Polarized pairing in neutron star crusts

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Early work done in collaboration
with Joe Carlson (LANL)

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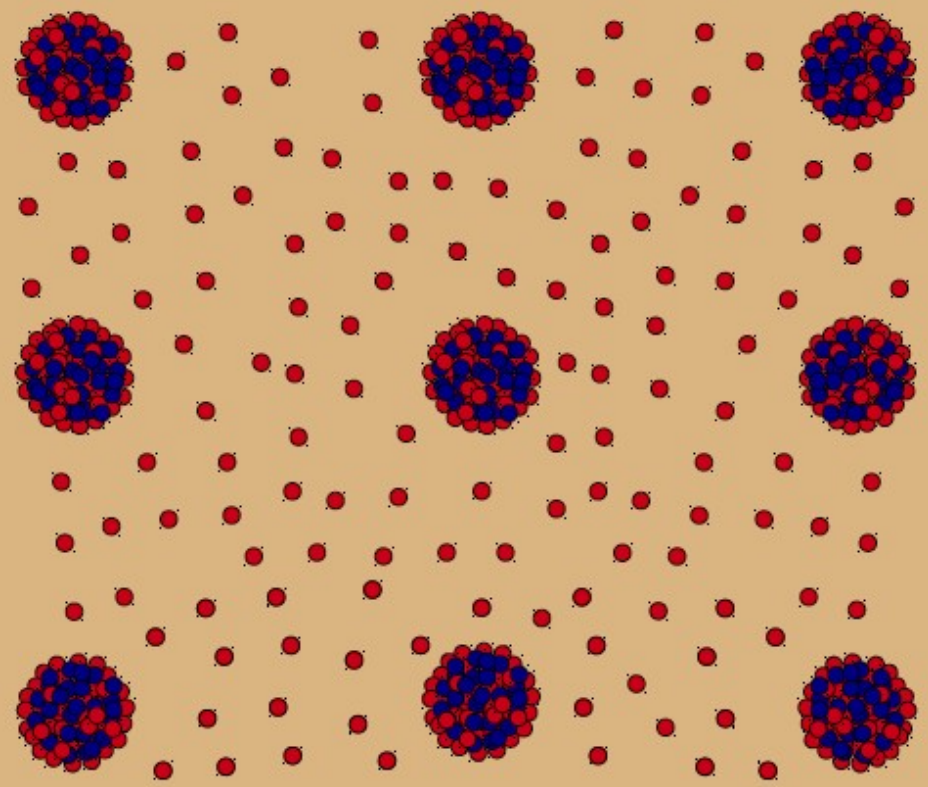
Neutron star inner crust

Low-density neutron matter in ion lattice

- Pairing significant: singlet in matter, possibly triplet in nuclei
- Nuclear toy problem of physical relevance



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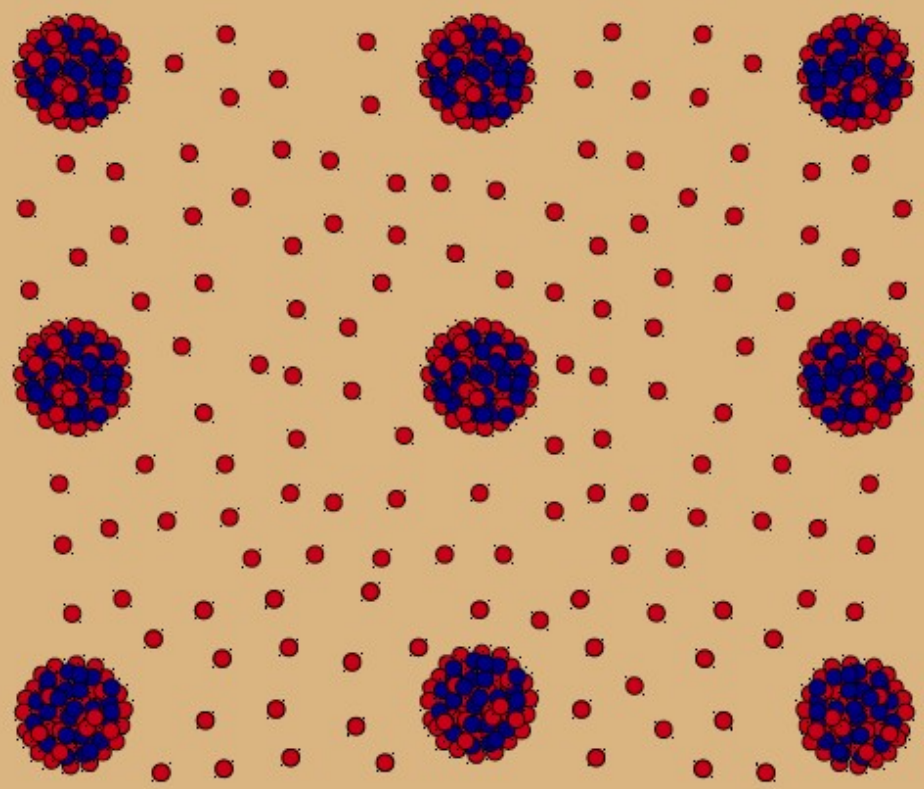


Properties

- Scattering length (a)
- Effective range (r_e)
- Density (ρ) or (k_F)

"Observables"

- Energy (E/E_{FG})
- Pairing gap (Δ/E_F)

The meaning of it all

- Negligible contribution to specific heat consistent with cooling of transients:

  E. F. Brown and A. Cumming. Astrophys. J. **698**, 1020 (2009).

- Young neutron star cooling curves depend on the magnitude of the gap:

  D. Page, J. M. Lattimer, M. Prakash, A. W. Steiner, Astrophys. J. **707**, 1131 (2009).

- Superfluid-phonon heat conduction mechanism viable:

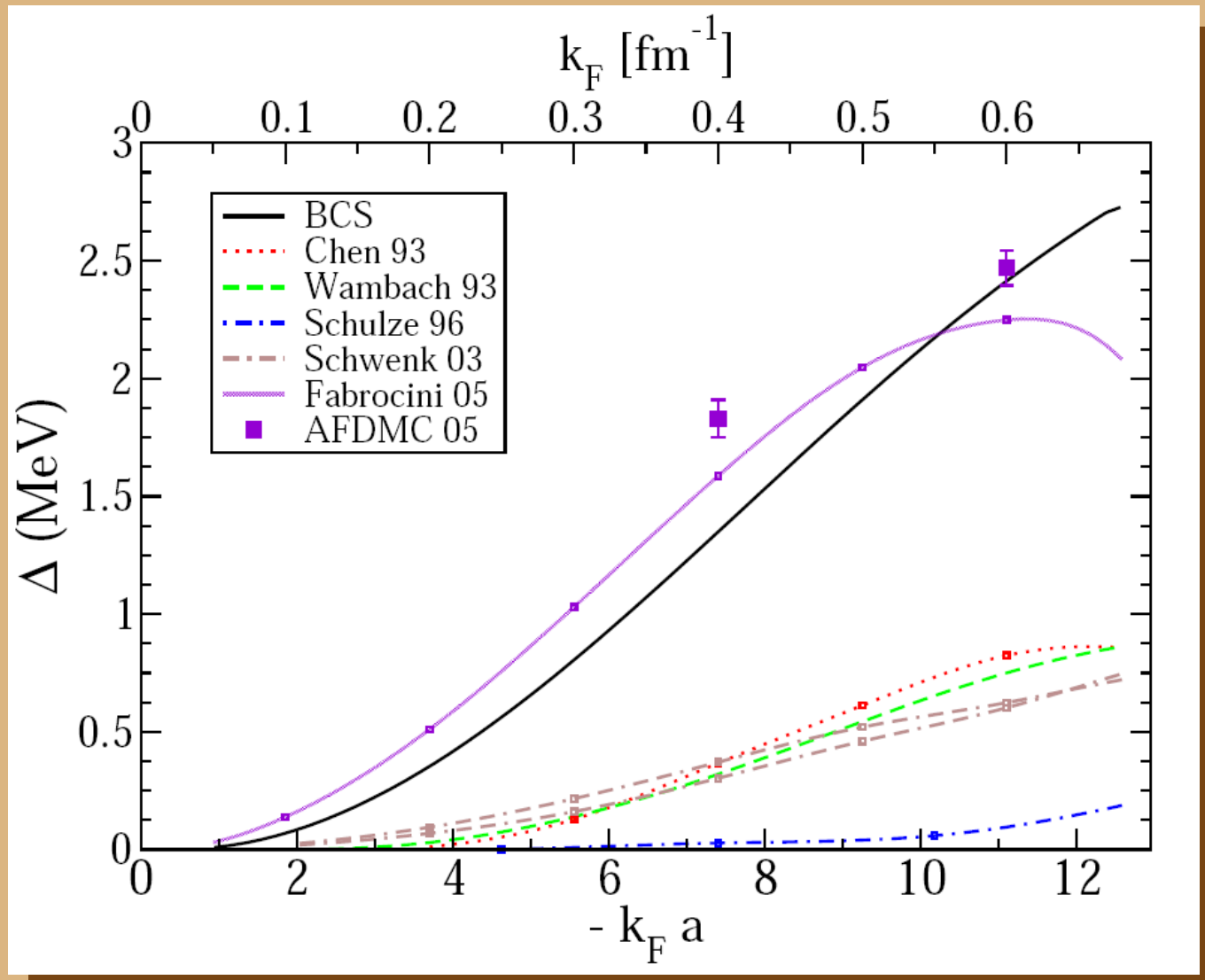
  D. Aguilera, V. Cirigliano, J. Pons, S. Reddy, R. Sharma, Phys. Rev. Lett. 102, 091101 (2009).

- Constraints to Skyrme-HFB calculations of neutron-rich nuclei:

  N. Chamel, S. Goriely, and J. M. Pearson, Nucl. Phys. A **812**, 27 (2008).

$^1S_0$ neutron matter pairing gap

No experiment → no consensus



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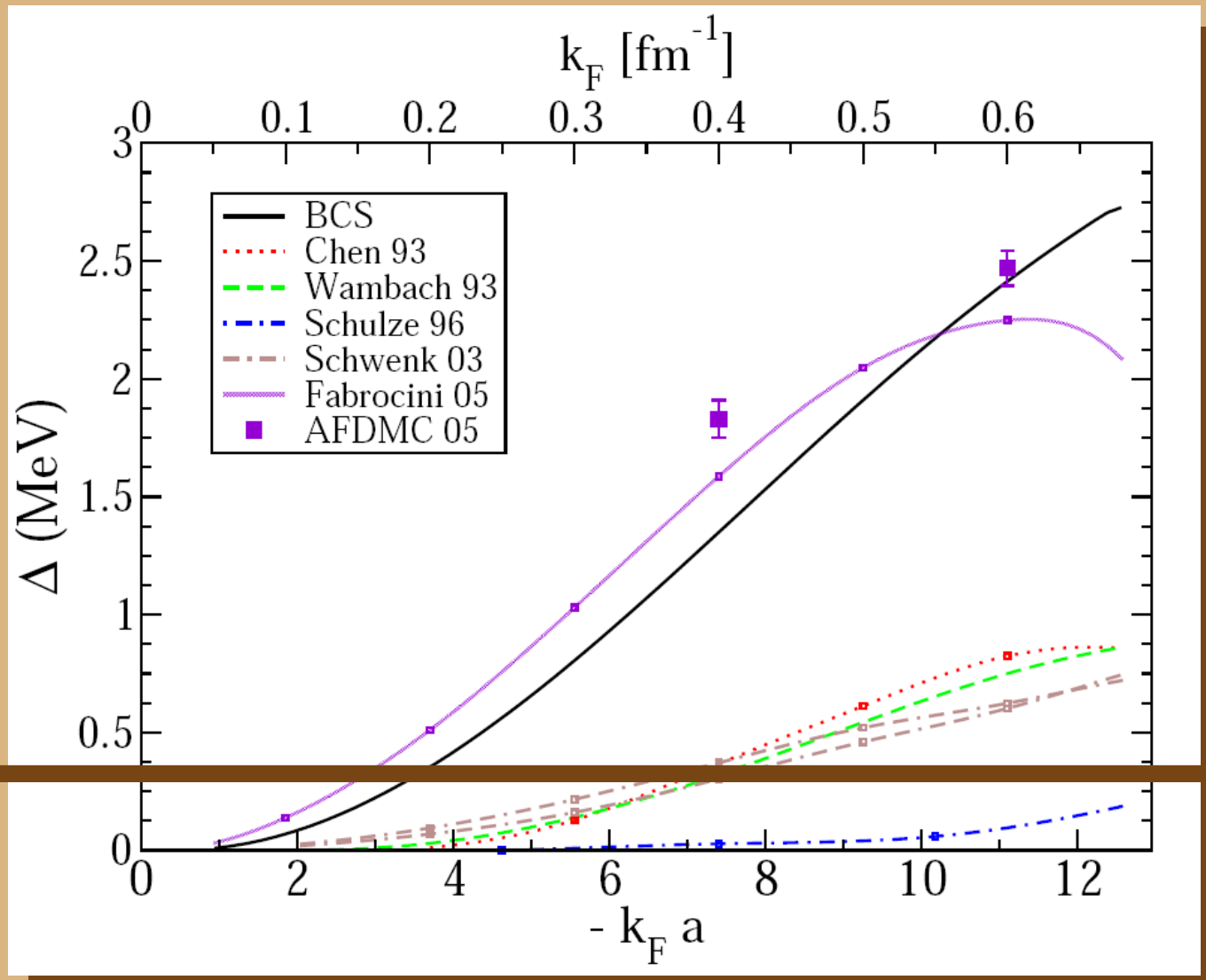

Cold atoms to the rescue

Theoretical many-body problem formulated by George Bertsch more than 10 years ago:

"What is the ground-state energy of a gas of spin-1/2 particles with infinite scattering length, zero range interaction?"

$$E = \xi E_{FG} \quad E_{FG} = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m}$$

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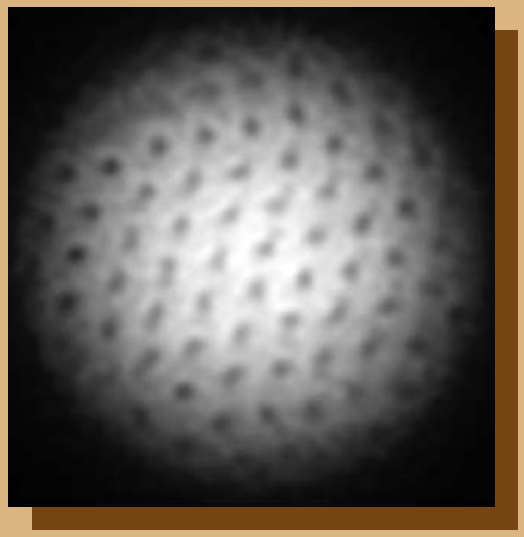
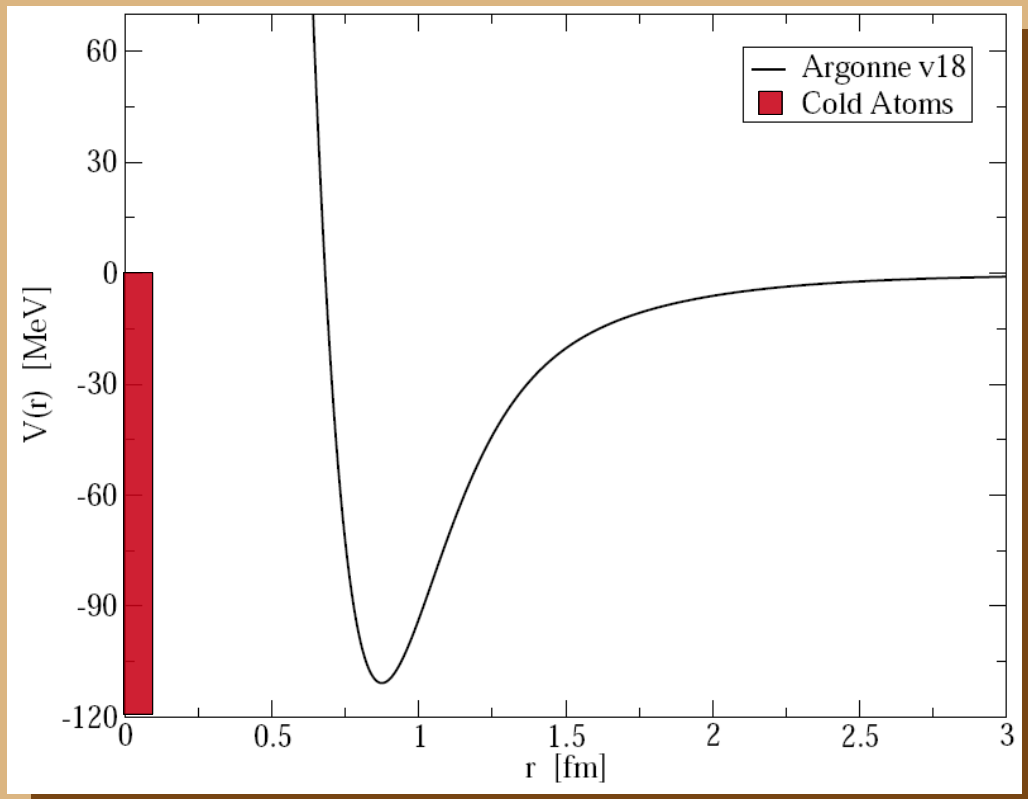
Now within _direct_ experimental reach!



Credit: Martin Zwierlein

Hamiltonian: unity in diversity

$$\mathcal{H} = -\frac{\hbar^2}{2m}\sum_{k=1}^{N}\nabla_k^2 + \sum_{i<j'} v(r_{ij'})$$



Neutron matter

$^1S_0$ channel of AV18 – later AV4

a = -18.5 fm, r_e = 2.7 fm

Cold atoms

modified Pöschl-Teller potential

a = tunable, r_e =tunable/infinitesimal

What do we know for sure?

Weak Coupling

Equation of state: $$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} k_F a + \frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a)^2$$

Pairing gap: $$\frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}} \Delta_{\mathrm{BCS}}$$

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Strong Coupling

Mean-field BCS is easy but unreliable:

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k}|V|\mathbf{k}'\rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}$$

Ab initio GFMC is difficult but accurate:

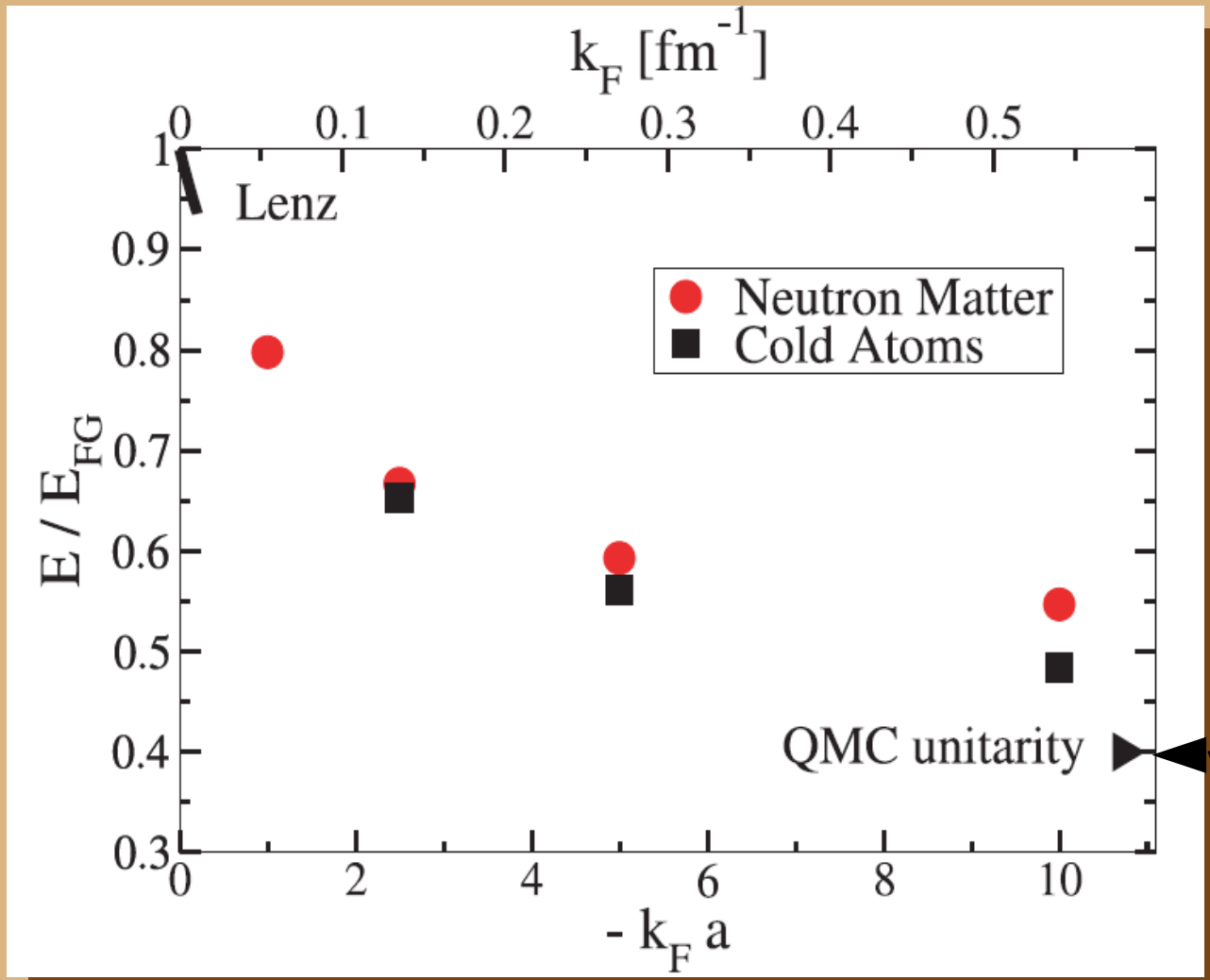
$$\Psi_V = \prod_{i<j} f(r_{ij})\ \mathcal{A}[\prod \phi(r_{ij})]$$

Equal Populations

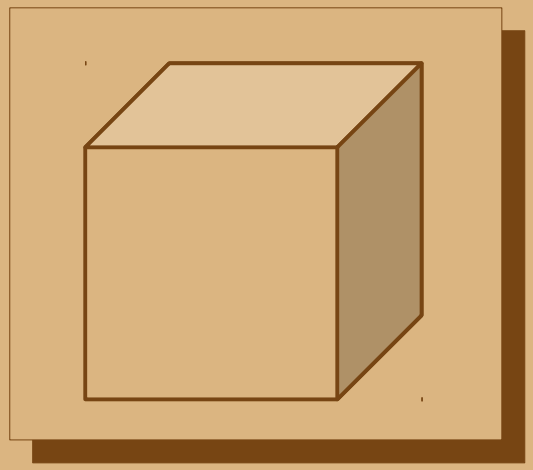
Equation of state

$$\uparrow\uparrow \quad + \quad \downarrow\downarrow$$

Equations of state: results

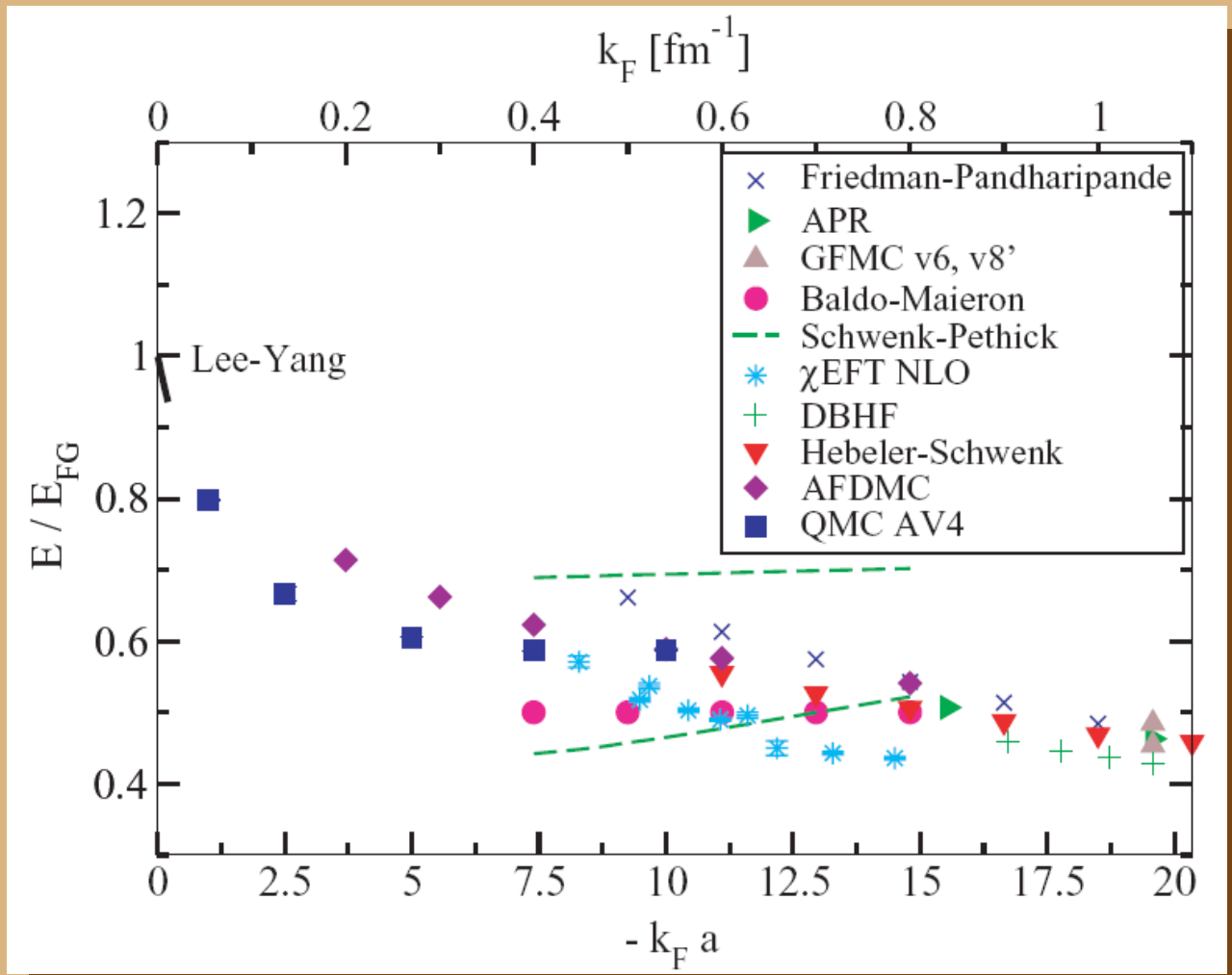


- Results identical at low density
- Range important at high density
- Duke and ENS experiments at unitarity

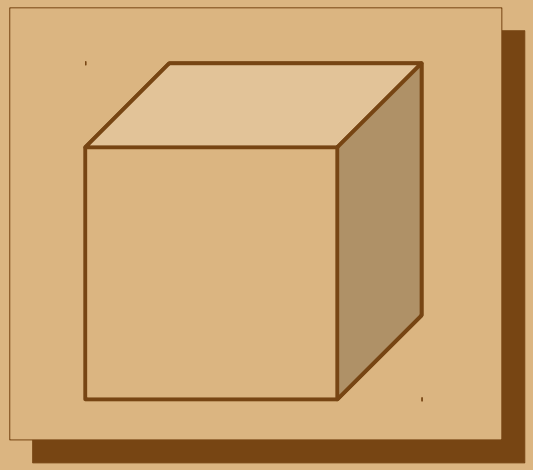


A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)

Equations of state: comparison



- Lowest densities on the market; agreement with Lee-Yang trend
- At higher densities all calculations are in qualitative agreement



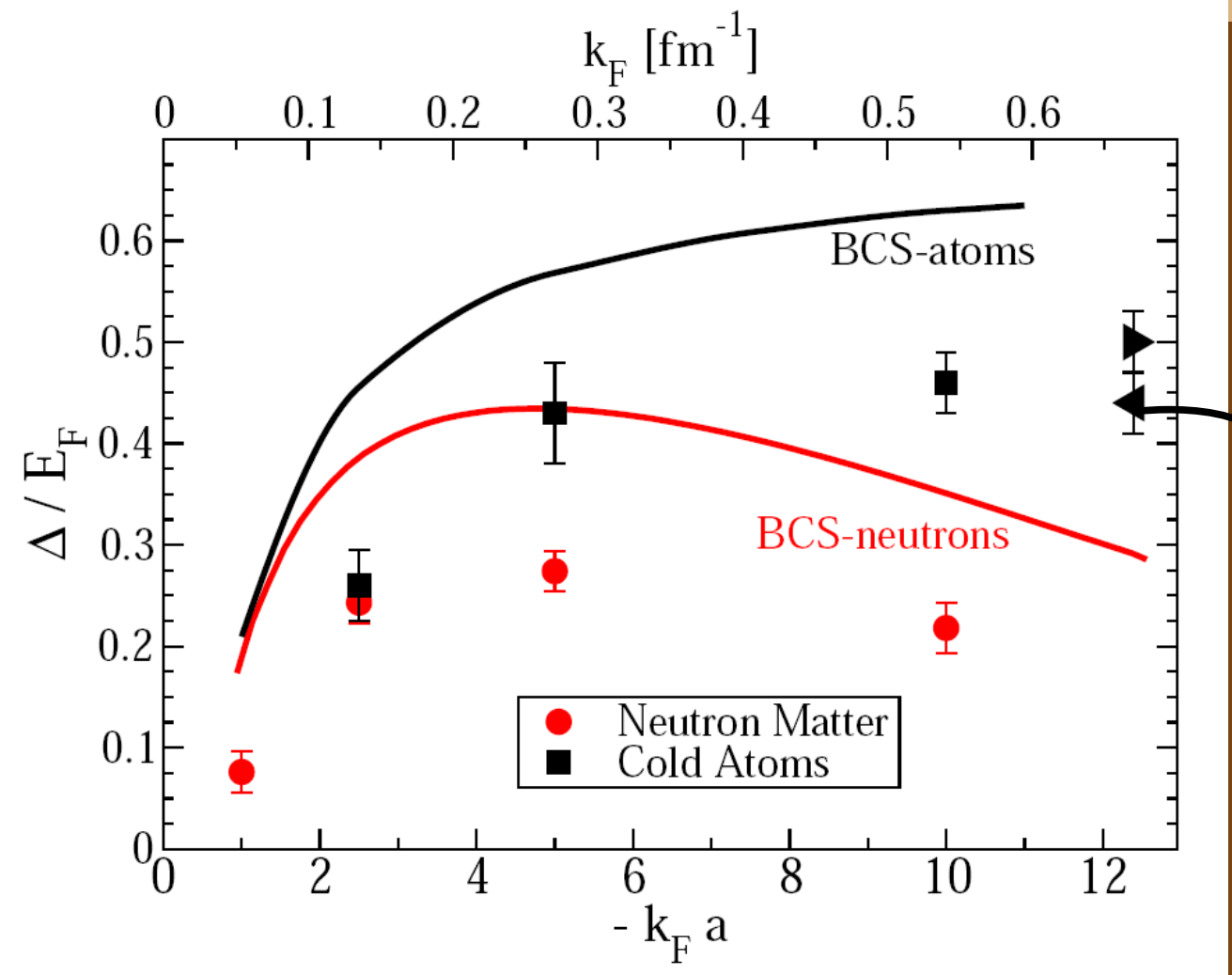
A. Gezerlis and J. Carlson, Phys. Rev. C **81**, 025803 (2010)

Odd-even staggering

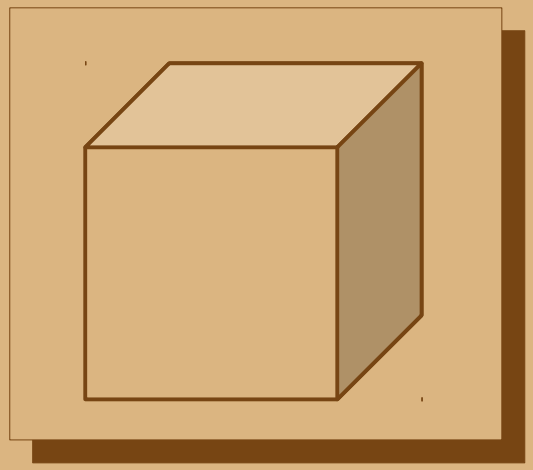
Pairing gap

↑↑↑ + ↓↓

Pairing gaps: results



- Results identical at low density
- Range important at high density
- Two independent MIT experiments at unitarity



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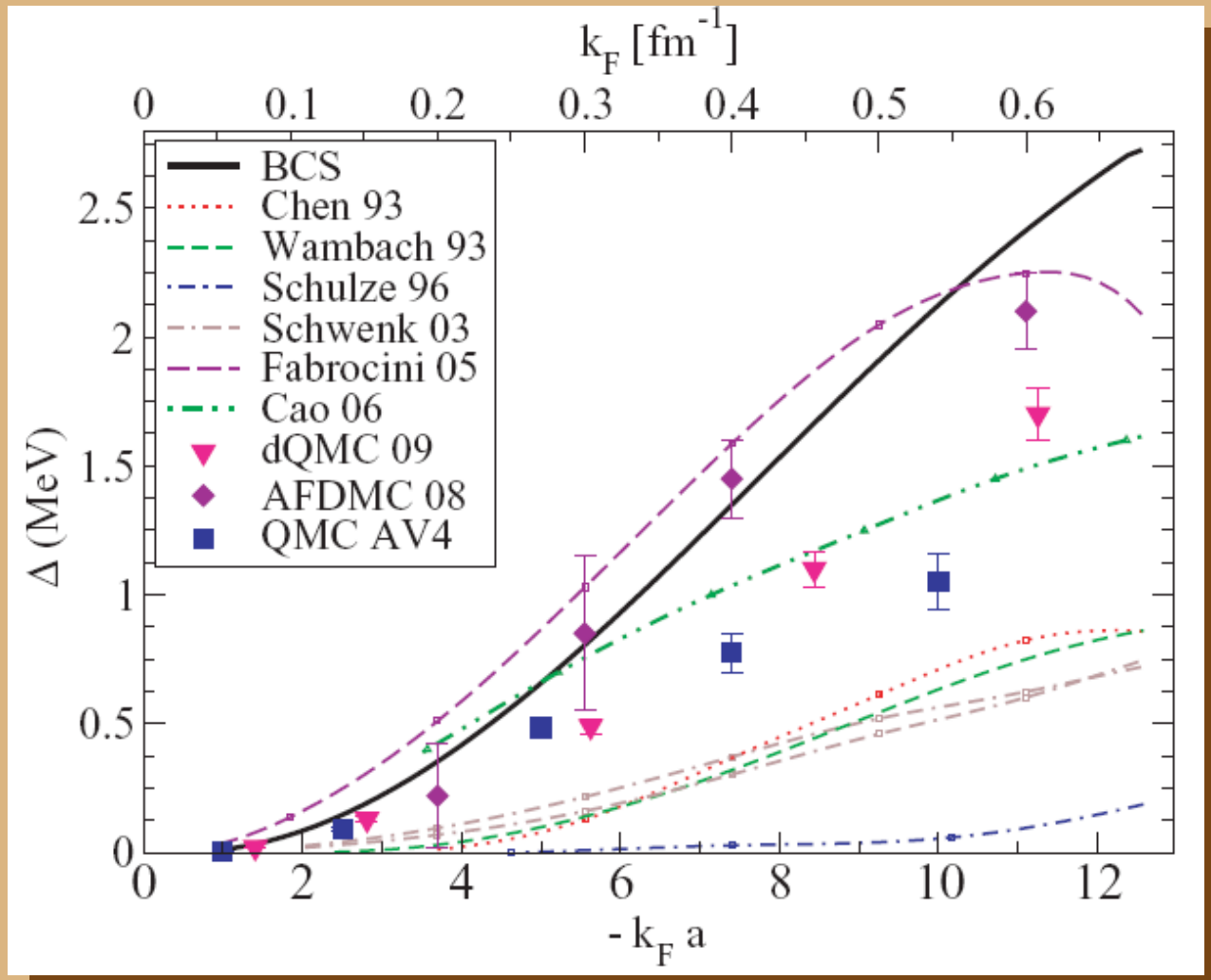
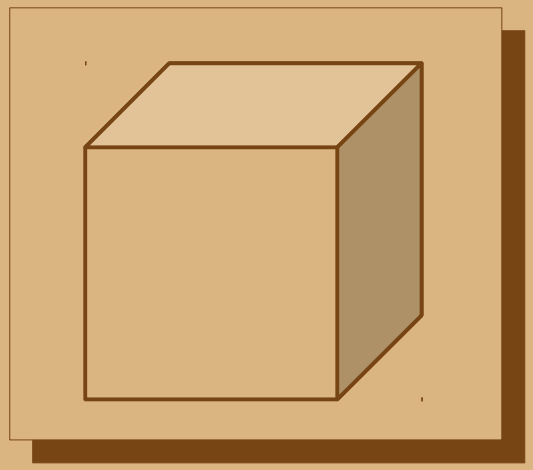
Pairing gaps: comparison



- Consistent suppression with respect to BCS; similar to Gorkov
- Disagreement with AFDMC studied extensively
- Emerging consensus?



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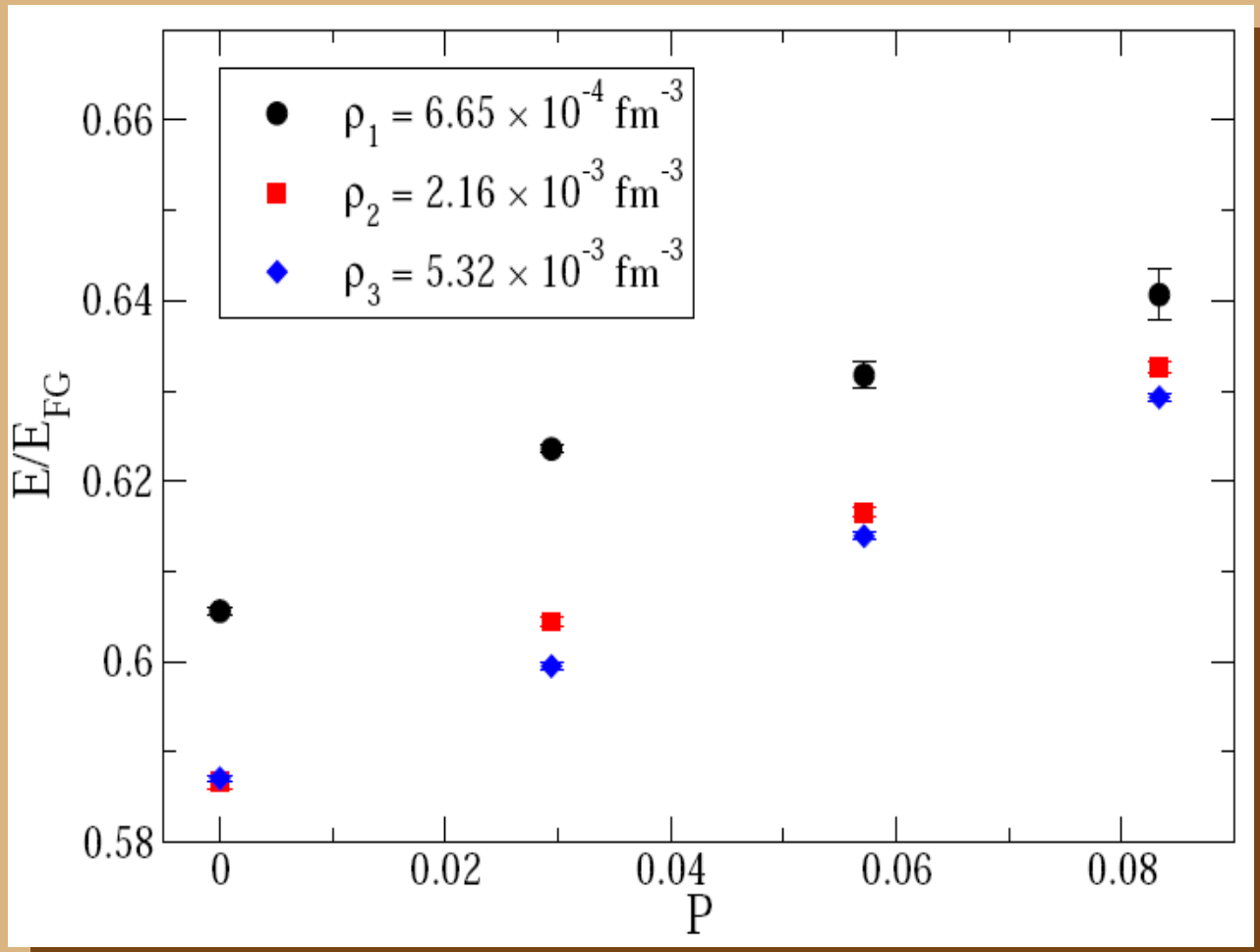
Unequal Populations

Energy versus polarization

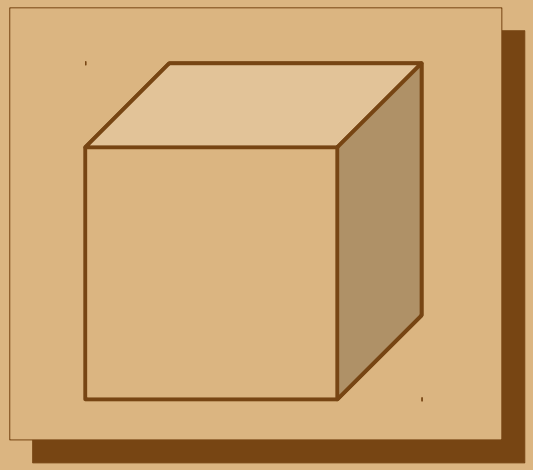
↑↑↑↑ + ↓↓

Polarized neutron matter: energy



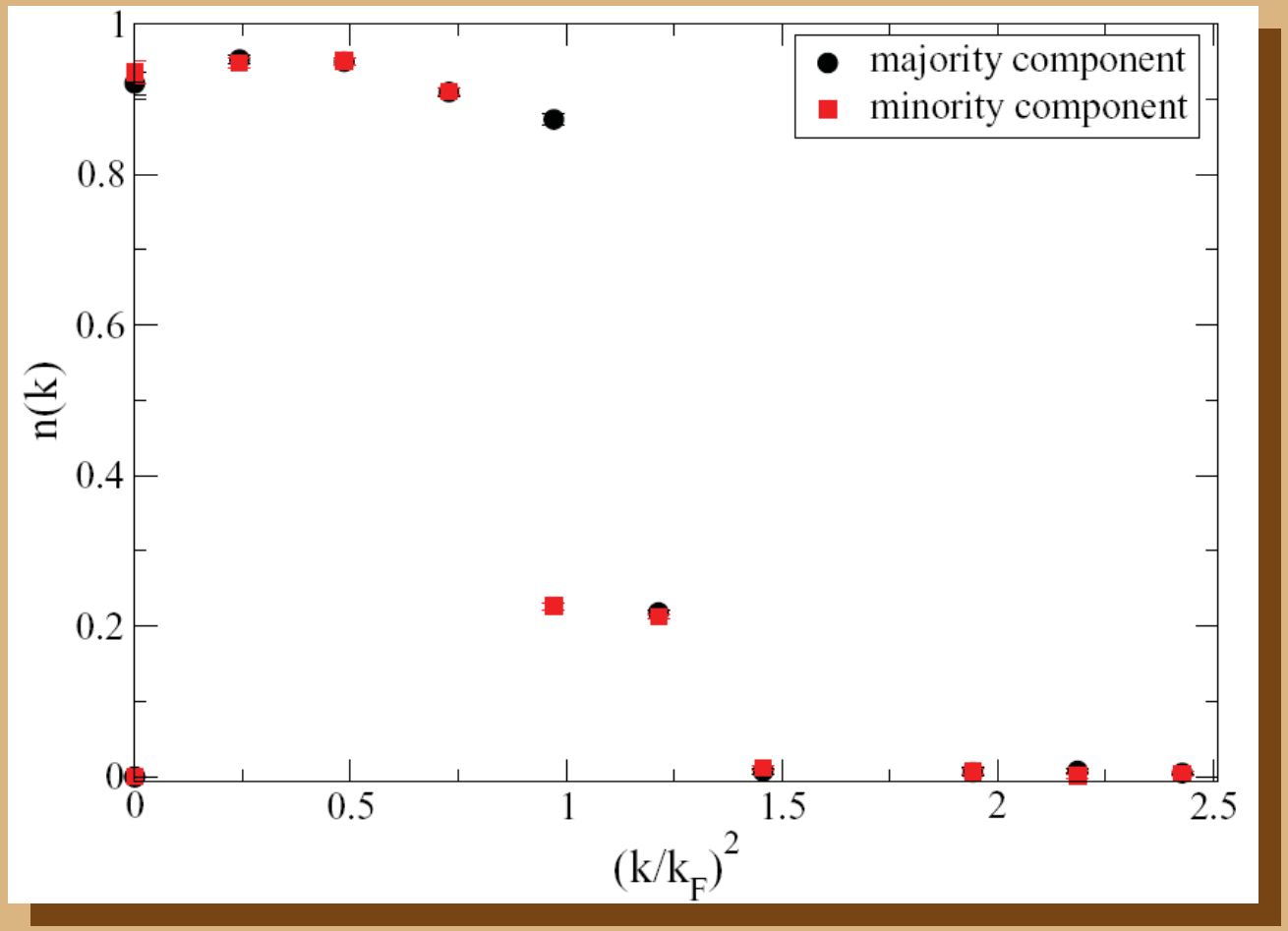
- Possibly relevant to magnetars and nuclear functionals
- Linear with respect to polarization:

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

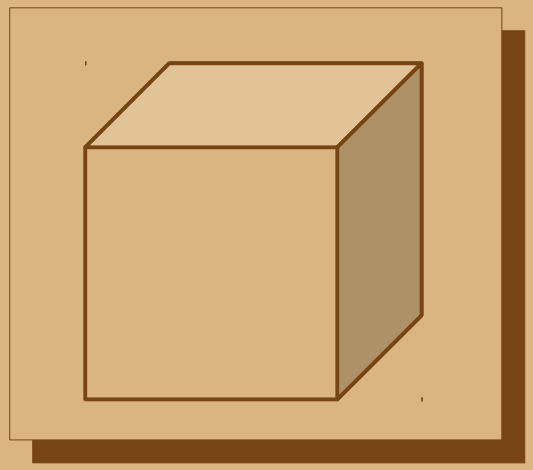


A. Gezerlis, Phys. Rev. C **83**, 065801 (2011)

Polarized neutron matter: momentum distribution



- Spread around k_F noticeable
- Known as “Fermi surface mismatch”



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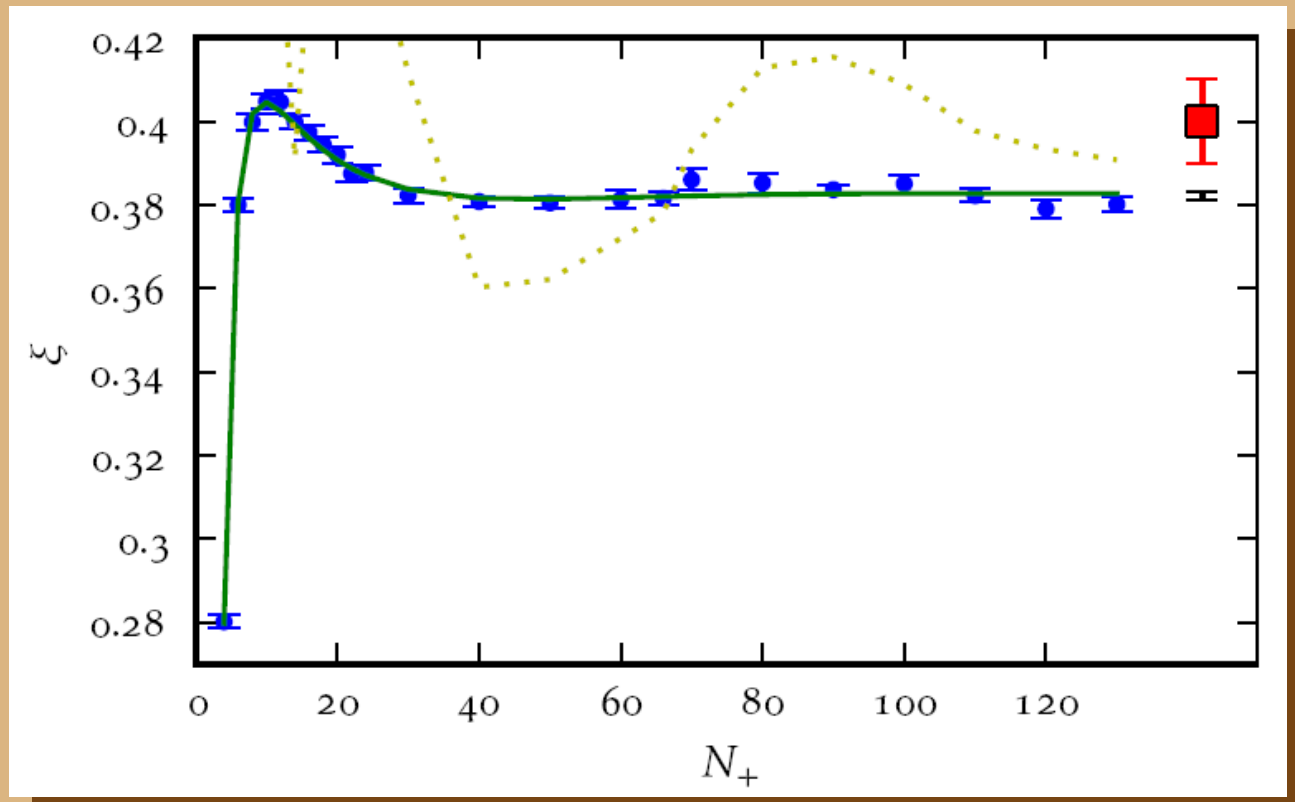
Conclusions

- Qualitative agreement on equation of state: benchmarking
- Emerging consensus (?) regarding the pairing gap
- Energy versus polarization is linear: benchmarking

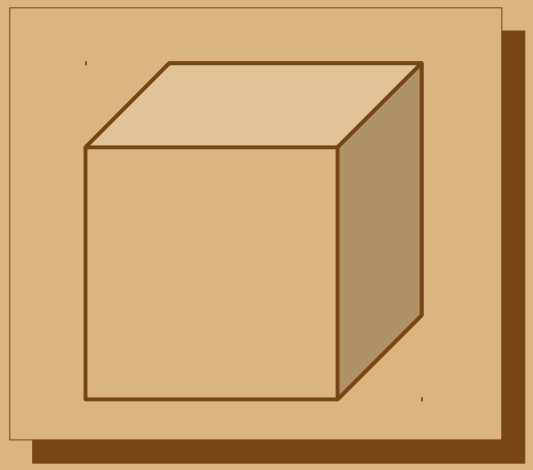
The Present Future

- Extreme polarizations: the polaron problem
- Highly asymmetric nuclear matter
- Static response of neutron matter

Energy shell effects: backup

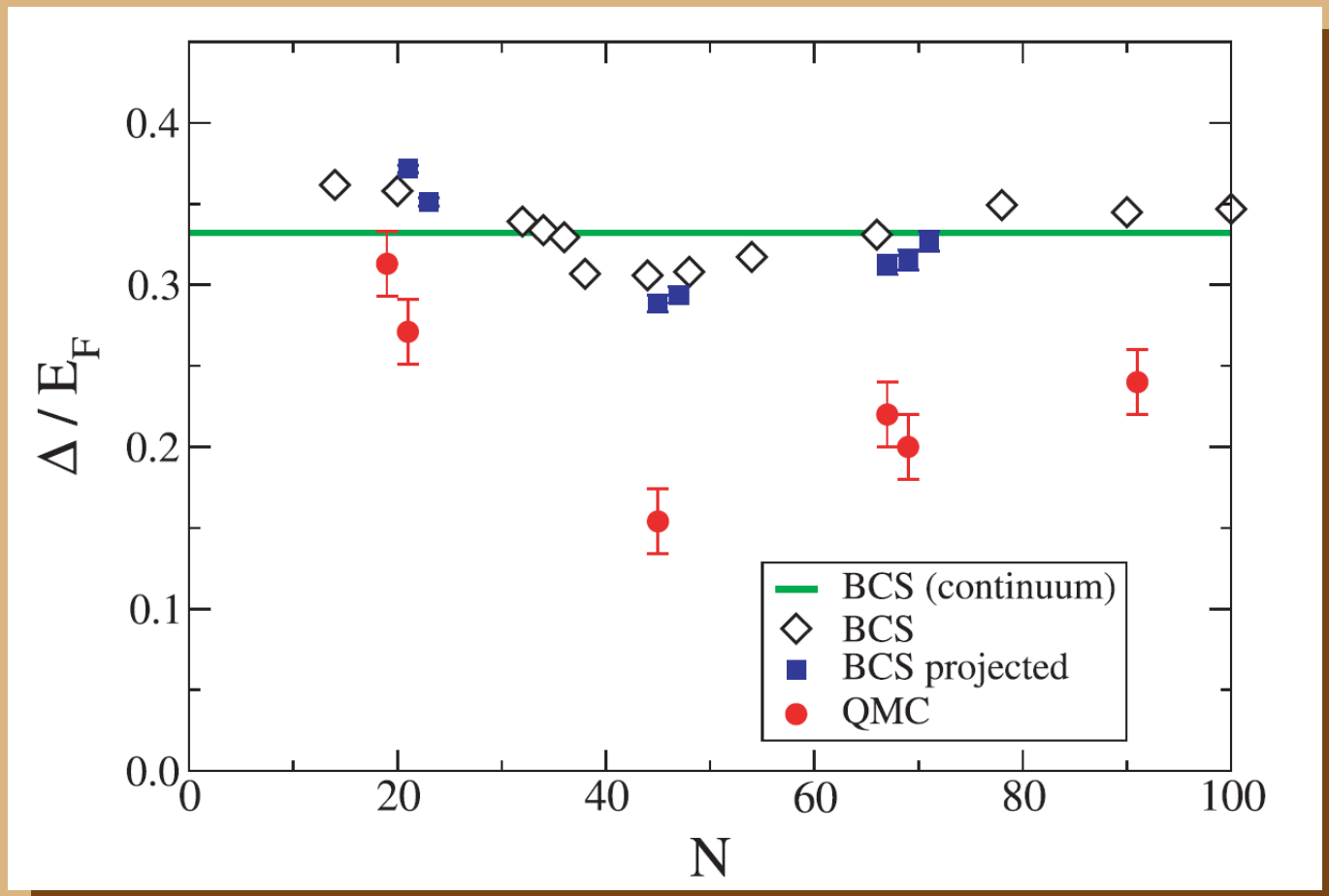


- No pairing term in the functional leads to large shell corrections
- All the while, we've only fit to QMC energies, *not* pairing gaps.



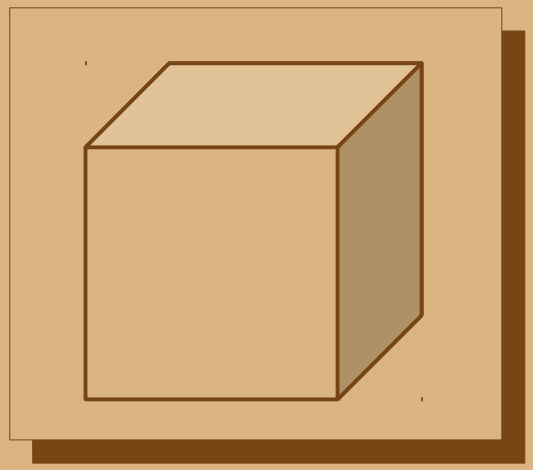
M. M. Forbes, S. Gandolfi, and A. Gezerlis, arXiv:1011.2197, submitted to *Phys. Rev. Lett.*

Pairing gap shell effects: backup



Neutron r_e = finite

Non-negligible finite-size effects: estimate the trend by comparing to the corresponding mean-field theory results.



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