Supplemental Information

Natalie G. Montoya | January 9, 2023

Calculations

This supplement details the calculations used to account for missile accuracy and estimate the number of silos that would survive a nuclear attack. Most was excerpted from Chapters 3 and 4 of the author's thesis: "No Winning Moves: Calculated Casualties and Damages of a Nuclear Attack on the United States by Russia for First and Second Strike Scenarios," Massachusetts Institute of Technology, Cambridge, MA, 2021. Modifications have been made as applicable to this article.

Distribution of Radial Error

Contrary to the saying "Close doesn't count except in horseshoes, hand grenades, and nuclear bombs," accuracy is critical in nuclear targeting, especially for hardened targets; therefore, error must be accounted for in order to estimate realistic damages rather than idealized ones. The prime example of this, as will be discussed in the next section, is determining the survivability of silos. By performing random sampling assuming a normal distribution of detonation points about the desired ground zero (DGZ), over repeated trials, the average number of silos expected to survive a specified attack can be estimated. In turn, the number of silos expected to survive determines what forces are available for a retaliatory strike.

For each warhead, actual detonation coordinates can be calculated using the DGZ, the circular error probable (CEP) of the missile, and the direction of approach—the full derivation of the equations is detailed in Edmundson's work in "The Distribution of Radial Error and Its Statistical Application in War Gaming" [1]. In order to simplify the computation and increase the processing speed, the calculation was modified to use SI units and coded in Python, which allows for the calculation of an entire attack at once. Each warhead's detonation coordinates were found using the following method.

Because the circular error probable is the known measure of accuracy for the missiles being considered, the radial error is calculated in two-dimensions (rather than three-dimensions, which uses the spherical error probable). First, the standard deviation σ is calculated as:

$$\sigma = \frac{CEP}{\sqrt{2\ln(2)}} \tag{1}$$

Wherein CEP is the median radial error and $\sqrt{2\ln(2)} = 1.1774$ is a statistical constant for a circular Gaussian distribution [2]. The change in distance in each dimension is then calculated as:

$$d_x = \sigma r_x \tag{2}$$

$$d_y = \sigma r_y \tag{3}$$

Wherein r_x and r_y are Gaussian random numbers from a standard normal distribution, and d_x and d_y are the displacements in each dimension. The radial displacement d and bearing θ are calculated using the Pythagorean theorem, and $\frac{\pi}{4}$ radians are added to the bearing in order to rotate the axis to align with the approximate direction of approach. The final detonation coordinates are calculated from the following equations [3]:

$$Lat_f = \arcsin(\sin(Lat_i)\cos\left(\frac{d}{R}\right) + \cos(Lat_i)\sin\left(\frac{d}{R}\right)\cos(\theta)) \tag{4}$$

$$Lng_f = Lng_i + \arctan\left(\frac{\sin(\theta)\sin(\frac{d}{R})\cos(Lat_i)}{\cos(\frac{d}{R}) - \sin(Lat_i)\sin(Lat_f)}\right) \tag{5}$$

Wherein Lat_i and Lng_i are the coordinates of the DGZ, R is the radius of the Earth[1], and Lat_f and Lng_f are the coordinates of the final detonation point. For the silo survivability application, the radial displacement d is the result used in calculations; however, the final coordinates found are useful for mapping applications.

Silo Survivability

One of the most critical metrics of a missile silo is its survivability—how likely it is to survive an attack—which depends on the hardness of the silo and the specifications of the attack warhead and missile. Though difficult to calculate, the number of silos expected to survive an attack can serve as a measure of both the destructive capabilities of the attacking arsenal and the vulnerability of the targeted arsenal; it also provides an estimate of how many missiles would survive a first strike to be used in a retaliatory attack.

There are two key radial distances that determine survivability—the displacement and the lethal radius. As previously detailed, the radial displacement of a warhead from the DGZ depends on the CEP of missile. The lethal radius (LR) is the distance from the detonation within which the target will be destroyed. The LR for a particular warhead and target can be calculated as:

$$LR = D_1 Y^{1/3} \tag{6}$$

Wherein D_1 is the characteristic silo hardness's overpressure radius[2] for a 1 kt surface burst and Y is the yield of the attacking warhead in kilotons [4]. For a single detonation, if the displacement of the warhead from the DGZ is less than the LR, the

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[1] $R = 6378.137$ km

[2] e.g., for an attack on the U.S. silos, which are rated at a hardness of 2,000 psi, the 2,000 psi overpressure radius would be used

target is destroyed.

Survivability is typically calculated one of three ways: with the probability of kill P_k, the probability of destroying a silo given a nearby detonation; the single-shot probability of kill SSP_k, the probability of kill for a single missile and warhead; and the multi-shot probability of kill $P_k(n)$, the probability of kill for multiple, independent warheads. The probability of kill can be calculated from one of the following equations [5, 2, 6]:

$$P_k = 1 - \exp\left(-\frac{LR^2}{2\sigma^2}\right) \tag{7}$$

$$P_k = 1 - \left(\frac{1}{2}\right)^{\left(\frac{LR}{CEP}\right)^2} \tag{8}$$

The SSP_k factors in the reliability of the missile R as:

$$SSP_k = RP_k \tag{9}$$

Eqs. 7, 8, and 9 give the probability of kill for a 1-on-1 attack. For n-on-1 attacks, $P_k(n)$ is calculated as:

$$P_k(n) = 1 - (1 - SSP_k)^n \tag{10}$$

However, these equations have limitations. First, they consider only one silo, not a strike on multiple silos, which would be the expected scenario in an attack. Second, the multi-shot probability assumes that each warhead attacking the silo has the same yield and CEP, which may not be the case if the warheads with the same specifications do not divide evenly. Most critically, the equation considers each warhead independently and ignores fratricide[3], which cannot be easily factored in because it

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[3] Fratricide refers to the destructive effects the first detonated warhead exerts on subsequently arriving warheads that can divert or destroy the subsequent warhead(s); these effects include thermal and nuclear radiation, winds, and debris and vary with the time between waves. This presents a challenge to the attacker because the longer the time between waves, the lower the fratricide effects, but the longer wait also gives the attacked party time to fire the silo-based missiles in a retaliatory strike leading to strikes on empty silos [7].

has never been tested or observed, so there is no empirical data to determine the magnitude of the effects and how they scale with the time elapsed between waves [7, 8].

In order to ameliorate the limitations of the theoretical calculation of silo survivability, this work determined silo survivability using a Monte Carlo simulation[4] coded in Python. Rather than evaluating the probability of a single silo surviving an attack, the program evaluates how many silos out of an entire ICBM force are expected to survive under a specified attack scenario. The input consists of entries for each warhead that includes the designation of the targeted silos, the detonation sequence (i.e., first, second, etc. warhead to hit that silo), the coordinates of the silo, the yield of the warhead, and the CEP of the missile. The detonation specifications are then used in multiple experiments of 10,000 trials in order to calculate the average number of silos that would survive the input attack.

For a single trial, the coordinates of the detonation point and the associated displacement from the DGZ due to missile inaccuracy of each warhead are found as detailed in the previous section. That displacement is then compared against the LR of the warhead as calculated in Eq. 6; if the displacement is less than the LR, the silo is counted as destroyed.

If the warhead is a subsequent detonation, fratricide is incorporated through random sampling by generating a random number in the range [0, 1); if the number is less than the fratricide rate, the subsequent warhead is assumed to miss its target, and the silo is marked as not destroyed. The process is repeated for all of the warheads in the attack. Next, the total number of silos that survive that trial are calculated and saved with each silo's designation used to prevent double-counting. Because there is no basis on which to determine the fratricide rate, repeated experiments were run at different fratricide rates in order to find both a range of surviving silos and the relation between the the fratricide rate and the number of silos that survive.

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[4] A Monte Carlo simulation estimates a value through random sampling in repeated trials.

For this analysis, missile reliability was assumed to be 100% in order to minimize the number of silos surviving each attack. This represents the worst-case scenario and models the lowest number of silos expected to survive for use in a retaliatory strike, thus making it a more cautious figure to use in developing defensive policy.

Each experiment consists of 10,000 trials, all at the same fratricide rate. After running all of the trials, the average number of surviving silos and the standard deviation are calculated. The results of the experiments were then plotted with error bars representing the 95% confidence interval, as seen in Fig. 1. The histograms of each experiment show a bounded Gaussian distribution, and the relation between the fratricide rate and the average number of surviving silos was found to be linear for a 2-on-1 attack, quadratic for a 3-on-1 up to a full deployed arsenal attack (4.86-on-1), and 3rd order for larger attacks.

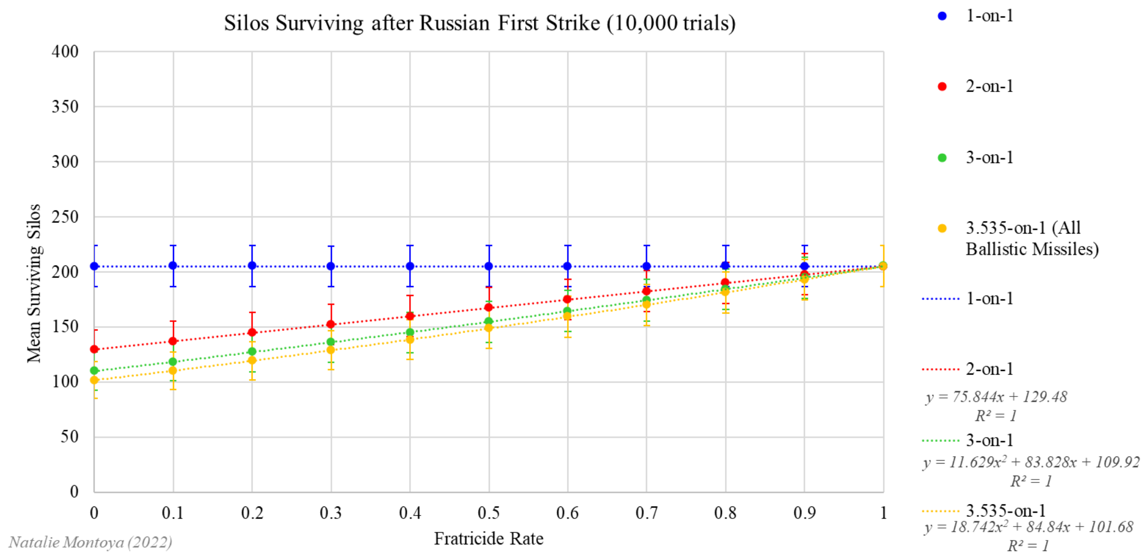

Figure 1: This plot depicts the number of U.S. ICBM silos out of the total 400 expected to survive an attack by Russian deployed ballistic missiles for 1-on-1 (blue), 2-on-1 (red), 3-on-1 (green), and all ballistic missiles (214 silos targeted 3-on-1 and 186 silos targeted 4-on-1; yellow) scenarios over a range of fratricide rates. The data point at each rate indicates the experimental mean from 10,000 trials and the error bars indicate the 95% confidence interval. The minimum number of surviving silos is 101.7107 ± 16.76150916 for an attack with all deployed ballistic missiles at a fratricide rate of 0%, and the maximum number of surviving silos is 205.4411 ± 18.50223826 for the 1-on-1 attack (i.e., fratricide rate of 100%). The trendlines' equations listed in the legend show the relationship between the fratricide rate and the number of silos expected to survive wherein y is the number of surviving silos and x is the fratricide rate.

Russian Arsenal

This section details the Russian arsenal used in this analysis. The arsenal specifications are based on the known status as of September 2022. The specifications of the Russian arsenal are critical parameters in developing strategies for targeting the United States and performing damage calculations because they define the technical capabilities and limitations of their force. As shown in Tab. 1, Russia's strategic nuclear forces currently include seven intercontinental ballistic missiles (ICBMs), three submarine-launched ballistic missiles (SLBMs), and two strategic bomber configurations, which differ in basing, range, accuracy, yield, and maximum load. Of those, the yield—the explosive energy measured as the equivalent mass of TNT that would produce the same amount of energy[5]—and the accuracy measured as the circular error probable (CEP)—the radial distance from the aiming point within which half of the warheads are expected to land [37, 38]—are of primary concern in calculating the blast effects. The basing determines the survivability of the missiles and, together with the range, the time of flight needed for the missile to reach its target(s). The maximum load denotes the number of warheads that an individual delivery system can carry. For bombers, that is simply how many air-launched cruise missiles (ALCMs) it can hold at full capacity. For ballistic missiles, the value corresponds to the maximum number of warheads that can be loaded on each missile that has multiple independently targeted reentry vehicles (MIRVs), though not all missiles are fully loaded. MIRVs must be taken into account when targeting because the warheads they carry are limited in how far apart they can travel[6], which imposes a geographic restriction.

Because official data on the composition of the Russian arsenal is not publicly available, the arsenal used in this work is based on the aggregate data from the March 2022 New START data exchange and assumptions made in light of ongoing

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[5] 1 kt = 4.184×10^{12} J

[6] 100 km cross-range, 200 km down-range [39]

ICBMs									
Russian Name	**NATO Name**	**Basing**	**Year**	**Range (km)**	**CEP (km)**	**Yield (kt)**	**Max Load**	**Launchers**	**Warheads**
RS-20V R-36M2	SS-18 Satan	silo	1988	11,500	0.22	800	10	40	71
Avangard	SS-19 Mod 4	silo	2019	9,000	0.22	HGV (800?)	1	6	6
RS-12M	SS-25 Sickle	mobile	1985	11,000	0.39	800	1	9	9
RS-12M2 Torpol-M	SS-27 Mod 1	silo	1997	11,500	0.35	800	1	60	60
RS12M1 Torpol-M	SS-27 Mod 1	mobile	2006	11,500	0.35	800	1	18	18
RS-24 Yars	SS-27 Mod 2	silo	2014	10,500	0.35	550	4	20	80
RS-24 Yars	SS-27 Mod 2	mobile	2010	10,500	0.35	550	4	153	530
Subtotal								**306**	**774**
SLBMs									
Russian Name	**NATO Name**	**Tubes/ SSBN**	**Year**	**Range**	**CEP (km)**	**Yield (kt)**	**Max Load**	**Launchers**	**Warheads**
R-29RM Sineva	SS-N-23 M1	16	2007	11,500+	0.5	500	4	16	64
R-29RM Layner	SS-N-23 M2	16	2014	8,300+	0.35	100	12	64	256
R-30 Bulava	SS-N-32	16	2014	8,000+	0.35	100	6	80	320
Subtotal								**160**	**640**
Ballistic Subtotal								**466**	**1414**
Strategic Bombers									
Russian Name	**NATO Name**	**ALCM**	**Year**	**ALCM Range**	**CEP (km)**	**Yield (kt)**	**Max Load**	**Launchers**	**Warheads**
Tu-95MS6/ MS16/MSM	Bear-H6/16	AS-15A/ AS-23B	1984/ 2015	5,000	0.025	250	6-16 or 14	55	448
Tu-160/M	Blackjack	AS-15B/ AS-23B	1987/ 2021	2,500	0.025	250	12	13	132
Assumed Strategic Bombers Counted as Deployed under New START and Warheads at Bases								*60*	*200*
Subtotal								**68**	**580**
Total								**534**	**1994**

Table 1: Estimated Russian Strategic Nuclear Forces as of Oct. 2020 [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]

modernization efforts [12, 9]. According to the March 2022 New START data exchange, Russia has 526 deployed ICBMs, SLBMs, and Heavy Bombers with 1,474 countable warheads [12]. Under New START, only one warhead is attributed to each bomber in the aggregate count; therefore, the actual number of warheads is several hundred greater than stated [40]. In this work, only ballistic missiles were used on account of delivery time, so the bomber loading is irrelevant.

The aggregate number of warheads does not correspond with the maximum loading of the ballistic missiles as it would exceed the limits set by the treaty, though which missiles are downloaded and by how much is unknown. For the SLBMs, all are assumed to be loaded with four warheads. The Layner (Liner) is an upgraded version of the Sineva with improved accuracy. The Delta IV class SSBNs in the Northern Fleet carry Layner missiles while the Delta IV class SSBN in the Pacific fleet carries Sineva missiles. The Borei class submarines in both fleets carry Bulava missiles [21]. On account of the modernization of the arsenal with SS-25 regiments being upgraded to SS-27 Mod2s, some SS-27 Mod2s, are assumed to be downloaded to three warheads instead of four in order to meet the treaty limits. The other ICBMs are assumed to be fully loaded except the oldest missiles, the SS-18s, which are gradually being retired. The rest of the warhead inventory distributed among the SS-18s, loaded at one or two warheads each to meet the reported New START totals.

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