18.S097: Introduction to Metric Spaces
IAP 2022
Credit: 3 units

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Course Description:
    Covers metrics, open and closed sets, continuous functions
    (in the topological sense), function spaces, completeness,
    and compactness. This class strives to bridge the gap
    between 18.100A and 18.100B material. Covering pages
    229-266 in Lebl's textbook (see below).

Lecture Structure:
    1.5 hour lectures TTh, Jan.4th-Jan.20 2022.

Textbook(s):
- Lebl's *Basic Analysis I: Introduction to Real Analysis,
  Volume 1*. This book is available as [a free PDF download](#).
- If wanted, for additional references, I suggest the
  following: *Topology* by Munkres and *Elementary Real Analysis*
  by Thomas, Bruckner, and Bruckner.

Prerequisites/Audience:
    18.100A/P is the intended prerequisite for this course.
    18.100B/Q will have covered the material in this class.
    This class is targeted towards students with a basic
    understanding of material covered in 18.100A.

Grading/Assignments:
    This class will be PNR with grading based on assignments
    and participation. There will be three PSETs, and there
    will not be any exams.

Office Hours:

    I will host office hours; the times will be based on
    students' availability.

Lecture Outline:

Lecture 1 (T):

    Motivation, definition, and intuition behind metric spaces.
    Redefining 18.100A/P in terms of metrics: open/closed sets,
    convergence, Cauchy sequences, and continuity.

Lecture 2 (Th):

    Some general theory of metric spaces regarding convergence,
    open and closed sets, continuity, and their relationship to
    one another.

Lecture 3 (T):

    Norms and analysis on finite sets (as motivation for
    compact sets). Topological compactness and sequential
    compactness. Today will focus purely on theorems regarding
    compact subsets of R^n. The Heine-Borel theorem and the
    Bolzano-Weierstrass theorem.

Lecture 4 (Th):

    Compact sets on general metric spaces. Sequential
    compactness is equivalent to topological compactness on
    metric spaces.

Lecture 5 (T):

    Compact is equivalent to totally bounded and complete. The
    Banach fixed point theorem and ODEs. The Arzela-Ascoli
    theorem.

Lecture 6 (Th):

    How these topics were motivated, and a preview of how these
    topics come up in later classes.