# ICRF Wave Simulation in the VASIMR Plasma

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An important motivation for the mathematical simulation in a Variable Specific Impulse Magnetoplasma Rocket (VASIMR)[1] is plasma heating by ion-cyclotron RF (ICRF) waves[2]. The current work emphasizes on results of the implementation of an iterative solver for the complex algebraic system. The suggested iterative method is Multigrid - Incomplete Cholesky Preconditioned Conjugate Gradient Squared solver. In the EMIR[3] code, the RF electric field, $\boldsymbol{E_{RF}}$, magnetic field, $\boldsymbol{B}_{RF}$, and RF antenna current density, $\boldsymbol{j_{ANT}}$, are expanded in a periodic Fourier sum in the azimuthal coordinate. Implicit time dependence is assumed as well as azimuthal symmetry of the equilibrium quantities, so that the fields and currents can be expanded into azimuthal modes: $\boldsymbol{E}_{RF}(r,f,z,t)=\sum_m \boldsymbol{E}_m(r,z)e^{imf-iwt}$, where $w$ is RF frequency. The RF fields are obtained by solving Maxwell's equations, written in harmonic form:

$$-\nabla\times\nabla\times\boldsymbol{E_{RF}}+(w^2/c^2)(\boldsymbol{E_{RF}}+(i/we)\boldsymbol{j_p})=-iwm\boldsymbol{j}_{ANT}.$$

In the current EMIR implementation, the plasma current density $\boldsymbol{j}_P$ is related to the electric field by a collisional cold plasma conductivity tensor $\boldsymbol{j}_p=\hat{s}\cdot\boldsymbol{E}_{RF}$. This equation can then be represented by system of independent equations with respect to $\boldsymbol{E}_m$ as suggested by Stix[4]: $-e^{-imf}\nabla\times\nabla\times\boldsymbol{E}_m e^{imf}+(w^2/c^2)\hat{\boldsymbol{K}}\cdot\boldsymbol{E}_m=-iwm\boldsymbol{j}_m$, where $\hat{\boldsymbol{K}}=\boldsymbol{I}+(i/we)\hat{s}$ is a cold plasma dielectric tensor: $\hat{\boldsymbol{K}}=\begin{pmatrix}K_\perp & -iK_f & 0\\ iK_f & K_\perp & 0\\ 0 & 0 & K_{||}\end{pmatrix}$ and $\boldsymbol{j}_m$ is the current density externally applied by an antenna. The entries of the dielectric tensor depend on the plasma density $n_i$ and the vacuum magnetic field $B_0$ and the driven frequency $w$ for a multiple-ion plasma as follows: $K_\perp=1-\sum_{l=e,i}w_{pl}^2/(w^2-w_{cl}^2)$, $K_f=\sum_{l=e,i}w_{ci}w_{pl}^2/(w^3-ww_{ci}^2)$, $K_{||}=1-\sum_{l=e,i}w_{pl}^2/w^2$, $w_{pl}^2=e^2n_l/(e_0\tilde{m}_l)$, $w_{cl}=eB_0/\tilde{m}_l$, where the sum is over the electrons and all ion species. Absorption is introduced in the cold plasma model by adding an imaginary collision frequency to the RF driven frequency, which is equivalent to adding an imaginary particle mass in the dielectric tensor elements: $\tilde{m}_l=m_l(1+ia_l)=m_l(1+in_l/w)$.

The implementation of the fast iterative solver for the described system is very challenging task, because the matrix is complex, ill-conditioned and non-symmetric.

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