18.S190: Introduction to Metric Spaces
IAP 2023
Credit: 3 units

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Course Description:

Covers metrics, open and closed sets, continuous functions (in the topological sense), function spaces, completeness, and compactness. This class strives to bridge the gap between 18.100A and 18.100B material. Covering pages 229-266 in Lebl's textbook (see below).

Lecture Structure:

1.5 hour lectures TTh, Jan.4th-Jan.20 2022.

Textbook(s):

- Lebl's *Basic Analysis I: Introduction to Real Analysis, Volume 1*. This book is available as a free PDF download.
- If wanted, for additional references, I suggest the following: *Topology* by Munkres and *Elementary Real Analysis* by Thomas, Bruckner, and Bruckner.

Prerequisites/Audience:

18.100A/P is the intended prerequisite for this course. 18.100B/Q will have covered the material in this class. This class is targeted towards students with a basic understanding of material covered in 18.100A.

Grading/Assignments:

This class will be PNR with grading based on assignments and participation. There will be three PSETs, and there will not be any exams.

Office Hours:

The times for OH will be based on students' availability.

Lecture Outline:

Lecture 1 (T):

Motivation, definition, and intuition behind metric spaces. Redefining 18.100A/P in terms of metrics: open/closed sets, convergence, Cauchy sequences, and continuity.

Lecture 2 (Th):

Some general theory of metric spaces regarding convergence, open and closed sets, continuity, and their relationship to one another.

Lecture 3 (T):

Norms and analysis on finite sets (as motivation for compact sets). Topological compactness and sequential compactness. Today will focus purely on theorems regarding compact subsets of R^n. The Heine-Borel theorem and the Bolzano-Weierstrass theorem.

Lecture 4 (Th):

Compact sets on general metric spaces. Showing sequential compactness is equivalent to topological compactness, which is equivalent to being totally bounded and complete (on metric spaces).

Lecture 5 (T):

Completions of metric spaces, motivating L^p spaces, Sobolev spaces, p-adic numbers, Banach spaces, and Hilbert spaces.

Lecture 6 (Th):

How these topics were motivated, and a preview of how these topics come up in later classes.