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A data-driven discrete simulation-based optimization algorithm for large-scale two-way car-sharing network design

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Abstract

This paper formulates a discrete simulation-based optimization (SO) algorithm for a family of large-scale car-sharing network design problems. We focus on the profit-optimal assignment of vehicle fleet across a network of two-way (i.e., round-trip) car-sharing stations. The proposed approach is a metamodel SO approach. A novel metamodel based on a mixed-integer program (MIP) is formulated. The metamodel is embedded within a general-purpose discrete SO algorithm. The proposed algorithm is validated with synthetic toy network experiments. The algorithm is then applied to a high-dimensional Boston case study using reservation data from a major US car-sharing operator. The method is benchmarked versus several algorithms, including stochastic programming. The experiments indicate that the analytical network model information, provided by the MIP to the SO algorithm, is useful both at the first iteration of the algorithm and across subsequent iterations. The solutions derived by the proposed method are benchmarked versus the solution deployed in the field by the car-sharing operator. Via simulation, the proposed solutions

improve those deployed with an average improvement of profit of 6% and of vehicle utilization of 3%.

The combination of the problem-specific analytical MIP with a general-purpose SO algorithm enables the discrete SO algorithm to: (i) address high-dimensional problems, (ii) become computationally efficient (i.e., it can identify good quality solutions within few simulation observations), (iii) become robust to the quality of the initial points and of the stochasticity of the simulator. More generally, the information provided by the MIP to the SO algorithm enables it to exploit problem-specific structural information. This leads to an algorithm with both asymptotic convergence guarantees as well as good short term performance (i.e., performance given few simulation observations). We view this general idea of combining analytical MIP formulations with general-purpose SO algorithms, or more broadly with general-purpose sampling strategies of high-resolution data, as an innovative and promising area of future research.

1. Introduction

In recent years, the most successful trend in the space of urban mobility services has been the widespread use of shared mobility services, such as ride-sharing, car-sharing, bike-sharing and, most recently, scooter-sharing (Shaheen and Chan 2016). Major technology companies have been behind the rapid growth of these shared services. The operators of these services collect abundant data of the usage of the vehicles and the behavior of the clients (or users). This data provides a high-resolution disaggregate description of the interaction of demand and supply. This paper is motivated by the following research question: how can we exploit the rich disaggregate information in this data to optimize the design and the operations of these new urban mobility services?

As is detailed below, the most common approach to address these optimization problems is to aggregate the data such as to estimate parameters of a mathematical program, such as a mixed-integer programming model (MIP) or a stochastic programming model (SP). These mathematical programs provide an aggregate description of both demand and of the interaction of demand and supply. This aggregate description enables their computational tractability and their scalability (i.e., their use for large-scale instances). Nonetheless, through this aggregation a wealth of information of the intricate interactions between demand and supply is lost.

A current trend among major technology companies is to design optimization methods that exploit the rich information in their disaggregate data. Companies are building high-resolution simulators of their services that sample directly from their disaggregate data and provide a disaggregate description of the performance of their services (e.g., Greenhall (2016)). Hence, the next generation of mobility optimization algorithms will increasingly perform optimization based on models that provide a disaggregate description of mobility. This paper addresses this need. It formulates a car-sharing optimization problem as a simulation-based optimization (SO) problem, and proposes a computationally efficient SO algorithm. We use a disaggregate car-sharing service simulator, which was developed in collaboration with Ford and with the car-sharing operator Zipcar (Fields, Osorio, and Zhou 2017). The simulator samples from disaggregate car-sharing reservation data to estimate (disaggregate) demand (i.e., it yields a set of desired reservations) and then provides a simple stochastic mapping of how this demand interacts with supply to yield disaggregate reservations (i.e., a final set of realized reservations). The proposed algorithm is an example of how abundant disaggregate mobility data can be used to perform large-scale (e.g., city-scale) optimization.

Car-sharing service optimization

Car-sharing has become a popular transportation mode in urban areas in the past decades. Its deployment, as of 2010, covered over 31,600 vehicles in over 1,100 cities in 26 countries with over 1 million members (Shaheen and Cohen 2013). The car-sharing literature has studied its potential to reduce the transportation cost of households (Duncan 2011), to complement private-vehicle ownership (Shaheen and Cohen 2013, Becker, Ciari, and Axhausen 2017) and public transportation systems (Chiraphadhanakul 2013 Chapter 4, Nair and Miller-Hooks 2014, Zhou 2015 Chapter 3), as well as to mitigate greenhouse gas emissions and total vehicle miles traveled (Firnkorn and Müller 2011, Shaheen and Cohen 2013).

The main types of car-sharing service are two-way (also known as round-trip), one-way station-based, free-floating and peer-to-peer. For full definitions, see for instance Schmöller et al. (2015). A station is a location with a certain number of vehicle-sharing parking spots. Two-way services consist of a set of vehicles parked at a set of fixed stations. In advance, customers reserve a vehicle for a given duration and a given start time. They then pick-up and drop-off the vehicle from the same predetermined station. Reservations can be made from several months in advance to minutes in advance. There is no upper limit on the duration of a reservation. As of July 2015, there were an estimated 1.17 million two-way service members along with 0.31 million one-way service members in the United States (NCSL 2017). This paper focuses on two-way services with an application to a Boston case study with Zipcar data. Zipcar is a major car-sharing service provider in the US. It is also one of the world's largest car-sharing service provider with operations in more than 500 cities worldwide. It has deployed over 12,000 vehicles around the world (Zipcar 2017). Currently, Zipcar offers two-way service, one-way station-based service and free-floating service. Round-trip is the primary service mode for Zipcar and the foundation of its business. Studying the optimization of its two-way service is critical for Zipcar's business.

The data we use in this paper consists of two-way car-sharing reservations made in the Boston metropolitan area. Each reservation contains detailed information, such as creation time (time at which the reservation was made), duration, start time, end time, station (location where the vehicle is to be picked up and dropped off), and other vehicle attributes. Hereafter, we use the term data-driven to emphasize that: (i) unlike most approaches, we do not aggregate the data, instead we sample directly from the disaggregate reservation data to yield a disaggregate description of latent demand; (ii) we then use the disaggregate latent demand as input to a (disaggregate) simulator that mimics the reservation process or behavior of individual clients (e.g., if their desired reservation is not available, they may consider opting out or opting for a reservation at another station or time); this yields a disaggregate set of realized (through simulation) reservations. The optimization problem studied in this paper is the optimal spatial allocation of a fleet of two-way car-sharing vehicles to a set of stations. This is a tactical decision that car-sharing operators typically make on a monthly basis. The corresponding optimization problem is solved offline. Detailed reviews of vehicle-sharing studies are given in Jorge and Correia (2013), Brandstätter et al. (2016). Table 1 summarizes some of the recent vehicle-sharing network design literature. The column "Optimization" indicates whether the method is analytical, simulation-based or a combination of both. The column "Context" specifies the type of vehicle-sharing service (one-way, two-way, free-floating) and the type of vehicle (bike, car). The column "Case study size" indicates, for the main case study of each paper, the number of sites (e.g., locations, regions, stations), the number of integer and the number of continuous variables (including both decision variables and auxiliary variables). Cells are left blank for cases where these numbers are not directly reported in the papers. The "Problem" column specifies the type of decisions the problem addresses.

Operational problems for one-way vehicle-sharing are often more intricate than their two-way counterparts (e.g., real-time versus offline problems). Hence, most recent studies have focused on one-way vehicle-sharing. More specifically, there is a vast literature on fleet rebalancing due to its importance for both one-way and free-floating car-sharing, its relevance to bike-sharing and most recently scooter-sharing, and also due to the interesting scientific challenges that it presents.

Car-sharing demand-supply interactions are intricate to model, yet are critical to account for when planning and operating car-sharing services. Studies of car-sharing demand include Millard-Ball et al. (2005), Stillwater, Mokhtarian, and Shaheen (2009), Ciari, Schuessler, and Axhausen (2013), De Lorimier and El-Geneidy (2013), Coll, Vandersmissen, and Thériault (2014), Ciari, Bock, and Balmer (2014), Ciari, Weis, and Balac (2016). The analytical modeling of demand involves accounting for how the distribution of demand varies as a function of space, time, user-specific attributes (e.g., value of time, willingness to walk, trip purpose) and other transportation system attributes (e.g., alternative travel modes for that user and that trip purpose). Moreover, for two-way car-sharing, the analytical modeling of the interaction of demand and supply is particularly difficult due to the often low supply capacity: there are typically few car-sharing parking spots

Table 1 Summary of recent related vehicle-sharing network design papers

Paper	Optimization		Context					Case study size			Problem			
	Analytical	Simulation-based	One-way	Two-way	Free-floating	Bike-sharing	Car-sharing	Site	Integer	Continuous	Site location	Fleet assignment	Station capacity	Other
Correia and Antunes (2012)	✓		✓				✓	75		0	✓	✓	✓	Rebalance fleet
Cepolina and Farina (2012)		✓	✓				✓	11	9	0			✓	Fleet size
Chiraphadhanakul (2013, Chapter 4)	✓		✓			✓		27	27		✓			Route user flow
Correia, Jorge, and Antunes (2014)	✓		✓				✓	116		0	✓	✓		Select trips
Nair and Miller-Hooks (2014)	✓		✓			✓	✓	64	4420	6295	✓	✓	✓	Route user flow
Boyacï, Zografos, and Geroliminis (2015)	✓		✓				✓	100	All together $\sim 10^5$		✓		✓	Determine fleet size, regions served by each station and number of relocation personnel, rebalance fleet
Deng (2015, Chapter 5)		✓	✓				✓	8	11	2		✓	✓	Determine fleet size, rebalance fleet
Jorge, Barnhart, and Correia (2015)	✓		✓	✓			✓	391		0		✓	✓	Select trips, rebalance fleet
O'Mahony (2015, Chapter 3)	✓		✓			✓		300				✓	✓	
Zhou (2015, Chapter 4)	✓		✓			✓		30	5133	$\sim 1.5 \times 10^7$	✓			Route user flow
Jian et al. (2016)	✓	✓	✓			✓		466	932	0		✓	✓	
Lu, Chen, and Shen (2017)	✓		✓	✓	✓		✓	9				✓	✓	Route user flow, rebalance fleet
He et al. (2017)	✓				✓		✓	61			✓			Route user flow, rebalance fleet
This paper	✓	✓		✓			✓	315	315	0		✓		

available at each station. Hence, if a user does not find a vehicle available at the desired time and station, the user may opt out of renting a vehicle (the demand is said to be lost, and the historical reservation data is aid to be truncated) or may opt into renting a nearby (e.g., in space, in time) reservation (the demand is said to spillover or spillback and the historical reservation is said to be censored). For a detailed description of truncation and censoring in the context of car-sharing, see Fields, Osorio, and Zhou (2018). The likelihood of truncation and of censoring can depend on user characteristics (e.g., willingness to walk, car ownership), on trip attributes (e.g., trip purpose) as well as on the general mobility system (e.g., availability of other competitive travel alternatives). Additionally, given this low supply capacity, it is important to account for the temporal order in which users make reservations. In other words, modeling the *first-come-first-reserve* principle (i.e., the fact that reservations are prioritized or processed in the order of their creation time) is important. Due to the difficulties of accurately modeling car-sharing demand, as well as demand-supply interactions, we propose to directly use disaggregate car-sharing reservation data that embeds a detailed description of the interaction of demand and supply.

The most popular approach to address vehicle-sharing (both car- and bike-sharing) network design problems across all service types (two-way, one-way, floating) is the use of analytical mixed integer programming (MIP). Studies with deterministic demand include Correia and Antunes (2012), Chiraphadhanakul (2013, Chapter 4), Correia, Jorge, and Antunes (2014), Nair and Miller-Hooks (2014), Zhou (2015, Chapter 3). Past work in the field has also accounted for demand uncertainty by using a parametric probability distribution for demand combined with optimization methods such as stochastic programming and robust optimization (O'Mahony 2015, Chapter 4, Lu, Chen, and Shen 2017, He et al., 2017).

While stochastic simulators enable a more detailed modeling of demand and supply uncertainties, and of demand-supply interactions, their use to address optimization problems of realistic dimensions remains intricate. In the context of vehicle-sharing, simulation tools have mostly been used to evaluate the performance of network designs obtained from analytical models, i.e., the simulator is used to perform what-if analysis (Cepolina and Farina 2012, O'Mahony 2015 Chapter 5, Ciari, Balac, and Balmer 2015). Various simulation studies that account for car-sharing (Ciari, Balmer, and Axhausen 2009, Ciari, Balac, and Axhausen 2016, Balac, Ciari, and Axhausen 2016, 2017) have been carried out with the MATSim transportation simulation software (MATSim 2018). Studies, such as Cepolina and Farina (2012) and Deng (2015, Chapter 5), have included the simulator as part of an optimization framework and have resorted to general-purpose black-box optimization algorithms such as simulated annealing and particle swarm optimization. The study of Jian et al. (2016) exploited problem-specific information to yield gradient-type information. Interestingly, Jian et al. (2016) use the solution of an analytical linear integer program as the initial solution for a simulation-based optimization algorithm. Such an approach is also used as benchmark method in the case studies of this paper. Of particular notice is the large-scale bike-sharing optimization instance studied in Jian et al. (2016), which considers a set of 466 stations.

Discrete simulation-based optimization

In this paper, in order to enable the direct use of disaggregate car-sharing reservation data for optimization, we formulate the problem as a discrete simulation-based optimization (SO) problem. The problem has a simulation-based objective function with discrete decision variables. Constraints are analytical (i.e., they are not simulation-based). The main challenges of addressing such problems are the following. There is no analytical expression available for the objective function, hence traditional (analytical) discrete optimization algorithms cannot be used. The objective function can only be estimated by running a set of stochastic simulation replications. Discrete SO problems inherit the curse of dimensionality of discrete analytical problems. Since simulation is used, the objective function is often an intricate (e.g., non-convex) function of the decision variables with several local optima.

There are a variety of discrete SO algorithms in the literature; recent reviews include Nelson (2010) and Hong, Nelson, and Xu (2015). Discrete SO algorithms include Convergent Optimization via Most-Promising-Area Stochastic Search (COMPASS) (Hong and Nelson 2006), Adaptive Hyperbox Algorithm (AHA) (Xu, Nelson, and Hong 2013), R-SPLINE (Wang, Pasupathy, and Schmeiser 2013), and cgR-SPLINE (Nagaraj 2014). Methods that aim to identify solutions with good performance at an early stage (i.e., within few simulations) include an extension of COMPASS known as the Industrial Strength COMPASS (ISC) (Xu, Nelson, and Hong 2010), as well as extension of AHA known as ISC-AHA (Xu, Nelson, and Hong 2013). Other common approaches to discrete SO problems include ranking-and-selection (R&S) techniques, such as Chick and Inoue (2001), Frazier, Powell, and Dayanik (2008). An R&S review can be found in Swisher, Jacobson, and Yücesan (2003).

Discrete SO algorithms are most often designed: (i) as general-purpose algorithms, i.e., they can be used to address a broad family of optimization problems, their use is not limited to transportation problems, and (ii) based on asymptotic convergence properties, there is limited focus on their short-term (i.e., small sample performance). The performance of these general-purpose discrete SO algorithms is typically illustrated with low-dimensional problems (e.g., around 20 decision variables). Few studies have reported higher-dimensional instances. The work of Xu, Nelson, and Hong (2013) reported experiments where AHA successfully addressed problems with up to 100 decision variables. Developing discrete SO algorithms that are suitable for high-dimensional problems remains a challenge. Past studies, such as Xu, Nelson, and Hong (2013), illustrate that for locally convergent general-purpose discrete SO algorithms, the quality of the final solution is sensitive to the quality of the initial solution. Hence, there is also an interest to develop algorithms with enhanced robustness to the quality of the initial solution.

There is a lack of studies that evaluate the performance of general-purpose discrete SO algorithms for high-dimensional problems and under tight computational budgets (i.e., within few simulation runs). Nonetheless, when using simulators to address optimization problems, practitioners often use the algorithms under tight computational budgets (e.g., they terminate the algorithm once a fixed time or a fixed number of iterations have elapsed). Hence, there is a need for computationally efficient algorithms. These are algorithms that provide solutions with improved performance (compared to initial solutions or solutions deployed in the field) within few simulation runs.

In transportation, fundamental optimization problems are naturally formulated as discrete problems. Additionally, realistic case studies quickly lead to high-dimensional instances. Hence, this paper designs a discrete SO algorithm that is both computationally efficient and suitable for high-dimensional problems. Moreover, the case studies of this paper, illustrate the robustness of the algorithm to the quality of the initial point.

This paper focuses on metamodel SO approaches. In past work, we have formulated metamodel SO algorithms for various continuous SO transportation problems (Osorio and Nanduri 2015, Chong and Osorio 2017, Zhang, Osorio, and Flötteröd 2017, Osorio, Chen, and Santos forthcoming, Osorio forthcoming). A recent review of metamodel SO methods appears in Osorio and Chong (2015). A more detailed description of commonly used metamodels is given in Section 2.2. To the best of our knowledge, the use of metamodel approaches for discrete SO has been limited to low-dimensional problems (with up to 15 decision variables). In the broader area of transportation (i.e., not limited to vehicle-sharing) discrete SO has been used in studies such as Jung et al. (2014), Chen et al. (2015), Sebastiani, Lüders, and Fonseca (2016), Jian et al. (2016), Boyacï, Zografos, and Geroliminis (2017).

This paper formulates a novel metamodel for a family of car-sharing SO problems. We then combine the metamodel with an existing general-purpose discrete SO algorithm, known as AHA, leading to a novel metamodel SO algorithm. The proposed algorithm preserves the asymptotic convergence properties of the general-purpose algorithm. More specifically, the proposed algorithm remains a locally convergent algorithm.

In this paper, we use a car-sharing network simulator (Fields, Osorio, and Zhou 2017), which relies on few demand modeling assumptions. Instead, it relies primary on sampling from disaggregate car-sharing reservation data. It provides a detailed description of the spatial-temporal distribution of demand as well as of demand-supply interactions. Unlike most methods that aggregate the data to fit aggregate model parameters, we use the data in disaggregate form. Hence, we refer to our method as a data-driven SO algorithm.

In Figure 1, we show the basic logic of the data-driven metamodel SO framework. The main idea is that historical disaggregate car-sharing reservation data (which represents potentially censored or truncated demand) is used to estimate disaggregate latent (i.e., uncensored and untruncated) demand (for a detailed description of this demand estimation methodology, see Fields, Osorio, and Zhou (2018)). For a given latent demand and a given supply solution (i.e., a given spatial allocation of the vehicle fleet), the simulator stochastically evaluates the performance of the solution, this yields a set of disaggregate reservations (which may have been censored). Every time new supply solutions are evaluated via simulation, the metamodel is updated (i.e., its parameters are fitted by using the current set of simulation observations) and then used to solve an analytical optimization problem. More specifically, the metamodel optimization problem is a mixed-integer program (MIP). While the analytical metamodel (i.e., the MIP) provides an aggregate description of demand and of demand-supply interactions, the simulator operates based on a disaggregate representation of demand (i.e., individual desired reservations) and of demand-supply interactions (i.e., individual realized reservations).

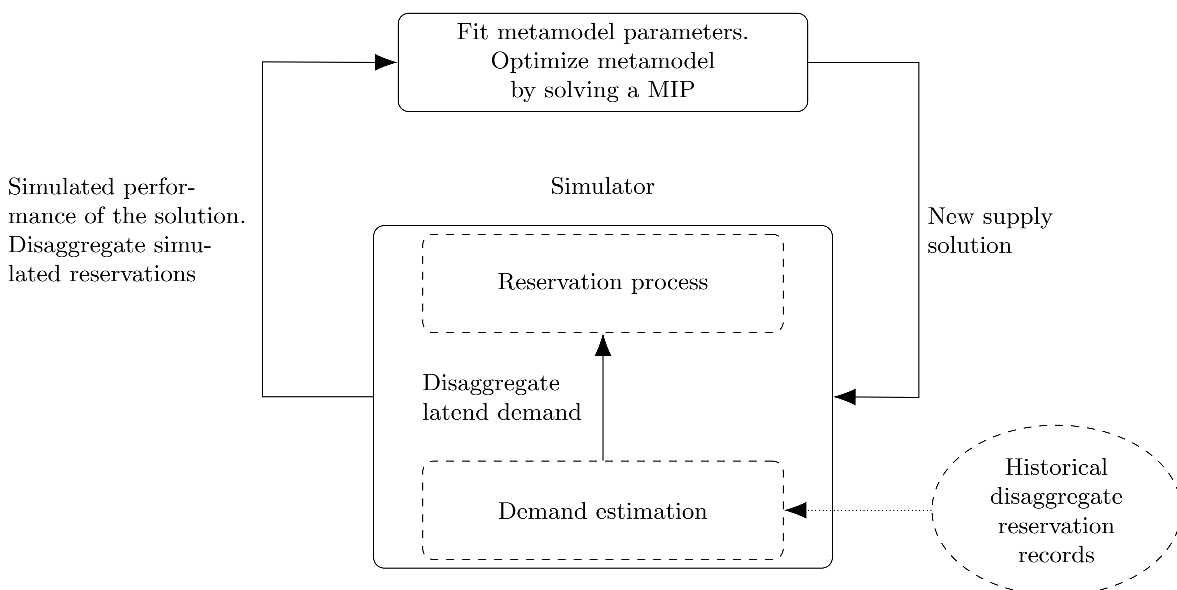


Figure 1 **Data-driven metamodel SO framework**

The contributions of this paper can be summarized as follows.

Data-driven technique The most traditional approach to car-sharing service optimization has been analytical optimization. This comes at the cost of a simplified description of demand and of demand-supply interactions. In this work, our goal is to acknowledge both the intricacy of a car sharing service (e.g., intricate demand distribution, intricate demand-supply interactions), as well as the availability of high-resolution data. Hence, we propose a method that relies heavily on the rich reservation data and uses limited modeling assumptions. The information captured in the data about the underlying demand distribution and demand-supply interactions is preserved and exploited at a disaggregate level. To the best of our knowledge, this is the first work to design an algorithm that preserves this high-resolution information of the data (i.e., does not merely aggregate the disaggregate data) for car-sharing network design optimization. Case studies with data from Zipcar's Boston market are carried out.

High dimensional discrete SO problems The proposed algorithm is suitable to address high-dimensional network design problems. In Section 3.3 and 3.4, we use it to address a Boston metropolitan area case study with 315 stations. General-purpose discrete SO algorithms have been extensively used to tackle problems with roughly 20 decision variables. Our enhanced scalability comes at the cost of proposing an algorithm tailored for a specific class of network design problems, while the general-purpose algorithms can be used for a broader class of general discrete SO problems. We achieve scalability by formulating and embedding within the general-purpose algorithm AHA information from a MIP. This yields the proposed discrete SO algorithm, which we call MetaAHA. The approach combines the merits of both analytical optimization methods (i.e., tractability and scalability) and of simulation-based optimization methods (i.e., we can sample directly from the disaggregate data to enable a detailed description of demand and of demand-supply interactions).

Computationally efficient algorithm The proposed algorithm is designed to identify good quality solutions within few iterations (i.e., when few simulation observations are available). This differs from most discrete SO literature which is focused on asymptotic performance. This efficiency is achieved through the novel metamodel formulation which embeds a non-simulation-based representation (a MIP formulation) of the network design problem. In other words, the simulator is no longer treated as a black box, instead analytical problem-specific information is embedded within the SO algorithm. The results of Section 3 indicate that this analytical structural information is the key to achieving computational efficiency. They also illustrated how the combination of the proposed metamodel along with a general-purpose discrete SO algorithm yields an algorithm with both good short-term and asymptotic performance properties. Moreover, the metamodel enables the general-purpose algorithm to become robust to the quality of the initial solution.

Metamodeling for discrete SO The main feature of the proposed algorithm is the formulation of a metamodel, (i.e., an analytical approximation of the simulation-based objective function) that has a functional form that is problem-specific. Such metamodel ideas for transportation problems have been successfully formulated for various continuous SO problems. This is the first paper that extends these ideas to the discrete SO setting. The paper shows that by using such metamodel ideas, high-dimensional discrete SO problems can be addressed in a computationally efficient way.

The paper shows how the proposed metamodel ideas enable general-purpose discrete SO algorithms to become more scalable (i.e., suitable for higher-dimensional problems). Since fundamental OR transportation optimization problems (e.g., routing) are naturally formulated as discrete optimization problems, the ideas of this paper lay the foundations for a variety of important and difficult transportation problems to be addressed efficiently with data-driven, or simulation-based, network models.

Section 2 formulates the proposed methodology. Its performance is evaluated and benchmarked in Section 3 with experiments on both synthetic toy networks and Boston networks. Conclusions are presented in Section 4. Algorithmic details are presented in Appendix A. The formulation of the SP model that is used as a benchmark in Section 3.4 is given in Appendix B. Additional implementation detailes are given in Appendix C.

2. Methodology

This section presents the proposed methodology. The network design problem is formulated in Section 2.1. The general metamodel SO framework is discussed in Section 2.2. The metamodel for the considered car-sharing network design problem is formulated in Section 2.3 and the proposed algorithm is described in Section 2.4. The car-sharing network simulator used in this paper as well as the role of the car-sharing data are summarized in Section 2.5.

2.1. Network design problem formulation

We consider a two-way car-sharing system from the perspective of the car-sharing operator. The network design problem is to assign a fleet of vehicles across a network of stations such as to maximize the expected profit. We also refer to this problem as the fleet assignment problem. The network design problem is studied for a given finite time horizon, which we refer to as the planning period. To formulate the problem, we introduce the following notation.

x_i	number of cars assigned to station i (decision variable);
$\boldsymbol{x}$	vector of x_i's for all $i \in \mathcal{I}$;
$R(\boldsymbol{x};\boldsymbol{q}_1)$	random variable representing the revenue;
$g(\boldsymbol{x};\boldsymbol{q}_1)$	expected profit (SO objective function);
c_i	cost, over the planning period, of a parking space at station i;
$\boldsymbol{q}_1$	exogenous simulation parameter vector (e.g., reservation pricing);
N^i	capacity of station i (i.e., number of parking spots);
X	total fleet size (i.e., number of cars to assign);
I	total number of stations;
$\mathcal{I}$	set of all stations, $\mathcal{I} = \{1, 2, \ldots, I\}$;
$\mathcal{F}$	feasible region.

The problem is formulated as follows:

$$\max_{\boldsymbol{x}} \quad g(\boldsymbol{x};\boldsymbol{q}_1) = E[R(\boldsymbol{x};\boldsymbol{q}_1)] - \sum_{i \in \mathcal{I}} c_i x_i \tag{1}$$

subject to

$$\sum_{i \in \mathcal{I}} x_i \leq X \tag{2}$$

$$x_i \leq N^i \quad \forall i \in \mathcal{I} \tag{3}$$

$$x_i \in \mathbb{Z}_+ \quad \forall i \in \mathcal{I}. \tag{4}$$

The objective function represents the expected profit for a given fleet assignment vector, $\boldsymbol{x}$. It is defined as the difference between the expected revenue $E[R(\boldsymbol{x}; \boldsymbol{q}_1)]$ and the costs. The expected revenue is a simulation-based function, estimates of which can be obtained via simulation. The cost parameters, c_i, are exogenous. In this work, they represent parking space leasing fees. Constraint (2) bounds the total number of cars assigned across all stations with the fleet size. Constraint (3) bounds the number of cars assigned to each station i with the space capacity of the station. The number of cars assigned to each station are assumed to be non-negative integers (Constraint (4)). Constraints (2)-(4) specify the feasible region, $\mathcal{F}$.

The expectation in the objective function accounts for the stochasticity in the simulation process. The simulator, which is summarized in Figure 1 and described in more detail in Section 2.5, combines a sampling procedure that samples from a set of car-sharing reservation data and an assigning procedure that determines whether a reservation request will be satisfied and how it will be satisfied. In other words, realizations of the revenue random variable R are obtained by sampling from car-sharing reservation data. The sources of uncertainty include: (i) a stochastic sampling process that samples from the historical reservation data to infer a set of disaggregate latent demand (or desired reservations); (ii) a stochastic description of how demand and supply interact and how truncation and censoring occur (e.g., probability with which a user for which his/her desired reservation is not available, decides to opt out of making a reservation or decides to find a substitute reservation).

The challenges of addressing discrete SO problems, such as Problem (1)-(4), were detailed in Section 1. Given these challenges, we propose an algorithm that at every iteration, uses the set of estimates of g obtained so far to formulate and solve an (approximate) analytical discrete problem that: (i) provides good quality solutions to the underlying SO problem, (ii) can be solved efficiently for high-dimensional instances, and (iii) can be solved with a variety of widely-used commercial solvers.

Note that the traditional approach to address this fleet assignment problem is to formulate it as a MIP (such a formulation is given in Section 2.3), or as a stochastic programming model. The problem would no longer be simulation-based, and hence large-scale instances could be solved efficiently. Nonetheless, this would come at the cost of using only aggregate information from the car-sharing reservation data (since the data would merely be used to fit a set parameters of the analytical mathematical programs) and of embedding a simplified description of demand-supply interactions, as is detailed below.

A stochastic programming model is used as a benchmark method in the case study of Section 3.4. Compared to its SP counterpart, the simulator provides a more detailed description of the truncation and of the censoring of demand. First, it satisfies the first-come-first-reserve principle, i.e., the desired reservations are processed in the order of their creation time (which is obtained from the historical data). Second, when a user's desired reservation is not available and he/she decides to consider alternate substitute reservations, the simulator considers a sequential process where feasible substitute reservations are ranked by a distance metric (for instance, available vehicles that are closer in spatial or temporal distance, are more likely to be a substitute than those further away). These aspects could be naturally formulated as analytical nonlinear functions, or based on analytical linear approximations. Nonetheless, the resulting SP would be less tractable and less scalable.

As is detailed below, the proposed methodology combines the advantages of using a simulation-based model (which allows for the use of more detailed models and historical data) and those of an analytical mathematical program (which allows for computational tractability and scalability).

2.2. General metamodel approach

Let us first briefly present the main ideas of the metamodel SO approach, which are based on the continuous SO framework of Osorio and Bierlaire (2013). To formulate the problem, we introduce the following notation.

k	iteration index of the SO algorithm;
m_k	metamodel at SO iteration k;
$\boldsymbol{\beta}_k$	vector of metamodel parameters at SO iteration k, element i is denoted $\beta_{k,i}$;
$\boldsymbol{z}$	vector of endogenous variables;
$\boldsymbol{q}_2$	vector of exogenous parameters;
g_A	approximation of g (Equation (1)) derived by the analytical network model;
h	constraints of the analytical network model.

The main idea of metamodel SO is to replace the simulation-based objective function (1) with an analytical approximation, which is known as the metamodel. In the metamodel literature, general-purpose functions (e.g., low-order polynomial functions, radial-basis functions, Kriging functions) are the most common choice both for continuous SO problems (Jones, Schonlau, and Welch 1998, Barton and Meckesheimer 2006, Wild, Regis, and Shoemaker 2008, Kleijnen, Van Beers, and Van Nieuwenhuyse 2010, Ankenman, Nelson, and Staum 2010) and for discrete SO problems (Xu 2012, Sun, Hong, and Hu 2014, Salemi 2014 Chapter 4, Xie, Frazier, and Chick 2016). They are chosen based on their mathematical properties. They are referred to as general-purpose functions because their choice does not depend on the specific problem formulation (i.e., their functional form is invariant to the choice of the objective function (1)). Nonetheless, due to this generality, their functional form does not embed any problem-specific structural information. Osorio and Bierlaire (2013) propose to formulate metamodels that embed problem-specific information. By doing so, the resulting SO algorithms have enhanced computational efficiency, scalability and robustness to both simulator stochasticity and to the quality of the initial solutions.

In this paper, we follow the idea of Osorio and Bierlaire (2013). The metamodel is defined by (5) as the sum of a problem-specific function (g_A) and a general-purpose linear function (term within parenthesis of (5)). The problem-specific function (g_A) is the analytical objective function of a mathematical program (more specifically of a mixed-integer linear fleet assignment problem), which embeds a simplified representation (compared to the simulator) of the mapping between the supply configuration ($\boldsymbol{x}$) and the expected profit (g of (1)). The goal of g_A is to provide a good analytical approximation of the simulation-based objective function for the considered problem. Nonetheless, this analytical approximation is not expected to be accurate (due to the more detailed and intricate models of demand and of supply embedded in the simulator, that are not accounted for in the mathematical program). Hence, the metamodel (5) can be thought of as the objective function of a MIP that is corrected for parametrically by both a scaling term (scalar $\beta_{k,0}$ of (5)) and an additive linear error term (term within parenthesis of (5)). To the best of our knowledge, this is the first work to consider a metamodel that combines both a problem-specific component and a general-purpose component for discrete SO problems.

At a given iteration k of the SO algorithm, we solve the following analytical problem, referred to as the metamodel optimization problem.

$$\max_{\boldsymbol{x},\boldsymbol{z}} \quad m_k(\boldsymbol{x},\boldsymbol{z};\boldsymbol{\beta}_k,\boldsymbol{q}_2) = \beta_{k,0}\, g_A(\boldsymbol{x},\boldsymbol{z};\boldsymbol{q}_2) + \left(\beta_{k,1} + \sum_{i\in\mathcal{I}} \beta_{k,i+1} x_i\right) \tag{5}$$

$$h(\boldsymbol{x},\boldsymbol{z};\boldsymbol{q}_2) = 0 \tag{6}$$

$$\boldsymbol{x} \in \mathcal{F}. \tag{7}$$

Since g_A is the objective function of a MIP, the corresponding constraints of the MIP are represented here through the function h of (6). These are formulated in detail in Section 2.3. The constraints of Section 2.3 consist of both equality and inequality constraints. They are summarized here as a set of equality constraints (they can equivalently be represented as a set of inequality constraints).

The metamodel optimization Problem (5)-(7) differs from the simulation-based optimization Problem (1)-(4) in that: (i) it replaces the (unknown) simulation-based objective function (g of (1)) with an analytical function (m_k of (5)); (ii) it has additional constraints (Eq. (6)). The main feature that has allowed us in the past to design efficient algorithms for continuous SO problems is the formulation of a metamodel that embeds an analytical and problem-specific approximation of $g(\boldsymbol{x})$. This is the key component of the approach, yet this is also where the main methodological challenge lies because it is necessary to formulate an analytical model that: (i) provides a good approximation of the intricate function $g(\boldsymbol{x})$, which as will be discussed in Section 2.3 is particularly difficult for this car-sharing context, (ii) is scalable (i.e., is suitable to address high-dimensional instances), and (iii) is computationally efficient. The latter is critical because the metamodel optimization problem is solved at *every* iteration of the SO algorithm. Hence, it should be sufficiently efficient to warrant the allocation of computing resources to solving it rather than to running the simulator (i.e., simulating new points or increasing the accuracy of the estimates of simulated points).

The metamodel (m_k of (5)) is a parametric function with parameter vector $\boldsymbol{\beta}_k$. The latter are fitted, at every iteration of the SO algorithm, by solving a problem that minimizes a least squares distance between metamodel predictions and simulation observations. For more details, see Problem (14) in Appendix A. As discussed above, the main challenge in this approach is the formulation of a computationally efficient and scalable problem-specific approximation of g, denoted here g_A. Let us now present the proposed formulation.

2.3. Car-sharing network design metamodel formulation

To formulate the analytical problem-specific component of the metamodel, g_A, which approximates the profit of a given network design strategy, we introduce the following additional notation.

d^i_{tl} number of customers that desire a reservation at station i with start time t and duration l;
r_{tl} revenue from a reservation with start time t and duration l;
p^{ij} discount to the revenue if a reservation is desired for station i but is fulfilled at (i.e., is made at) station j;
z^i_{tl} number of customers that make a reservation at station i with start time t and duration l;
z^{ij}_{tl} number of customers that desire to make a reservation at station i with start time t and duration l but make an adjusted reservation at station j with start time t and duration l;
$t_{\max}$ number of one-hour reservation start time intervals during the planning period (e.g., for an n-day planning period, $t_{\max} = n \times 24$);
$l_{\max}$ maximum reservation duration;
$\mathcal{I}_i$ set of stations "near" station i, including station i;
$\mathcal{L}$ set of reservation durations (in hours), $\mathcal{L} = \{1, 2, \ldots, l_{\max}\}$;
$\mathcal{T}$ set of reservation start time interval indices, $\mathcal{T} = \{1, 2, \ldots, t_{\max}\}$;
$\mathcal{T}_1(t, l)$ set of reservation start times for reservations with duration l that are ongoing at time t (i.e., they start prior to t and have not finished at time t).

The vector $\boldsymbol{z}$ defined in Section 2.2 consists of all variables $\{z^i_{tl}\}$ and $\{z^{ij}_{tl}\}$. The function g_A is formulated as:

$$g_A(\boldsymbol{x}, \boldsymbol{z}; \boldsymbol{q}_2) = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}_i} \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}} p^{ij} r_{tl} z^{ij}_{tl} - \sum_{i \in \mathcal{I}} c_i x_i. \tag{8}$$

This function is defined as the difference between the total revenue and the total cost. Note that in the total revenue expression, we give a discount (p^{ij}) for reservations that are adjusted (i.e., the initial desired reservation was not feasible because a car was not available). This allows us to account for the impact on revenue of demand spillback (i.e., demand censoring). Note that demand spillback and loss are described in a more detailed and disaggregate manner in the simulator (see. Section 2.5).

The auxiliary variable z_{tl}^{ij} is related to the decision vector $\boldsymbol{x}$ through the analytical network model, which is denoted by h in Equation (6) and is defined as follows.

$$\sum_{j\in\mathcal{I}_i} z_{tl}^{ji} = z_{tl}^{i} \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L} \tag{9}$$

$$\sum_{j\in\mathcal{I}_i} z_{tl}^{ij} \le d_{tl}^{i} \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L} \tag{10}$$

$$\sum_{l\in\mathcal{L}} z_{tl}^{i} + \sum_{l\in\mathcal{L}} \sum_{t'\in\mathcal{T}_1(t,l)} z_{t'l}^{i} \le x_i \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T} \tag{11}$$

$$z_{tl}^{i} \in \mathbb{R}_+ \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L} \tag{12}$$

$$z_{tl}^{ij} \in \mathbb{R}_+ \qquad \forall i\in\mathcal{I}, \forall j\in\mathcal{I}_i, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}, \tag{13}$$

where $\mathcal{T}_1(t,l) = \{t'\in\mathcal{T} : t'+1 \le t \le t'+l-1\}$. Equation (9) states that z_{tl}^{i}, the number of reservations at station i with start time t and duration l, is the sum of all desired reservations at station j (with start time t and duration l) that were shifted to station i. Note that $i\in\mathcal{I}_i$, hence this summation includes the reservations that were desired and also made at station i (with start time t and duration l). Equation (10) is a demand constraint. The right-hand side is the total demand for station i with start time t and duration l. The left hand side considers the set of reservations with a preference for station i start time t and duration l. This summation includes reservations where: (i) the preference was available and was made, (ii) the preference was not available and the reservation was adjusted and made at a neighboring station j (with the same start time t and the same duration l). For a given fleet assignment, the difference between the right-hand side and the left-hand side represents the lost demand for reservations at station i with start time t and duration l. The left-side of the Constraint (11) consists of two terms. The first term represents the total number of reservations at station i that start at time t. The second term represents the total number of reservations at station i that have started prior to time t and are still ongoing. Hence, Constraint (11) ensures that at station i and time t, the number of reserved cars (left-side of the inequality) is bounded above by the number of cars assigned to station i. Constraints (12) and (13) assume non-negative real values for the auxiliary variables (z_{tl}^{i} and z_{tl}^{ij}). The use of real-valued auxiliary variables, rather than integer variables, contributes to the computational efficiency of this analytical approximation. In this model, the exogenous parameters are d_{tl}^{i}, r_{tl}, c_i, p^{ij}, $t_{\max}$ and

$l_{\max}$. Together they form the exogenous parameter vector $\boldsymbol{q}_2$. The endogenous variables are z_{tl}^{i}, z_{tl}^{ji} and x_i. The exogenous parameters r_{tl}, c_i, p^{ij} and $l_{\max}$ are directly estimated from the data and in consultation with Zipcar staff. Note that we have $0 \leq p^{ij} \leq 1$ and $p^{ii} = 1$ for all $i \in \mathcal{I}$ and $j \in \mathcal{I}_i$. A discussion on the simplifications of this analytical model compared to the simulator is given in Section 2.5.

The demand parameters (d_{tl}^{i}) are estimated by sampling from the historical reservation data to estimate latent demand. For a description of the demand sampling step, see Section 2.5. Since this data sampling is stochastic, the case studies of Section 3 consider experiments based on different realizations of the sampling (i.e., they consider different latent demand estimates). This serves to evaluate the impact of varying demand on the performance of the proposed method. Moreover, the SP, which is used as one of the benchmark algorithms in Section 3.4, considers a set of latent demand realizations. This data sampling process yields a set of disaggregate desired reservations, which are then aggregated to estimate the aggregate parameters d_{tl}^{i}. The simulator, unlike the analytical MIPs (metamodel MIP or SP), uses the disaggregate demand without aggregating it.

For any station $i \in \mathcal{I}$, if we assume the maximum number of stations in the neighborhood of i, $\mathcal{I}_i$, is smaller than a constant W, i.e., $|\mathcal{I}_i| \leq W$, then the number of auxiliary variables of the metamodel is in the order of $O\left(W|\mathcal{I}||\mathcal{T}||\mathcal{L}|\right)$, and the number of constraints is in the order of $O\left(|\mathcal{I}||\mathcal{T}||\mathcal{L}|\right)$. Hence, by bounding the duration of the planning period and the maximum duration of a reservation, the number of variables and the number of constraints increase linearly with the number of stations. This contributes to the scalability of the model. In summary, the metamodel optimization problem is a mixed-integer linear model, which can be solved in a computationally efficient way with a variety of standard solvers. For the case studies of Section 3.2, the MIP (5)-(7) is solved on average in 2.4 seconds for the the Boston South End network and in 35.1 seconds for the large-scale Boston network of Section 3.3.

2.4. Discrete SO algorithm: MetaAHA

The proposed metamodel is embedded within a general-purpose discrete SO algorithm. We have chosen the Adaptive Hyperbox Algorithm (AHA) of Xu, Nelson, and Hong (2013). As discussed in Section 1, AHA has been used to solve high-dimensional problems with a decision vector of dimension 100. AHA is a locally convergence random search algorithm. We use the term *current iterate*, denoted $\boldsymbol{x}_k$, to refer to the point considered to have best performance at iteration k. The name AHA stems from the sampling, at every iteration, from a region which is the intersection of the feasible region and the *hyperbox* (for more details, see Appendix A). The latter is centered at the current iterate with a size that is updated, at every iteration, based on the performance of the current iterate and of its neighbors. Let $\mathcal{H}_k$ denote the hyperbox at iteration k. The proposed algorithm, denoted MetaAHA, is an extension of AHA. Algorithm 1 presents MetaAHA.

Algorithm 1 MetaAHA

Initialization:

— Initialize parameters: iteration index $k = 1$, hyperbox $\mathcal{H}_1 = \{\boldsymbol{x} : 0 \leq x_i \leq N^i, \forall i \in \mathcal{I}\}$, set n (the number of solutions to simulate per iteration). Set the number of randomly sampled solution in each iteration $r = n - 2$. For the metamodel parameter vector $\boldsymbol{\beta_1}$, set $\beta_{1,0} = 1$ and $\beta_{1,i} = 0$ ($\forall i \geq 1$).

Step 1: identify the set of n points to sample

— Step 1a: obtain r points in $\mathcal{F} \cap \mathcal{H}_k$ based on the asymptotically uniform sampling mechanism of AHA.

— Step 1b: obtain 1 point, denoted $\boldsymbol{x}_k^{\text{meta}}$, as the solution to the metamodel optimization Problem (5)-(7).

— Step 1c: obtain 1 point, denoted $\boldsymbol{x}_k^{\text{meta-hyper}}$, as the solution to the metamodel optimization Problem (5)-(7) with the additional constraint that the point belongs to $\mathcal{H}_k$.

Step 2: simulation

— Following the procedure of AHA: simulate the points identified in Step 1; simulate $\boldsymbol{x}_{k-1}$ (for $k > 1$); select the point with best performance $\boldsymbol{x}_k$ (i.e., update the current iterate); update the hyperbox.

Step 3: check for algorithm termination

— Step 3a: test if $\boldsymbol{x}_k$ is a local optimum following the procedure of AHA. If so, stop.

— Step 3b: if the total number of iterations exceeds the maximum number of iterations (i.e., if the computational budget is depleted), stop.

Step 4: metamodel update

— Step 4a: for any simulated point $\boldsymbol{x}$ that has not been evaluated by the analytical network model, evaluate it (i.e., for a given $\boldsymbol{x}$, maximize $g_A(\boldsymbol{x}, \boldsymbol{z})$ of Equation (8) over $\boldsymbol{z}$ subject to Constraints (9)-(13)).

— Step 4b: use all simulation observations collected so far to fit the metamodel parameter $\boldsymbol{\beta}_k$ (i.e., solve the least squares Problem (14) defined in Appendix A).

Step 5: update iteration counter

— Set $k = k + 1$, proceed to Step 1.

Each iteration k of the algorithm consists of 4 main steps. Step 1 identifies the set of points to simulate. These can be new points that have not been simulated before or points that have already been simulated and for which we will run additional simulation replications. Step 2 simulates these points. Step 3 checks whether termination criteria are satisfied. Step 4 uses the set of all simulation observations collected so far and updates the fit of the metamodel. Additional algorithmic details and a flowchart summary of MetaAHA are given in Appendix A.

Algorithm AHA is obtained from MetaAHA by omitting Steps 1b, 1c , 3b and 4; and setting r (of Step 1a) to n (while for MetaAHA $r = n-2$). Steps 1b and 1c solve mixed-integer programs. These steps yield solutions to MIPs. Hence, they exploit problem-specific analytical structural information provided by the analytical network design model. This information enables the algorithm to: (i) identify points with good performance within few, or even no, simulation runs because the analytical network design model can be solved without available simulation observations, and (ii) become less sensitive to the quality of the initial sample. This sensitivity to the quality of the initial sample has been identified and discussed in past AHA work (Xu, Nelson, and Hong 2013). While both Steps 1b and 1c exploit this problem-specific analytical information, Step 1c does so within the hyperbox, leading to the identification of local points with good performance, while Step 1b does so in the entire feasible region, leading to the identification of global points with good performance. In Step 2, we determine the number of replications to simulate for each point (this is done based on the approach of AHA, which is also described in this paper in Appendix A), we simulate the points and then update both the hyperbox and the current iterate. In Step 3b, if the computational budget is depleted, then the algorithm is terminated without convergence. This serves to reflect the most common way in which these algorithms are used in practice.

Note that MetaAHA does not change the main building blocks of the basic algorithm AHA. It merely complements it by adding a problem-specific sampling strategy which is based on the use of the metamodel. Hence, AHA's asymptotic local optimality guarantee is preserved. MetaAHA illustrates how a variety of general-purpose discrete SO algorithms can be complemented with such problem-specific sampling strategies to improve their robustness to the quality of the initial points as well as their short-term (i.e., small sample) performance. For practitioners, who typically use these algorithms under tight computational budgets, this has the potential to improve the performance of these general-purpose algorithms.

2.5. Two-way car-sharing simulator

We summarize here the main ideas underlying the simulator. For more details on the specification of the simulator as well as on its validation, see Fields, Osorio, and Zhou (2017). The simulator takes as input disaggregate historical reservation data, estimated daily demand per station (i.e., total daily number of reservations desired per station), a fleet assignment strategy, and yields as output a set of realized reservations (reservations actually made) with the corresponding network-wide profit.

More specifically, the simulation process consists of two main parts, as summarized in Figure 1. The first step, referred to as the demand sampling step, samples from the data such as to (approximately) obtain a set of desired reservation requests (i.e., reservations that users would ideally desire to make). These reservations can be thought of as realizations of latent demand. Hence, we distinguish between realized demand (an empirical distribution of which is given by the dataset) and latent demand. The second step, referred to as the reservation simulation step, considers a given latent demand (i.e., a given set of desired reservations) and simulates the reservation process as follows. It ranks, and then sequentially processes, the desired reservations by increasing creation time. For a given reservation, if a car is available (at the desired station and during the desired time interval), then the reservation is made. Otherwise, with a given probability the client will either not make a reservation (this is referred to as lost demand) or it will consider an "adjacent" reservation, which is either at a nearby station or at a nearby start time (this is referred to as demand spillback; it accounts for demand censoring). The probability depends on the distance between the initially desired reservation and the considered adjacent reservation. Once a given reservation is made, other users cannot use the same car at any time during this reservation period. This procedure mimics the first-come-first-reserve process.

The most important input to the simulator is the set of historical disaggregate reservation data. In this work, we use Zipcar data. For each reservation observation in the dataset, the following attributes are used: station (this is both the pick-up and the drop-off location), start time, duration and reservation creation time (i.e., the timestamp of when the reservation was made). Additionally, based on information available online we have estimated reservation revenues. The time resolution of the simulator is based on that of the data which is 30 minutes. This means that reservation durations and reservation start times are defined in 30 minute increments.

A main feature of this simulator is that this reservation process simulation is based on a handful of parameters, which are estimated from the data. Additionally, there are few modeling assumptions, which were made in consultation with Zipcar staff. They include the probability of considering an adjacent reservation and the formulation of a distance metric between reservations. Each of the two steps of the simulation process described above (i.e., demand sampling and reservation simulation) are stochastic. In other words, the generation of a set of desired reservation requests is stochastic and the mapping of a desired reservation to a realized (or even a lost) reservation is also stochastic.

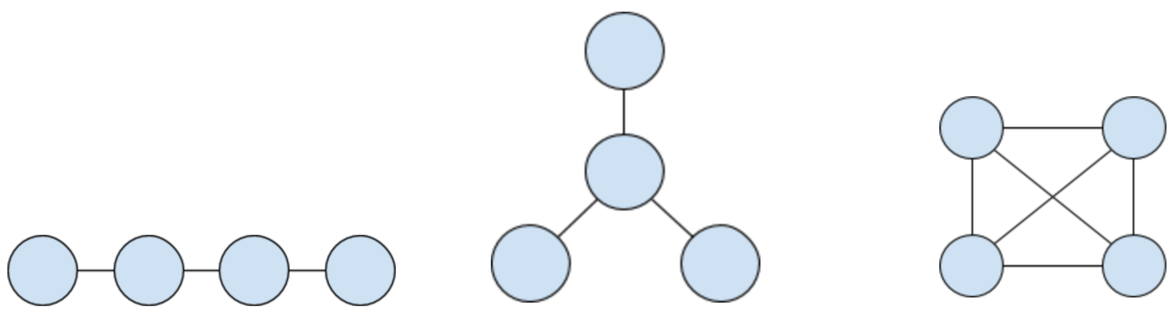
We now present the main simplifications of the analytical network model compared to the simulator. These simplifications contribute to the formulation of an efficient analytical metamodel. First, the analytical model does not enforce the first-come-first-reserve rule of the simulator. In other words, for a given set of reservation requests, they will not be processed by increasing order of reservation creation time. Instead, the set of reservations that leads to highest (metamodel) profit will be realized regardless of their respective creation times. Second, the analytical model allows for reservations to be adjusted in space (i.e., change of station) but not in time (i.e., the start time of a desired reservation cannot change). Third, the adjustment process is simplified. For a given reservation, the simulator checks whether it is available, and if not with a certain probability it considers to either not make any reservation (leading to lost demand) or to attempt a nearby (in space and time) reservation (leading to demand spillback). The simulator iterates on these steps (i.e., a given client may attempt to make several reservations before deciding on a final reservation or before deciding not to make a reservation). In the analytical model, there is no sequential reservation process. Instead, demand spillback is approximated through the discounted revenue parameter, p^{ij} of Eq. (8). Fourth, the simulator considers a time resolution of 30 minutes (i.e., reservation start times and durations are defined in 30 minute increments), while the analytical model considers a time resolution of 1 hour.

3. Case studies

In this section, we apply MetaAHA to optimize the design of two-way car-sharing systems. Section 3.1 considers a low-dimensional problem with synthetic toy networks. Sections 3.2-3.4 consider high-dimensional problems for Zipcar's Boston market. We study its two-way services for two Boston areas: (i) an area of downtown Boston known as South End (Section 3.2) and (ii) a larger network that includes 23 zipcodes of the Boston metropolitan area (they include Allston, Arlington, Boston, Brighton, Brookline, Cambridge, Charlestown, Chelsea, Medford and Somerville) (Section 3.3 and 3.4). All experiments are conducted on a machine with 125GB RAM with an Intel Xeon E5-2630 v3 processor.

3.1. Synthetic toy networks

The goal of these low-dimensional synthetic experiments is to evaluate the quality of the analytical approximation (g_A of Equation (8), which is provided by the analytical network model) of the simulation-based objective function (g of Equation (1)). We consider 3 networks with topologies that are simple and are representative of subnetwork topologies of Zipcar's Boston network. The 3 networks are displayed in Figure 2. Each circle represents a car-sharing station. Each network has four stations. Recall from Section 2.3 that, in the analytical model, when the desired reservation of a user is not available, he/she may consider a substitute chosen from a set of neighbors that are defined as spatially nearby locations (this set was denoted $\mathcal{I}_i$ in Section 2.3). In other words, these are stations where the demand can spillover. In Figure 2, for a given station i, its set of neighbors or substitute stations, $\mathcal{I}_i$, is the set of stations that are connected with an edge to station i. Hence, the three network topologies of Figures 2a, 2b and 2c consider, respectively, a loosely connected network of stations, where no stations share any neighbors; a centralized network, where all stations have the center station as a neighbor; and a fully connected network, where all stations are neighbors with all other stations. Each station has a capacity of 6 vehicles (i.e., N^i of Equation (3) equals 6), the fleet size is unlimited (i.e., X of Equation (2) takes any value such that $X \geq 24$). Hence, the feasible region is $\{x \in [0,6]^4 \cap \mathbb{Z}^4\}$, which contains 2401 feasible solutions. The data used for simulation is the Zipcar reservation data related to such a subnetwork. The planning period is an 8-day period in July 2014 (July 10 to July 17).



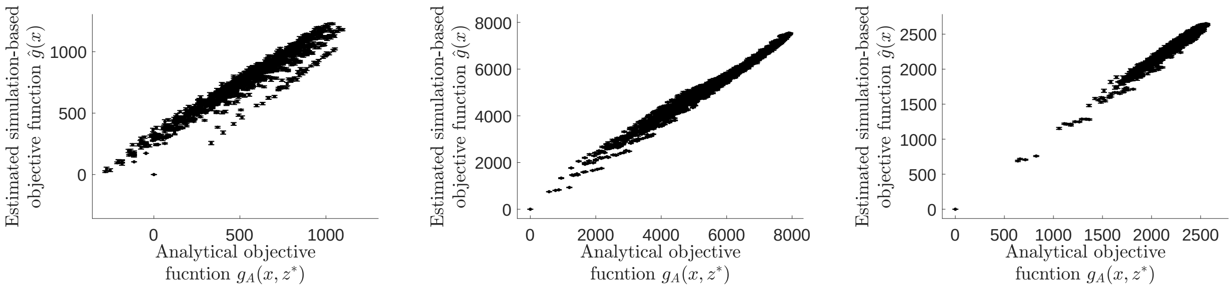
(a) Toy network 1 (b) Toy network 2 (c) Toy network 3

Figure 2 Toy network topologies

For each network, we generate a group of 10 demand scenarios. A demand scenario consists of the desired reservations generated through the demand sampling step described in Section 2.5. The use of various demand scenarios serves to account for demand stochasticity. For a given point, $\boldsymbol{x}$, one simulation replication (i.e., one simulation-based realization of its performance) is defined as the average simulated performance over the 10 demand scenarios. For a given point, $\boldsymbol{x}$, the final estimate of its simulation-based performance, $\hat{g}(\boldsymbol{x})$, is obtained as the average over 50 simulation replications. For the analytical model, we generate a different demand scenario to estimate its exogenous parameters (d^i_{tl} of Equation (10)). For a given point $\boldsymbol{x} \in \mathcal{F}$, the analytical objective function, $g_A(\boldsymbol{x}, \boldsymbol{z}^*)$, is obtained by maximizing Equation (8) over $\boldsymbol{z}$ subject to Constraints (9)-(13).

Each plot of Figure 3 considers one network and displays the analytical objective function, $g_A(\boldsymbol{x}, \boldsymbol{z}^*)$, along the x-axis and the estimated simulation-based objective function, $\hat{g}(\boldsymbol{x})$, with a corresponding 95% interval along the y-axis. The confidence intervals are barely visible. Each plot displays the 2401 feasible solutions.

For all three plots, there is a positive linear correlation between the analytical approximations, $g_A(\boldsymbol{x}, \boldsymbol{z}^*)$, and the simulation-based estimates, $\hat{g}(\boldsymbol{x})$. This indicates that for all three representative network topologies the analytical network model provides a good approximation of the simulation-based objective function.



(a) Toy network 1 (b) Toy network 2 (c) Toy network 3

Figure 3 Comparison of the analytical objective function value with the estimated simulation-based objective function value for toy networks

3.2. Boston South End network

We now consider the South End neighborhood in downtown Boston. A map of the area is displayed in Figure 4. The 23 stations over which we optimize are displayed with red circles. The planning period is July 10-17, 2014. During this period the average fleet size is 101 cars (i.e., $X = 101$). Based on consultation with Zipcar, we set the station capacity, N^i, to 16. We compare the performance of MetaAHA and AHA. This comparison serves to evaluate the added value of complementing AHA with information from the analytical problem-specific network model. The maximum number of algorithm iterations, K, is set to 40. At every iteration, the number of points to be simulated is set to 10 (i.e., $n = 10$).

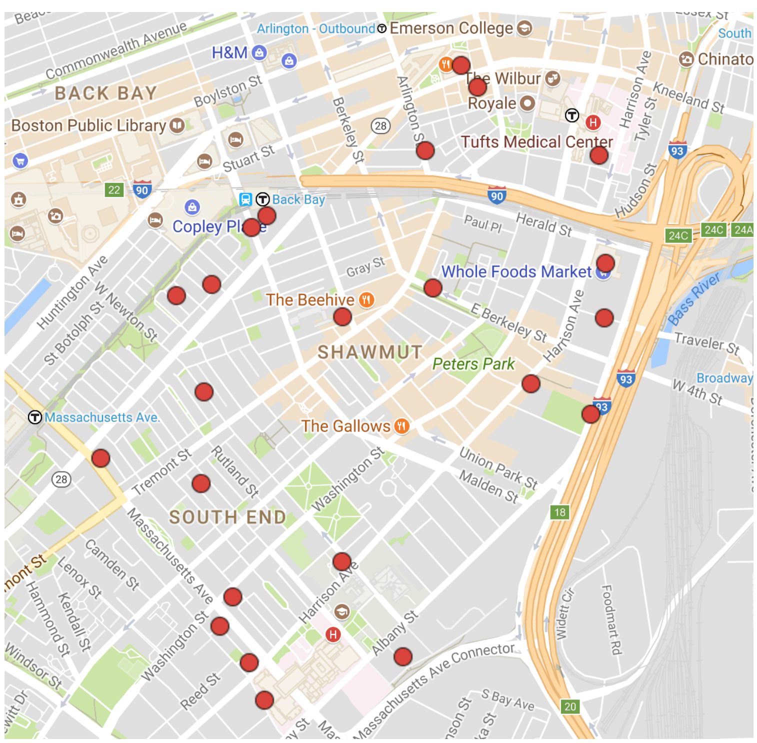
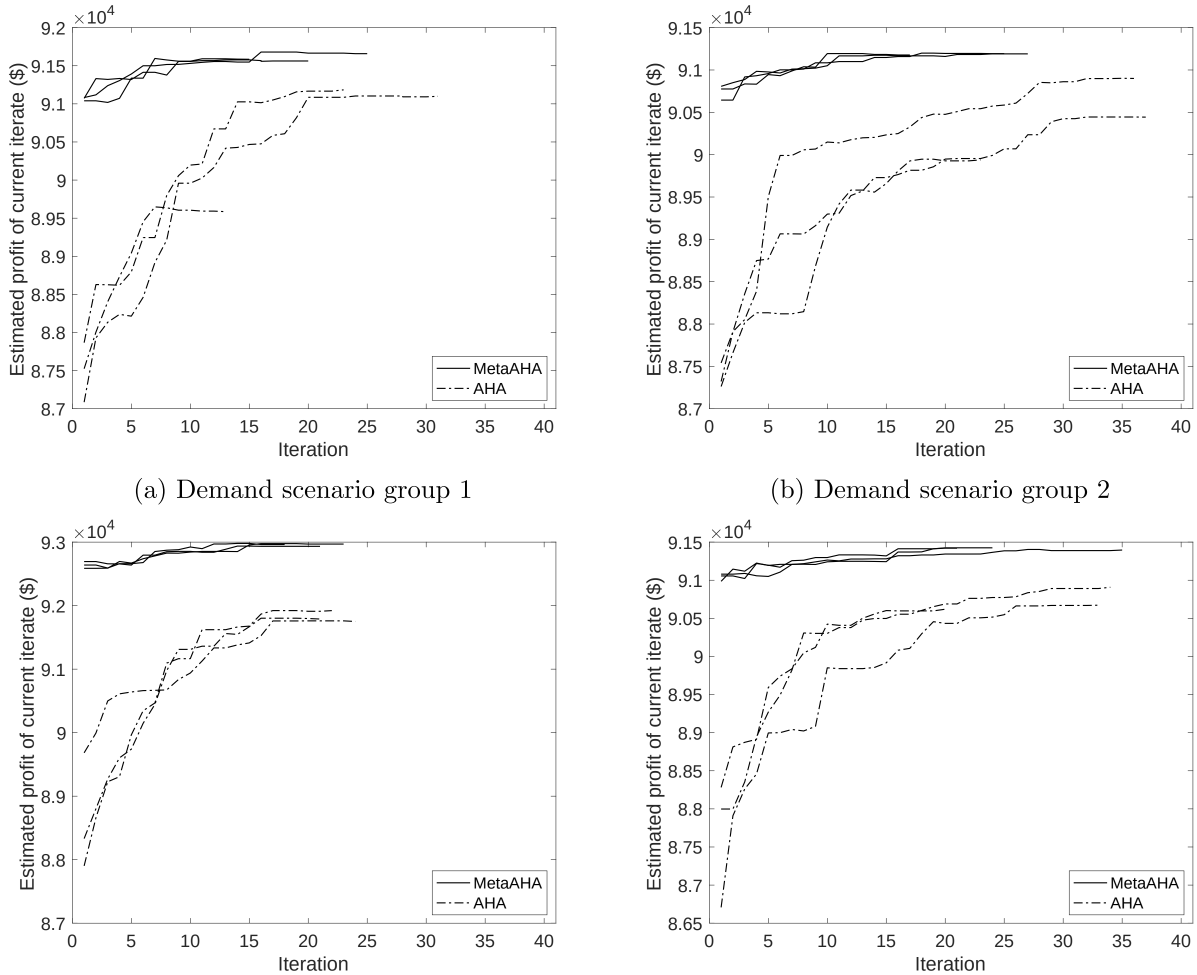


Figure 4 Zipcar stations in Boston South End neighborhood (map data: Google Maps (2017b))

To account for the stochasticity of demand, we proceed as in Section 3.1. We consider a group of 10 demand scenarios. For a given point, $\boldsymbol{x}$, one simulation replication (i.e., one simulation-based realization of its performance) is defined as the average simulated performance over the 10 demand scenarios. Figure 5 contains four plots. Each plot considers a different group of 10 demand scenarios. As in Section 3.1 for each group of demand scenarios, one additional demand scenario is used to estimate the exogenous parameters of the analytical network model.



(a) Demand scenario group 1 (b) Demand scenario group 2

(c) Demand scenario group 3 (d) Demand scenario group 4

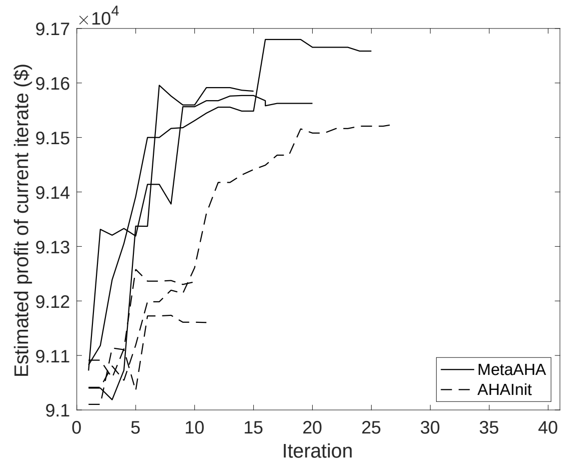
Figure 5 MetaAHA vs. AHA: objective function estimate of the current iterate across iterations

Each plot displays the iteration index along the x-axis and the performance estimate of the current iterate (i.e., simulation-based estimate of the objective function of the best point) along the y-axis. The range of the y-axis differs across the plots. Each plot illustrates, for a given demand scenario group, the difference in performance of the two methods. Each plot displays 6 lines: 3 solid (resp. dashed) lines that represent 3 MetaAHA (resp. AHA) runs. For all plots, we observe the following main trends. First, MetaAHA identifies points with good performance from the first iteration, while the points initially sampled by AHA do not have good performance. Actually, for all six runs of MetaAHA, the best point identified in the first iteration corresponds to the solution of the analytical network design problem (i.e., maximize g_A of Equation (8) over both $\boldsymbol{x}$ and $\boldsymbol{z}$ subject to Constraints (2)-(4) and (9)-(13)). This shows the added value of the analytical structural information provided by g_A. Note that the initial points sampled by AHA are obtained from an asymptotically uniform sampling distribution for integral points from compact polyhedrons as defined in Hong and Nelson (2006). This general-purpose sampling method allows AHA to ensure asymptotic convergence properties, yet since it lacks problem-specific information, it is not designed to provide good quality initial solutions. Second, as the iterations advance, AHA identifies points with improved performance. This is consistent with the experiments and observations in Xu, Nelson, and Hong (2013), which show that AHA is an efficient algorithm for a broad class of discrete SO problems. Nonetheless, it is outperformed throughout by MetaAHA. Third, MetaAHA shows a slight improvement across iterations, yet it is not as significant as that of AHA. Fourth, the performance of the final solution derived by MetaAHA (i.e., the current iterate at the final iteration) is similar across the 3 MetaAHA runs, while final solutions have higher variability in performance for the 3 AHA runs. This indicates that MetaAHA is less sensitive to the stochasticity of the simulator. This may be attributed to the structural analytical information provided by the problem-specific network design model (g_A).

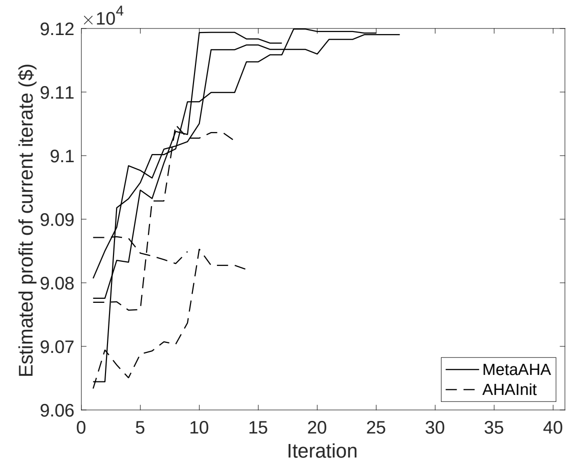
Note that in Figure 5 all lines terminate prior to iteration 40. This occurs if a current iterate is considered to be a local optimum (Step 3a of Algorithm 1). To limit the premature convergence of AHA, Xu, Nelson, and Hong (2013) have combined it with the multi-start ISC framework (Xu, Nelson, and Hong 2010). Also, most lines are not monotonically non-decreasing. This can occur when running additional simulation replications of the current iterate leads to a lower objective function estimate (which can itself lead to a change of the current iterate).

These results indicate the ability of the metamodel approach to: (i) improve the robustness of the algorithm to the quality of the initial points, (ii) identify good solutions within very few iterations, and (iii) lead to low variability across the performance of the derived final solutions. These are all trends that have been observed in our past metamodel work for continuous SO transportation problems (Zhang, Osorio, and Flötteröd 2017, Chong and Osorio 2017, Osorio, Chen, and Santos forthcoming, Osorio forthcoming).

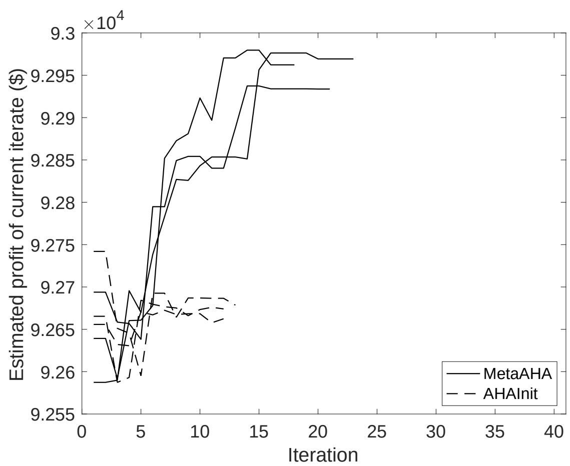
The results of Figure 5 indicate that a suitable approach would be to include in the initial sample of AHA the solution proposed by the analytical network design problem (i.e., the solution that maximizes g_A of Equation (8) over both $\boldsymbol{x}$ and $\boldsymbol{z}$ subject to Constraints (2)-(4) and (9)-(13)), and then to use the traditional AHA algorithm for all other iterations. Let AHAInit denote this approach. We now carry out a comparison of MetaAHA with AHAInit. This comparison serves to evaluate the added value of using analytical network model information across the iterations of AHA, rather than limiting the use of this analytical model to the first iteration. We use the same experimental design as for Figure 5. Figure 6 display four plots. Each plot considers a given group of 10 demand scenarios for the simulator and one demand scenario for the analytical model. The solid (respectively, dashed) lines represent MetaAHA (resp. AHAInit). The range of the y-axis differs across the plots.



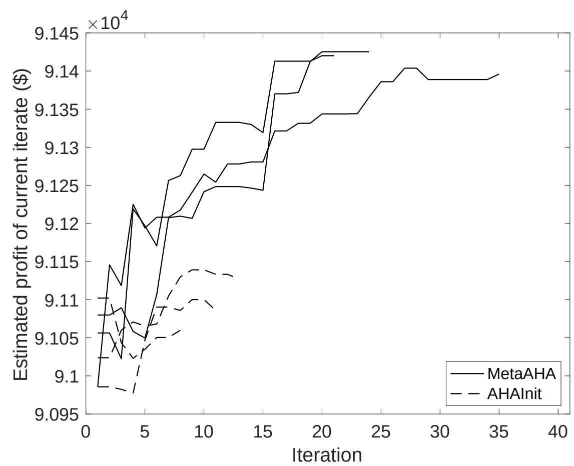
(a) Demand scenario group 1



(b) Demand scenario group 2



(c) Demand scenario group 3



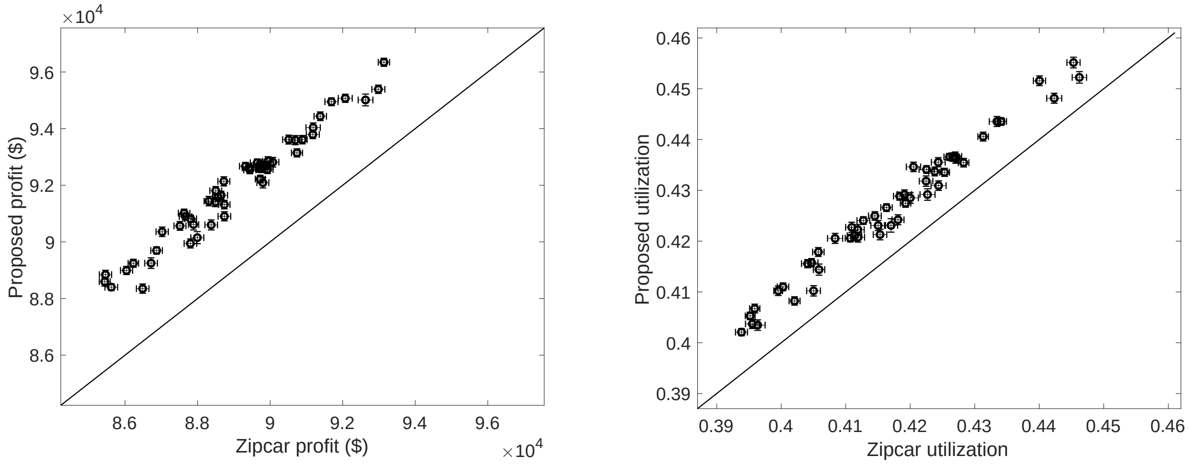
(d) Demand scenario group 4

Figure 6 MetaAHA vs. AHAInit: objective function estimate of the current iterate across iterations

The following trends are common to the four plots. First, MetaAHA outperforms AHAInit across all iterations. This reveals the added value of the metamodel m_k which combines the analytical network design information g_A with the simulation information. In other words, using the analytical network design model g_A to initialize a general-purpose algorithm contributes to its efficiency, yet there is even further added value of using the analytical information across iterations. Second, AHAInit tends to converge more quickly to a local optimum. Often, this local optimum has performance that is similar to that of the point obtained by solving the analytical network design problem (i.e., the point obtained by maximizing g_A subject to Constraints (2)-(4) and (9)-(13)).

For the 12 MetaAHA runs of Figure 6 (i.e., 3 runs for each of the 4 plots), there are a total of 87 instances where the current iterate is updated. Recall that for MetaAHA a current iterate can be of 3 types: (i) it can be a solution to the metamodel optimization problem solved in the entire feasible region (i.e., Step 1b of Algorithm 1, which yields points denoted $\boldsymbol{x}^{\text{meta}}$), (ii) it can be a solution to the metamodel optimization problem solved in the intersection of the entire feasible region and the hyperbox (i.e., Step 1c of Algorithm 1, which yields points denoted $\boldsymbol{x}^{\text{meta-hyper}}$), or (iii) it can be obtained from random sampling (i.e., Step 1a of Algorithm 1, which yields points denoted $\boldsymbol{x}^{\text{sampled}}$). Note that a point can be both of type $\boldsymbol{x}^{\text{meta}}$ and of type $\boldsymbol{x}^{\text{meta-hyper}}$. This occurs when the solution to the metamodel optimization problem in the entire feasible region is located in the hyperbox. Of the 87 different current iterates of the 12 MetaAHA runs in Figure 6, more than two thirds (i.e., 71.3% or 62 points) are of type $\boldsymbol{x}^{\text{meta}}$ or $\boldsymbol{x}^{\text{meta-hyper}}$, while less than one third (28.7% or 25 points) are of type $\boldsymbol{x}^{\text{sampled}}$. In other words, two thirds of the current iterates are obtained by using the structural information of the analytical network model. For the 12 final best solutions returned by the MetaAHA runs, 9 of them are identified by solving the metamodel and 3 of them by random sampling. Moreover for the 12 runs of MetaAHA, we simulated 2364 points. Only 18.1% of the simulated points are obtained by solving the metamodel (429 points), while the remaining 81.9% are obtained by random sampling. Hence, even though the points derived by metamodel evaluations represent only 18.1% of the total set of sampled points, they lead to 75% of the final solutions and 71.3% of the current iterates. This highlights the added value of the structural information provided by the analytical MIP. Among the 62 current iterates obtained by using structural analytical information, 21 are of type $\boldsymbol{x}^{\text{meta}}$ and 47 are of type $\boldsymbol{x}^{\text{meta-hyper}}$ (note that 6 points are both of type $\boldsymbol{x}^{\text{meta}}$ and $\boldsymbol{x}^{\text{meta-hyper}}$). This shows that both the global (i.e., in the entire feasible region) and the local (i.e., in the hyperbox) information of the analytical network model help to identify points with improved performance. Recall that the metamodel is fitted after every iteration, hence the metamodel optimization problems solved across iterations differ and hence their solutions may differ. It is through this fitting process that the metamodel combines information from the simulator with information from the analytical network model. The high number of distinct current iterates identified by the metamodels illustrates the added value, across iterations, of combining the analytical information with the simulated information.

Figure 7 compares the performance of the best fleet assignment identified by MetaAHA (the proposed strategy) with that used by Zipcar during the planning period of interest. The final proposed (or “best”) MetaAHA solution is defined as follows. We consider a set of 50 new demand scenarios. For all 12 solutions derived by MetaAHA (i.e., 3 algorithmic runs for each of the 4 plots of Figure 6), we estimate the average (over the 50 demand scenarios, each scenario is simulated with 50 replications) performance. The proposed solution is that with the best (i.e., largest) average performance.



(a) Profit (b) Utilization

Figure 7 **Comparison of the Zipcar fleet assignment with the proposed assignment for the Boston South End network**

Figure 7 displays two plots. The left plot compares the profit estimates of the two assignments. The right plot compares them according to vehicle utilization. Both of these metrics are important for Zipcar. For each plot, the x-axis considers the Zipcar assignment and the y-axis considers the MetaAHA proposed assignment. Each plot displays 50 points, which correspond to 50 demand scenarios. For each demand scenario, we estimate the performance based on 50 simulation replications. The performance estimate of each point is displayed along with a, barely visible, 95% confidence interval along each direction. Both the left and the right plots indicate that for all 50 demand scenarios the proposed plan yields improved performance, and this across all 50 demand scenarios. Compared to Zipcar's fleet assignment, the proposed solution yields an average improvement of profit of 3.2% and of vehicle utilization of 2.2%. Recall that these estimates are obtained via simulation. Hence, they do not state that the proposed method outperforms the Zipcar method when deployed in the field.

3.3. Boston area network - comparison versus AHAInit

In this section, we consider a larger area of the Boston metropolitan area. This serves to evaluate the performance of MetaAHA for a high-dimensional problem. We consider a network of 315 stations distributed throughout 23 zipcodes that span over Allston, Arlington, Boston, Brighton, Brookline, Cambridge, Charlestown, Chelsea, Medford and Somerville. The map of Figure 8 displays the stations as red circles. We consider the same planning period as before. The station capacity, N^i, is set to 16, based on consultation with Zipcar. Historical data indicates that, during this planning period, there are an average of 894 cars assigned to these stations, i.e., $X = 894$. We proceed as before and consider a group of 10 demand scenarios. One additional demand scenario is used to estimate the exogenous parameters of the analytical model. We set the maximum number of iterations to 40 (i.e., $K = 40$) and the number of points to simulate per iteration to 70 (i.e., $n = 70$).

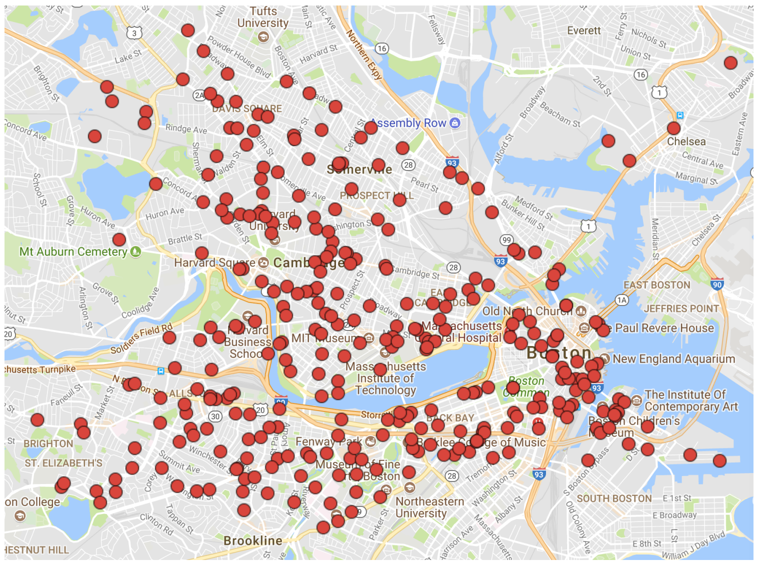


Figure 8 **315 Zipcar stations in Boston area (map data: Google Maps (2017a))**

Figure 9 displays the results of 8 MetaAHA runs (solid lines) and 8 AHAInit runs (dashed lines). Only 3 of the 16 runs deplete the computational budget (i.e., they stop at iteration 40). They correspond to 3 MetaAHA runs. More specifically, the 8 MetaAHA runs stop at iterations 13, 14, 24, 33, 33, 40, 40 and 40. Those of AHAInit stop at iterations 14, 15, 15, 19, 20, 24, 33 and 38. All 8 runs of AHAInit yield final solutions with similar objective estimates. Seven out of the 8 MetaAHA final solutions are better than all 8 AHAInit final solutions.

Figure 10 compares the performance of the best solution identified by MetaAHA with the fleet assignment strategy used by Zipcar. To evaluate the performance of a given fleet assignment strategy (that proposed by MetaAHA or that of Zipcar), we proceed as before. We generate 50 demand scenarios. For each of the 8 final solutions derived by MetaAHA and for each demand scenario, we run 50 simulation replications to estimate the average profit per solution. The solution with the highest average simulated profit is selected as the proposed solution. Figure 10 displays two plots: the left plot considers profit and the right plot considers vehicle utilization. For each plot, the x-axis considers the Zipcar assignment and the y-axis considers the proposed assignment. Each plot displays 50 points which correspond to the 50 demand scenarios. Each point estimate is displayed

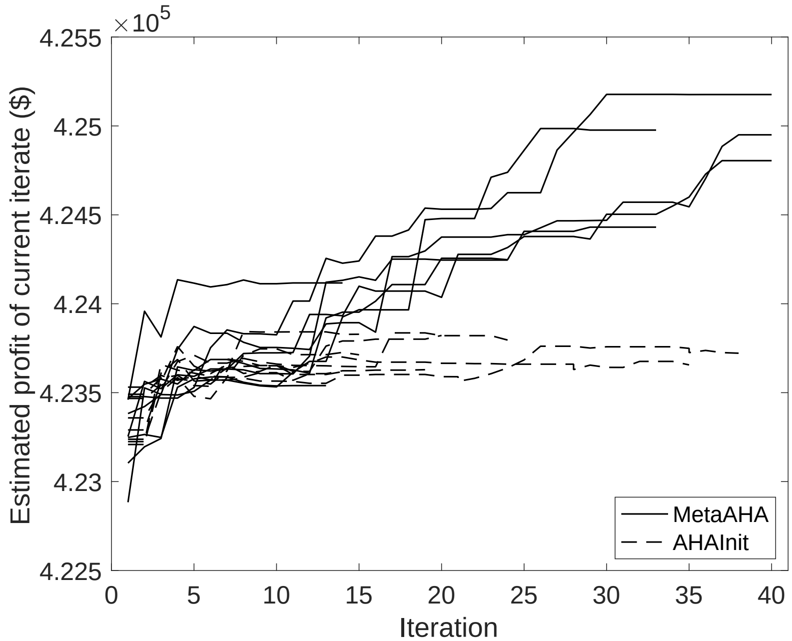
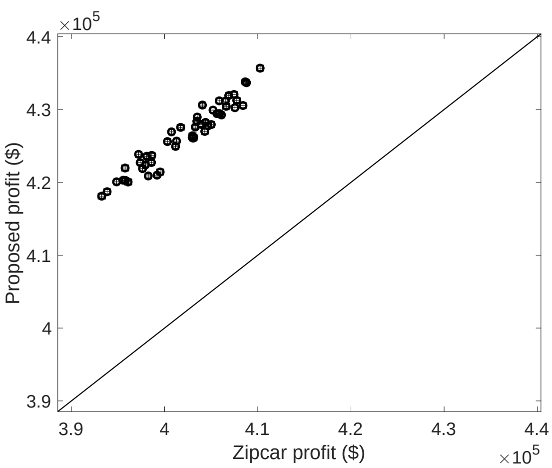
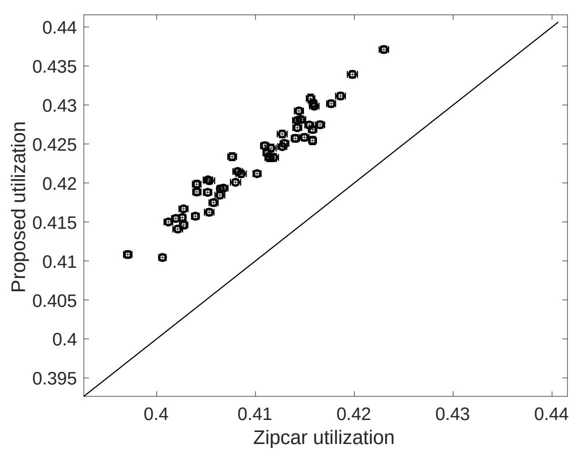


Figure 9 **MetaAHA vs. AHA: objective function estimate of the current iterate across iterations**

along with a 95% confidence interval along both directions. The confidence intervals, which are barely visible, are computed based on 50 replications. For both plots, the 50 points, which represent 50 different demand scenarios, are above the diagonal. Compared to Zipcar's fleet assignment, the proposed solution yields an average improvement of profit of 6% and of vehicle utilization of 3.1%. Moreover, for all 50 demand scenarios, the proposed strategy improves both the profit and the fleet utilization. Again, note that this comparison is based on simulated performance, which may not reflect field performance.



(a) Profit



(b) Utilization

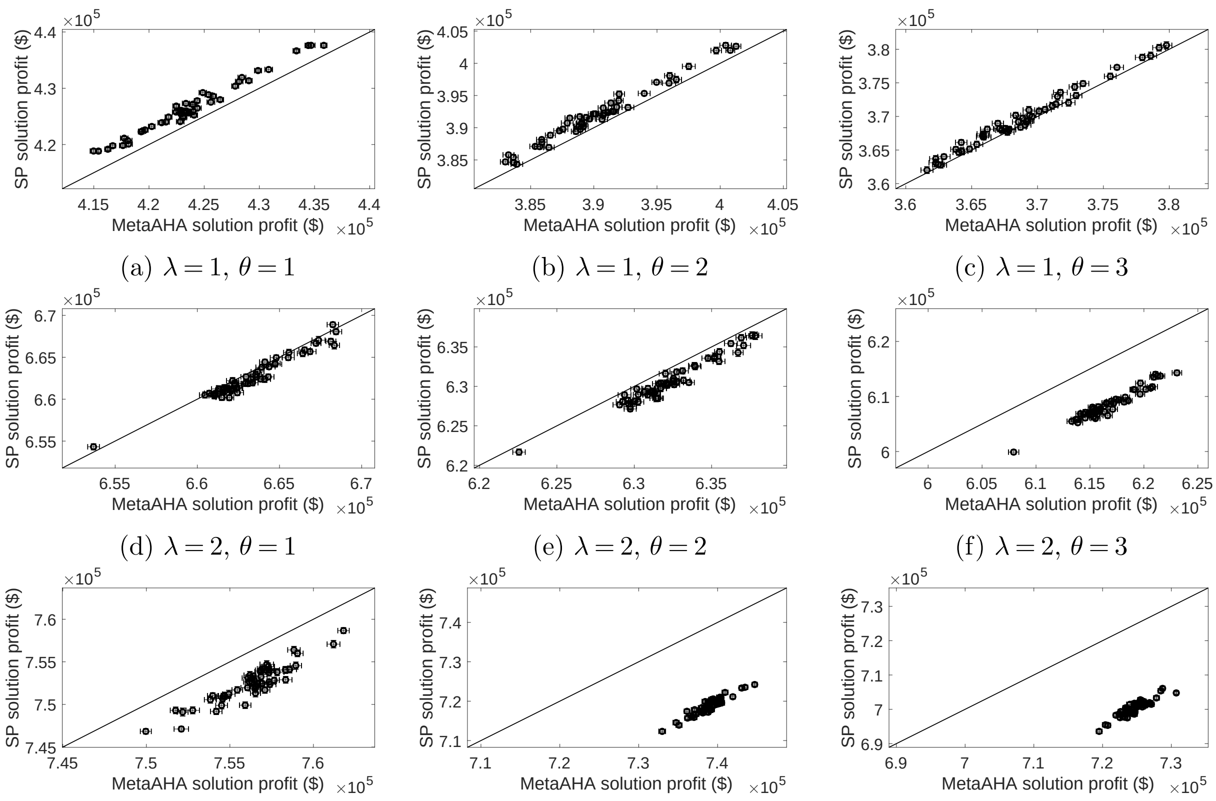
Figure 10 **Comparison of the Zipcar fleet assignment with the proposed assignment for the Boston area network**

3.4. Boston area network - comparison versus stochastic programming

As mentioned in Section 1, the most common approach to study the fleet assignment problem is the use of analytical optimization methods, such as mathematical programs. In this section, we benchmark the performance of the proposed approach to stochastic programming (SP), which accounts for demand uncertainty. We consider a two-stage SP, where the second stage accounts for demand scenario realizations. The SP formulation is given in Appendix B. We use the same Boston area network as that of Section 3.3. We consider a set of 9 experiments with varying levels of demand and of cost. Demand is scaled by a factor: $\lambda \in \{1,2,3\}$, cost (i.e., term c_i of Eq.(1)) is scaled by a factor: $\theta \in \{1,2,3\}$. For each experiment (i.e., a given value of (λ,θ)), we proceed as follows. We consider one demand scenario to estimate the exogenous parameters of MetaAHA, and 3 additional demand scenarios for SP and for the simulator. Hence, the SP model and the simulator used as part of MetaAHA have the same demand input. When evaluating a solution via simulation (within MetaAHA), the simulated estimate of the objective function is obtained as the average over the 3 demand scenarios. We solve the SP model to obtain one SP solution. We run MetaAHA 3 times (in order to account for the stochasticity of both the simulator and the algorithm) to obtain 3 MetaAHA solutions. We use the same MetaAHA algorithmic parameters as in Section 3.3 (i.e., $K = 40$ and $n = 70$). For each final solution (SP or MetaAHA), we simulate its performance considering 50 demand scenarios and 50 simulation replications per demand scenario (for a total of 2500 simulations per solution). We compare the performance of the SP solution to that of the best MetaAHA solution, which is defined as that with the best simulated profit over the 2500 simulations.

Figure 11 displays 9 plots, one for each experiment. The demand scaling factor λ (resp. cost scaling factor θ) is constant across columns (resp. rows) and increases across rows (resp. columns). Each plot displays 50 points, which correspond to each of the 50 demand scenarios. The y-axis (resp. x-axis) displays the estimated, via simulation, mean profit over 50 replications using the SP (resp. best MetaAHA) solution. Each point has a 95% confidence interval along both coordinate directions. These intervals are barely visible. Each plot also displays the diagonal line defined by $y = x$.

For $\lambda = 1$ (i.e., top row of plots: Figures 11a, 11b, 11c), all 50 points are above the diagonal line. In other words, the SP solution outperforms that of MetaAHA. More specifically, the SP solution improves, on average, the profit by 0.65%, 0.43% and 0.21%, for $\theta = 1, 2$ and 3, respectively. This trend is reversed for the other 2 rows of plots. For $\lambda = 2$ (i.e., second row of plots), MetaAHA outperforms SP by an average of 0.1%, 0.26% and 1.36%, for $\theta = 1, 2$ and 3, respectively. For $\lambda = 3$ (bottom row of plots), MetaAHA outperforms SP, on average, by 0.51%, 2.80% and 3.48%, for $\theta = 1, 2$ and 3, respectively. For demand levels λ of 2 or 3 (i.e., second or third row of plots), as the cost level θ increases, so does the amount by which MetaAHA outperforms SP. For a given cost level (i.e., a given column of plots), as the demand level increases (i.e., from the top row to the bottom row), so does the amount by which MetaAHA outperforms SP.



(a) $\lambda = 1,\ \theta = 1$ (b) $\lambda = 1,\ \theta = 2$ (c) $\lambda = 1,\ \theta = 3$

(d) $\lambda = 2,\ \theta = 1$ (e) $\lambda = 2,\ \theta = 2$ (f) $\lambda = 2,\ \theta = 3$

(g) $\lambda = 3,\ \theta = 1$ (h) $\lambda = 3,\ \theta = 2$ (i) $\lambda = 3,\ \theta = 3$

Figure 11 Comparison of the average profit, considering 50 demand scenarios, of the SP solution and of the best MetaAHA solution

Figure 12 considers the same 9 levels of demand and cost. It evaluates the ability of SP to approximate the simulation-based objective function. Each point of Figure 12 considers a given feasible solution and displays along the x-axis the simulated estimate of its objective function value and along the y-axis its SP objective function value. Each plot considers a set of the following 30 feasible solutions: the SP optimal solution (displayed as a blue circle), the best MetaAHA solution (displayed as a red triangle), and a set of 28 randomly sampled solutions (black crosses) that are in the neighborhood of the line connecting the SP optimal solution and the best MetaAHA solution (the sampling process is detailed in Appendix C). The diagonal line, $y = x$, is also displayed. For each solution, the SP objective function value and the simulation-based objective function estimate are based on the same 3 demand scenarios. These demand scenarios are the same as those used to derive the SP solution of the previous analysis. The simulation-based objective function estimate is obtained as the average across 50 replications of each of these 3 demand scenarios (i.e., each estimate involves $150 = 50 \times 3$ simulation evaluations). Figure 13 differs from Figure 12 in that the y-axis of each plot displays the metamodel objective function value. The value of the metamodel parameter $\boldsymbol{\beta}$ is that of the last iteration of the MetaAHA run which generated the best MetaAHA solution. Each of the 9 subplots of Figure 12 have the same axis limits as the corresponding subplot in Figure 13. Hence, the subplots are directly comparable across Figures 12 and 13.

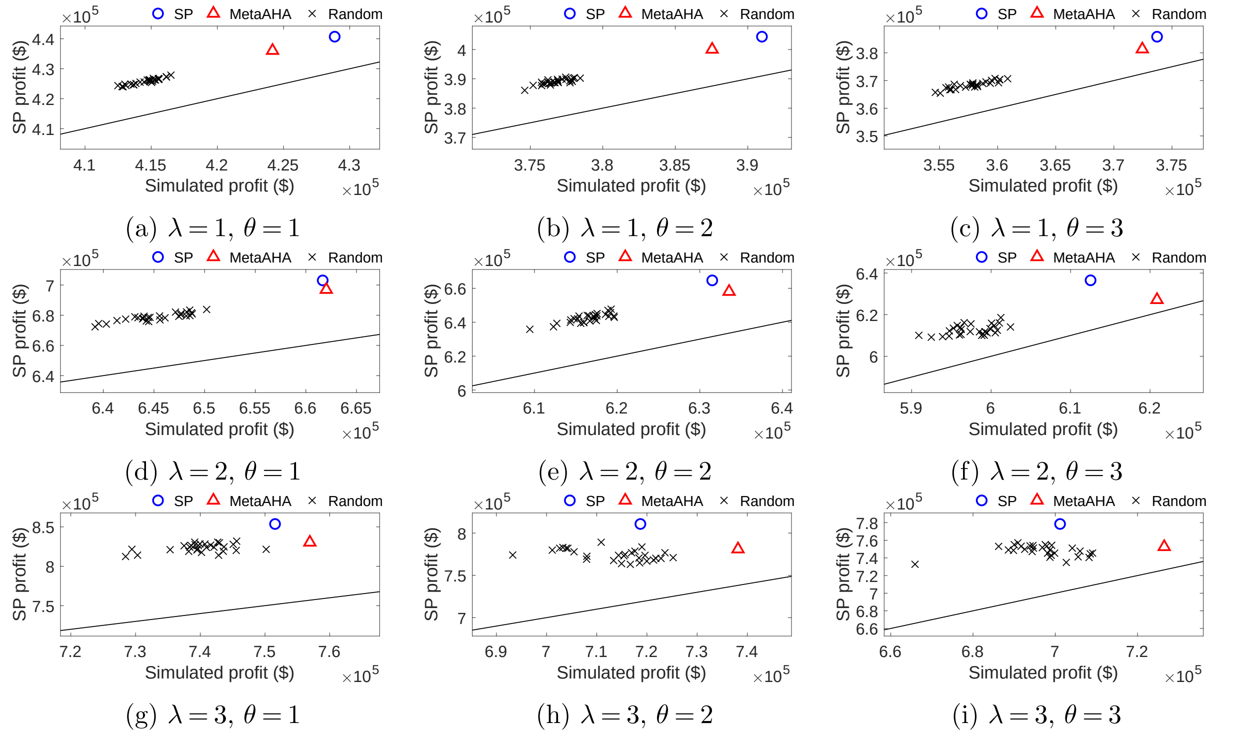
For all plots of Figure 12, all points are above the diagonal line, i.e., SP tends to overestimate the simulated profit for all cost and demand levels. For $\lambda = 1$, the SP objective function exhibits a positive linear correlation with the simulated estimate. This also occurs for $(\lambda, \theta) \in \{(2,1),(2,2)\}$. Hence, the SP model correctly ranks the performance of the feasible solutions, and hence is an adequate tool for optimization. Nonetheless for high values of demand and of cost (i.e., all plots with $\lambda = 3$, as well as the plot $(\lambda, \theta) = (2,3)$), there is no longer a positive linear correlation.

For all plots of Figure 13, all points are close to the diagonal line. This indicates that the metamodel approximates well the simulation-based objective function. The positive linear correlation also highlights the ability of the metamodel to correctly rank the performance of the solutions, and hence its adequacy for optimization. Unlike Figure 12, this correlation trend holds even for high levels of demand and of supply.

These figures indicate the ability of the metamodel approach to approximate the simulation-based objective function even as the demand-supply interactions become more intricate to model (i.e., when both demand levels and cost levels are high). Recall that the main simplifications of the MIPs (both the metamodel MIP and the SP) compared to the simulator are the lack of the first-come-first-reserve principle, as well as the coarse description of demand spillback (for a description of these simplifications, see the last paragraph of Section 2.5). Hence, we expect the ability of the MIPs to approximate the simulator's objective function to deteriorate as the demand and cost levels increase. This is illustrated by comparing Figures 12 and 13. Moreover, the results of Figure 13 indicate that a simple linear parametric correction to the MIP (through the metamodel parameter $\boldsymbol{\beta}$) suffices to correct for these simplifications. In other words, we need not resort to the formulation of a more intricate analytical optimization problem (e.g., with nonlinear functions to describe spillback in more detail).

4. Conclusions

This paper formulates a discrete SO algorithm for a family of large-scale car-sharing network design problems. The approach is a metamodel SO approach. A novel metamodel is formulated, which is based on a MIP formulation. The metamodel is embedded within a general-purpose discrete SO algorithm. The proposed algorithm is validated with synthetic toy network experiments. The metamodel approximations of profit are shown to have a positive linear correlation with the

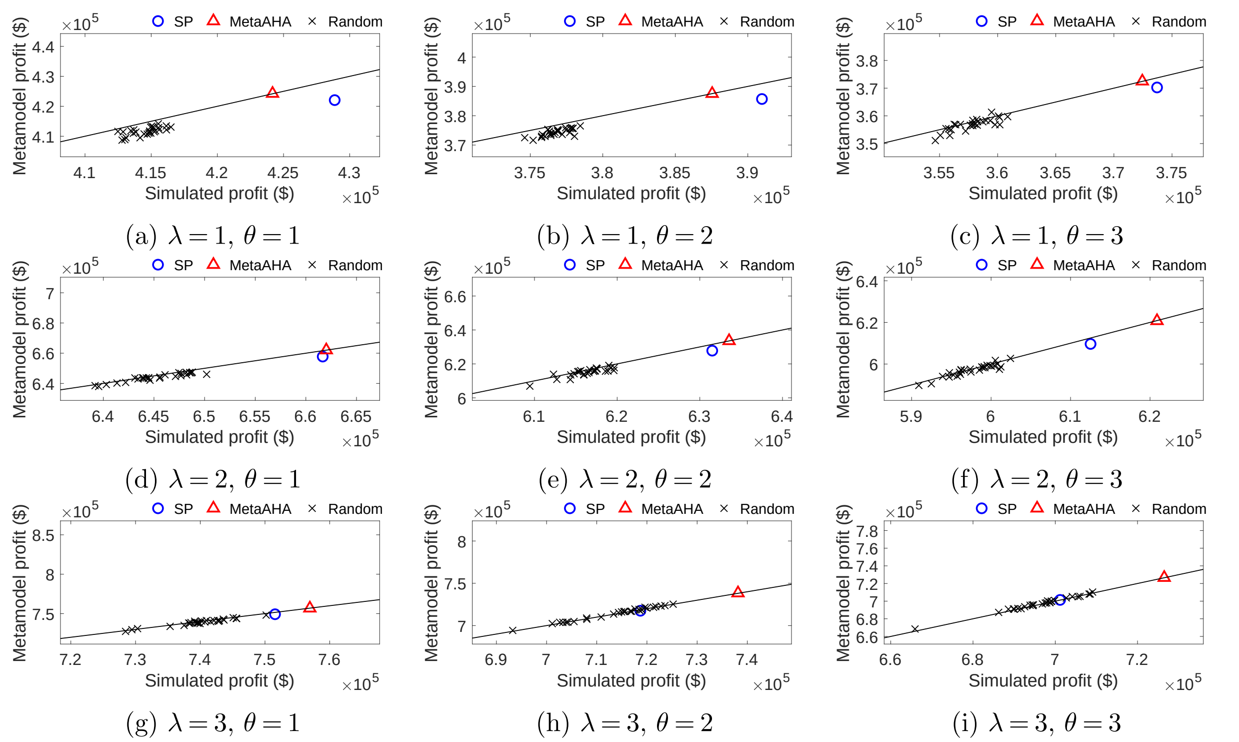


(a) $\lambda = 1,\ \theta = 1$ (b) $\lambda = 1,\ \theta = 2$ (c) $\lambda = 1,\ \theta = 3$

(d) $\lambda = 2,\ \theta = 1$ (e) $\lambda = 2,\ \theta = 2$ (f) $\lambda = 2,\ \theta = 3$

(g) $\lambda = 3,\ \theta = 1$ (h) $\lambda = 3,\ \theta = 2$ (i) $\lambda = 3,\ \theta = 3$

Figure 12 Comparison of the objective functions of the SP model and of the simulation model across various demand levels and cost levels.



(a) $\lambda = 1,\ \theta = 1$ (b) $\lambda = 1,\ \theta = 2$ (c) $\lambda = 1,\ \theta = 3$

(d) $\lambda = 2,\ \theta = 1$ (e) $\lambda = 2,\ \theta = 2$ (f) $\lambda = 2,\ \theta = 3$

(g) $\lambda = 3,\ \theta = 1$ (h) $\lambda = 3,\ \theta = 2$ (i) $\lambda = 3,\ \theta = 3$

Figure 13 Comparison of the objective functions of the metamodel and of the simulation model across various demand levels and cost levels.

higher resolution simulation-based profit estimates. The algorithm is then applied to several Boston case studies using Zipcar car-sharing reservation data, including a high-dimensional problem. The method is first benchmarked versus two types of algorithms that differ only in their use of the analytical MIP information: one benchmark algorithm (AHA) does not use any analytical network information (i.e., no MIP information), the second benchmark algorithm (AHAInit) uses the MIP information only to identify an initial solution but not throughout the optimization process. The experiments indicate that the analytical network model information is useful both at the first iteration of the algorithm and across iterations. The solutions derived by the proposed method are also benchmarked versus the Zipcar deployed solution. Via simulation, the proposed solutions outperform those deployed, both in terms of profit and vehicle utilization. This holds for all considered demand scenarios. We also benchmark MetaAHA versus stochastic programming (SP). SP outperforms the proposed approach for low levels of demand and of cost. As demand and cost levels increase, so does the occurrence of demand spillback and the importance of accounting for the first-come-first-reserve principle. In these cases, the SP approach is outperformed by the metamodel approach.

The combination of the problem-specific analytical MIP with a general-purpose SO algorithm enables the discrete SO algorithm to: (i) address high-dimensional problems, (ii) become computationally efficient (i.e., it can identify good quality solutions within few simulation observations), (iii) become robust to the quality of the initial points and to the stochasticity of the simulator. More generally, the information provided by the MIP to the SO algorithm enables it to exploit problem-specific structural information. Hence, the simulator is no longer treated as a black box.

We view this general idea of combining analytical MIP formulations with general-purpose SO algorithms, or more broadly with general-purpose sampling strategies, as an innovative and promising area of future research. With the increase in the availability and the resolution of transportation data comes the potential to address more intricate formulations of traditional transportation optimization problems (e.g., formulations with a more detailed probabilistic data-driven description of demand). This paper illustrates how the traditional MIP formulations that exist can be coupled with high-resolution data, a sampling (or simulation) strategy, and a general-purpose SO algorithm, to address this next generation of transportation problems.

There is a wide-variety of general-purpose discrete SO algorithms. As general-purpose algorithms, they can be used to address a broad class of problems. Nonetheless, they are rarely designed such as to achieve good short term performance (i.e., good performance within few simulation runs). This paper illustrates how the scalability, computational efficiency and robustness of these SO algorithms can be enhanced such as to enable them to address realistic transportation problems.

The proposed approach performs optimization preserving the disaggregate information in the data (rather than limiting its use to fitting aggregate demand parameters). This leads to a data-driven approach that exploits the rich information of demand and of demand-supply interactions embedded in the data. Nonetheless, this also limits its use for car-sharing markets where data is unavailable or unreliable. In particular, it is not directly applicable for new markets where data has not yet been collected. Extensions of ongoing interest include the use of MIPs to enable the design of real-time discrete SO problems.

Acknowledgments

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Appendix A: Algorithmic details

In this section, we present algorithmic details of MetaAHA. The algorithmic steps refer to Algorithm 1. In Step 2 of the algorithm, the number of simulation replications to run for a given point $\boldsymbol{x}$ up until and including iteration k, denoted $N_k(\boldsymbol{x})$, is computed based on the approach of AHA (Xu, Nelson, and Hong 2013). It is given by $N_k(\boldsymbol{x}) = \min\{5, \lceil 5(\log k)^{1.01} \rceil\}$. If at a given iteration k, the number of simulation replications of point $\boldsymbol{x}$ obtained from previous iterations is greater or equal to $N_k(\boldsymbol{x})$, then we do not evaluate additional replications.

In Step 2 of the algorithm, the hyperbox is updated based on the following AHA approach (Xu, Nelson, and Hong 2013). Let $\boldsymbol{x}_k$ denote the current iterate at iteration k, with the ith element denoted $x_{k,i}$. Let $\mathcal{E}(k)$ denote the set of points that have been simulated up until and including iteration k. The hyperbox is defined (or updated) at iteration k as $\mathcal{H}_k = \{\boldsymbol{x} : l_{k,i} \le x_i \le u_{k,i}, \forall i \in \mathcal{I}\}$. The bounds $l_{k,i}$ and $u_{k,i}$ are defined as follows.

$$l_{k,i} = \max_{\boldsymbol{x} \in \mathcal{E}(k) \setminus \{\boldsymbol{x}_k\}} \{x_i : x_i < x_{k,i}\}, \forall i \in \mathcal{I}.$$

If $l_{k,i}$ is empty, then set $l_{k,i} = 0$. Similarly,

$$u_{k,i} = \min_{\boldsymbol{x} \in \mathcal{E}(k) \setminus \{\boldsymbol{x}_k\}} \{x_i : x_i > x_{k,i}\}, \forall i \in \mathcal{I}.$$

If $u_{k,i}$ is empty, then set $u_{k,i} = N^i$.

Step 4b of the algorithm fits the metamodel parameter $\boldsymbol{\beta}_k$ by solving the below least squares problem, which is formulated and discussed in more detail in Osorio and Bierlaire (2013).

$$\min_{\boldsymbol{\beta}_k} \sum_{\boldsymbol{x} \in \mathcal{E}(k)} \left[w_k(\boldsymbol{x}) \left(\hat{g}(\boldsymbol{x}; \boldsymbol{q}_1) - m_k(\boldsymbol{x}, \boldsymbol{z}; \boldsymbol{\beta}_k, \boldsymbol{q}_2) \right) \right]^2 + \left(w_0(\beta_{k,0} - 1) \right)^2 + \sum_{i=1}^{|\mathcal{I}|+1} (w_0 \beta_{k,i})^2, \tag{14}$$

where w_0 is a fixed scalar weight, $\hat{g}(\boldsymbol{x}; \boldsymbol{q}_1)$ represents the simulated estimate of the profit function for point $\boldsymbol{x}$, and the weight $w_k(\boldsymbol{x})$ function is defined as $w_k(\boldsymbol{x}) = 1/(1 + \|\boldsymbol{x} - \boldsymbol{x}_k\|_2)$. The least squares problem minimizes a weighted distance between the simulated profit estimates $\hat{g}$ and the metamodel predictions m_k. Each point is weighted by a distance function that gives more weights to points that are closer to the current iterate, such as to improve the local (i.e., close to the current iterate) fit of the metamodel. The additional terms in the least squares problem are included such as to ensure a full rank least-squares matrix.

Figure 14 provides a flowchart summary of the MetaAHA algorithm.

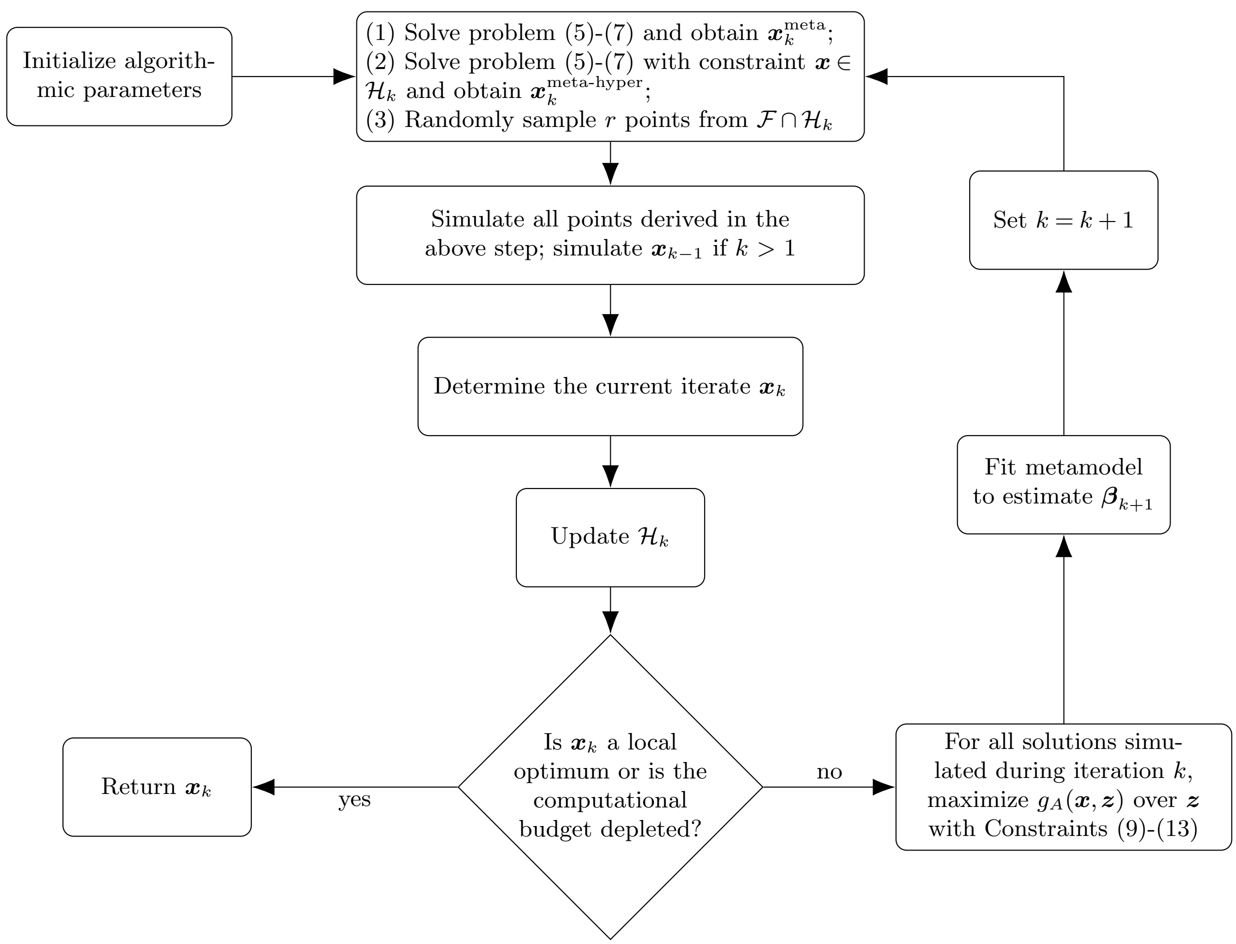

Figure 14 **MetaAHA Steps**

Appendix B: Stochastic programing (SP) formulation

To formulate the SP model, we use the notation of the main manuscript and introduce the following notation.

Q	number of demand scenarios;
$d_{tl}^{i(q)}$	number of customers that desire a reservation at station i with start time t and duration l in demand scenario q;
$z_{tl}^{i(q)}$	number of customers that make a reservation at station i with start time t and duration l in demand scenario q;
$z_{tl}^{ji(q)}$	number of customers that desire to make a reservation at station j with start time t and duration l but make an adjusted reservation at station i with start time t and duration l in demand scenario q;
$\boldsymbol{z}$	vector that combines all variables $\{z_{tl}^{i(q)}\}$ and $\{z_{tl}^{ji(q)}\}$;
$\pi^{(q)}$	probability of scenario q, set to $1/Q$;
$\boldsymbol{q}_3$	vector of exogenous parameters;
g_{SP}	analytical approximation of g (Equation (1)) derived by the SP model.

We view the network design strategy $\boldsymbol{x}$ as the first-stage decision variables, and the demand-supply interaction $\boldsymbol{z}$ as the second-stage decision variables. The SP problem is formulated as follows.

$$\max_{\boldsymbol{x},\boldsymbol{z}} \quad g_{SP}(\boldsymbol{x},\boldsymbol{z};\boldsymbol{q}_3) = \sum_{q=1}^{Q} \pi^{(q)} \left(\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{I}_i} \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} p^{ij} r_{tl} z_{tl}^{ij(q)} \right) - \sum_{i\in\mathcal{I}} c_i x_i, \tag{15}$$

subject to

$$\sum_{j\in\mathcal{I}_i} z_{tl}^{ji(q)} = z_{tl}^{i(q)} \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}, \forall q\in\{1,2,\ldots,Q\}, \tag{16}$$

$$\sum_{j\in\mathcal{I}_i} z_{tl}^{ij(q)} \le d_{tl}^{i(q)} \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}, \forall q\in\{1,2,\ldots,Q\}, \tag{17}$$

$$\sum_{l\in\mathcal{L}} z_{tl}^{i(q)} + \sum_{l\in\mathcal{L}} \sum_{t'\in\mathcal{T}_1(t,l)} z_{t'l}^{i(q)} \le x_i \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall q\in\{1,2,\ldots,Q\}, \tag{18}$$

$$z_{tl}^{i(q)} \in \mathbb{R}_+ \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}, \forall q\in\{1,2,\ldots,Q\}, \tag{19}$$

$$z_{tl}^{ij(q)} \in \mathbb{R}_+ \qquad \forall i\in\mathcal{I}, \forall j\in\mathcal{I}_i, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}, \forall q\in\{1,2,\ldots,Q\}, \tag{20}$$

$$\boldsymbol{x}\in\mathcal{F}, \tag{21}$$

where $\mathcal{T}_1(t,l) = \{t'\in\mathcal{T} : t'+1 \le t \le t'+l-1\}$. In this model, the exogenous parameters are $d_{tl}^{i(q)}$ and $\pi^{(q)}$, as well as r_{tl}, c_i, p^{ij}, $t_{\max}$ and $l_{\max}$ defined in Section 2.3, represented by the vector $\boldsymbol{q}_3$. The objective function (15) is the expected revenue over all scenarios minus the cost. Constraints (16)-(20) are the equivalent of their MIP counterparts Constraints (9)-(13), respectively. Constraint (21) is equivalent to Constraint (7).

Appendix C: Sampling of feasible solutions for the experiments of Figures 12 and 13

For a given demand scenario, let $\boldsymbol{x}^{(1)}$ be the SP solution and $\boldsymbol{x}^{(2)}$ be the best MetaAHA solution of the experiments of Section 3.4. We use the following procedure to generate a solution near the line connecting the SP optimal solution and the best MetaAHA solution:

- Step 1: generate $u \sim U(0,1)$, where $U(0,1)$ is the standard uniform distribution.

- Step 2: let $\tilde{\boldsymbol{x}} = \boldsymbol{x}^{(1)} + u\left(\boldsymbol{x}^{(2)} - \boldsymbol{x}^{(1)}\right)$.
- Step 3: build a hyperbox $\mathcal{H}(\tilde{\boldsymbol{x}}) = \{\boldsymbol{x} : \tilde{x}_i - 2 \le x_i \le \tilde{x}_i + 2, \forall i \in \mathcal{I}\}$.
- Step 4: randomly sample a point $\boldsymbol{x}$ from $\mathcal{H}(\tilde{\boldsymbol{x}}) \cap \mathcal{F}$ using the uniform sampling distribution of AHA (Xu, Nelson, and Hong 2013).

References

Ankenman B, Nelson BL, Staum J, 2010 *Stochastic Kriging for simulation metamodeling. Operations Research* 58(2):371–382.

Balac M, Ciari F, Axhausen KW, 2016 *Evaluating the influence of parking space on the quality of service and the demand for one-way carsharing: a Zürich area case study. Proceedings of the 95th Annual Meeting of the Transportation Research Board.*

Balac M, Ciari F, Axhausen KW, 2017 *Modeling the impact of parking price policy on free-floating carsharing: case study for Zürich, Switzerland. Transportation Research Part C* 77:207–225.

Barton RR, Meckesheimer M, 2006 *Metamodel-based simulation optimization.* Henderson SG, Nelson BL, eds., *Handbooks in operations research and management science: simulation*, volume 13, 535–574 (Amsterdam, Netherlands: Elsevier).

Becker H, Ciari F, Axhausen KW, 2017 *Comparing car-sharing schemes in Switzerland: User groups and usage patterns. Transportation Research Part A* 97:17–29.

Boyacï B, Zografos KG, Geroliminis N, 2015 *An optimization framework for the development of efficient one-way car-sharing systems. European Journal of Operational Research* 240(3):718–733.

Boyacï B, Zografos KG, Geroliminis N, 2017 *An integrated optimization-simulation framework for vehicle and personnel relocations of electric carsharing systems with reservations. Transportation Research Part B* 95:214–237.

Brandstätter G, Gambella C, Leitner M, Malaguti E, Masini F, Puchinger J, Ruthmair M, Vigo D, 2016 *Overview of optimization problems in electric car-sharing system design and management.* Dawid H, Doerner KF, Feichtinger G, Kort PM, Seidl A, eds., *Dynamic Perspectives on Managerial Decision Making: Essays in Honor of Richard F. Hartl*, volume 22, 441–471 (Cham, Switzerland: Springer International Publishing).

Cepolina EM, Farina A, 2012 *A new shared vehicle system for urban areas. Transportation Research Part C* 21(1):230–243.

Chen X, Zhu Z, He X, Zhang L, 2015 *Surrogate-based optimization for solving a mixed integer network design problem. Transportation Research Record* 2497(1):124–134.

Chick SE, Inoue K, 2001 *New two-stage and sequential procedures for selecting the best simulated system. Operations Research* 49(5):732–743.

Chiraphadhanakul V, 2013 *Large-scale analytics and optimization in urban transportation: Improving public transit and its integration with vehicle-sharing services.* Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

Chong L, Osorio C, 2017 *A simulation-based optimization algorithm for dynamic large-scale urban transportation problems. Transportation Science* 52(3):637–656.

Ciari F, Balac M, Axhausen KW, 2016 *Modeling carsharing with the agent-based simulation MATSim: state of the art, applications, and future developments. Transportation Research Record* 2564(1):14–20.

Ciari F, Balac M, Balmer M, 2015 *Modelling the effect of different pricing schemes on free-floating carsharing travel demand: a test case for Zurich, Switzerland. Transportation* 42(3):413–433.

Ciari F, Balmer M, Axhausen KW, 2009 *Concepts for large-scale carsharing system: Modeling and evaluation with agent-based approach. Proceedings of the 88th Annual Meeting of the Transportation Research Board.*

Ciari F, Bock B, Balmer M, 2014 *Modeling station-based and free-floating carsharing demand: test case study for Berlin. Transportation Research Record* 2416(1):37–47.

Ciari F, Schuessler N, Axhausen KW, 2013 *Estimation of carsharing demand using an activity-based microsimulation approach: model discussion and some results. International Journal of Sustainable Transportation* 7(1):70–84.

Ciari F, Weis C, Balac M, 2016 *Evaluating the influence of carsharing stations location on potential membership: a Swiss case study. EURO Journal on Transportation and Logistics* 5(3):345–369.

Coll MH, Vandersmissen MH, Thériault M, 2014 *Modeling spatio-temporal diffusion of carsharing membership in Québec City. Journal of Transport Geography* 38:22–37.

Correia GHA, Antunes AP, 2012 *Optimization approach to depot location and trip selection in one-way carsharing systems. Transportation Research Part E* 48(1):233–247.

Correia GHA, Jorge DR, Antunes DM, 2014 *The added value of accounting for users flexibility and information on the potential of a station-based one-way car-sharing system: an application in Lisbon, Portugal. Journal of Intelligent Transportation Systems* 18(3):299–308.

De Lorimier A, El-Geneidy AM, 2013 *Understanding the factors affecting vehicle usage and availability in carsharing networks: A case study of communauto carsharing system from Montréal, Canada. International Journal of Sustainable Transportation* 7(1):35–51.

Deng Y, 2015 *Design and management of mobility-on-demand (MOD) transportation systems considering demand uncertainty and flexibility - a simulation-based approach.* Ph.D. thesis, National University of Singapore, Singapore.

Duncan M, 2011 *The cost saving potential of carsharing in a US context. Transportation* 38(2):363–382.

Fields E, Osorio C, Zhou T, 2017 *A data-driven car sharing simulator for inferring latent demand.* Technical report, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA, available at: `http://web.mit.edu/osorioc/www/papers/fields17Sim.pdf`.

Fields E, Osorio C, Zhou T, 2018 *A data-driven method for reconstructing a distribution from a truncated sample with an application to inferring car-sharing demand.* Technical report, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA, available at: `https://web.mit.edu/osorioc/www/papers/fieOsoZho_carShareDem.pdf`.

Firnkorn J, Müller M, 2011 *What will be the environmental effects of new free-floating car-sharing systems? The case of Car2go in Ulm. Ecological Economics* 70(8):1519–1528.

Frazier PI, Powell WB, Dayanik S, 2008 *A knowledge-gradient policy for sequential information collection. SIAM Journal on Control and Optimization* 47(5):2410–2439.

Google Maps, 2017a *315 Zipcar stations in Boston area.* `https://drive.google.com/open?id=1HggY_4-dOuYU71lhnudL8WqeSJA&usp=sharing`, accessed: 2017-09-22.

Google Maps, 2017b *Zipcar stations in Boston South End neighborhood.* `https://drive.google.com/open?id=1hOvbRIjfZJjF5L3OfoAThiq0_p8&usp=sharing`, accessed: 2017-09-22.

Greenhall A, 2016 *Experimentation in a ridesharing marketplace.* `https://eng.lyft.com/https-medium-com-adamgreenhall-simulating-a-ridesharing-marketplace-36007a8a31f2`, accessed: 2018-08-15.

He L, Mak HY, Rong Y, Shen ZJM, 2017 *Service region design for urban electric vehicle sharing systems. Manufacturing & Service Operations Management* 19(2):309–327.

Hong LJ, Nelson BL, 2006 *Discrete optimization via simulation using COMPASS. Operations Research* 54(1):115–129.

Hong LJ, Nelson BL, Xu J, 2015 *Discrete optimization via simulation.* Fu MC, ed., *Handbook of simulation optimization*, volume 216, 9–44 (New York, NY, USA: Springer).

Jian N, Freund D, Wiberg HM, Henderson SG, 2016 *Simulation optimization for a large-scale bike-sharing system.* Roeder TMK, Frazier PI, Szechtman R, Zhou E, Huschka T, Chick S, eds., *Proceedings of the 2016 Winter Simulation Conference*, 602–613 (Piscataway, NJ, USA: IEEE Press).

Jones DR, Schonlau M, Welch WJ, 1998 *Efficient global optimization of expensive black-box functions. Journal of Global optimization* 13(4):455–492.

Jorge D, Barnhart C, Correia GHA, 2015 *Assessing the viability of enabling a round-trip carsharing system to accept one-way trips: Application to Logan Airport in Boston. Transportation Research Part C* 56:359–372.

Jorge D, Correia GHA, 2013 *Carsharing systems demand estimation and defined operations: A literature review. European Journal of Transport and Infrastructure Research* 13(3):201–220.

Jung J, Chow JY, Jayakrishnan R, Park JY, 2014 *Stochastic dynamic itinerary interception refueling location problem with queue delay for electric taxi charging stations. Transportation Research Part C* 40:123–142.

Kleijnen JP, Van Beers W, Van Nieuwenhuyse I, 2010 *Constrained optimization in expensive simulation: Novel approach. European Journal of Operational Research* 202(1):164–174.

Lu M, Chen Z, Shen S, 2017 *Optimizing the profitability and quality of service in carshare systems under demand uncertainty. Manufacturing & Service Operations Management* 20(2):162–180.

MATSim, 2018 *MATSim: multi-agent transport simulation.* `https://www.matsim.org`, accessed: 2018-08-14.

Millard-Ball A, Murray G, Ter Schure J, Fox C, Burkhardt J, 2005 *Car-sharing: Where and how it succeeds*. Technical Report TCRP Report 108, Transportation Research Board of the National Academies, Washington, D.C., USA.

Nagaraj KS, 2014 *Stochastically constrained simulation optimization on mixed-integer spaces*. Ph.D. thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA.

Nair R, Miller-Hooks E, 2014 *Equilibrium network design of shared-vehicle systems. European Journal of Operational Research* 235(1):47–61.

NCSL, 2017 *Car sharing - state laws and legislation.* `http://www.ncsl.org/research/transportation/car-sharing-state-laws-and-legislation.aspx`, NCSL stands for National Conference of State Legislatures. Accessed: 2018-08-12.

Nelson BL, 2010 *Optimization via simulation over discrete decision variables*. Hasenbein JJ, Gray P, Greenberg HJ, eds., *Tutorials in Operations Research: Risk and Optimization in an Uncertain World*, volume 7, 193–207 (Hanover, MD, USA: INFORMS).

O'Mahony ED, 2015 *Smarter tools for (Citi) bike sharing*. Ph.D. thesis, Cornell University, Ithaca, New York, USA.

Osorio C, forthcoming *Dynamic OD (origin-destination) calibration for large-scale network simulators. Transportation Research Part C* Available at `http://web.mit.edu/osorioc/www/papers/osoDynamicOD.pdf`.

Osorio C, Bierlaire M, 2013 *A simulation-based optimization framework for urban transportation problems. Operations Research* 61(6):1333–1345.

Osorio C, Chen X, Santos BF, forthcoming *Simulation-based travel time reliable signal control. Transportation Science* Available at `http://web.mit.edu/osorioc/www/papers/osoCheSanReliableSO.pdf`.

Osorio C, Chong L, 2015 *A computationally efficient simulation-based optimization algorithm for large-scale urban transportation problems. Transportation Science* 49(3):623–636.

Osorio C, Nanduri K, 2015 *Energy-efficient urban traffic management: A microscopic simulation-based approach. Transportation Science* 49(3):637–651.

Salemi PL, 2014 *Gaussian Markov random fields and moving least squares for metamodeling and optimization in stochastic simulation.* Ph.D. thesis, Northwestern University, Evanston, Illinois, USA.

Schmöller S, Weikl S, Müller J, Bogenberger K, 2015 *Empirical analysis of free-floating carsharing usage: The Münich and Berlin case. Transportation Research Part C* 56:34–51.

Sebastiani MT, Lüders R, Fonseca KVO, 2016 *Evaluating electric bus operation for a real-world BRT public transportation using simulation optimization. IEEE Transactions on Intelligent Transportation Systems* 17(10):2777–2786.

Shaheen S, Chan N, 2016 *Mobility and the sharing economy: Potential to facilitate the first-and last-mile public transit connections. Built Environment* 42(4):573–588.

Shaheen SA, Cohen AP, 2013 *Carsharing and personal vehicle services: Worldwide market developments and emerging trends. International Journal of Sustainable Transportation* 7(1):5–34.

Stillwater T, Mokhtarian P, Shaheen S, 2009 *Carsharing and the built environment: Geographic information system-based study of one US operator. Transportation Research Record* 2110(1):27–34.

Sun L, Hong LJ, Hu Z, 2014 *Balancing exploitation and exploration in discrete optimization via simulation through a Gaussian process-based search. Operations Research* 62(6):1416–1438.

Swisher JR, Jacobson SH, Yücesan E, 2003 *Discrete-event simulation optimization using ranking, selection, and multiple comparison procedures: A survey. ACM Transactions on Modeling and Computer Simulation* 13(2):134–154.

Wang H, Pasupathy R, Schmeiser BW, 2013 *Integer-ordered simulation optimization using R-SPLINE: Retrospective search with piecewise-linear interpolation and neighborhood enumeration. ACM Transactions on Modeling and Computer Simulation* 23(3):1–24.

Wild SM, Regis RG, Shoemaker CA, 2008 *ORBIT: Optimization by radial basis function interpolation in trust-regions. SIAM Journal on Scientific Computing* 30(6):3197–3219.

Xie J, Frazier PI, Chick SE, 2016 *Bayesian optimization via simulation with pairwise sampling and correlated prior beliefs. Operations Research* 64(2):542–559.

Xu J, 2012 *Efficient discrete optimization via simulation using stochastic Kriging.* Laroque C, Himmelspach J, Pasupathy R, Rose O, Uhrmacher AM, eds., *Proceedings of the 2012 Winter Simulation Conference*, 1–12 (Piscataway, NJ, USA: IEEE Press).

Xu J, Nelson BL, Hong JL, 2010 *Industrial strength COMPASS: A comprehensive algorithm and software for optimization via simulation. ACM Transactions on Modeling and Computer Simulation* 20(1):1–29.

Xu J, Nelson BL, Hong LJ, 2013 *An adaptive hyperbox algorithm for high-dimensional discrete optimization via simulation problems. INFORMS Journal on Computing* 25(1):133–146.

Zhang C, Osorio C, Flötteröd G, 2017 *Efficient calibration techniques for large-scale traffic simulators. Transportation Research Part B* 97:214–239.

Zhou T, 2015 *Network design for integrated vehicle-sharing and public transportation service*. Master's thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

Zipcar, 2017 *Zipcar officially launches in Worcester, Massachusetts*. `http://www.zipcar.com/press/releases/worcester`, accessed: 2017-09-26.