Transport Coefficients of Deconfined Strongly Interacting Matter:

From Weak to Strong Coupling

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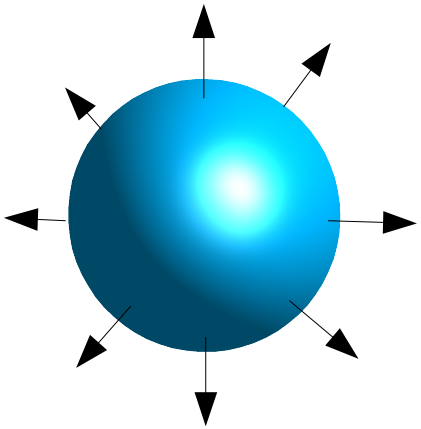
Motivation – Bulk and Shear Viscosities

system w/o conserved charge number density or particle-antiparticle symmetric systems

$$\partial_\mu T^{\mu\nu} = 0$$

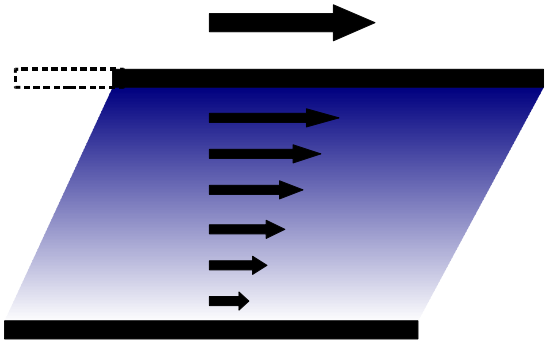
$$T^{\mu\nu} = T^{\mu\nu}_{(0)}(\epsilon, P; u^\mu) + T^{\mu\nu}_{(1)}(\zeta, \eta; u^\mu, \partial_\nu u^\mu)$$

ζ : bulk (volume) viscosity



change in volume **BUT** constant shape/form

η : shear viscosity



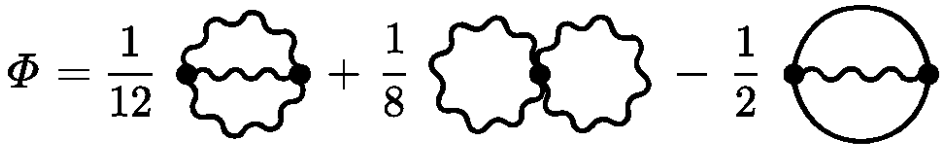
change in shape **BUT** constant volume

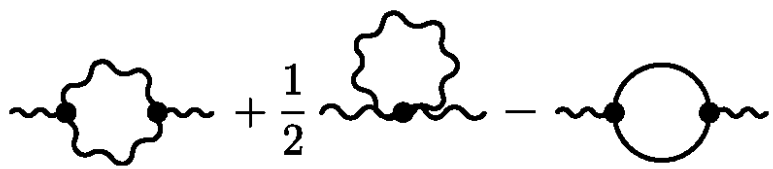
application of ideal hydrodynamics modelling heavy-ion collisions at RHIC and LHC suggests at most small dissipative effects; viscous calculations confirm this

Quasiparticle Modell (QPM)

QPM based on Φ- functional approach to QCD:

$$\frac{\Omega[D,S]}{T} = \frac{1}{2}\mathrm{Tr}\left[\ln D^{-1} - \Pi D\right] - \mathrm{Tr}\left[\ln S^{-1} - \Sigma S\right] + \Phi[D,S]$$

$\Phi =$ 

$\Pi =$  $, \ \Pi = 2\frac{\delta\Phi}{\delta D}$

$\Sigma =$  $, \ \Sigma = -\frac{\delta\Phi}{\delta S}$

→ modell for equilibrium thermodynamics ➔ corresponding energy-momentum tensor:

$$T^{\mu\nu}_{(0)}(T) = \sum_i \int \frac{d^3\vec{p}}{(2\pi)^3 E_i(T)} p^\mu p^\nu f_i^{(0)} + g^{\mu\nu} B[\{\Pi_j(T)\}]$$

for excitations with medium-modified dispersion relations (thermal mass) $E_i^2(T) = \vec{p}^{\,2} + \Pi_i(T)$

Effective Kinetic Theory

- self-consistent generalization of $T^{\mu\nu}_{(0)}$ to non-equilibrium systems:
- space-time dependence of $T(x)$ implies $E = E(x)$ is a functional of the distribution function $f(x,p)$
- to assure **basic relations**:
  - $\partial_\mu T^{\mu\nu}(x) = 0$
  - $\delta\langle T^{00}\rangle/\delta f(x,p) = E$ (Fermi liquids)
  - in thermal equilibrium: $\epsilon + P = T\dfrac{\partial P}{\partial T}$

one generalizes (in case of a one-component system) to

$$\boxed{T^{\mu\nu}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} p^\mu p^\nu f(x,p) + g^{\mu\nu} B[\Pi(x)]}$$

kinetic term **potential term**

$T^{\mu\nu}$ closely related to **effective kinetic equation of Boltzmann-Vlasov type** for the single-particle distribution function $f(x,p)$: $(\mathcal{L} + \mathcal{V})f = \mathcal{C}[f]$

- above conditions satisfied **if** $\dfrac{\partial B}{\partial \Pi} = -\dfrac{1}{2}\displaystyle\int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} f(x,p)$ related to form of Vlasov-term

Bulk and Shear Viscosity Coefficients

for quasiparticle systems

Bulk and Shear Viscosities

- decompose $T^{\mu\nu}$ and compare w/ definition: $T^{\mu\nu}_{(1)} = \zeta\, \Delta^{\mu\nu} \partial_\alpha u^\alpha + \eta\, S^{\mu\nu}{}_{\alpha\beta} \partial^\alpha u^\beta$

bulk viscosity:

$$\zeta = \frac{1}{T} \int \frac{d^3\vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} \times \left\{ \left[\left(\frac{pu}{T} \right)^2 - \frac{1}{2T} \frac{\partial \Pi}{\partial T} \right] T^2 v_s^2 + \frac{1}{3} [p^2 - (pu)^2] \right\}^2$$

shear viscosity:

$$\eta = \frac{1}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} [p^2 - (pu)^2]^2$$

cf. Chakraborty, Kapusta (2010)
& MB, Kämpfer, Redlich (2009,'10,'11)

differences: Excitations with constant vs. thermal mass

bulk viscosity:

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cf. Gavin (1985)

Relaxation Time

- collision processes relevant for **shear** and **bulk viscosities** different; **assumption**: same τ, independent of $|\vec{p}|$

- concentrate on SU(3): 2 ◄► 2 gluon-gluon scatterings

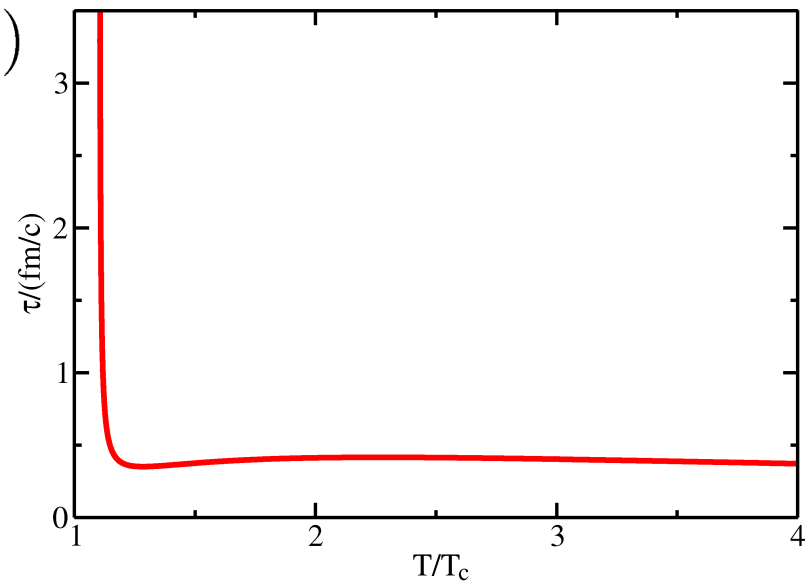
  parametrically

$$\tau^{-1} \sim T G^4(T) \ln(a/G^2(T))$$

  based on perturbative considerations

  cross section depends crucially on ratio of maximum to minimum momentum transfer $\sim a$

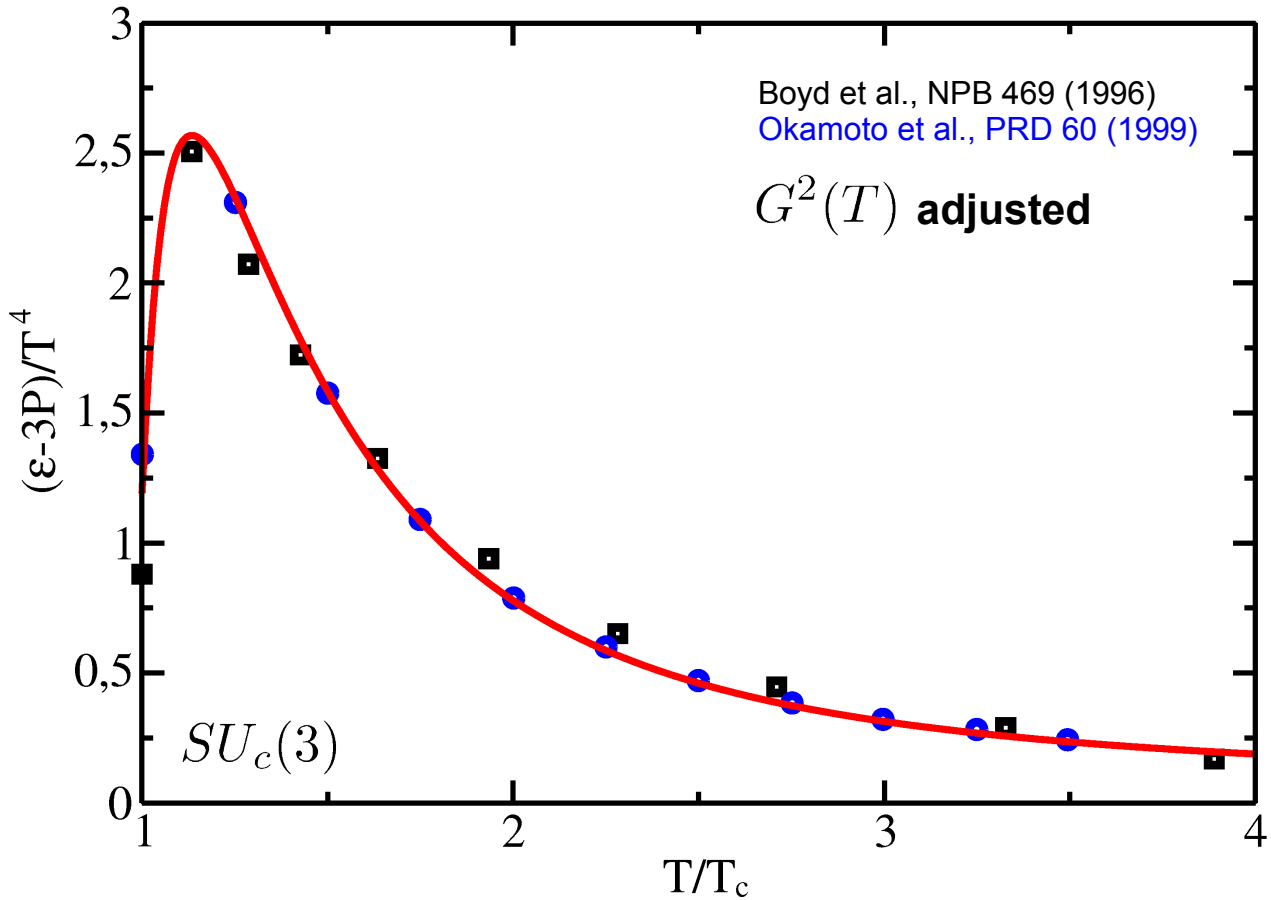
  cf. Heiselberg (1993)



- parametric dependencies of pQCD results for ζ and η on coupling and temperature reproduced at large T

Adjust Parameters in Thermal Equilibrium – $SU_c(3)$

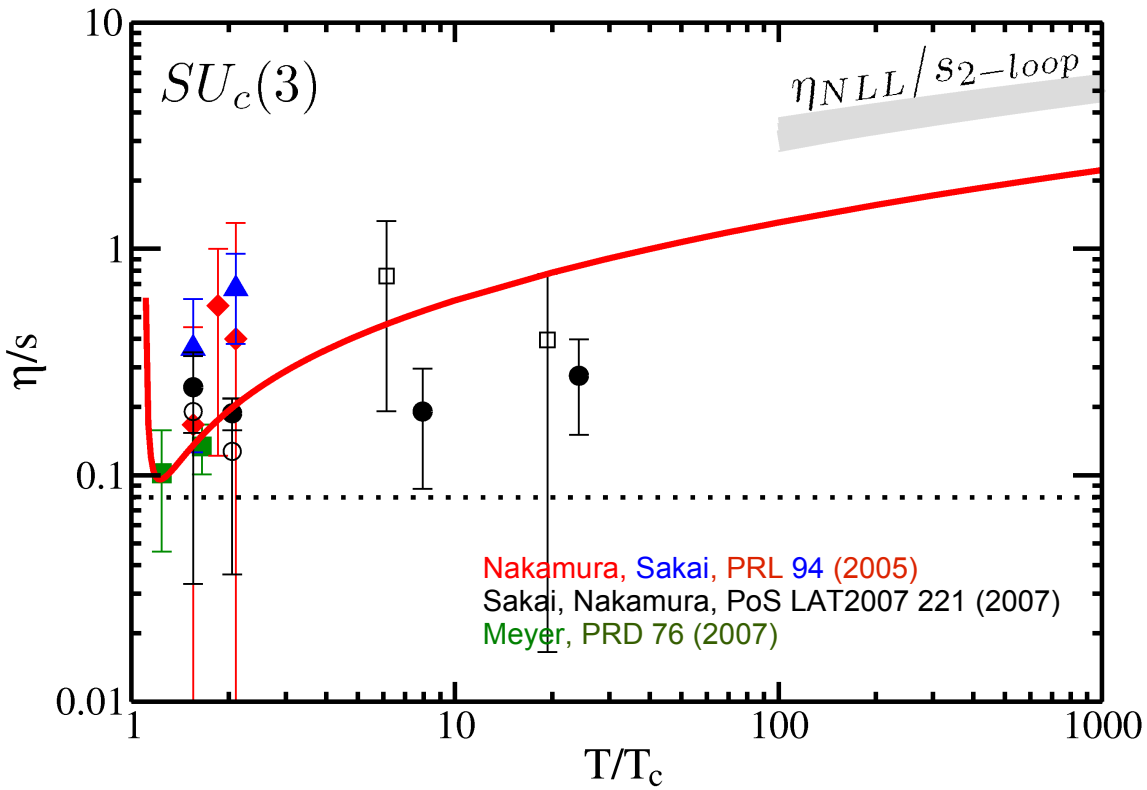
$$\Pi_g(T) = \frac{1}{2}T^2 G^2(T)\ , \quad \text{where} \quad G^2(T) = \frac{48\pi^2}{11 N_c \log\left(\frac{\lambda(T-T_s)}{T_c}\right)^2}$$



→ maximum around $T/T_c \simeq 1.15$

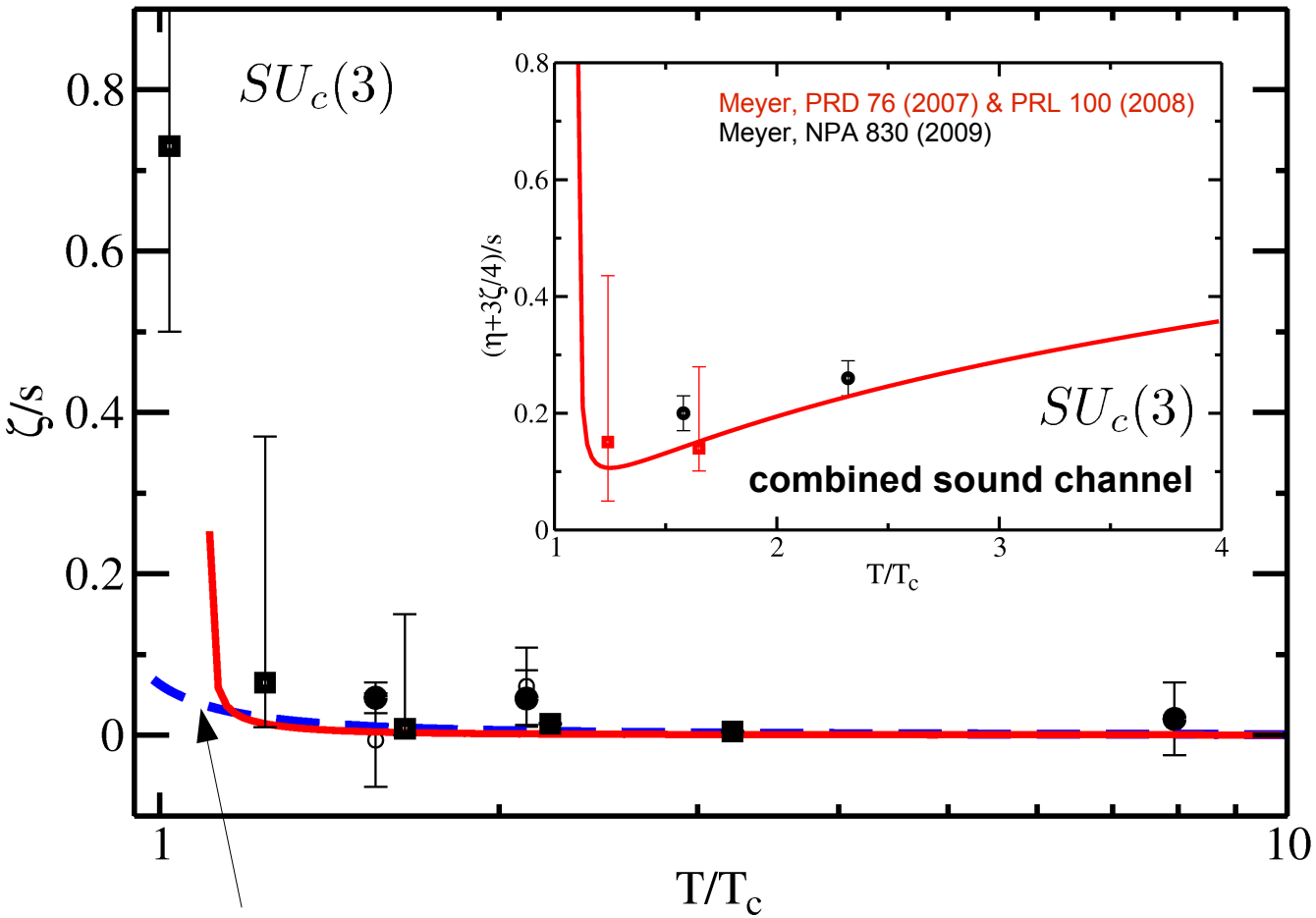
Quantitative Results – Specific Shear Viscosity

→ **behaviour close to T_c driven by τ**



cf. MB, Kämpfer & Redlich (2009, 2011)

Quantitative Results – Specific Bulk Viscosity



holographic QCD,
cf. Gürsoy et al. (2009)

cf. MB, Kämpfer & Redlich (2011)

Ratio of Bulk to Shear Viscosities

Bulk to Shear Viscosity Ratio

Big Theoretical Motivation: Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

- Kovtun-Son-Starinets bound: $\eta/s \geq 1/(4\pi)$

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Buchel bound: $$(\zeta/\eta)_B \geq 2\left(\frac{1}{k} - v_s^2\right)$$

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  Buchel bound: $(\zeta/\eta)_B \geq 2\left(\frac{1}{k} - v_s^2\right)$

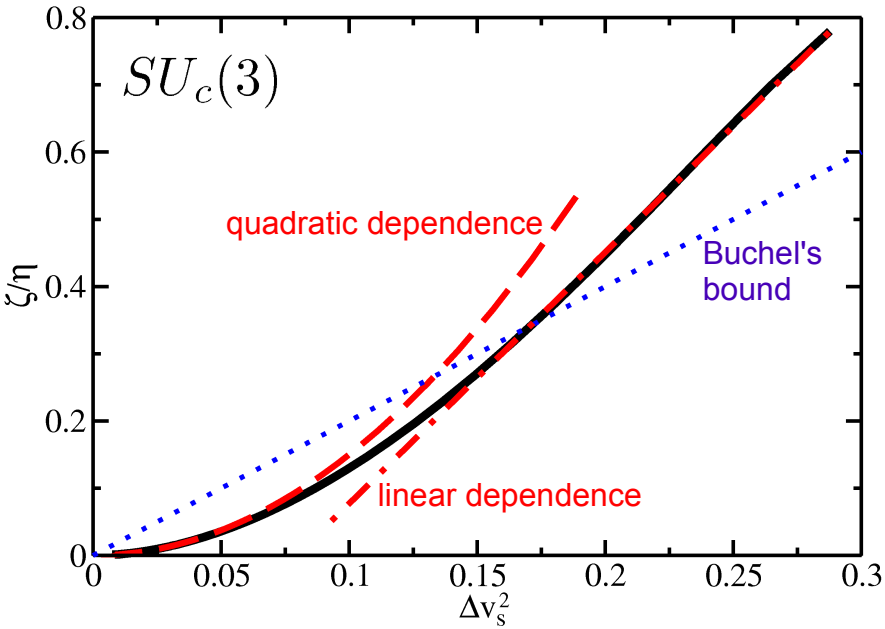
  - specific strongly coupled but nearly conformal theories (AdS/CFT) $\zeta/\eta \sim \Delta v_s^2 \equiv \left(\frac{1}{3} - v_s^2\right)$
  - for scalar theory or photons in hot fluid parametrically correct also in pQCD (weak coupling) $\zeta/\eta = 15\left(\Delta v_s^2\right)^2$

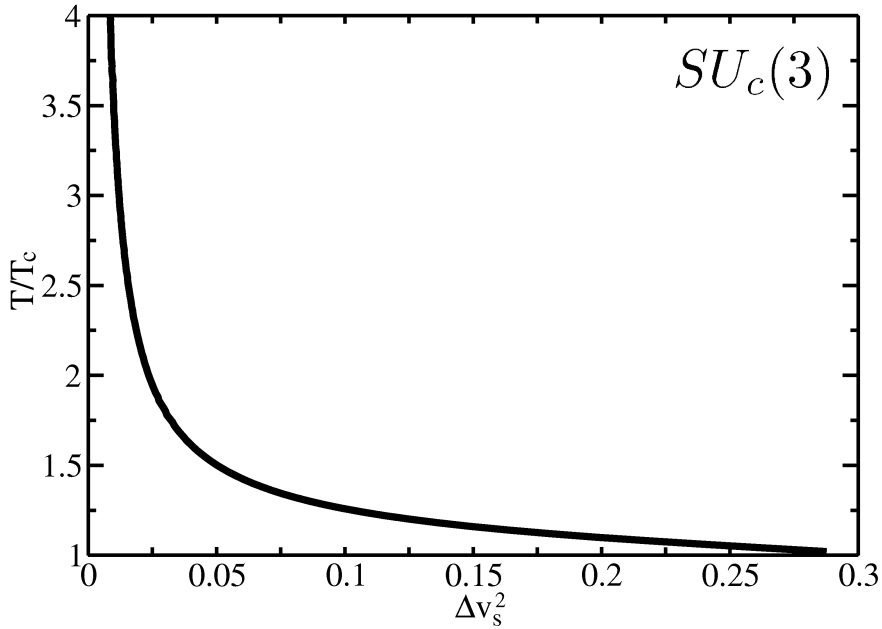
- might expect that there is a gradual change from one behaviour to the other as a function of temperature

Bulk to Shear Viscosity Ratio

$$\frac{\zeta}{\eta} = 15\left(\Delta v_s^2\right)^2 - 30\,\Delta v_s^2\,(\Pi - \tilde{a})\,v_s^2\frac{\mathcal{I}_0}{\mathcal{I}_{-2}} + 15(\Pi - \tilde{a})^2\left(v_s^2\right)^2\frac{\mathcal{I}_2}{\mathcal{I}_{-2}}\,,$$

$$\tilde{a} = T^2\frac{\partial \Pi}{\partial T^2}\;,\;\mathcal{I}_k \ldots \text{ momentum integrals}$$





- at asymptotically large T: $v_s^2 = \frac{1}{3} + \frac{5}{36}bT\frac{dG^2}{dT} + \mathcal{O}\left(G^2T\frac{dG^2}{dT}\right)$
- for $T \to T_c$: $\Delta v_s^2 \to 1/3$

➡ linear dependence on Δv_s^2 and Buchel's bound satisfied for $T \leq 1.15\,T_c$

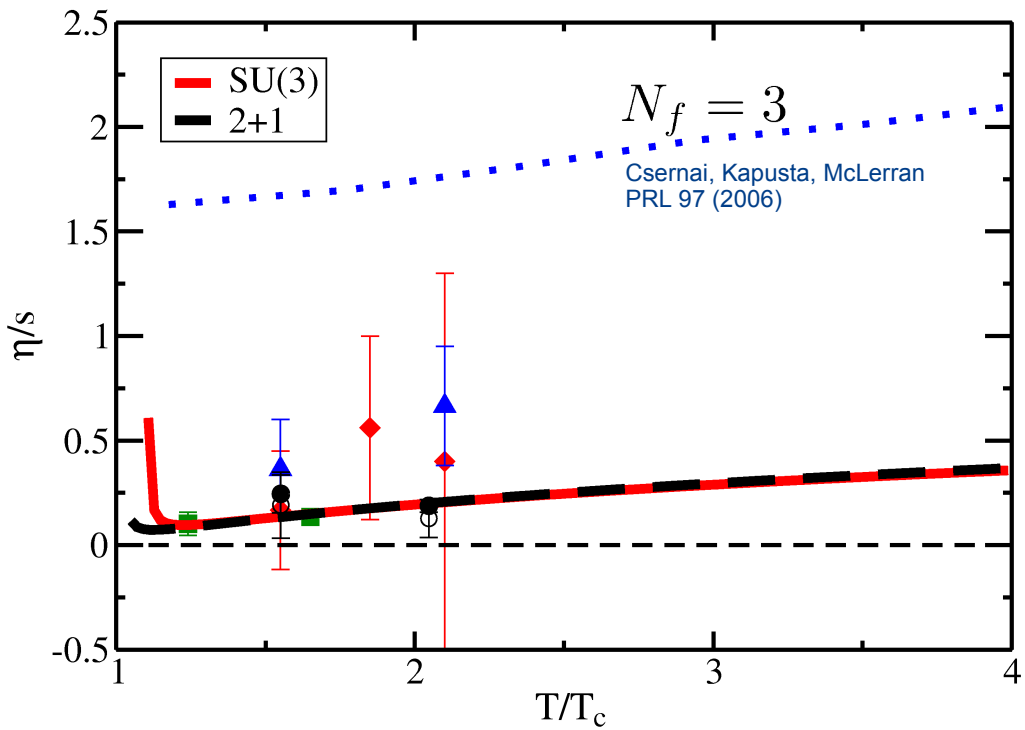
Estimating the QGP Specific Shear Viscosity

inclusion of quark degrees of freedom by assuming that relations between gluon and quark sector known from perturbative regime hold close to T_c

→ **leading-order estimate:**

$$\eta = \eta_g + \eta_q \text{ (additive)}$$

$$\eta_q \simeq 2.2 \frac{(1 + 11N_f/48)}{(1 + 7N_f/33)} N_f \, \eta_g$$



→ **mild overall increase with T; still small at $3T_c$**

Conclusions

- knowing QCD transport coefficients important for understanding the behaviour of strongly interacting matter observed in high-energy nuclear collisions

➡ **picture: excitations with effective thermal mass**

- inclusion of mean field term in energy-momentum tensor necessary for self-consistency of the approach
- follows from kinetic equation of Boltzmann-Vlasov type

- influence of a medium-dependent effective mass in dispersion relation minor on η <u>but</u> prominent in ζ

- fairly nice agreement w/ available lQCD data ($SU_c(3)$); specific shear viscosity as small as $1/4\pi$

- ratio of bulk to shear viscosities exhibits both quadratic and linear dependence on conformality measure; turning point located at the maximum in the scaled interaction measure

Energy Loss Parameter

relation between η/s and averaged transverse momentum transfer squared per unit distance of an energetic parton $\hat{q}$

cf. Majumder, Müller, Wang (2007)

$$\eta \sim \frac{1}{3}\rho \langle p \rangle \lambda$$

$$\Rightarrow \hat{q} \simeq \frac{1}{12}\frac{\rho}{s}\langle p \rangle \langle \hat{s} \rangle \left(\frac{\eta}{s}\right)^{-1}$$

underlying assumption:
interaction between energetic parton and medium is of same structure and strength as interaction among thermal excitations