QCD thermodynamics on the lattice

A. Bazavov [HotQCD collaboration]

Brookhaven National Laboratory

July 28, 2011

HotQCD collaboration

A. Bazavov (BNL)
T. Bhattacharya (LANL)
M. Cheng (LLNL)
N. H. Christ (Columbia)
C. DeTar (Utah)
H.-T. Ding (BNL)
S. Gottlieb (Indiana)
R. Gupta (LANL)
P. Hegde (BNL)
U. Heller (APS)
C. Jung (BNL)
F. Karsch (BNL-Bielefeld)
E. Laermann (Bielefeld)
L. Levkova (Utah)
R.D. Mawhinney (Columbia)
S. Mukherjee (BNL)
P. Petreczky (BNL)
D. Renfrew (Columbia)
C. Schmidt (FIAS)
R. A. Soltz (LLNL)
W. Soeldner (Regensburg)
R. Sugar (UCSB)
D. Toussaint (Arizona)
W. Unger (Zurich)
P. Vranas (LLNL)

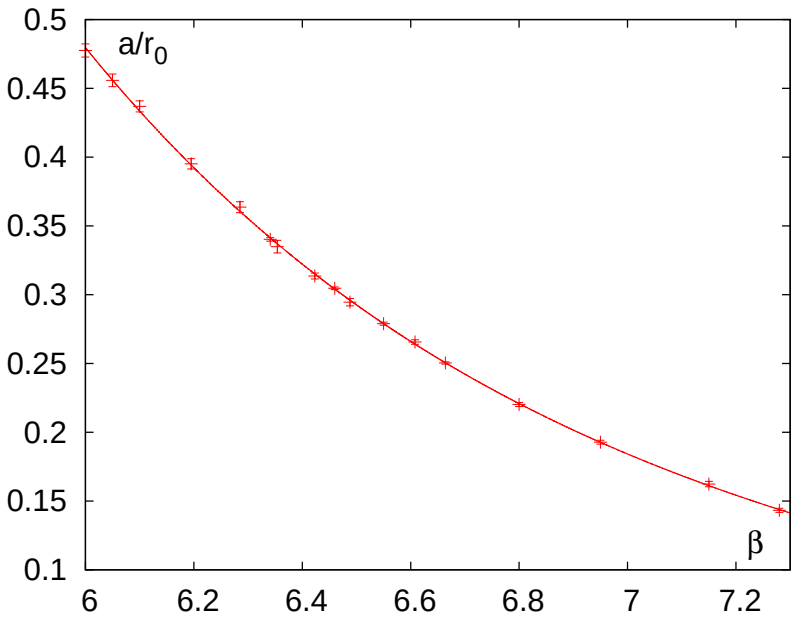
Introduction

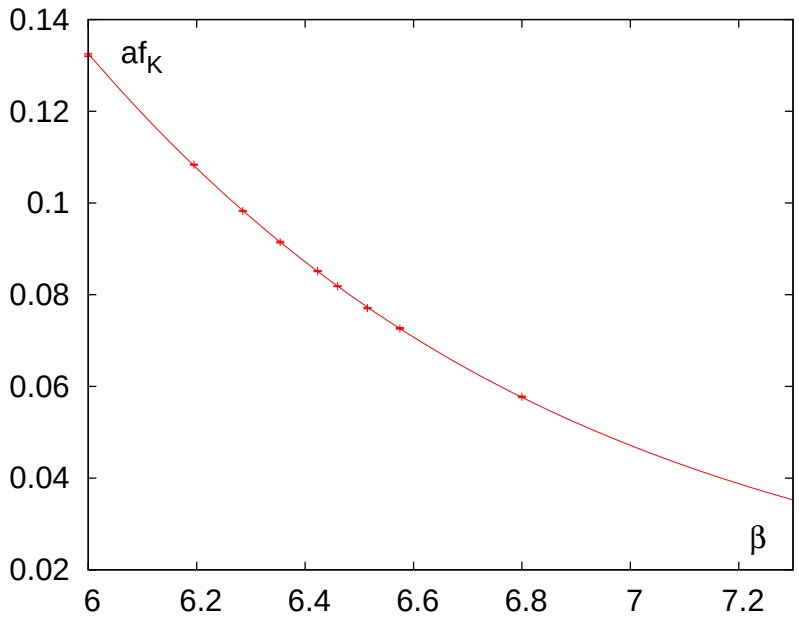
- We would like to study the phase diagram of QCD.
- At high enough temperatures and/or densities a transition from ordinary hadronic matter to the quark-gluon plasma phase takes place.
- Experimentally this transition is being studied at RHIC and a subject of future study of the LHC heavy-ion program.
- The physics of the hadronic phase and the transition region is non-perturbative $\Rightarrow$ requires lattice QCD techniques.
- Chiral symmetry plays important role in understanding the phase diagram of strongly interacting matter.
- It is known that at the physical values of the light and strange quark masses at $\mu = 0$ the transition is a smooth crossover.
- What is meant by T_c: define pseudo-critical temperature in a way related to the chiral transition, that reduces to the critical temperature in the chiral limit.
- The disconnected chiral susceptibility is sensitive to the singular part of the free energy and diverges in the chiral limit. We define the pseudo-critical temperature as the location of the peak of the disconnected chiral susceptibility.

Lattice setup

- 4D hypercubic lattice $N_s^3 \times N_\tau$.
- Temperature is set by compactified temporal dimension: $T = 1/(N_\tau a)$, lattice spacing a is varied.
- Use $N_\tau = 6$, 8 and 12, fixed $N_s/N_\tau = 4$.
- 2+1 flavors of staggered fermions, actions: p4, asqtad and HISQ/tree.
- Lines of constant physics (LCP): tune bare quark masses with the lattice spacing so that pion and kaon masses are fixed.
- Evaluate QCD path integrals stochastically, using Monte Carlo techniques.

HISQ/tree: scale setting



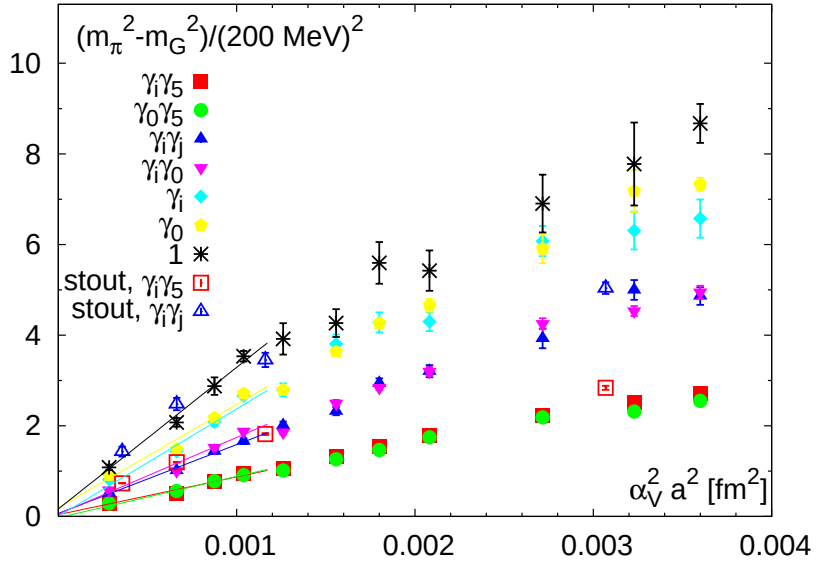


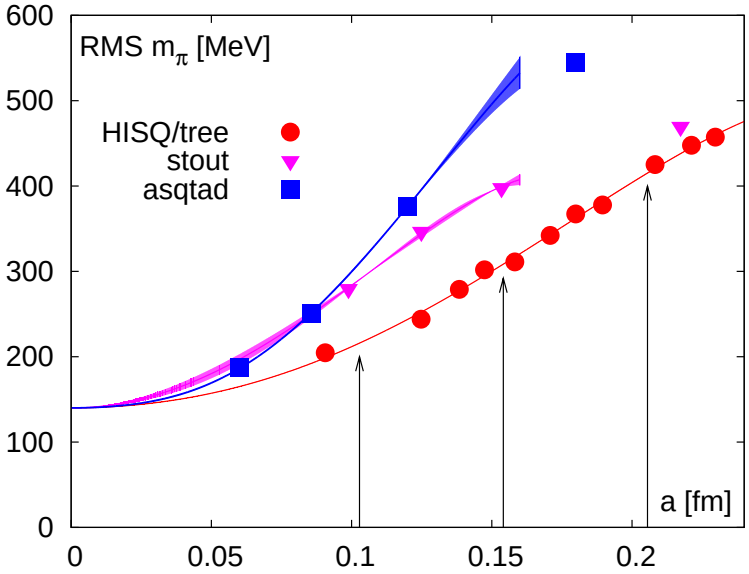
- We set the lattice spacing using the parameters r_0 and r_1 that are fixed by the slope of the static quark anti-quark potential:

$$\left(r^2 \frac{dV_{q\bar{q}}(r)}{dr}\right)_{r=r_0} = 1.65, \quad \left(r^2 \frac{dV_{q\bar{q}}(r)}{dr}\right)_{r=r_1} = 1.$$

- Alternatively, we set the scale by measuring the kaon decay constant f_K.

HISQ/tree: pion taste splittings



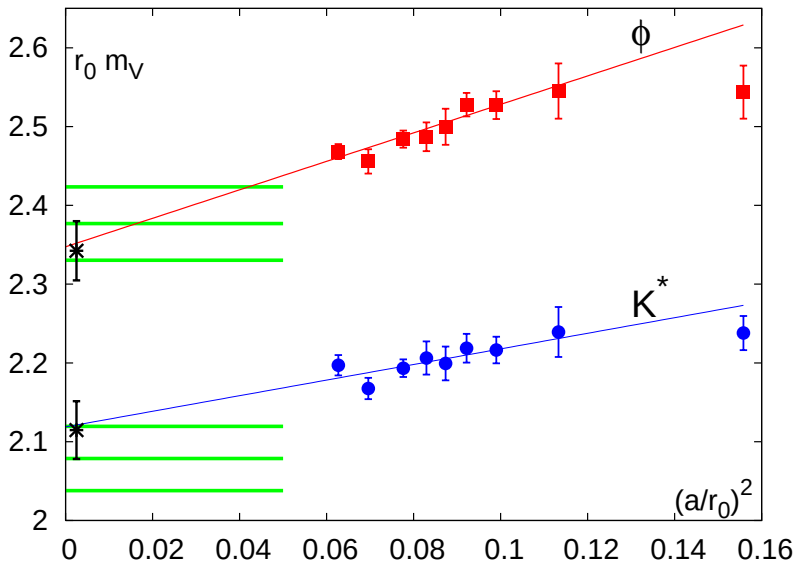


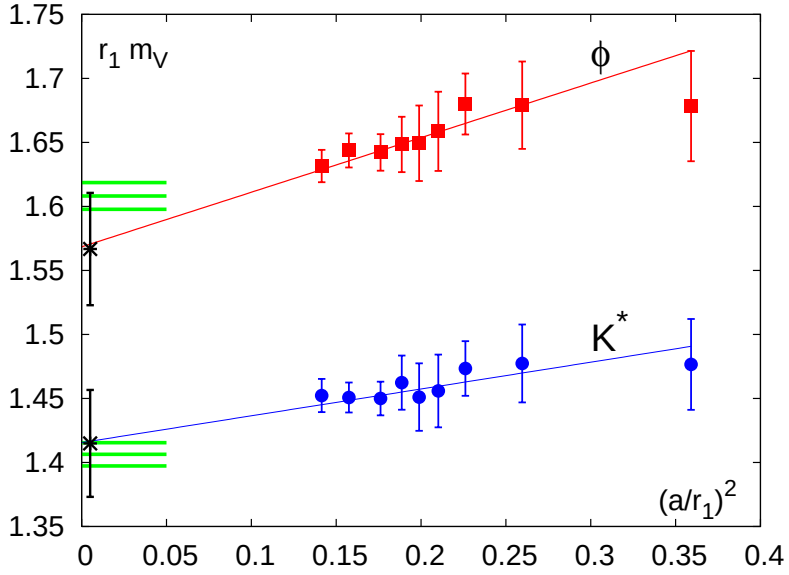
- Left: HISQ/tree pion mass splittings (i.e. quadratic differences in mass between the Goldstone pion and higher pion multiplets).
- Right: root-mean-squared (RMS) pion mass for HISQ/tree, stout and asqtad, defined as

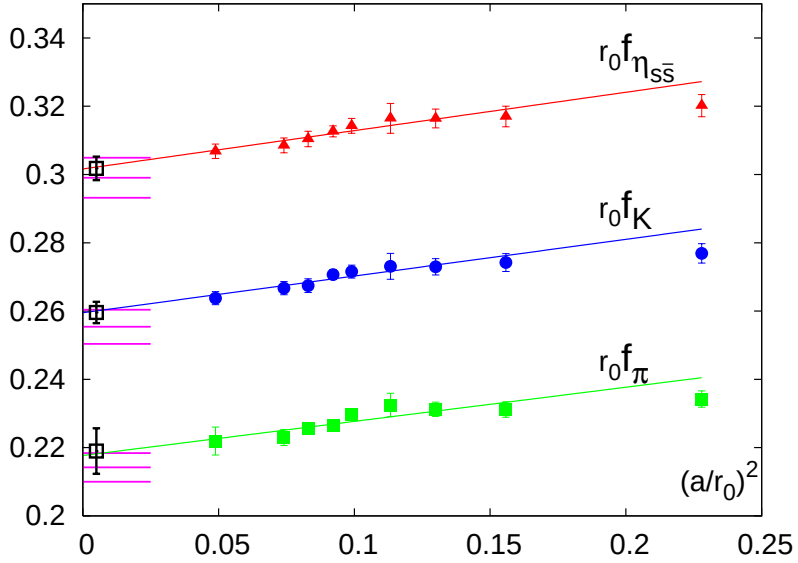
$$m_\pi^{RMS} = \sqrt{\frac{1}{16}\left(m^2_{\gamma_5} + m^2_{\gamma_0\gamma_5} + 3m^2_{\gamma_i\gamma_5} + 3m^2_{\gamma_i\gamma_j} + 3m^2_{\gamma_i\gamma_0} + 3m^2_{\gamma_i} + m^2_{\gamma_0} + m^2_1\right)}.$$

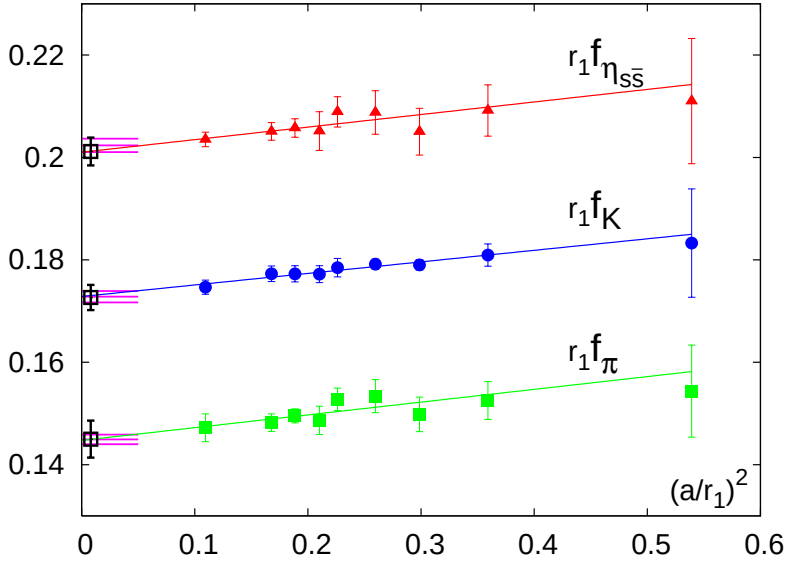
- Right: vertical arrows indicate lattice spacings where $T = 160$ MeV at $N_\tau = 12$, 8 and 6.

HISQ/tree: hadron spectrum









Chiral observables

Chiral condensate:

$$\langle\bar{\psi}\psi\rangle_{q,x} = \frac{1}{4}\frac{1}{N_\sigma^3 N_\tau}\mathrm{Tr}\langle D_q^{-1}\rangle, \quad q = l, s, \quad x = 0, \tau \ ,$$

The susceptibility:

$$\chi_{m,l}(T) \quad = \quad 2\frac{\partial\langle\bar{\psi}\psi\rangle_l}{\partial m_l} = \chi_{l,disc} + \chi_{l,con} \ ,$$

$$\chi_{m,s}(T) \quad = \quad \frac{\partial\langle\bar{\psi}\psi\rangle_s}{\partial m_s} = \chi_{s,disc} + \chi_{s,con} \ ,$$

$$\chi_{q,disc} \quad = \quad \frac{n_f^2}{16 N_\sigma^3 N_\tau}\left\{\langle\left(\mathrm{Tr}D_q^{-1}\right)^2\rangle - \langle\mathrm{Tr}D_q^{-1}\rangle^2\right\} \ ,$$

and

$$\chi_{q,con} \quad = \quad -\frac{n_f}{4}\mathrm{Tr}\sum_x\langle\, D_q^{-1}(x,0)D_q^{-1}(0,x)\,\rangle \ , \quad q = l, s.$$

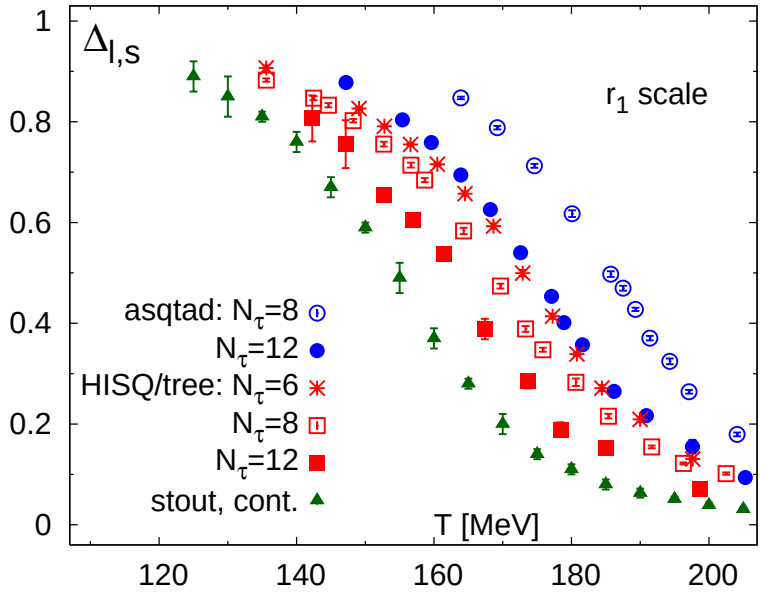
The renormalized condensate:

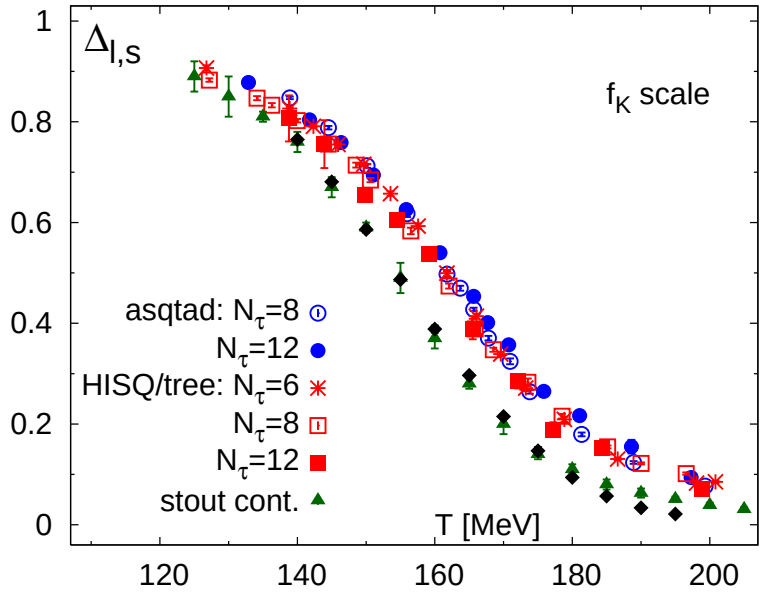
$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,\tau} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,\tau}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

or

$$\Delta_l^R(T) = const + m_s r_0^4(\langle\bar{\psi}\psi\rangle_{l,\tau} - \langle\bar{\psi}\psi\rangle_{l,0}).$$

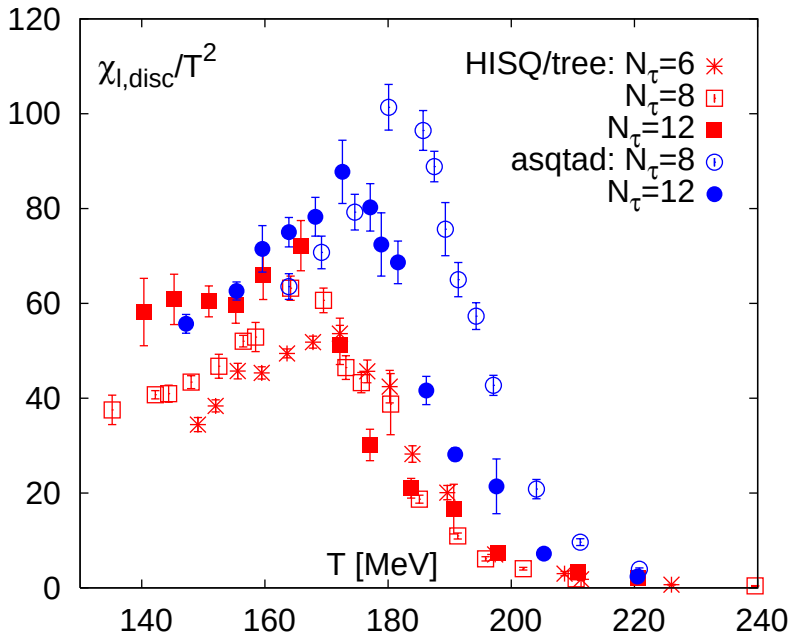
Chiral condensate





- Left: the renormalized chiral condensate with r_1 scale.
- Right: the renormalized chiral condensate with f_K scale.

HISQ/tree: chiral susceptibility



- The disconnected chiral susceptibility for the asqtad and HISQ/tree action.
- Going to finer lattice spacing or more improved action shifts the peak to lower temperatures.
- The pseudo-critical temperature, T_c, is defined from the peak of the disconnected chiral susceptibility, relying on $O(N)$ scaling analysis.

Chiral condensate (scaling)

- In the limit of vanishing light quark masses and for sufficiently large strange quark mass QCD is expected to have a second order phase transition in $O(4)$ universality class.
- Staggered fermions preserve only a part of the chiral symmetry, thus, the relevant universality class in the chiral limit at non-zero lattice spacing is $O(2)$.
- In the numerical analysis the difference between $O(2)$ and $O(4)$ is rather small, so we refer to scaling properties as $O(N)$ scaling.
- Previous studies with the p4 action demonstrated that even for non-vanishing light quark masses, provided they are small enough, universal scaling properties can be used to define pseudo-critical temperature.
- $O(N)$ scaling analysis has been extended to asqtad and HISQ/tree.
- The multiplicatively renormalized chiral condensate (the order parameter in the chiral limit):

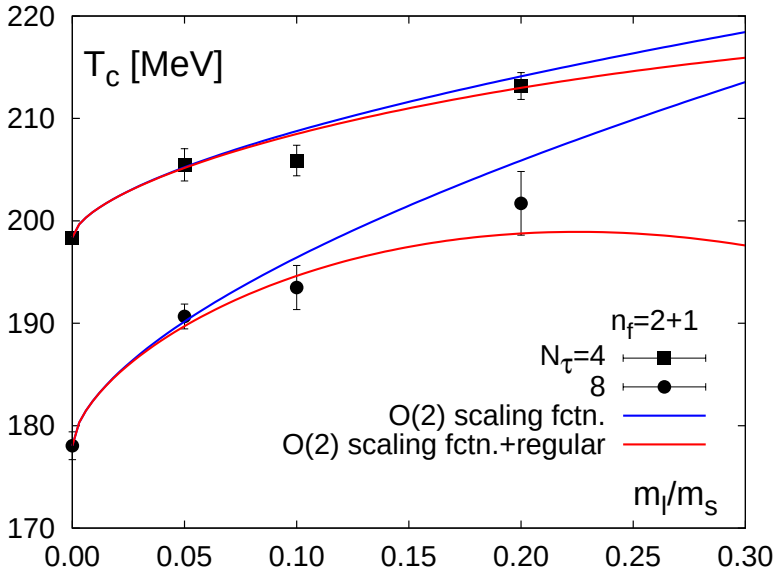
$$M_b = \frac{m_s}{T^4}\langle\bar{\psi}\psi\rangle_l.$$

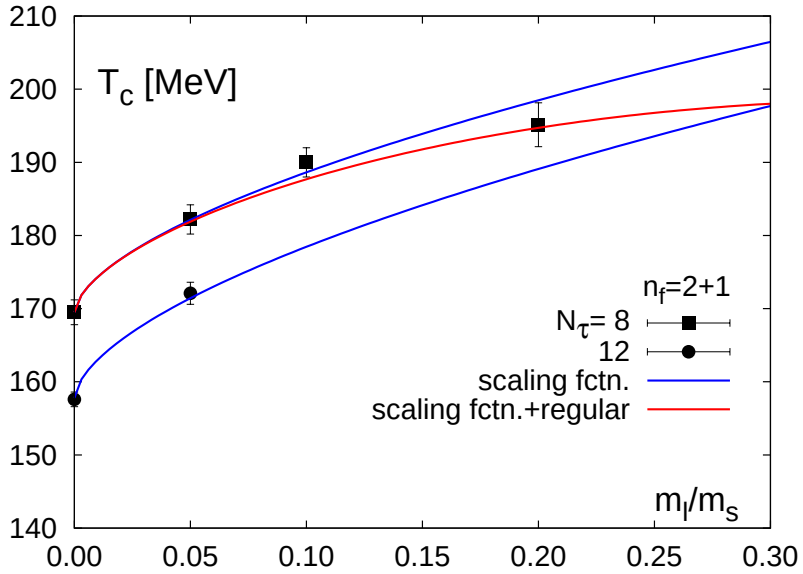
- At sufficiently low mass the chiral condensate is described by a universal scaling function f_G plus additional scaling violating terms:

$$M_b(T, m_l, m_s) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + a_t \Delta T H + b_1 H,$$

$$H = \frac{m_l}{m_s}, \qquad \Delta T = \frac{T - T_c^0}{T_c^0}, \qquad h = H/h_0, \qquad t = \Delta T/t_0.$$

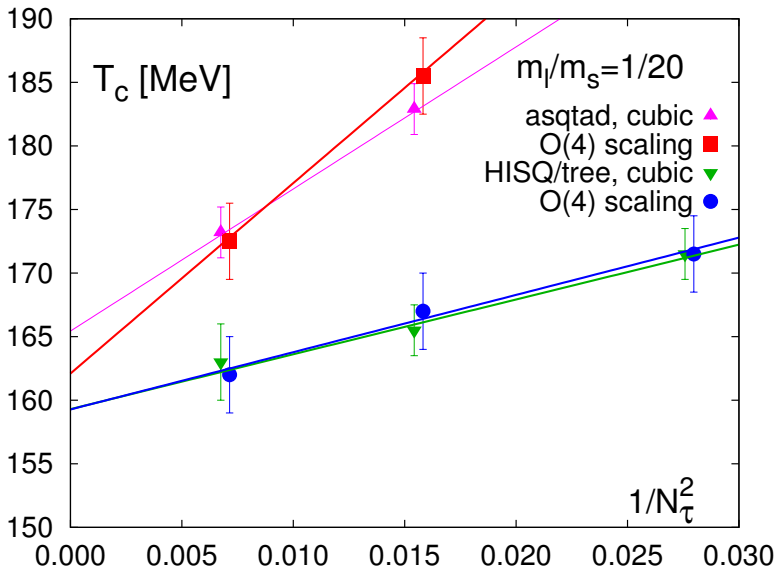
$O(N)$ scaling analysis

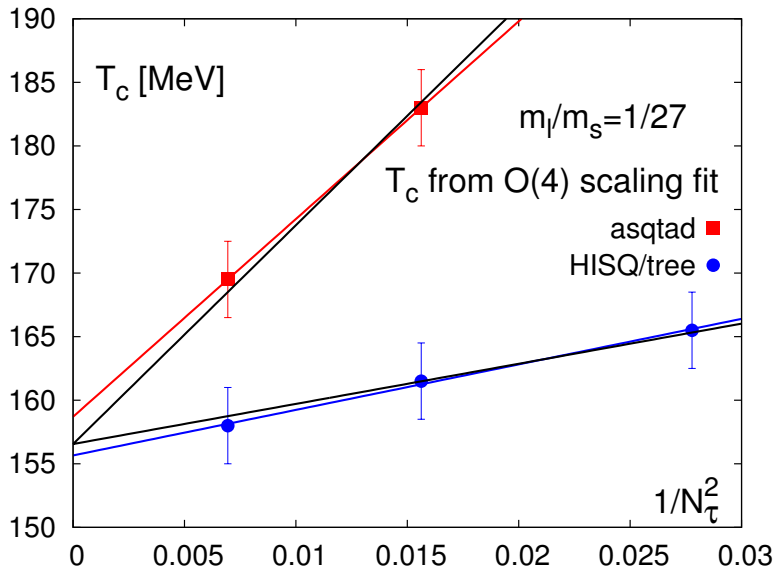




- The pseudo-critical temperature as function of light quark mass at $N_\tau = 4$ and 8 for the p4 action (left) and $N_\tau = 8$ and 12 for the asqtad action (right).
- Points represent the pseudo-transition temperature T_c determined from the peak of the disconnected chiral susceptibility.
- Lines are predictions from the fits to the chiral condensate.

T_c determination





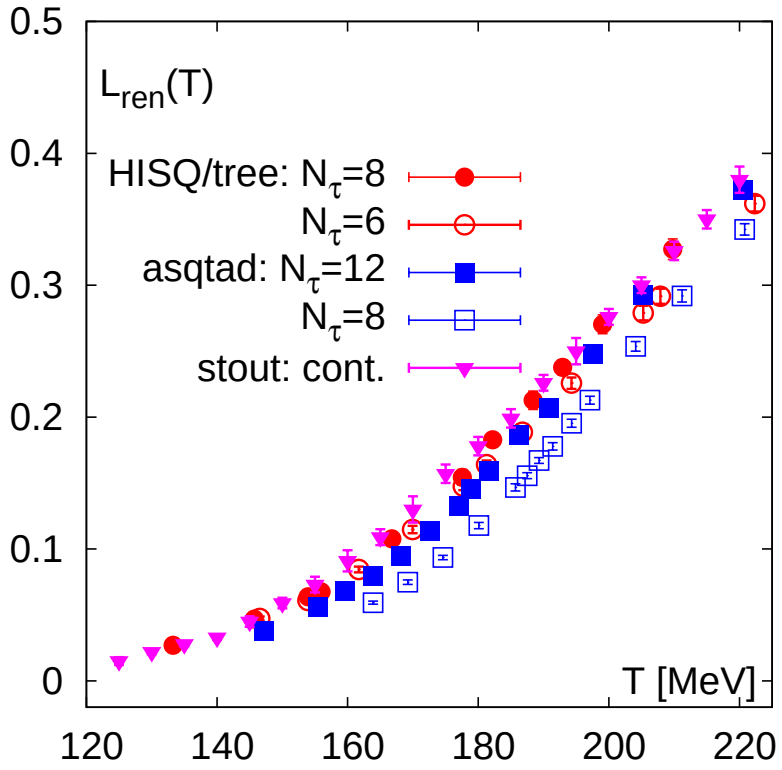
- Left: comparison of extrapolation of T_c determined from the peak of the disconnected chiral susceptibility and from the fits of the chiral condensate.
- Right: T_c extrapolation using fits of the chiral condensate at $m_l = m_s/27$.
- Based on the combined fit of the asqtad $N_\tau = 8$, 12 and HISQ/tree $N_\tau = 6$, 8, 12 data our preliminary result is:

$$T_c = 157 \pm 4 \pm 3 \pm 1 \text{ MeV}.$$

- Errors: statistical, systematic, scale setting.
- We combine first and second in quadrature and add the third, so alternatively:

$$T_c = 157 \pm 6 \text{ MeV}.$$

Deconfinement: renormalized Polyakov loop



- Related to the free energy of a static quark anti-quark pair at infinite separation:

$$L_{ren}(T) = \exp(-F_\infty(T)/(2T)).$$

- The renormalization constant

$$z(\beta) = \exp(-c(\beta)/2),$$

where $c(\beta)$ is the additive renormalization of the static potential.

$$L_{ren}(T) = z(\beta)^{N_\tau} L_{bare}(\beta), \qquad L_{bare}(\beta) = \left\langle \frac{1}{3}\mathrm{Tr} \prod_{x_0=0}^{N_\tau-1} U_0(x_0,\vec{x}) \right\rangle .$$

- The increase of $L_{ren}(T)$ (and decrease of $F_\infty(T)$) is related to the onset of screening at higher temperatures.

Deconfinement: fluctuations of conserved charges

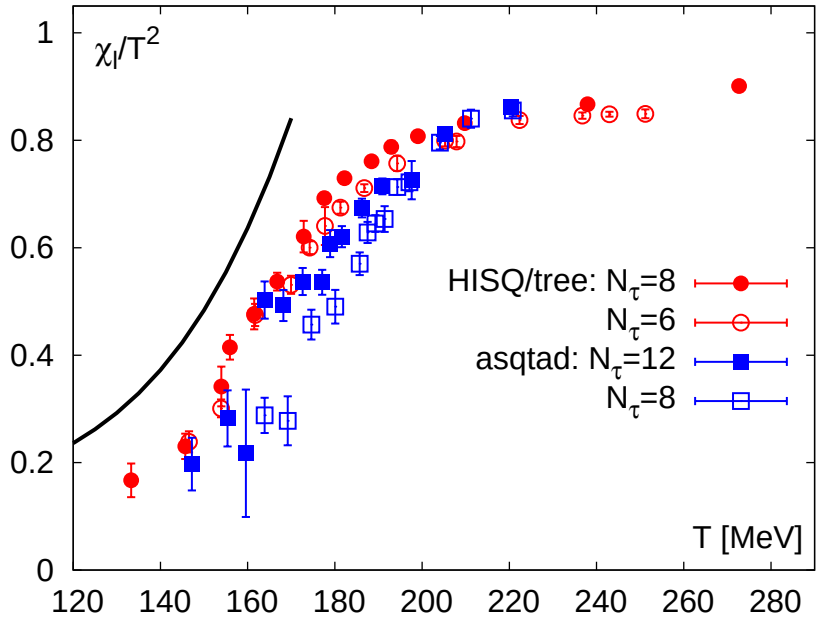
- Fluctuations and correlations of conserved charges:

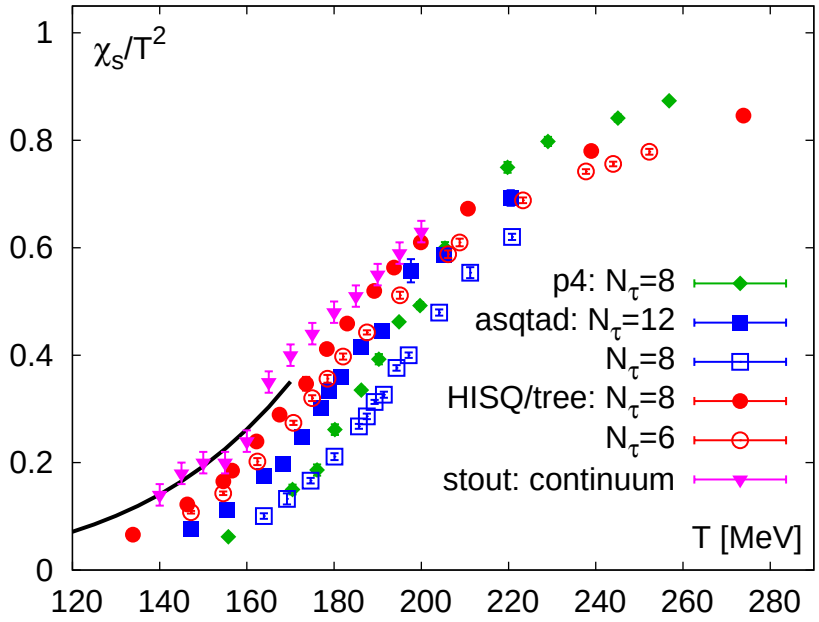
$$\frac{\chi_i(T)}{T^2} = \frac{1}{T^3 V} \frac{\partial^2 \ln Z(T,\mu_i)}{\partial(\mu_i/T)^2}\bigg|_{\mu_i=0},$$

$$\frac{\chi_{11}^{ij}(T)}{T^2} = \frac{1}{T^3 V} \frac{\partial^2 \ln Z(T,\mu_i,\mu_j)}{\partial(\mu_i/T)\partial(\mu_j/T)}\bigg|_{\mu_i=\mu_j=0}.$$

- Consider light and strange quark number susceptibility.
- At low temperatures they are carried by massive hadrons and their fluctuations are suppressed.
- At high temperatures they are carried by quarks and therefore can signal deconfiment.

Deconfinement: fluctuations of conserved charges





- The light (left) and strange (right) quark number susceptibility[1], comparison with the hadron resonance gas (HRG) model (solid line).
- Quark number susceptibilities rapidly rise in the transition region and approach the ideal gas limit (up to the cut-off effects).
- The light quark number susceptibility is carried by the lightest states (pions) and therefore is more sensitive to the taste symmetry breaking effects resulting in poorer agreement with HRG.

[1] Stout data from: Borsanyi et al. [Budapest-Wuppertal collaboration], JHEP09(2010)073

Trace anomaly

- The anomalous part of the energy-momentum tensor:

$$\frac{\epsilon - 3p}{T^4} = \frac{\Theta_{\mu\mu}(T)}{T^4} = \frac{\Theta_G^{\mu\mu}(T)}{T^4} + \frac{\Theta_F^{\mu\mu}(T)}{T^4},$$

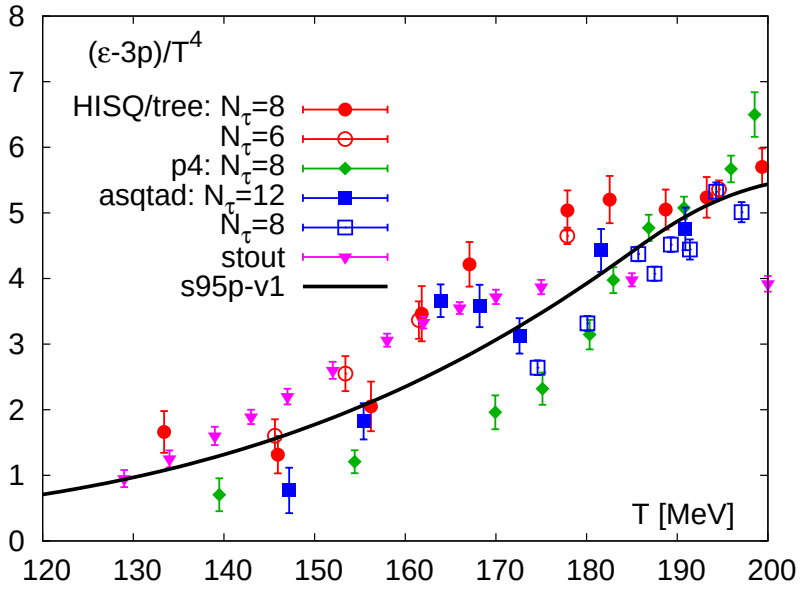
$$\frac{\Theta_G^{\mu\mu}(T)}{T^4} = R_\beta \left[\langle s_G \rangle_0 - \langle s_G \rangle_\tau\right] N_\tau^4 \ ,$$

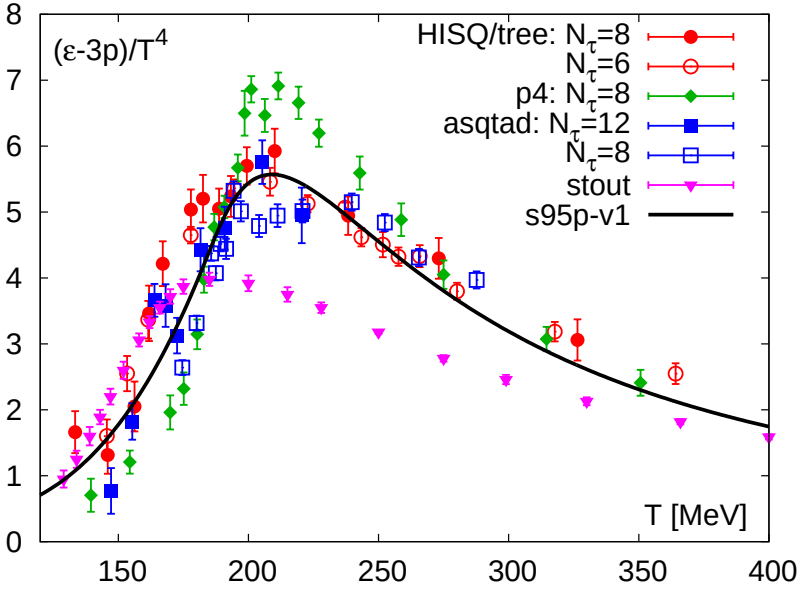
$$\frac{\Theta_F^{\mu\mu}(T)}{T^4} = -R_\beta R_m \left[2\hat{m}_l \left(\langle \bar{\psi}\psi \rangle_{l,0} - \langle \bar{\psi}\psi \rangle_{l,\tau}\right) + \hat{m}_s \left(\langle \bar{\psi}\psi \rangle_{s,0} - \langle \bar{\psi}\psi \rangle_{s,\tau}\right)\right] N_\tau^4 \ ,$$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{\hat{m}_l(\beta)}\frac{d\hat{m}_l(\beta)}{d\beta}.$$

- Requires subtraction of ultra-violet divergencies (noisy on the lattice, gets worse for finer lattices).

Trace anomaly ($m_l/m_s = 1/20$)





- The trace anomaly at $m_l/m_s = 0.05$ for p4, asqtad and HISQ.
- Pressure and other thermodynamic quantities can be derived from the trace anomaly.
- At low temperatures HISQ results agree with stout[2] (left).
- At high temperatures p4, Asqtad and HISQ agree (as expected), but substantial disagreement with stout is observed (right).
- The solid curve is a parametrization based on the HRG model and lattice data[3].

[2] Borsanyi et al., JHEP11 (2010) 077
[3] Huovinen and Petreczky, Nucl. Phys. A 837, 26 (2010)

Conclusions

- We have extended previous studies with the asqtad and HISQ/tree staggered fermion formulations to $N_\tau = 12$, $m_l/m_s = 1/20$.
- The HISQ/tree action has largely reduced cutoff effects and makes possible to get closer to the continuum limit with currently available computational resources.
- At about the physical light quark masses the chiral transition can be described in terms of the universal $O(N)$ scaling.
- Preliminary[4] result for the pseudo-critical temperature defined through the disconnected chiral susceptibility (sensitive to the singular behavior in the chiral limit) is $T_c = 157 \pm 6$ MeV.
- Cut-off effects on the lattice are manifested in a heavier than physical hadron spectrum. Going to finer lattices and/or more improved actions (i.e. HISQ) makes the spectrum more realistic and therefore better agreement between lattice and the Hadron Resonance Gas model is observed.
- HISQ/tree $N_\tau = 8$ and asqtad $N_\tau = 12$ results for the trace anomaly agree with the stout data in the low-temperature region.
- In 250-350 MeV range p4, asqtad and HISQ/tree results agree, but disagree with stout.

[4] HotQCD collaboration, publication in preparation