237
$\overline{x}_3$	3.88	3.95	0.0180	0.0042
$\overline{x}_4$	7.46	7.21	-0.0335	-0.0071
$\overline{x}_5$	42.35	42.95	0.0142	0.0086
$\overline{x}_6$	1.82	1.71	-0.0604	-0.0022
$\overline{x}_7$	2.81	2.80	-0.0036	-0.0001
$\overline{X}$	123.97	125.18	0.0098	0.0004

The quantities shown in Table 6.20, show that the equivalent eight-exponential-machine line obtained using the representation method provides very close estimates for the values of the performance measures of the original system.

The estimate of mean production rate $\hat{P}$ is within 0.12% from the actual value P. The estimates for mean buffer level show errors ranging from 0.3 to approximately 7%, when measured relative to the actual mean buffer level, and from 0.01 to 2.4%

when measured relative to buffer size. The total inventory in the line is estimated with less than 1% error.

In Chapter 7 we provide an additional example of the use of the representation techniques introduced in this chapter.

Chapter 7

Heterogeneous Transfer Lines — An Application of the Representation Method

In Chapter 6 we describe a method for representing a heterogeneous k-machine transfer-line as a k-exponential-machine transfer line. In this chapter we examine a situation in which the representation tools of the previous chapter can be applied.

In the case we consider in this chapter, machines are unreliable and can experience exponentially distributed failures and repairs. In addition to these failures, processing at the machines may be interrupted in order to perform a pre-scheduled operation.

We show how the tools of the previous chapter can be used to model machine set-ups. In a manufacturing line, machines have to be prepared for operation. During set-ups, the machine is prevented from processing material. We consider two cases. The first corresponds to frequent and fast set-ups. The second case correspond to a change in the machine in order to serve one or more different lines. In this case we will refer to the machine as a *shared machine*.

This application requires some modeling before the representation is applied. We start by describing how it can be represented as a heterogeneous transfer line. Then we present a detailed example of the use of the representation method for analyzing set-ups in a transfer line.

7.1 Definition of a Transfer Line with Failures and Set-Ups

Machines in a transfer line need to be prepared for operation. This preparation of the machine or *set-up* may occur at the beginning of the working day, or after a certain amount of material has been processed. It may be the case that a particular machine can process more than one product type; in this case, it might have to be set up every time the product that is being processed changes.

A set-up can be viewed as an interruption of processing in the particular machine. One could model set-ups using the models available to model failuires, using the mean time between set-up and their mean duration as parameters. However, set-ups usually occur in a predictable manner. Consequently, a model that accounts for the deterministic fashion of this events would be preferred.

In this section we consider transfer lines whose machines, in addition to experiencing exponentially distributed failures and repairs, have to be set up after a certain amount of material has been processed. We consider the time needed to complete the set-up to be predetermined and fixed.

Each machine M_i^S in this model can be described by the following parameters:

- Failure, Repair and Processing Rates, p_i, r_i and μ_i. This parameters correspond to those of machines in a continuous-material transfer line as described in Section 2.1.1.
- Material Flow between Set-ups, $\mathcal{P}_i^S$. This is the amount of material that has to be processed between two consecutive set-ups of the machine.

- Set-up Time, $\mathcal{R}_i^S$. This is the time that takes to complete set-up of machine i. This time is predetermined and fixed.

There exist a direct correspondence between parameters $\mathcal{P}_i^S$ and $\mathcal{R}_i^S$ defined above and parameters $\mathcal{P}_i$ and $\mathcal{R}_i$, defined for the deterministically failing machine in Section 6.2.1.

The set-up model we define here allows each machine to experience both exponentially distributed and deterministic failures and repairs. However, in Section 6.2.1, we defined as a heterogeneous transferline a line where some machines experience exponentially distributed failures and repairs while others fail and get repaired deterministically. Before we can apply the methods introduced in Chapter 6, we need to model the set-up transfer line defined above as a heterogeneous transfer line.

7.2 Application of the Representation Tools to the Analysis of Set-ups

We need to model each machine with failures and set-ups using machines with only one kind of failure. We can do this by replacing each machine with failures and set-ups by a pair of machines, one representing the exponentially distributed failure and repair behavior, the other representing the set-up (deterministic) interruption behavior. That is, for each machine M_i^S, construct a pair of machines, M_i^E and M_i^D, as follows:

- **Exponentially-Failing Machine** Machine M_i^E experiences only the exponentially distributed failures and repairs of the original machine M_i^S. Thus, M_i^E has parameters p_i, r_i and μ_i.

- **Deterministically-Failing Machine** Machine M_i^E experiences only the deterministic interruptions of the original machine M_i^S. Thus, M_i^D has parameters $\mathcal{P}_i$, $\mathcal{R}_i$ and μ_i.

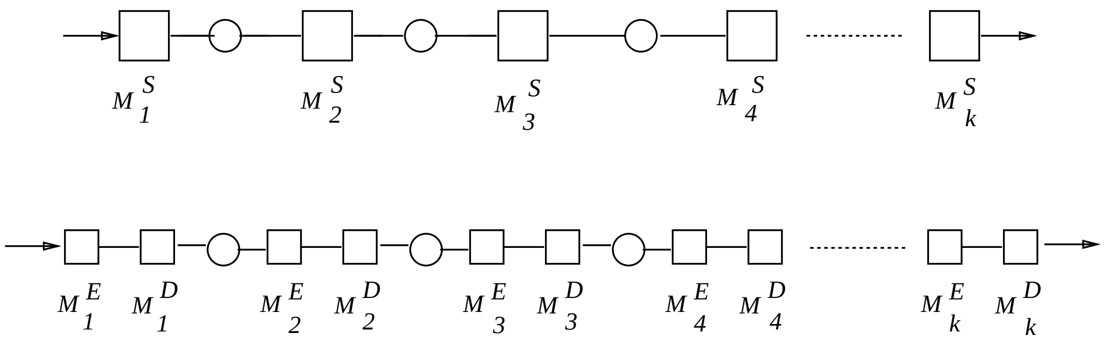

Figure 7-1: Representation of Machines with Setup as a Heterogenenous Transfer Line

The original k-machine line, of the form $\{M_1^S, B_1, M_2^S, B_2, ..., M_{k-1}^S, B_{k-1}, M_k^S\}$ is replaced by a heterogeneous $2k$-machine line of the form:

$$\{M_1^E, M_1^D, B_1, M_2^E, M_2^D, B_2, ..., M_{k-1}^E, M_{k-1}^D, B_{k-1}, M_k^E, M_k^D\}$$

where there is no buffer separating the two machines that represent each original machine with set-ups. Figure 7-1 illustrates this representation.

Once the machines with set-ups M_i^S have been represented as two consecutive machines in a heterogeneous transfer line, we can use the methods of Section 6 to represent the heterogeneous line as a exponential-machine transfer line.

Example 7.1. Fast set-up In this example we consider a five-machine transfer line, where each individual machine experiences exponentially distributed failures and repair and is set-up after a fixed amount of material is processed. The parameters of this line are shown in Table 7.1.

Table 7.1. Example 7.1: Original line with set-ups

	Parameters					
Machine	r_i	p_i	$\mathcal{R}_i^S$	$\mathcal{P}_i^S$	μ_i	N_i
M_1^S	0.1531	0.0144	1.3	10.0	1.00	21
M_2^S	0.1874	0.0144	1.1	11.0	1.00	16
M_3^S	0.1014	0.0102	0.9	10.0	1.00	24
M_4^S	0.1781	0.0164	1.2	12.0	1.00	17
M_5^S	0.1587	0.0137	1.0	10.0	1.00	n/a

Each machine M_i^S is replaced by a pair of machines, M_i^E and M_i^D. We obtain the heterogeneous transfer line whose parameters are shown in Table 7.2.

Table 7.2. Example 7.1: Heterogeneous Line						
	Parameters					
Machine	$\mathcal{R}_i^S$,r_i	$\mathcal{P}_i^S$, p_i	e_i	ρ_i	μ_i	N_i
M_1^E	0.1531	0.0144	1.00	0.9140	0.9140	0
M_1^D	0.7692	0.1000	1.00	0.8850	0.9872	21
M_2^E	0.1874	0.0144	1.00	0.9286	0.9276	0
M_2^D	0.9090	0.0909	1.00	0.9090	0.9090	16
M_3^E	0.1014	0.0102	1.00	0.8942	0.8942	0
M_3^D	1.1000	0.1000	1.00	0.9166	0.9166	24
M_4^E	0.1781	0.0164	1.00	0.9157	0.9157	0
M_4^D	0.8333	0.0833	1.00	0.9091	0.9091	17
M_5^E	0.1587	0.0137	1.00	0.9205	0.9205	0
M_5^D	1.0000	0.1000	1.00	0.9091	0.9091	n/a

Once the heterogeneous $2k$-machine transfer line has been constructed we can apply the tools presented in Chapter 6 for representing this heterogeneous line as a $2k$-exponential-machine transfer line.

For each of the machines with deterministic failures and repairs, M_i^D, we construct the two-machine transfer line with machines M_i^D, M_{i+1}^E and buffer B_i. Tables 7.3 through 7.7 shows the parameters for the five two-machine lines generated in this step.

Table 7.3. Example 7.1. Heterogeneous Two-Machine Line for M_1^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_1^D	0.7692	0.1000	1.00	0.8850	0.9872	21
M_2^E	0.1874	0.0144	1.00	0.9286	0.9276	0

Table 7.4. Example 7.1. Heterogeneous Two-Machine Line for M_2^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_2^D	0.9090	0.0909	1.00	0.9090	0.9090	16
M_3^E	0.1014	0.0102	1.00	0.8942	0.8942	0

Table 7.5. Example 7.1. Heterogeneous Two-Machine Line for M_3^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_3^D	1.1000	0.1000	1.00	0.9166	0.9166	24
M_4^E	0.1781	0.0164	1.00	0.9157	0.9157	0

Table 7.6. Example 7.1. Heterogeneous Two-Machine Line for M_4^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_4^D	0.8333	0.0833	1.00	0.9091	0.9091	17
M_5^E	0.1587	0.0137	1.00	0.9205	0.9205	0

Table 7.7. Example 7.1. Heterogeneous Two-Machine Line for M_5^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_5^D	1.0000	0.100	1.00	0.9091	0.9091	10000
	1.0	0.0000	1.000	1.00	1.0000	n/a

Since machine M_5^D is the last machine in the line, we construct its corresponding heterogeneous two-machine-line replacing the second machine by a perfecly reliable machine, with processing rate higher than the processing speed of M_5^D. This is done to represent the behavior at the end of the line, where there is never blockage.

We use the search algorithm presented in Section 6.2.1 to obtain the exponentially-failing machines that replace the deterministic one in the equivalent exponential-machine line. Tables 7.8 through 7.12 show the equivalent two-exponential-machine lines corresponding to the heterogeneous two-machine lines whose parameters are shown in Tables 7.3 through 7.7.

Table 7.8. Example 7.1. Equivalent Two-Machine Line for M_1^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_1^D	8.2692	1.0750	1.00	0.8850	0.9872	21
M_2^E	0.1874	0.0144	1.00	0.9286	0.9276	0

Table 7.9. Example 7.1. Equivalent Two-Machine Line for M_2^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_2^D	6.2508	0.6251	1.00	0.9090	0.9090	16
M_3^E	0.1014	0.0102	1.00	0.8942	0.8942	0

Table 7.10. Example 7.1. Equivalent Two-Machine Line for M_3^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_3^D	1.9160	0.1724	1.00	0.9166	0.9166	24
M_4^E	0.1781	0.0164	1.00	0.9157	0.9157	0

Table 7.11. Example 7.1. Equivalent Two-Machine Line for M_4^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_4^D	3.9780	0.3978	1.00	0.9091	0.9091	17
M_5^E	0.1587	0.0137	1.00	0.9205	0.9205	0

Table 7.12. Example 7.1. Equivalent Two-Machine Line for M_5^D						
	Parameters					
i	$\mathcal{R}_1^{-1}$, r_2	$\mathcal{P}_1^{-1}$, p_2	μ_i	e_i	ρ_i	N_i
M_5^D	1.0000	0.100	1.00	0.9091	0.9091	10000
	1.0	0.0000	1.000	1.00	1.0000	n/a

Replacing the equivalent exponential machines in the original heterogeneous ten-machine-line, we obtain the equivalent ten-exponential-machine transfer line. The parameters of this line are shown in Table 7.13.

Table 7.13. Example 7.1: Heterogeneous Line						
	Parameters					
i	r_i	p_i	e_i	ρ_i	μ_i	N_i
1	0.1531	0.0144	1.00	0.9140	0.9140	0
2	8.2692	1.0750	1.00	0.8850	0.9872	21
3	0.1874	0.0144	1.00	0.9286	0.9276	0
4	6.2508	0.6251	1.00	0.9090	0.9090	16
5	0.1014	0.0102	1.00	0.8942	0.8942	0
6	1.9160	0.1724	1.00	0.9166	0.9166	24
7	0.1781	0.0164	1.00	0.9157	0.9157	0
8	3.9780	0.3978	1.00	0.9091	0.9091	17
9	0.1587	0.0137	1.00	0.9205	0.9205	0
10	1.0000	0.1000	1.00	0.9091	0.9091	n/a

Table 7.14 shows the performance measures obtained with the original heterogeneous transfer line and those obtained with the approximately equivalent exponential-machine line.

Table 7.14. Example 7.1: Performance Measures.				
	Model		Errors	
Measure	Original	Simplified	$\mathcal{E}_P$	
P	0.68354	0.68511	0.0023	
Measure			$\mathcal{E}_{\overline{x}i}$	$\mathcal{E}_{Ni}$
$\overline{x}_1$	11.29	11.68	0.0345	0.0186
$\overline{x}_2$	7.68	7.69	0.0010	0.0006
$\overline{x}_3$	9.95	10.83	0.0884	0.0366
$\overline{x}_4$	5.10	4.79	-0.0608	0.0182
$\overline{X}$	34.02	34.99	0.0285	0.0124

The results in Table 6.20 show that the equivalent ten-exponential-machine line obtained using the representation method provides very close estimates for the values of the performance measures of the original system with setups and failures.

The estimate of mean production rate $\hat{P}$ is within 0.22% from the actual value P. The estimates for mean buffer level show errors ranging from 0.1 to approximately 9%, when measured relative to the actual mean buffer level, and from 0.06 to 3.7% when measured relative to buffer size. The total inventory in the line is estimated with less than 3% error, with respect to the actual mean total buffer level and with an error just above 1% when measured with respect to total buffer space.

Chapter 8

Conclusions

8.1 Summary of Results

In this thesis we develop a set of modeling and analysis tools for two classes of transfer lines. The methods we introduce in this thesis allow the treatment of a larger set of transfer lines with the analytical tools already available to model continuous-material transfer lines with exponentially distributed failures and repairs.

This thesis examines a set of tools for the representation of two classes of continuous-material transfer lines, multiple-event-frequency transfer lines and heterogeneous transfer lines.

We summarize separately the results obtained from the analysis of each of these classes transfer line. In Section 8.1.1 we report the results on the event-frequency decomposition of transfer line, and in Section 8.1.2 we present the results of our analysis of heterogeneous transfer lines.

8.1.1 Event Frequency Decomposition

We examine continuous-material transfer line characterized by the fact that events occur at significantly different frequencies at different machines. This event frequency induces a hierarchical decomposition of the transfer line.

In this thesis we define the set of transfer lines that we characterize as multiple-frequency transfer line and define the corresponding event hierarchy.

We develop a method for the representation of a two-frequency, two-machine transfer line at the higher level of the hierarchy. This method replaces the high-event-frequency machine by a perfectly reliable machine with reduced maximum processing rate μ, equal to the isolated mean production rate of the original high-event-frequency machine.

We show that this representation of the line provides close estimates of the production rate P of the original line. We examine the estimates of mean buffer level $\overline{x}$ and find that these are not as accurate as those for production rate, with errors reaching as much as 16%.

We characterize the differences in the buffer level trajectory of the original and the simplified (reliable) machine and, on the basis of this characterization, propose a method for improving the estimate for mean buffer level provided by the simplified single-frequency, two-machine line. We examine the performance of this correction using results from a large number of ran domly generated cases.

In Chapter 4, we extend the representation method to F-frequency, k-machine lines. As for two-frequency, two-machine lines, the estimates for mean production rate P are very accurate, while the estimates for individual buffer levels and mean total buffer level are significantly less accurate. Errors for the mean total buffer level $\overline{X}$ show errors of as much as 7%.

In Chapter 5 we apply the decomposition method to the analysis of another two classes of transfer lines. We consider transfer lines whose machines can experience several failure modes, and transfer lines whose machine are grouped in cell. The tools developed for frequency decomposition are shown to be useful in modeling these two situations.

8.1.2 Heterogeneous Continuous-Material Transfer Lines

We examine the cases of continuous-material transfer lines whose machine can experience failures and repairs either according to an exponential process or in a deterministic manner. In particular, we focus in transfer lines in which some of the machine fail (stop processing) after having processed a predetermined and fixed amount of material.

First, we analyze the two-machine case, where one machine experiences deterministic failures and repairs and the other experiences exponentially distributed events. We show that there exists an exponential machine line from whose analysis we obtain close estimates of performance measures of the original heterogeneous line.

We also show that the equivalent two-exponential-machine line can be used in obtaining an equivalent k-exponential-machine that replaces a heterogeneous k-machine line. We develop an algorithm for obtaining this equivalent line and show the performance of the method with an example.

Finally, we show how the representation method for heterogeneous machines can be used to analyze transfer lines whose machines experience exponentially distributed failures and repairs and deterministic set-ups.

8.2 Further Research

There is significant research to be done in the area of modeling more general transfer line and, in particular, the classes of transfer line we consider in this thesis.

In the case of the frequency decomposition, a better characterization of the buffer trajectories for the two-machine case could be obtained. Such a closer characterization would allow for a better correction of the estimates provided by the simplified line for mean buffer levels.

In the case of longer lines, a correction mechanism, possibly similar to the developed for the two-machine case, can be developed, allowing for the correction of the estimates of individual and total mean buffer levels.

For the heterogeneous transfer line problem, we observe that the method used to find the equivalent two-exponential-machine line involves a search in the set of transfer line that satisfy preserves the same mean isolated production rate and maximum processing rate of the machines in the original line.

A first step can be the development a better search algorithm, more efficient than the linear incremental search we suggest in this thesis. A further advance can be represented by an analytical characterization, or approximate characterization, of how the parameters of the equivalent machine are influenced by those of the original line. Such a characterization will allow us to obtain the equivalent machine without time-consuming searches.

Appendix A

Simulation Software

A.1 The software

The preformance measures for k-machine continuous-material transfer lines, for $k > 2$ reported in this thesis were obtained using a continuous-time continuous-material transfer line simulator written by this author.

This software simulates heterogeneous k-machine transfer lines. That is, it allows each machine in the line to have a different failure and repair distribution and rate. The simulator can be used to obtain performance measures, like mean buffer level and production rate, and to generate sample cumulative production or buffer level trajectories.

Figures 3-12, comparing the buffer level trajectory of the original and the simplified machine line in Chapter 3, and 6-2, showing sample processing rate for individual deterministic and expontial machines were generated using this simulator.

A.2 Result Reported

Although in this thesis we always present a single quantity for each performance measure, the reported results always correspond to the mean value of the particular performance measure over a set of at least 5 simulations.

Each simulation was run enough time to provide 95% confidence intervals around the means that were not wider than 2% on each side. This applies for both production rate and mean buffer levels.

Bibliography

[1] M. Burman (1995), "New Results in Flow Line Analysis", PhD Dissertation, M.I.T, Cambridge Ma.

[2] Y. Dallery, R. David, and X.-L. Xie (1987), "Approximate Analysis of Transfer Lines with Unreliable Machines and Finite Buffers," Technical Report LAG Number 87-64, June, 1987, Grenoble.

[3] S. B. Gershwin (1994), *Manufacturing Systems Engineering,* Prentice-Hall, 1994.

[4] Suri, R and Fu, B. "On Using Continuous Flow Lines for Performance Estimates of Discrete Production Lines" *t Journal of Discrete Event Dynamic Systems*, 1992.