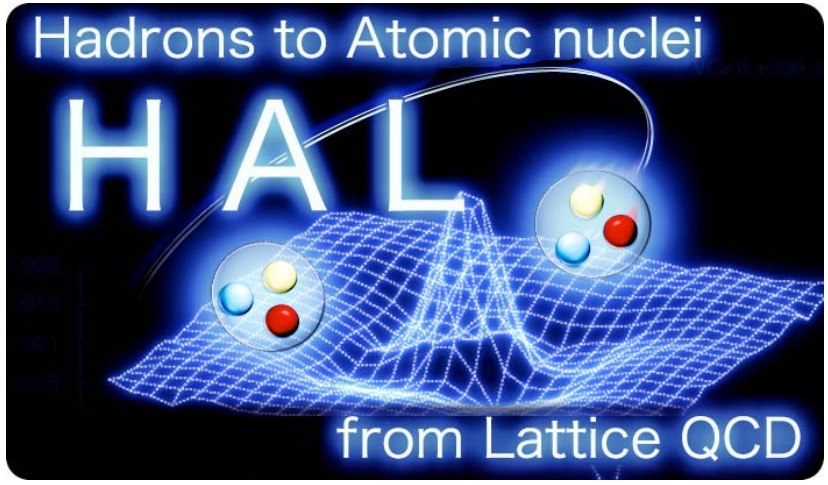
The Lattice QCD study of the Three Nucleon Force

Takumi Doi

(CNS, Univ. of Tokyo)

for HAL QCD Collaboration

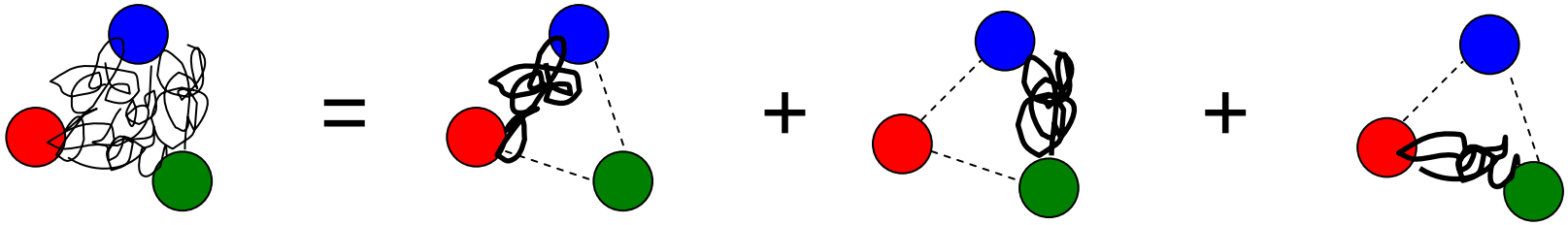


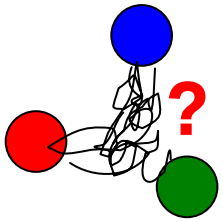
S. Aoki, N. Ishii, K. Sasaki (Univ. of Tsukuba)
T. Hatsuda (Univ. of Tokyo & RIKEN)
Y. Ikeda (Tokyo Inst. Tech.)
T. Inoue (Nihon Univ.)
K. Murano (RIKEN)
H. Nemura (Tohoku Univ.)

arXiv:1106.2276 [hep-lat]

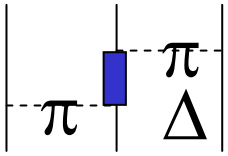
Importance of Three Nucleon Force (3NF)

- **3NF** ← potential among 3N which **cannot** be reduced to pair-wise 2N potential





(3NF)



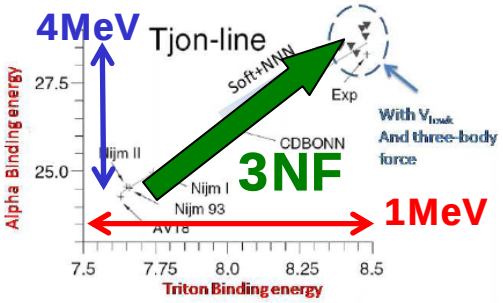
Fujita-Miyazawa(1957)

Importance of Three Nucleon Force (3NF)

- Precise few-body calc:
  NN force cannot reproduce B.E.

  δB.E.= 0.5-1MeV for ^{3}H
  δB.E.= 2-4 MeV for ^{4}He

  Attractive 3NF necessary



Nogga et al., PRL85(2000)944

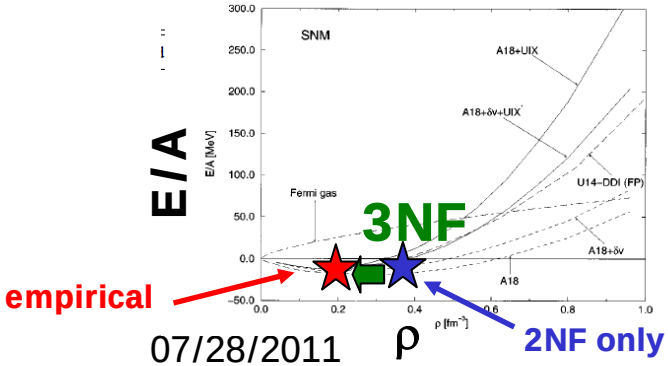
- Saturation density/energy of nuclear matter also requires 3NF

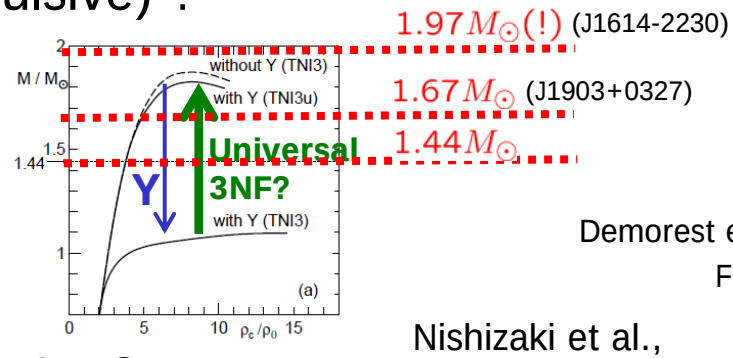
  Repulsive 3NF also necessary

  A.Akmal et al., PRC58(1998)1804

  Takatsuka et al., PTPS174(2008)80

  - EOS of neutron star
    - Flavor universal 3BF (repulsive) ?



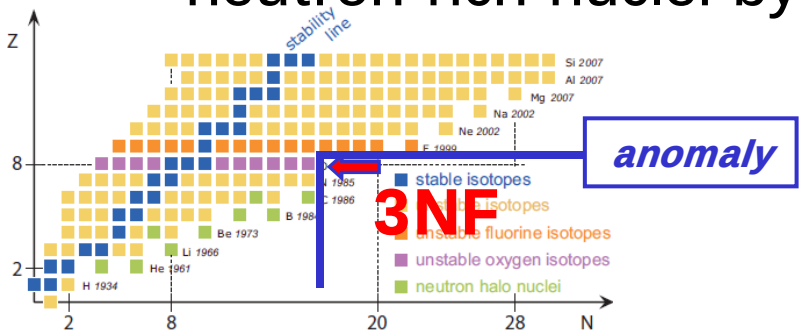


Demorest et al. (2010)
Freire (2009)

Nishizaki et al., PTP108(2002)703

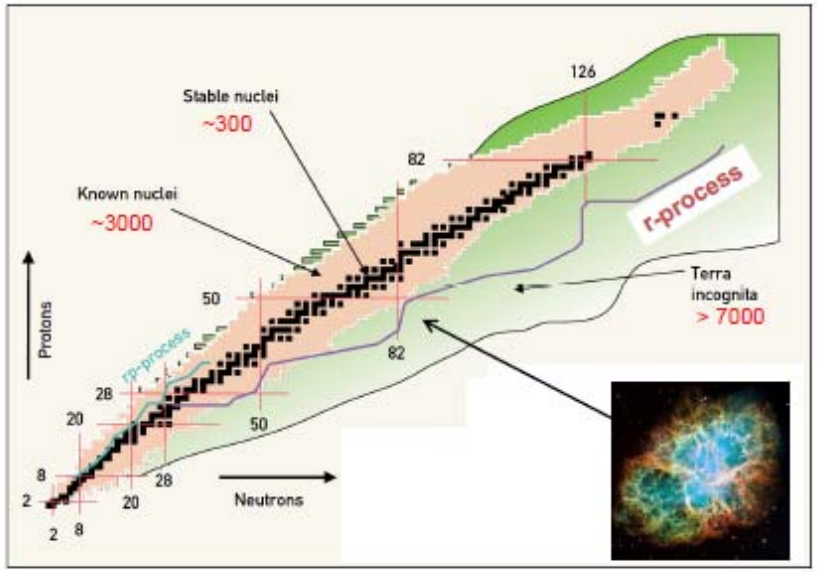
Importance of Three Nucleon Force (3NF)

- The effect on the nuclear chart
  - Anomaly in drip line and nontrivial magic number in neutron rich nuclei by 3NF



drip line: $^{28}O \rightarrow {}^{24}O$

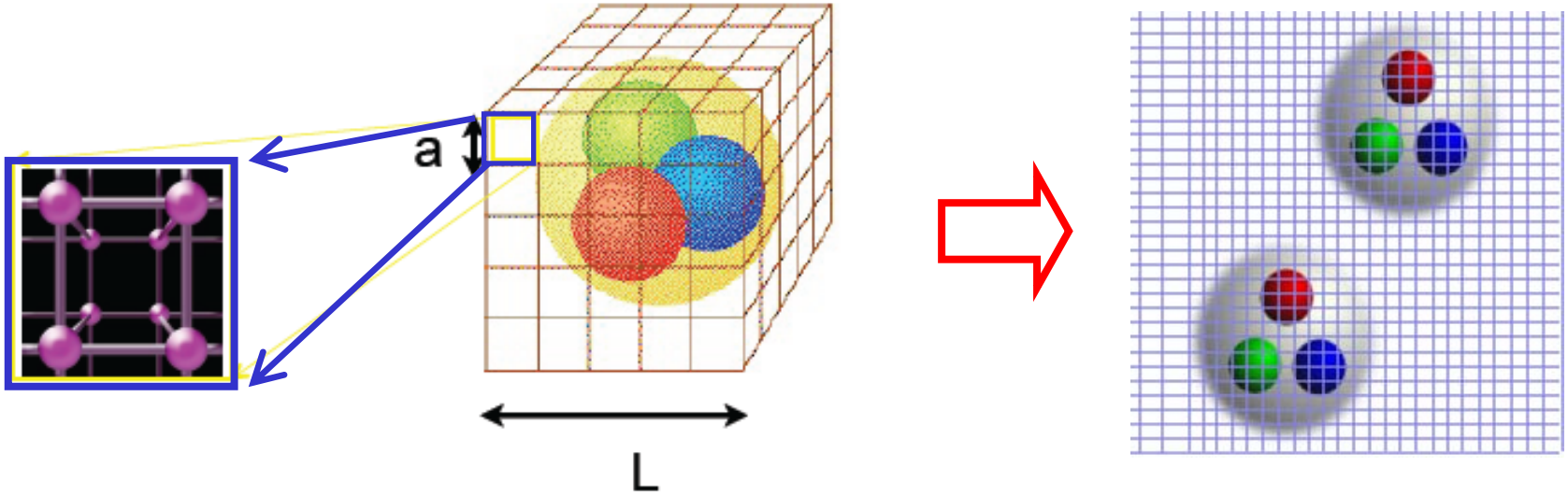
nontrivial magic number
N=28 for $_{20}Ca$



Nucleosynthesis by Supernova

T.Otsuka et al., PRL105(2010)032501
J.D.Holt et al., arXiv:1009.5984

Lattice QCD as 1st principle calc



- well-defined statistical system (finite a and L)
- gauge invarinat
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

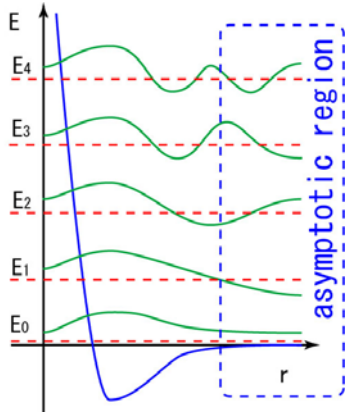
Nuclear Force from Lattice QCD [HAL QCD strategy]

- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)
- Nambu-Bethe-Salpeter(NBS) wave function

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)|2N\rangle$$

  - Key concept: asymptotic region ←→ phase shift

$$(\nabla^2 + k_\delta^2)\psi(\vec{r}) = 0, \quad r > R$$



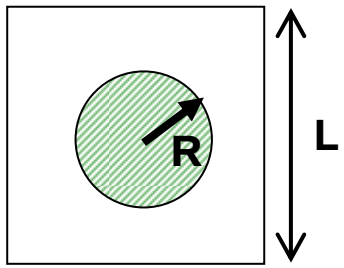
Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504

- Define the potential at interaction region

$$(\nabla^2 + k_\delta^2)\psi(\vec{r}) = \int d\vec{r'} \boxed{U(\vec{r},\vec{r'})} \psi(\vec{r'}), \quad r < R$$

  - Non-local, but E-independent potential



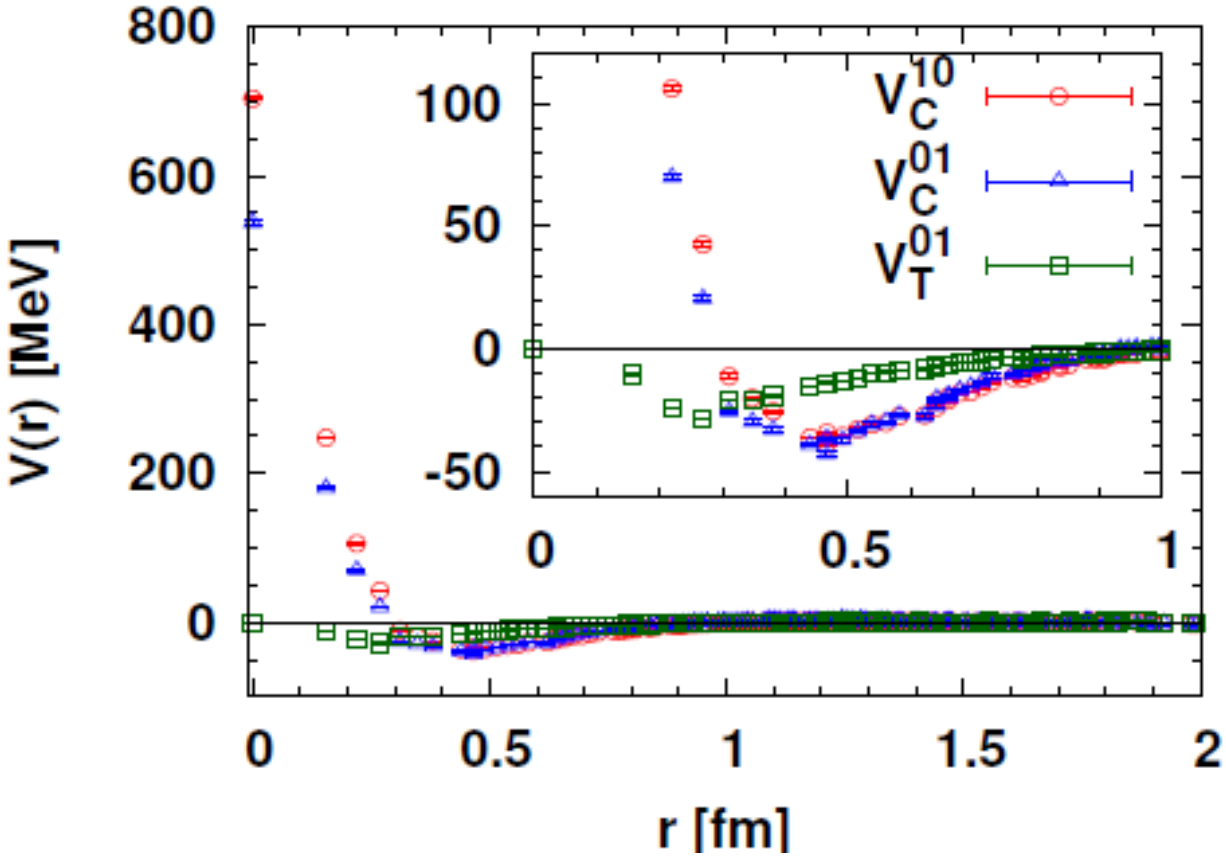
- Velocity expansion Okubo-Marshak(1958)

$$\boxed{U(\vec{r},\vec{r'})} = \underset{\text{LO}}{V_c(r)} + \underset{\text{LO}}{S_{12}V_T(r)} + \underset{\text{NLO}}{\vec{L}\cdot\vec{S}V_{LS}(r)} + \underset{\text{NNLO}}{\mathcal{O}(\nabla^2)}$$

  - Truncation in expansion introduces E-dep (only practically), but we can **improve order by order**

Aoki-Hatsuda-Ishii
PTP123(2010)89

2N potentials (parity-even) from Lattice QCD



Nf=2 (CP-PACS)
16^3x32, L=2.5fm
1/a=1.27GeV

M(π)=1.13GeV

How can we tackle 3NF in Lattice QCD ?

c.f. pioneering lat calc of B.E. ^{3}He(=^{3}H), ^{4}He
T.Yamazaki et al., arXiv:0912.1383

- **In the case of 2N system…**
  - Calc 4pt func ➔NBS amp.

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r};t)N(\vec{x};t)|2N\rangle$$

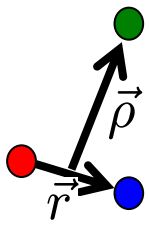
$$|2N\rangle = \bar{N}_{src}(t=0)\bar{N}_{src}(t=0)|0\rangle$$

➔ $(E - H_0)\psi(\vec{r})$

$$= [V_c(r) + S_{12}V_T(r) + \cdots]\psi(r)$$

- **Extention to 3N system**
  - Calc 6pt func ➔NBS amp. of 3N

$$\psi(\vec{r},\vec{\rho}) = \langle 0|\boxed{N(\vec{x}+\vec{r})}\boxed{N(\vec{x})}\boxed{N(\vec{x}+\vec{r}/2+\vec{\rho})}|3N\rangle$$



  - Obtain 3NF through

$$(E - H_0^r - H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\underline{\textstyle\sum_{i<j} V_{ij}(\vec{r}_{ij})} + V_{3NF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

by 2N calc

  - Difficulty(1): volume factor
    - 2N: naïve O(L^6) calc ➔ O(L^3 log L^3)
    - 3N: naïve O(L^9) calc ➔ O(L^6 log L^6)

O(10^4-10^5) factor

  - Difficulty(2): naïve calc of quark dof grows in factorial (~N_u! N_d!)
    - 2N: O(L^3) X N_{wick} X color/spinor loops
    - 3N: O(L^6) X N_{wick} X color/spinor loops

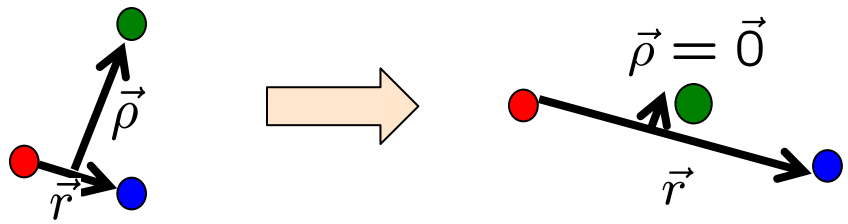
O(L^3) X O(4000) = O(10^7-10^8) factor

3NF is exceptionally challenging problem !

How can we tackle 3NF in Lattice QCD ? (cont'd)

- Calculation for **fixed 3D-configuration** of 3N system
  - **Direct access to 3NF is possible !**
  - ➔ We can explore the various features of 3NF (spin/isospin/spacial, etc.)
  - Huge calc cost (**O(10^2-10^3) factor** compared to 2N)
  - We study **linear setup**

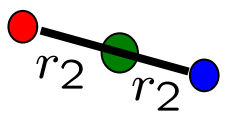
We consider Triton channel



$$(\vec{r}_2 \equiv \vec{r}/2)$$

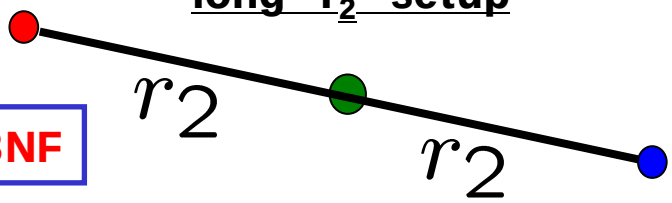
  - **Linear setup** with various distance **"r_2"**

short "r_2" setup

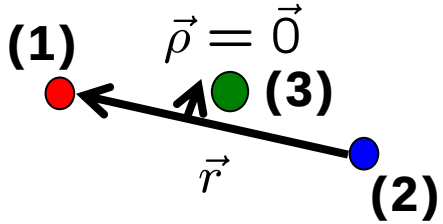


Study r_2-dependence of 3NF

long "r_2" setup



Features of Linear setup for ^{3}H



- Simplified coupled channel analysis possible
  - The vector to 3rd particle $\vec{\rho} = \vec{0}$
  - ➔ $L^{(1,2)\text{-pair}} = L^{\text{total}} = 0$ or 2 only
  - ➔ Possible **bases are only three**, which can be labeled by **1S0, 3S1, 3D1 for (1,2)-pair**

Schrodinger Eq.

$$\hat{H}_0 \begin{pmatrix} \psi_{(^1S_0)} \\ \psi_{(^3S_1)} \\ \psi_{(^3D_1)} \end{pmatrix} + \underbrace{\begin{pmatrix} V \\ (V_{2N} + V_{3NF}) \end{pmatrix}}_{\text{3x3 Matrix}} \begin{pmatrix} \psi_{(^1S_0)} \\ \psi_{(^3S_1)} \\ \psi_{(^3D_1)} \end{pmatrix} = E \begin{pmatrix} \psi_{(^1S_0)} \\ \psi_{(^3S_1)} \\ \psi_{(^3D_1)} \end{pmatrix}$$

Kinetic energy

Parity-odd potential Issue

- However, in order to determine TNF in **3x3 coupled channel**, we need information of parity-odd potential
  - Although (1,2)-pair is L=even, (3,1),(2,3)-pair have L=odd components
- Parity-odd potential from lattice QCD (still) in progress
  - ➔3X3 channel, but unknown $V_C^{I,S=0,0}$,$V_C^{I,S=1,1}$,$V_T^{I,S=1,1}$,TNF(s)

$$\hat{H}_0 \begin{pmatrix} \psi_{(^1S_0)} \\ \psi_{(^3S_1)} \\ \psi_{(^3D_1)} \end{pmatrix} + \begin{pmatrix} & V & \\ & (V_{2N} + V_{3NF}) & \\ & & \end{pmatrix} \begin{pmatrix} \psi_{(^1S_0)} \\ \psi_{(^3S_1)} \\ \psi_{(^3D_1)} \end{pmatrix} = E \begin{pmatrix} \psi_{(^1S_0)} \\ \psi_{(^3S_1)} \\ \psi_{(^3D_1)} \end{pmatrix}$$

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$: (P = odd)

unknown

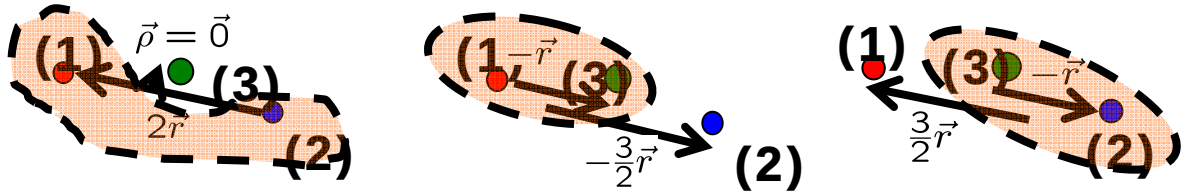
Target to be determined

Solution using "symmetric" wave function

- Rotate the basis $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right) \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric



- ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed

All pair P=even

$$\hat{H}_0 \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \hat{V}_{3NF} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric
  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only 3NF unknown</u>**
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(3NF) unknown

$$\begin{pmatrix} H_0 \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} + \begin{pmatrix} V_{2N} \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} + \begin{pmatrix} V_{3NF} \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} = E\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix}$$

No V(P=odd)

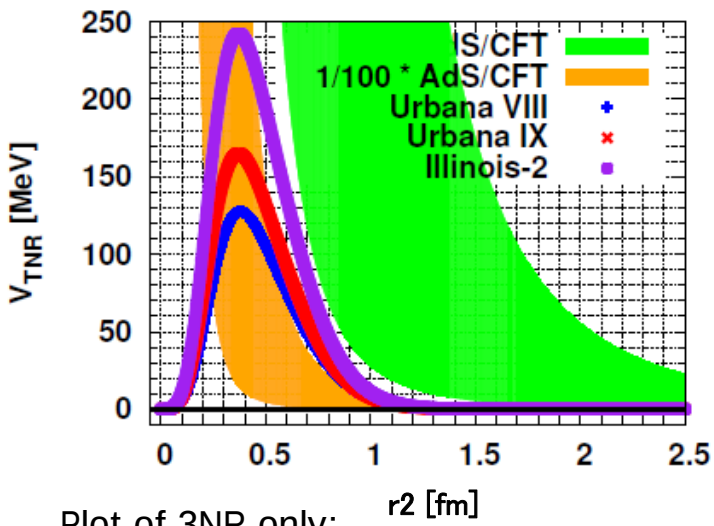
Target to be determined

(L^2-dep ignored)

- ➔ Even without parity-odd V, we can determine one 3NF
  - This methodology works for any fixed 3D-conf other than linear

Repulsive 3NF (3NR)

- We determine 3NF effectively represented by a scalar/isoscalar functional form
  - c.f. phenomenological 3NF to reproduce saturation point of nuclear matter, etc.



Plot of 3NR only:
there is cancellation from 3NA

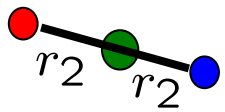
$$V_{3NF} = V_{2\pi E} + (V_{3\pi R}) + V_{3NR}$$

Urbana/Illinois

$$V_{3NR} = U_0 \sum_{cyc} T^2(r_{12}) T^2(r_{13})$$

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} T_{cut}(r)$$

AdS/CFT: $V_{3NF} = +\text{const.} \cdot \frac{1}{r^4}$



K.Hashimoto, N.Iizuka
JHEP 1011 (2010) 058

Lattice calculation setup

- Nf=2 dynamical clover fermion + RG improved gauge configs (CP-PACS)
  - 598 configs X 32 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 X 32 lattice, L=2.5fm
  - Kappa(ud)=0.13750
    - $M(\pi)$ = 1.13GeV
    - M(N) = 2.15GeV (Mπ L=14)
    - $M(\Delta)$ = 2.31GeV
  - Techniques
    - **Automatic Wick contraction code** to handle 4 up- and 5 down-quarks
    - **Non-rela limit op** is used to create 3N state **at source**

$$N^{src} = \epsilon_{abc}(u_a^T C\gamma_5 \tfrac{1+\gamma_4}{2} d_b)\tfrac{1+\gamma_4}{2} u_c$$

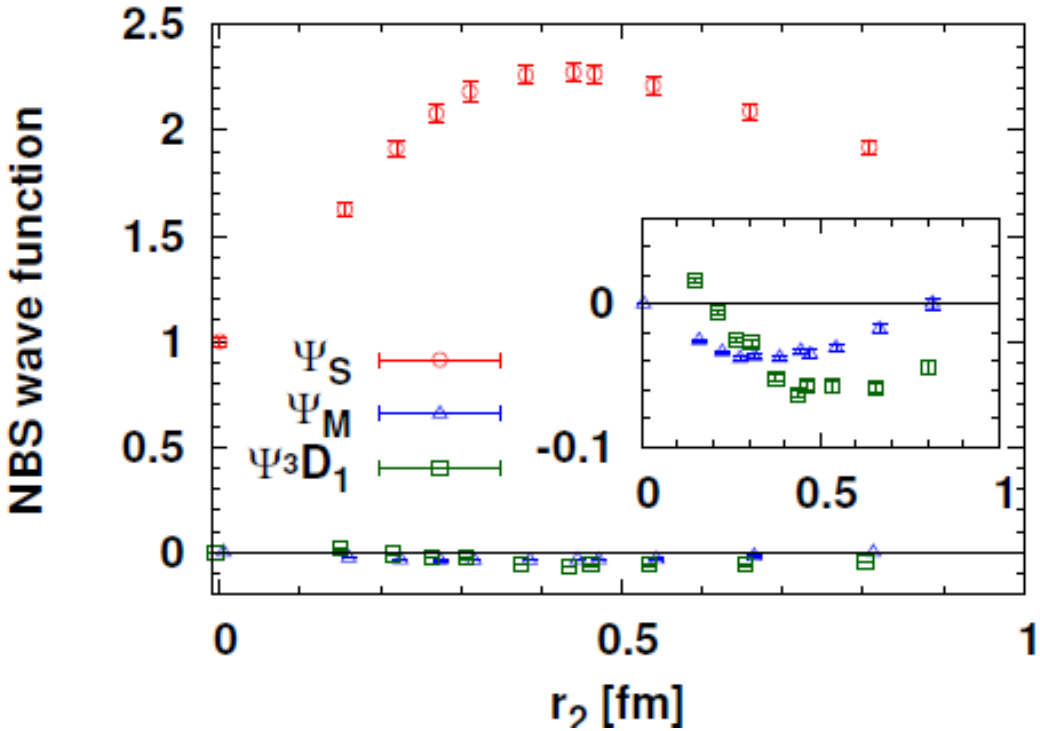
➔ Factor of 2^3=8 faster

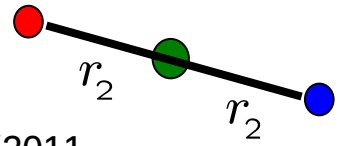
T2K@Tsukuba

BGL@KEK

CP-PACS Coll. S. Aoki et al., Phys. Rev. D65 (2002) 054505

Results for wave functions



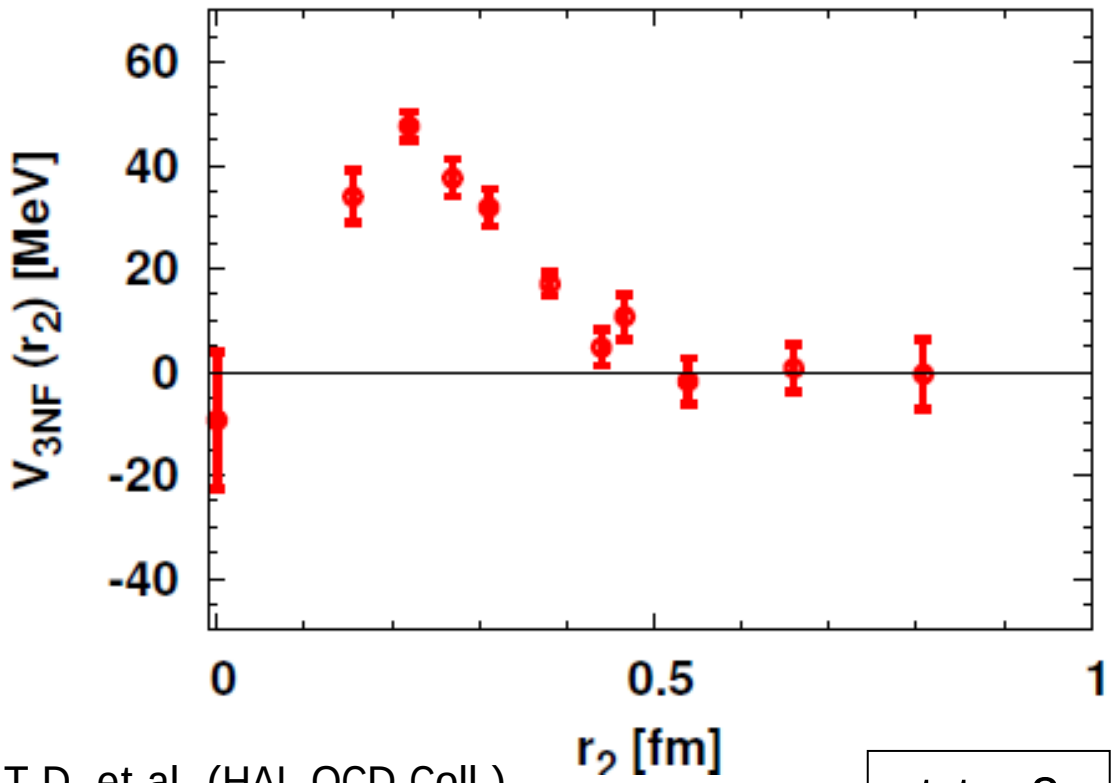


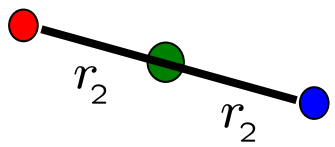
Red: Ψ_S
Blue: Ψ_M
Green: Ψ_{3D1}

Ψ_S overwhelms the wave function:

➔ Indication of the dominance of all S-wave component, higher waves suppressed

Genuine Three Nucleon Force





short-range repulsive 3NF !

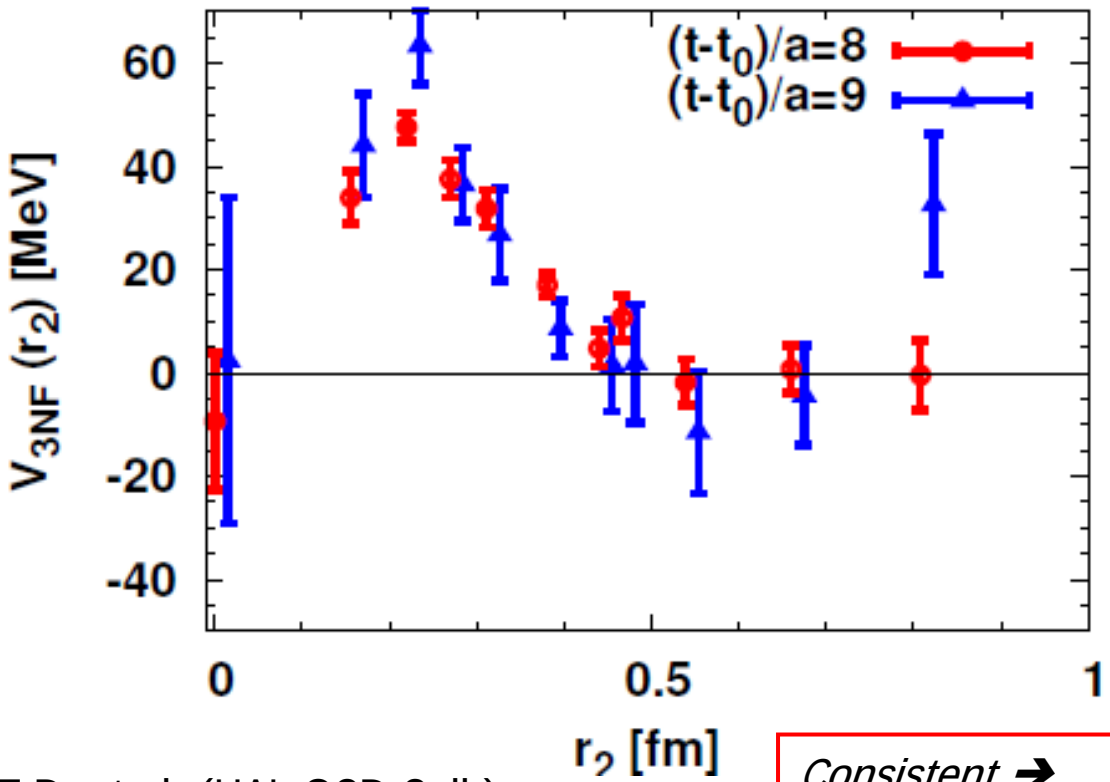
Huge Impact on physics of high density matters, EoS, Neutron Star, SuperNova, …

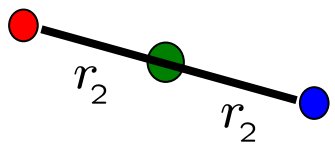
T.D. et al. (HAL QCD Coll.)
arXiv:1106.2276 [hep-lat]

$t-t_0=8$

$M(\pi)=1.13$GeV

Check on sink time dependence





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Huge Impact on physics of high density matters, EoS, Neutron Star, SuperNova, …

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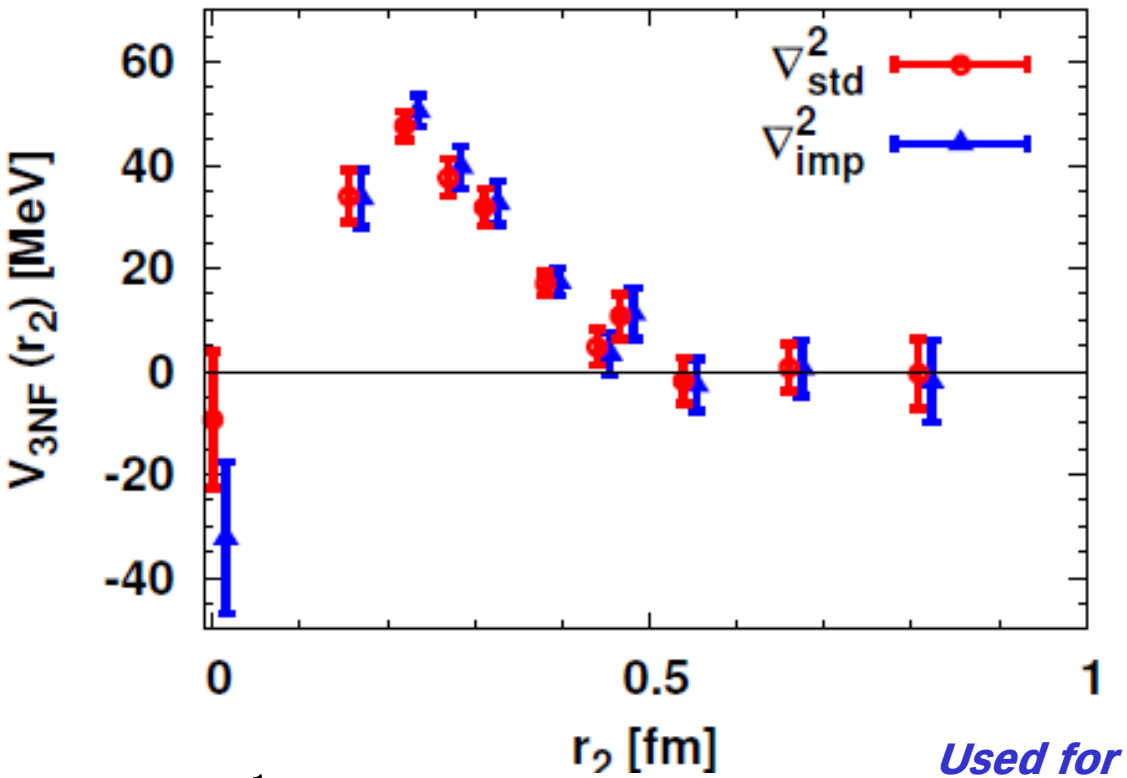
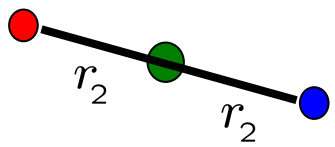
Consistent ➔ Saturated to G.S.

M(π)=1.13GeV

(further improvement in progress)

Studies on discretization error





Comparison with
Improved Laplacian op.

Discretization error in
Laplacian op. is small

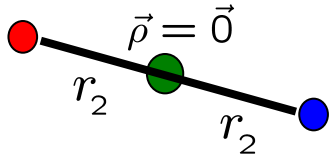
Used for Kinetic energy

$$\nabla^2_{std} f(\vec{x}) = \frac{1}{a^2}\sum_i [f(\vec{x}+a_i) + f(\vec{x}-a_i) - 2f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^2)$$

$$\nabla^2_{prc} f(\vec{x}) = \frac{1}{12a^2}\sum_i [-(f(\vec{x}+2a_i) + f(\vec{x}-2a_i)) + 16(f(\vec{x}+a_i) + f(\vec{x}-a_i)) - 30f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^4)$$

Summary/Outlook

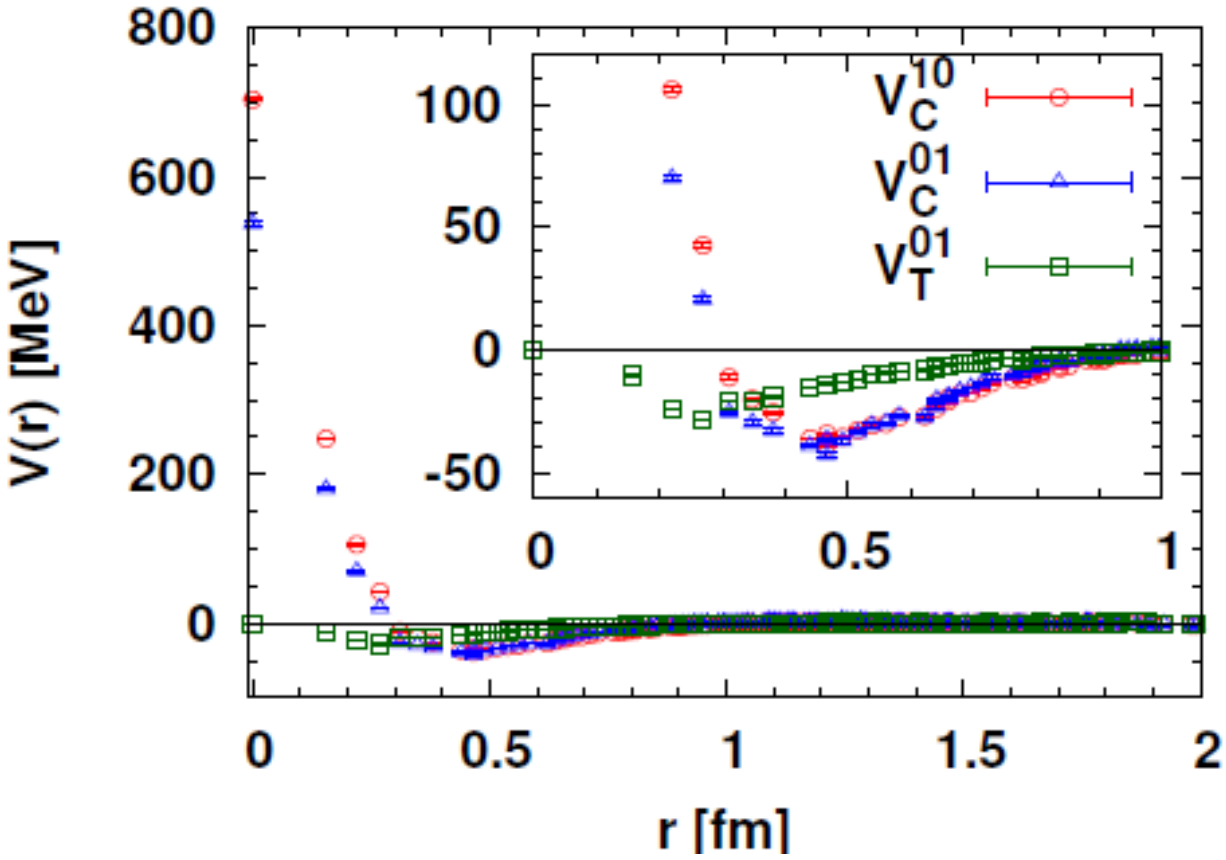
- We have performed the **Lattice QCD** study of the **Genuine Three Nucleon Force (3NF)**
  - Wave function with anti-symmetric in spin/isospin for any 2N pair
    - 2N subtraction is possible using only parity-even potentials
  - We have calculated **linear setup** of 3N (^{3}H) system
    - System is reduced to 3X3 coupled channel
  - Nf=2 dynamical clover fermion at mπ= 1.13 GeV
    - **Repulsive 3NF at short distance**, further studies ongoing



- Outlook
  - Finer lattices and lighter masses, larger volumes
  - More independent 3NFs using parity-odd potential ➔ FM, chEFT
  - Other 3D-conf of 3N, such as triangle ➔ spacial information of 3NF
  - In future: other channel, I=3/2 [hard to access by scatt. exp]
    Extend the flavor space SU(2) ➔ SU(3) : Astrophysics (e.g., Neutron Star)

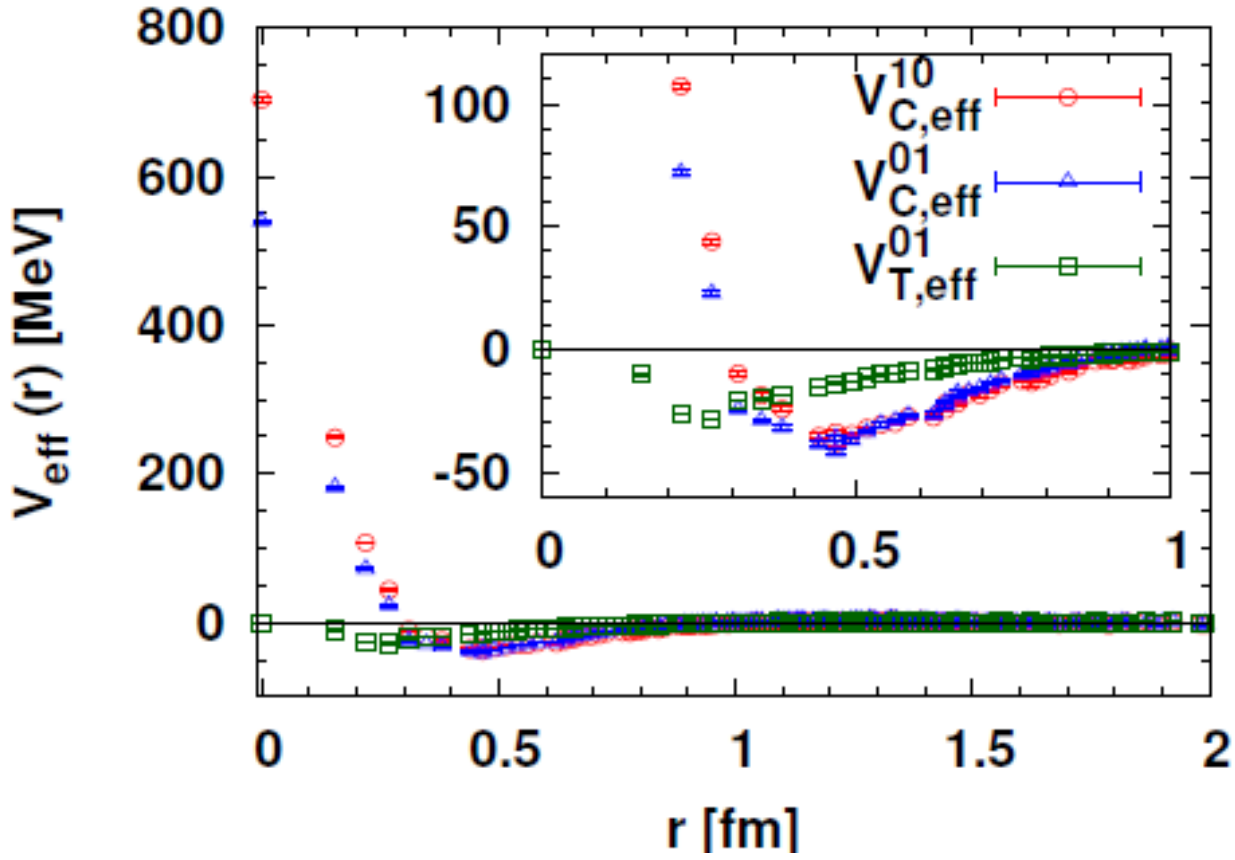
Backup Slides

Parity-even 2N potentials (input)



M(π)=1.13GeV

Effective 2N potential in 3N



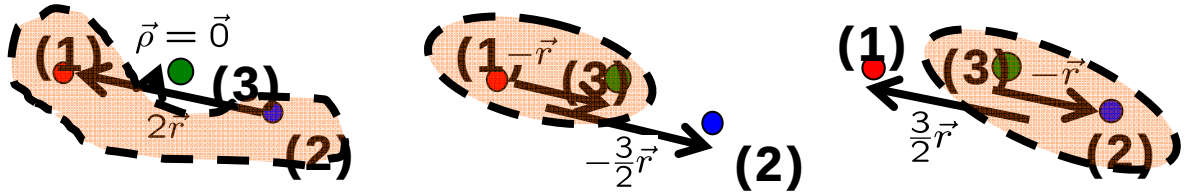
M(π)=1.13GeV

Solution using "symmetric" wave function

- Rotate the basis $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right) \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric



$$\begin{aligned}
|\psi_S\rangle \;=\; 1/\sqrt{6}\Big[& -(p_\uparrow n_\uparrow - n_\uparrow p_\uparrow) n_\downarrow && (\leftarrow I=0, S=1)\\
& -(n_\uparrow n_\downarrow - n_\downarrow n_\uparrow) p_\uparrow && (\leftarrow I=1, S=0)\\
& +1/2(p_\uparrow n_\downarrow + n_\uparrow p_\downarrow - p_\downarrow n_\uparrow - n_\downarrow p_\uparrow) n_\uparrow && (\leftarrow I=1, S=0)\\
& +1/2(p_\uparrow n_\downarrow - n_\uparrow p_\downarrow + p_\downarrow n_\uparrow - n_\downarrow p_\uparrow) n_\uparrow\Big] && (\leftarrow I=0, S=1)
\end{aligned}$$

- ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed

Explicit formula for the potential matrix

- The potential matrix for the 2N part in 3x3 coupled channel in linear setup can be written as:

$$V_{2N} = \begin{pmatrix} \begin{array}{c} +V_C^{10}(r)+V_C^{01}(r) \\ +\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} +\frac{1}{2}V_C^{10}(r)-\frac{1}{2}V_C^{01}(r) \\ -\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} -2V_T^{01}(r) \\ +2V_T^{01}(2r) \end{array} \\ \hline \begin{array}{c} +\frac{1}{2}V_C^{10}(r)-\frac{1}{2}V_C^{01}(r) \\ -\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} +\frac{3}{4}V_C^{00}(r)+\frac{1}{4}V_C^{10}(r)+\frac{1}{4}V_C^{01}(r)+\frac{3}{4}V_C^{11}(r) \\ +\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} +V_T^{01}(r)-3V_T^{11}(r) \\ +2V_T^{01}(2r) \end{array} \\ \hline \begin{array}{c} -2V_T^{01}(r) \\ +2V_T^{01}(2r) \end{array} & \begin{array}{c} +V_T^{01}(r)-3V_T^{11}(r) \\ +2V_T^{01}(2r) \end{array} & \begin{array}{c} +\frac{1}{2}V_C^{01}(r)+\frac{3}{2}V_C^{11}(r)-V_T^{01}(r)-3V_T^{11}(r) \\ +V_C^{01}(2r)-2V_T^{01}(2r) \end{array} \end{pmatrix} \quad (4.2)$$

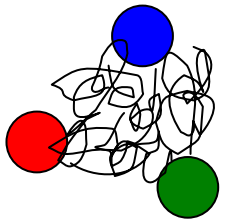
$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

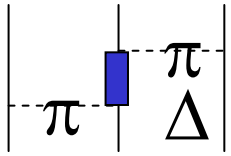
$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$: (P = odd)

No V(P=odd)

Importance of Three Nucleon Force (3NF)

- **3NF** ← potential among 3N which **cannot** be reduced to pair-wise 2N potential
  - Important role in B.E. of light-nuclei
  - Nucleus-nucleus scattering, Ay puzzle ?
  - Saturation point of nuclear matter
  - **EoS** of high density matter ➔ Neutron Star, SuperNova
  - Properties of neutron rich nuclei ➔ Nucleosynthesis





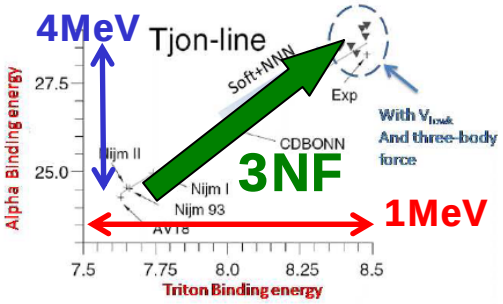
Fujita-Miyazawa(1957)

Importance of Three Nucleon Force (3NF)

- Precise few-body calc:
  NN force cannot reproduce B.E.

  δB.E.= 0.5-1MeV for ^{3}H
  δB.E.= 2-4 MeV for ^{4}He

  Attractive 3NF necessary



Nogga et al., PRL85(2000)944

- Saturation density/energy of nuclear matter also requires 3NF
  - EOS of neutron star
    - Flavor universal 3NF (repulsive) ?

  Repulsive 3NF also necessary

  A.Akmal et al., PRC58(1998)1804

  Takatsuka et al., PTPS174(2008)80

- The effect on the nuclear chart T.Otsuka et al., PRL105(2010)032501
  - anomaly in drip line and magic numbers by 3NF
- Ay puzzle in N-d, N-A scatt., etc. (3NF may worsen the situation)

Importance of Three Nucleon Force (3NF)

- Precise few-body calc:
  - e.g. benchmark calc of ^{4}He by 7 methods (NN only)

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

➔ **0.5% prec. for B.E.**

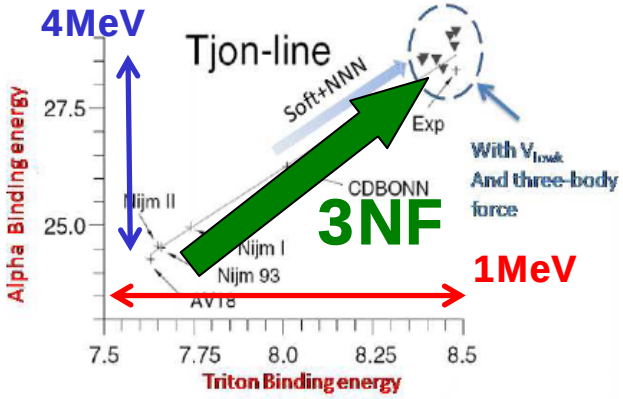
H.Kamada et al., PRC64(2001)044001

- 2N force cannot reproduce B.E.

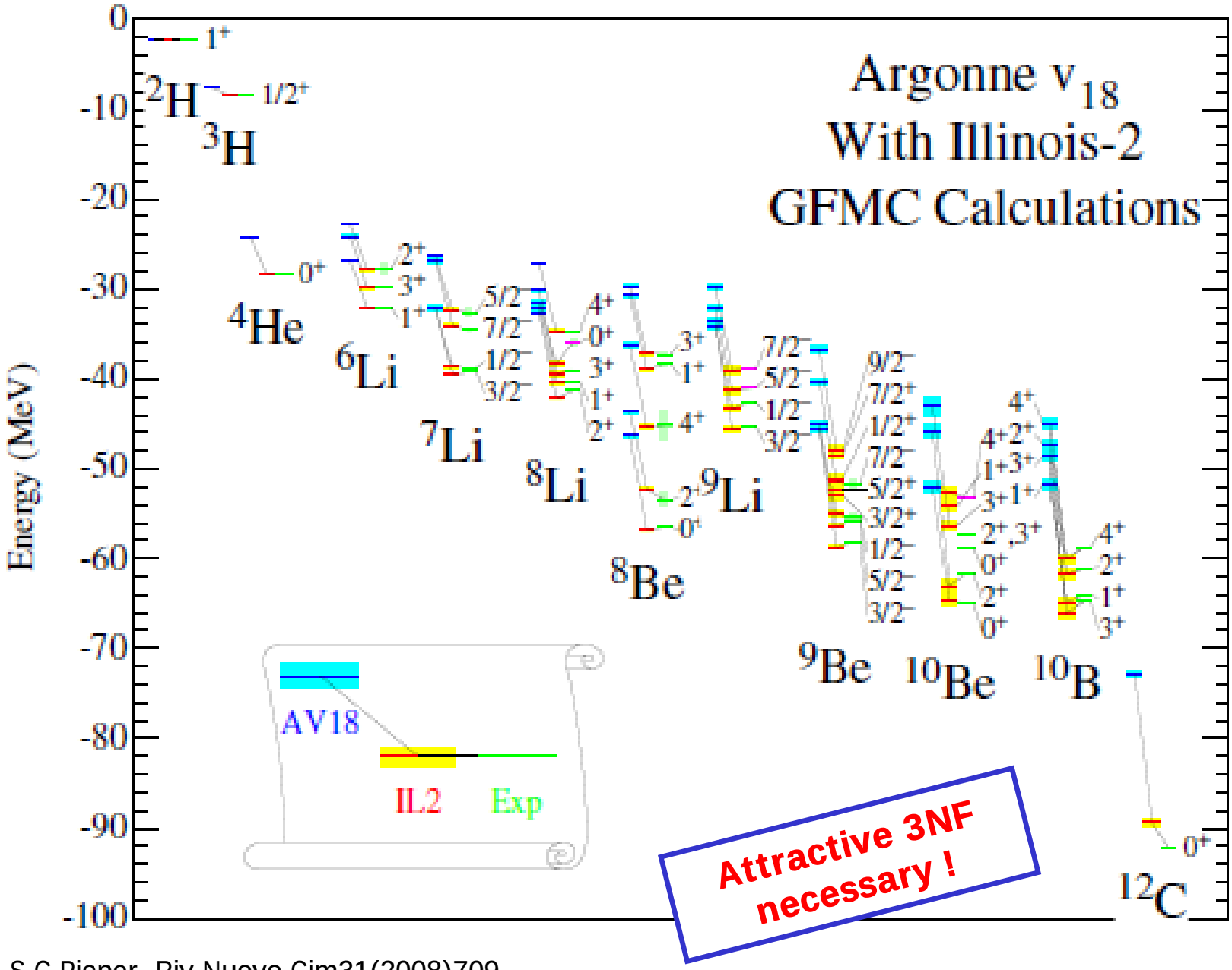
δB.E.= 0.5-1MeV for ^{3}H
δB.E.= 2-4 MeV for ^{4}He

missing

➡ Attractive 3NF necessary



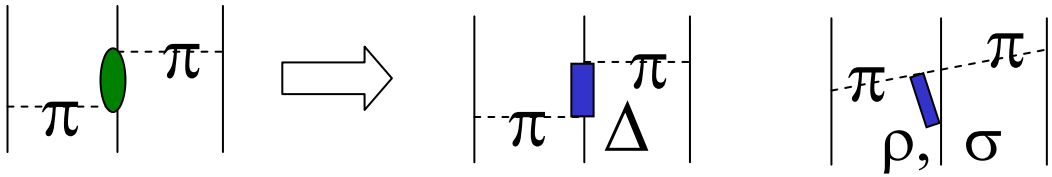
Nogga et al., PRL85(2000)944



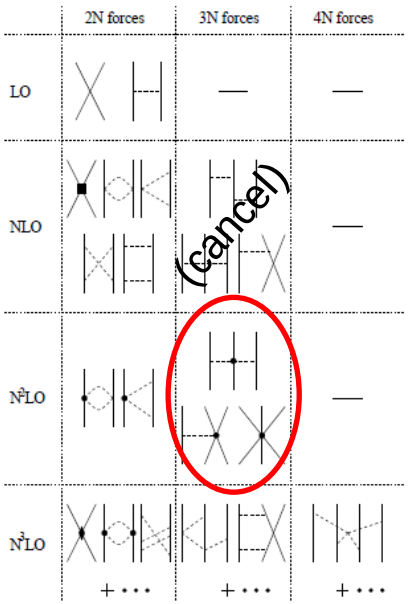
S.C.Pieper, Riv.Nuovo.Cim31(2008)709
arXiv:0711.1500

Three Nucleon Force (3NF)

- It is natural to expect the existence of 3NF
- It is very nontrivial to determine 3NF from QCD
- 2πE-3NF Fujita-Miyazawa, PTP17(1957)360
  - Off-energy-shell πN scatt

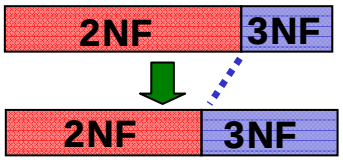


- EFT expansion ➔ 3NF appears at NNLO order
- Phenomenological short-range repulsion is necessary
- 2πE-3NF too attractive, often suppressed (artificially) by form factor
- NB: the combination of (2NF,3NF) ➔ observables



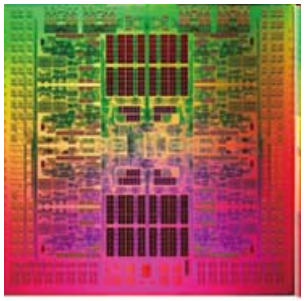
U.v.Kolck, PRC49(1994)2932

Epelbaum, Prog.Part.Nucl.Phys.57(06)654



Japan's next gen computer

- K computer at Kobe, Japan
  - 10PFlops (2012)



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K (Kei) = 10^{16} = 10 Peta