Lecture 21: Optimal Routing

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Optimal Routing

- View routing as a "global" optimization problem

- Assumptions:
  - The cost of using a link is a function of the flow on that link
  - The total network cost is the sum of the link costs
  - The required traffic rate between each source-destination pair is known in advance
  - Traffic between source-destination pair can be split along multiple paths with infinite precision

- Find the paths (and associated traffic flows) along which to route all of the traffic such that the total cost is minimized

Formulation of optimal routing

- **Let Dij (fij) be the cost function for using link (i,j) with flow fij**
  - **Fij is the total traffic flow along link (i,j)**
  - **Dij() can represent delay or queue size along the link**
  - **Assume Dij is a differentiable function**

- **Let D(F) be the total cost for the network with flow vector F**

- **Assume additive cost: D(F) = $\text{Sum}_{(ij)}$ Dij (fij)**

- **For S-D pair w with total rate r_w**
  - **P_w is the set of paths between S and D**
  - **X_p is the rate sent along path $p \in P_w$**

$$S.t. \sum_{p \in P_w} X_p = r_w, \quad \forall w \in W \qquad fij = \sum_{all\ p\ containing\ (i,j)} X_p$$

Formulation continued

- **Optimal routing problem can now be written as:**

$$Min\, D(F) \;\; S.t. \sum_{p \in P_w} X_p = r_w, \quad \forall w \in W$$

$$\Rightarrow \; Min \sum_{(i,j)} D_{(i,j)} \left[\sum_{p\, contains\, (i,j)} X_p \right] \; s.t. \sum_{p \in P_w} X_p = r_w, \quad \forall w \in W$$

Optimal routing solution

- Let $dD(*)/dx_p$ be the partial derivative of D with respect to X_p
- Then,
- $D'_{xp} = dD(*)/dx_p = Sum_{(i,j)\in p}\ D'_{(I,j)}$
  - Where $D'_{(i,j)}$ is evaluated at the total flow corresponding to x_p
- D'_{xp} consists of first derivative lengths along path p

Optimal routing solution continued

- Suppose now that $X^* = \{x^*_p\}$ is an optimal flow vector for some S-D pair w with paths P_W
- Any shift in traffic from any path p to some other path p' cannot possibly decrease the total cost (since X* is assumed optimal)
- Define Δ as the change in cost due to a shift of a small amount of traffic (δ) from some path p with x*p > 0 to another path p'

$$\Delta = \delta \frac{\partial D(X^*)}{\partial x_{p'}} - \delta \frac{\partial D(X^*)}{\partial x_p} \geq 0 \Rightarrow \frac{\partial D(X^*)}{\partial x_{p'}} \geq \frac{\partial D(X^*)}{\partial x_p}, \quad \forall p' \in P_w$$

- Optimality conditions (necessary and sufficient):
  - optimal flows can only be positive on paths with minimum first derivative lengths
  - All paths along which r_w is split must have same first derivative lengths

Example

Example, continued

Routing in the Internet

- Autonomous systems (AS)
  - Internet is divided into AS's each under the control of a single authority
- Routing protocol can be classified in two categories
  - Interior protocols - operate within an AS
  - Exterior protocols - operate between AS's

- Interior protocols
  - Typically use shortest path algorithms
    - Distance vector - based on distributed Bellman-ford
    - link state protocols - Based on "distributed" Dijkstra's

Distance vector protocols

- Based on distributed Bellman-Ford
  - Nodes exchange routing table information with their neighbors
- Examples:
  - Routing information protocols (RIP)
    - Metric used is hop-count (dij=1)
    - Routing information exchanged every 30 seconds
  - Interior Gateway Routing Protocol (IGRP)
    - CISCO proprietary
    - Metric takes load into account
    - Dij ~ $1/(\mu - \lambda)$ (estimate delay through link)
    - Update every 90 seconds
    - Multi-path routing capability

Link State Protocols

- Based on Dijkstra's Shortest path algorithm
  - Avoids loops
  - Routers monitor the state of their outgoing links
  - Routers broadcast the state of their links within the AS
  - Every node knows the status of all links and can calculate all routes using dijkstra's algorithm
    - Nonetheless, nodes only send packet to the next node along the route with the packets destination address. The next node will look-up the address in the routing table

- Example: Open Shortest Path First (OSPF) commonly used in the internet

- Link State protocols typically generate less "control" traffic than Distance-vector

Inter-Domain routing

- Used to route packets across different AS's
- Options:
  - Static routing - manually configured routes
  - Distance-vector routing
    - Exterior Gateway Protocol (EGP)
    - Border Gateway Protocol (BGP)
- Issues
  - What cost "metric" to use for Distance-Vector routing
    - Policy issues: Network provider A may not want B's packets routed through its network or two network providers may have an agreement
    - Cost issues: Network providers may charge each other for dlivery of packets

Bridges, Routers and Gateways

- A Bridge is used to connect multiple LAN segments
  - Layer 2 routing (Ethernet)
  - Does not know IP address
  - Varying levels of sophistication
    - Simple bridges just forward packets
    - smart bridges start looking like routers
- A Router is used to route connect between different networks using network layer address
  - Within or between Autonomous Systems
  - Using same protocol (e.g., IP, ATM)
- A Gateway connects between networks using different protocols
  - Protocol conversion
  - Address resolution
- These definitions are often mixed and seem to evolve!

Bridges, routers and gateways