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A Formal Theory of Strategy

Eric Van den Steen

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A Formal Theory of Strategy

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Abstract

What makes a decision strategic? When is strategy most important? This paper studies the structure and value of strategy (in its everyday sense), starting from a (functional) definition of strategy as '*the smallest set of (core) choices to optimally guide the other choices.*' This definition captures the idea of strategy as the core of a – potentially flexible and adaptive – intended course of action. It coincides with the equilibrium outcome of a 'strategy formulation game' where a person can – at a cost – look ahead, investigate, and announce a small set of choices to the rest of the organization.

Starting from that definition, the paper studies what makes a decision 'strategic' and what makes strategy important, considering commitment, irreversibility, and persistence of a choice; the presence of uncertainty (and the type of uncertainty); the number and strength of interactions and the centrality of a choice; its level and importance; the need for specific capabilities; and competition and dynamics. It shows, for example, that irreversibility does not make a decision more strategic but makes strategy more valuable, that long-range strategies will be more concise, why a choice what *not* to do can be very strategic, and that a strategy 'bet' can be valuable. It shows how strategy creates endogenously a hierarchy among decisions. And it also shows how understanding the structure of strategy may enable a strategist to develop the optimal strategy in a very parsimonious way.

JEL Codes: D70, L20, M10

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1 Introduction

Judging from the more than 70,000 management books on the topic (Kiechel 2010), strategy is an issue of great interest to business. But the importance of strategy – in its everyday meaning – goes beyond business: a central bank needs a strategy to fight a financial crisis and a health agency needs a strategy to fight an epidemic. But what makes a decision strategic? How do you determine whether some set of decisions constitutes a strategy? And why does strategy matter? The existing definitions in the literature – such as Andrews' definition as 'the goals of the firm and the pattern of policies and programs designed to achieve those goals' – provide little concrete guidance on these foundational questions. But the questions really matter: How do you find a strategy if you don't know what you're looking for? And why would you look for one if you don't know why it matters?

The purpose of this paper is to develop a formal economic theory of strategy[1] that captures existing ideas in the management literature but that is formalized in a way that permits analysis and transparent interpretations. The analysis explores the foundational questions what makes decisions strategic and what makes strategy important, considering a range of factors such as commitment, uncertainty, irreversibility, the level of a decision, and more. (It is, however, explicitly *a* theory of strategy and thus not intended to be either exclusive.) While motivated by a business setting, the paper studies a generic project so that the ideas apply more broadly, though some sections will focus on competitive strategy.

A theory of strategy needs to build on a clear definition of strategy. The definitions common in the literature, however, are mostly *descriptive* ('what strategy looks like'), which makes them hard to use for analysis.[2] I therefore start from a functional definition ('what strategy does') as *'the smallest set of (core) choices to optimally guide the other choices.'*[3] This definition captures the idea that strategy is the core of a – potentially flexible and adaptive – intended course of action and that it provides each decision maker with just enough guidance and with just enough of the full picture to ensure consistency across decisions, both over time or at a point in time. Note that strategy, so defined, generates *endogenously* a hierarchy of decisions, with more 'strategic' decisions guiding subordinate decisions. (In equilibrium, the strategic choices will turn out to be, among other things, high-level choices, such as the choice of product scope or target customer.) Developing a functional definition rather than starting from the existing descriptive ones is key to this analysis.

This definition can be motivated by asking what characterizes an 'absence of strategy'. When

[1] In the body of the paper, I will use the term 'strategy' always in its everyday sense, rather than its game-theoretic sense. Whereas the proofs use both meanings, the meaning will be clear from the context.

[2] Defining a gun by describing it without mentioning that it is 'something to shoot with' is not practical.

[3] The term 'optimally' reflects the need to consider dynamics, flexibility, and cost-benefit trade-offs. This definition is also useful for practice and educational purposes (Van den Steen 2012a).

people say (or complain) that 'this organization lacks a strategy' they usually mean that the organization took a number of actions that each made sense on its own but that did not make sense together, i.e., that lack a unifying logic. Strategy thus ensures, like a plan, that all decisions fit together, over time and at a point in time. This fits with the Oxford Dictionaries Online definition of strategy as 'a plan of action designed to achieve a long-term or overall aim' and Mintzberg's (1987) statement that 'to almost anyone you care to ask, *strategy is a plan* [emphasis in original] – some sort of consciously intended course of action, a guideline [...].'[4] But a strategy is *not* a detailed plan of action; it is a plan of action boiled down to its most essential choices and decisions. It is 'minimal with a maximal effect.' That leads to this definition as 'the smallest set of (core) choices to optimally guide the other choices.'

The paper observes – and formally shows – that this definition of strategy coincides with the equilibrium outcome of a game that captures a 'planned' strategy process where someone takes a step back, collects information, and develops an overall direction for the organization. In the model, a group of people each make a choice that affects a common project. Each person has only 'local' information about her own decision and interactions, and knows little about others' decisions. Without a strategist, this would result in the piecemeal outcome, characteristic of a 'lack of strategy': each decision is optimal on a standalone basis but without alignment across decisions. The analysis then allows one person ('the strategist') to collect information and announce a set of choices. In equilibrium, the announcement will be exactly a 'strategy' as defined above. By linking the definition of strategy to a concrete process, this connection provides a transparent basis and micro-foundations for the formal analysis.

The paper then uses this to analyze the structure and importance of such a strategy. Which decisions will be strategic – i.e., within this endogenous hierarchy of decisions, which decisions will be investigated and announced to guide others? And when is strategy most important?

The first set of results are about the role of persistence, commitment, and irreversibility in strategy. These matter not only for their practical relevance but also because commitment and irreversibility are one of the few, if not the only, characteristics that have been explicitly identified in the management literature as making a decision strategic, in particular in Ghemawat's (1991) seminal work. The paper carefully defines each of these characteristics and then shows the following:

- Irreversibility does not necessarily make a decision more strategic, and may even make it less strategic. But it always increases the value of strategy. And it makes decisions with which that decision interacts more strategic.

[4]'Decisions fitting together' and 'guiding towards an objective' are two sides of the same coin. Mintzberg (1987) goes on to provide 9 other perspectives on strategy. The current paper takes the perspective of 'almost anyone you care to ask,' to cite Mintzberg.

- Persistence makes decisions more strategic. One implication is that, in volatile environments, it is often optimal to build a strategy around stable factors or around internal factors that are more under the control of the firm, such as resources or capabilities. Another implication is that longer-term strategies and strategy for start-ups will be more concise.

- The ability to commit may make a decision more strategic, but only when there are no other drivers of persistence. Automatic commitment (upon announcement) may actually make a decision less strategic.

I will discuss how these results confirm some, but also differ from some of Ghemawat's (1991) insights.

I then consider the effects of uncertainty. Intuitively, uncertainty seems to make strategy both more valuable (because it can give direction in the face of uncertainty) but also less valuable (because uncertainty makes it more difficult to get the strategy right). The formal analysis shows that it is important to distinguish two kinds of uncertainty. First, *prior* uncertainty – uncertainty before the strategist investigates – makes a choice more strategic and strategy more valuable. Moreover, I show that prior uncertainty is a complement to the strength of interactions, which implies that such uncertainty matters *not* because uncertainty makes it difficult to find the right decision, but because uncertainty makes it hard to predict what others will do and thus to align. One important implication is that high-level generic choices, such as 'maximize shareholder value' or 'be the preferred service provider' are not good strategic choices because there is little uncertainty about them (unless they go against the expected direction). Another important implication is that choices what *not* to do can be very strategic as they often convey a lot of information. Second, *residual* uncertainty – after the state and interactions have been investigated – makes a choice less strategic and reduces the value of strategy. The best strategic choices have clear implications. I also show that a 'strategy bet' – where a company commits to a strategy despite facing very high uncertainty – may be valuable, especially when internal alignment is important. This fits the observation that high-tech firms often talk in terms of strategy bets.

A third set of results is that decisions with a large number of strong interactions are more strategic – and make strategy more valuable – especially if those decisions' interactions do not overlap with those of other strategic decisions. The intuition is that such choices can provide effective guidance for many other choices at once. A key implication of this result is that both more *central* decisions and, more importantly, *higher-level* decisions will be more strategic. This fits the casual observation that effective strategies tend to specify high-level central choices, such as scope and distinctive value proposition. Another implication is that strategy guides towards a pattern of choices but is *itself* not necessarily a pattern of choices: it can consist of a single choice. The result on overlap implies that a company's strategy will often specify one or two choices per business function (such as marketing or production).

The paper further shows that strategy may also improve investments in resources or capabilities that are specific to a particular course of action. The reason is that strategy makes clear which investments will pay off. Hence, choices on which such investments depend are more strategic and strategy is more valuable in the presence of such specific investments.

I also illustrate how the model can be used for analyzing competitive strategy and strategy dynamics. With respect to competitive strategy, the analysis shows that decisions may be more *or less* strategic when they influence competitors' actions, depending on the direction of the influence. Choices about strategic complements are more 'strategic' (in this sense) than choices about strategic substitutes. The model also helps to identify 'strategic rivals,' i.e., competitors that must be endogenized in a strategic analysis. With respect to dynamics, the model shows that optimal strategy will often be dynamic but that learning may make a choice both more *or less* strategic.

A final important insight is that understanding the structure of strategy may enable a strategist to find the optimal strategy without a comprehensive optimization and that such strategy can be very parsimonious. In particular, in a simple example with 1000 choices where all choices interact equally with each other, the strategist only investigates up to 6 or 7 states when investigation is free, and even less when it is costly, and announces these choices as the strategy. In fact, when all players have a common objective, a strategy that investigates just *one* state and announces *one* choice is sufficient for this setting. This shows how strategy can be a very effective tool to find and give direction to an organization.

This perspective on strategy also has implications for leadership and for organization design. With respect to leadership, strategy as defined here is also – in some very precise sense – the smallest set of decisions that needs to be decided centrally to get consistency, tying this definition of strategy back to its etymological origin as the decisions that need to be under the authority of the overall commander. In a companion paper, Van den Steen (2013b) builds on this analysis to explore the role of people in strategy formulation: how does it matter *who* develops the strategy. From an organization design perspective, it is of particular interest that strategy creates *endogenously* a tree-like hierarchy out of an essentially horizontal network of choices. Hence, I conjecture that the drivers that make a decision strategic may also be drivers of hierarchy and organization design.

I discuss some important limitations and boundary conditions of the analysis immediately after the model description, as that discussion will be more clear with the model as background.

Literature The economics literature closest to this paper is probably Milgrom and Roberts' (1992) insightful, though informal, discussion of how coordination through strategy and coordination through prices differ. Their discussion of the Hurwicz criterion is also related to some of the ideas in this paper. But they do not formally define strategy or study the nature of strategy, i.e., what decisions are strategic, or what determines the value of a strategy.

Also closely related within the economics literature is a stream of research on the organizational effects of specific (strategy) choices. This literature does not define strategy but equates it implicitly with a choice of direction by the manager and then studies how specific choices may be optimal. Rotemberg and Saloner (1994, 1995) show that a narrow strategy – equated with favoring certain projects – can provide incentives for effort and reduce conflict. Zemsky (1994), which is close to Section 3.5 and discussed there, shows that a commitment to a strategy – equated with a choice of project – can create incentives for investments in skills. Mailath, Nocke, and Postlewaite (2004) show that human capital specific to a strategy – equated with a choice of business – may, for example, make mergers unattractive. Van den Steen (2005), which is close to Section 3.1 and discussed there, shows that a manager's commitment to a specific choice – through her beliefs and communicated through strategy (Van den Steen 2001)) – gives direction to the rest of the organization.[5] All these papers differ in a number of ways from the current paper. First, these papers do not – either formally or explicitly – define strategy, but equate it with a choice of direction. Equating strategy with a CEO's choice of direction raises the issue that many of a CEO's choices (of direction) are not considered strategic (at all) and, in the other direction, that strategy is not always developed by the CEO. Second, and closely related, none of these papers considers the nature of strategy – i.e., which decisions are strategic – or comparative statics on what makes strategy important. Instead, these papers focus on demonstrating *in principle some benefit* of having a (particular kind of) strategy. But all papers are consistent with this paper in the sense that their insights could be derived in a variation on the model of Section 2.

In the management literature, studies of what makes a decision strategic are limited. The seminal work of Ghemawat (1991) on irreversibility and commitment will be discussed in Subsection 3.1,

[5]More indirectly related is the economic literature on vision and leadership, as vision and strategy are closely related (Rotemberg and Saloner 2000, Van den Steen 2001). This literature also often informally refers to a choice of direction as 'a strategy' (Rotemberg and Saloner 2000, Van den Steen 2001, Ferreira and Rezende 2007, Bolton, Brunnermeier, and Veldkamp 2012). Rotemberg and Saloner (2000) showed that vision could improve *incentives for effort* and discussed the relationship between vision and strategy. Most relevant to the current paper, because of the focus on direction setting and coordination, are Van den Steen (2001, 2005) – which showed that vision (in the sense of strong beliefs) could give a firm *direction* and *coordination* and also discussed the relationship between vision and strategy – and Hart and Holmstrom (2002), Van den Steen (2010), and Bolton, Brunnermeier, and Veldkamp (2012) – which all developed theories of the effect of vision on coordination and also sometimes refer to a choice of direction as a strategy. Of all these, Rotemberg and Saloner (2000) is the only one to explicitly motivate their use of the term 'strategy', pointing out that their model results in some 'pattern of decisions,' consistent with Andrews' definition of strategy. Another difference is that, by nature, that literature assumes a manager with biased preferences or differing priors, while this paper shows that strategy does not depend on that. Starting with Brandenburger and Stuart (1996) and MacDonald and Ryall (2004), there is also a small but growing literature on 'competitive advantage,' which is a central concept for strategy, but it is not focused on the nature or role of strategy itself.

in particular how the results here confirm some predictions but differ on others. In his discussion, Ghemawat (1991) actually points out that even von Clausewitz (1833) side-stepped the question which decisions are strategic by referring to 'common usage.' The other related contributions are experience-based lists or rules of thumb on what elements should be specified as part of a strategy, such as Andrews (1971), Bower et al. (1995), Saloner et al. (2001) and Collis and Rukstad (2008), which will be discussed in detail Section 4 when relating this definition to the literature. The fact that Collis and Rukstad (2008) received one of the most coveted awards in the business press illustrates the importance of the question what makes a decision strategic.

The general management literature on strategy, such as Andrews (1971) or Porter (1980), often defines strategy but not in a way that is conducive to a formal analysis. Section 4 shows how this paper formalizes key elements of these definitions. Moreover, some of the results, such as the importance of interactions, relate to ideas suggested in the management literature.

The academic management literature on strategy, such as Bower (1970), Mintzberg and Waters (1985), Hambrick and Mason (1984), Levinthal (1997), Rivkin and Siggelkow (2003), has mainly focused on the process by which strategy takes shape in organizations, with particular attention to the non-planned and non-analytical aspects. Within this literature, the paper closest to the current one is Ghemawat and Levinthal (2008) who investigate the effect of bounded rationality on strategy (relatively informally defined): they extend the NK model to investigate how many choices must be set at optimal value, when the other choices are determined by boundedly rational search, to approach the optimum. Their analysis thus evaluates how bounded rationality affects the effectiveness of such a strategy-like process. They show that this approach can be quite effective even in the very rugged landscapes of the NK model, especially when setting central choices rather than random choices, and they analyze the effect of misspecifications. Their process has similarities with the equilibrium outcome of the game of Section 2 – and thus with strategy as defined in this paper – but also important differences. For one thing, while they simply posit the process, this paper derives it endogenously as an equilibrium outcome. Second, this equilibrium analysis shows that, in fact, the strategic choices will often *not* be first-best, but second-best given the cost of investigation. Third, because they immediately (and informally) equate being strategic with being central – and do not define 'being strategic' or 'strategy' independently from 'being central' – they do not draw conclusions on what makes a decision strategic or what is exactly a strategy, but focus on how well the boundedly rational model performs relative to first best. This very different focus (on the impact of bounded rationality) is also reflected in the fact that they compare the performance of the model with the first-best outcome, while this paper compares the outcome with strategy to that without strategy, thus focusing on the value of strategy rather than the effect of bounded rationality. The current paper complements that work: instead of researching how the actual processes deviate from 'strategy as deliberate planning', it takes that idea and fleshes it out, yielding complementary insights for strategy. In the other direction, their work also complements this paper as it shows that the model is also sensible in a boundedly rational world. In NK terms, a strategy as defined here would be the smallest set of dimensions to fix in order to guide optimally towards a basin of attraction; and the question is then which dimensions are strategic based on their characteristics – such as commitment, persistence, centrality, and uncertainty – and how much does strategy improve over simply baseline search.[6] Finally, while different in approach and focus, the discussions of strategy in Ghemawat (1991) and Casadesus-Masanell and Ricart (2010) raised many of the questions that I study in this paper.

From a more structural perspective, this paper is also somewhat related to the literature on, or closely related to, team theory (Marschak and Radner 1972, Geanakoplos and Milgrom 1991, Radner and Van Zandt 1992, Garicano 2000, Dessein and Santos 2006, Alonso, Dessein, and Matouschek 2008, Dessein, Galeotti, and Santos 2012) as many of the results have also been derived in a team theory model (Van den Steen 2012c, Van den Steen 2013b). This paper essentially suggests strategy as an alternative solution to team theory problems. But the agency-based approach of this paper implies that team theory is not a fundamental feature. Moreover, there are some important differences with the existing literature, such as the fact that investigations can be dynamic and the focus on very different questions – what core choices to announce or fix as a central intervention. Van den Steen (2012c) discusses this in more depth.

The contribution of this paper is to develop a formal theory of strategy, focused on what makes decisions strategic and what makes strategy important. In doing so, it derives new results and insights on the nature and role of strategy and also develops a functional definition of strategy that permits formal analysis and that clearly distinguishes between a strategy and either a full plan or just a set of important decisions.

The next section describes the model, which Section 3 uses to study what decisions are strategic and what makes strategy important. Section 4 backs this up by formalizing the definition of 'strategy,' relating it to the literature, and showing that it is the equilibrium outcome of the model of Section 2. Section 5 discusses and concludes. All proofs are in Appendix.

2 Model

This paper studies a setting in which a group of people are engaged in a common project and must make (sequential or simultaneous) choices that affect the project's outcome. The basic research

[6] I thank Jan Rivkin for this interpretation.

question is the nature, properties, and value of 'a strategy' (in the everyday sense of the word).

Payoff Structure Formally, consider a project that generates revenue Π, which depends on K choices $\{C_1, \ldots, C_K\}$. Each choice C_k selects a course of action from an infinite set $\mathcal{C}_k$ of alternatives, i.e., $C_k \in \mathcal{C}_k = \{c_k^1, \ldots, c_k^f, \ldots\}$.[7] The project revenue Π depends *both* on whether the choices are correct by themselves (on a standalone basis) *and* on whether the choices *align* correctly. With respect to C_k being correct on a standalone basis, there is a finite subset $T_k \subset \mathcal{C}_k$ of $N \geq 1$ alternatives that are correct (and the others wrong): choice C_k is correct if and only if $C_k \in T_k$ and it is wrong otherwise. With respect to C_k and C_l aligning correctly, there is a set $T_{kl} \subset \mathcal{C}_k \times \mathcal{C}_l$ of pairs (c_k^f, c_l^h) that are correct (and the others wrong) with each c_k^f being part of N such pairs (and analogously for c_l^g). So C_k and C_l are aligned correctly iff $(C_k, C_l) \in T_{kl}$. I will refer to the T_k and T_{kl} as respectively choice states and interaction states and use $\mathcal{T}_k$ and $\mathcal{T}_{kl}$ for the sets of all possible states. The revenue Π is then an increasing function of the choices being correct on a standalone basis and of the choices interacting correctly. In particular, the project revenue has the following parametric form:

$$\Pi = \sum_{k=1}^{K} \alpha_k I_k + \sum_{k=1}^{K} \sum_{l=1}^{k-1} \gamma_{kl} I_{kl}$$

where $I_k = I_{C_k \in T_k}$ is the indicator function whether choice C_k is correct, $\alpha_k > 0$ is the parameter that measures the importance of the choice, $I_{kl} = I_{(C_k, C_l) \in T_{kl}}$ is the indicator function whether the choices C_k and C_l are aligned correctly, and γ_{kl} measures the importance of the interaction. In its simplest form, γ_{kl} is just a fixed exogenous parameter with $\gamma_{kl} \geq 0$. I will, however, consider later a slightly more general form to capture the fact that high performance sometimes requires *simultaneously* a good standalone choice *and* alignment of other decisions. The interaction states T_{kl} capture what is often called 'internal alignment' while the choice states T_k capture 'external alignment' (e.g. Bower et al. (1995)). The choice labels c_k^f are arbitrary and have no particular meaning or order. For example, nothing would change if the $\{c_k^1, \ldots, c_k^f, \ldots\}$ labels on some particular choice were permutated and/or renamed to $\{x_k^1, \ldots, x_k^f, \ldots\}$.

The project – consisting of the set of K decisions and $K!$ potential interactions – is partitioned into K (decision) tasks Z_k, each containing one decision C_k and a number of its interactions. For each such task, there is a project participant P_k who is responsible for that task: P_k makes the choice C_k, with each participant having at most one task. Apart from these K project participants,

[7] Formally, I will assume – when necessary – that $\mathcal{C}_k$ has M elements with $M \to \infty$. The results for an alternative case with $K > 2$ and $M = 2$ are available from the author. Results for $K = 2$ and $M = 2$ can be found in Van den Steen (2012c). If $N = 1$, then T_k consists of one choice that is correct and T_{kl} is a bijection between $\mathcal{C}_k$ and $\mathcal{C}_l$.

there will also be a strategist S whose role is discussed below.

Beliefs All players, including S, know the parameters α_k and γ_{kl},[8] but have initially – at the start of the game – no knowledge of the states T_k or T_{kl}. In particular, each player starts with a prior belief that the T_k and T_{kl} are independent random draws from the sets of all possible states $\mathcal{T}_k$ and $\mathcal{T}_{kl}$ with all states being equally likely.[9] (Section 3 will study the effect of public/initial information by introducing an up-front public signal about one of the choice states.) The empirical probability distribution of the states and interactions is also that each of the possible states is equally likely. The players thus happen to have a common prior belief that moreover happens to be the true empirical distribution. Van den Steen (2013b) – which studies how it matters *who* the strategist is – allows for differing priors, which seems appropriate for settings where strategy matters.

Whereas all players start with uninformative priors, each project participant P_k will get – in the course of the game per the timing below – *local* information about his choice. In particular, each participant P_k gets a signal $\theta_k \in \mathcal{T}_k$ for his own choice state T_k and a signal $\theta_{kl} \in \mathcal{T}_{kl}$ for each of the interactions in his task, with θ_k and θ_{kl} being correct with commonly known respective probabilities $p_k > .5$ and $p_{kl} \geq .5$ (and completely uninformative otherwise). But P_k does not get any (direct) signal about any other choice state T_l ($l \neq k$) or about any other interaction states T_{lm}.[10] (If he makes no relevant inference from the strategist's announcements, then P_k thus keeps his prior beliefs about these T_l and T_{lm}.) Let $\theta = (\theta_k; \theta_{kl})_{k,l \in K, l<k}$ denote the vector of all potential signals.

If the strategist S – per the timing below – decides to investigate a choice state T_k (resp. an interaction state $T_{kl} \in Z_k$), she gets a signal τ_k (resp. τ_{kl}) that is a garbling of θ_k (resp. θ_{kl}) and correct with probability q_k (resp. q_{kl}). In particular, for $T_k, T_{kl} \in Z_k$, τ_k and τ_{kl} equal θ_k and θ_{kl} with probability $1 - \Delta_k$ and are completely random with probability Δ_k. This garbling can capture either the fact that local decision makers have more information or, more importantly, that the underlying state – and thus the optimal choice – can change over time (given that the participant gets his signal after the strategist does). As with the signals θ, P_k (and only P_k) also observes S's signals about his task Z_k.[11]

Timing The timing of the game is indicated in Figure 1. At the start of the game, the strategist decides which states to investigate. If the strategist investigates some choice state T_k (resp. some interaction state T_{kl}), she thus gets the signal τ_k (resp. τ_{kl}) about that state. After receiving the

[8] In fact, all that matters is that participants know *their own* α_k and γ_{kl}.

[9] Formally, $\#C_k = M \to \infty$. Hartigan (1983) showed that improper priors are consistent for conditional (probability) statements.

[10] The results would not be affected, but the analysis more involved, if P_k observed all his interactions T_{kl}.

[11] All that matters for the proofs is whether P_k is aware that the signal may have changed since S observed it. The alternative that the participant is not aware of any changes gives very similar results but with added complexity.

1 Strategy formulation

a. The strategist decides which states T_k and T_{kl} to investigate.
b. When she investigates a state T_k or T_{kl}, the strategist receives a signal τ_k or τ_{kl}. She can then either return to 1a or continue to 1c.
c. The strategist can announce a set of choices C_k.

2 Strategy implementation

a. Each participant P_k receives the signals θ_k and θ_{kl} about (only) the choice state T_k and interaction states T_{kl} of her task.
b. Each participant makes his or her choice (sequentially without observing others' choices or simultaneously).
c. Payoffs are realized.

Figure 1: Timing of basic game

signal, she can decide whether to investigate another state, and so on, or to continue. The cost of investigating I states is $c_I(I)$, with $c_I(.)$ a strictly increasing function. Based on the signals from all investigations, the strategist can then announce, through cheap talk, one or more choices. The announcements are costless but players have lexicographic preferences: when otherwise indifferent, they prefer less announcements.[12]

In stage 2a of the game, each participant P_k gets his or her local information (i.e., the θ_k and θ_{kl} signals). In stage 2b, all participants then make their choices. To capture the setting of a large organization, participants are assumed to make their choices either sequentially over time without observing each others' choices or simultaneously, to capture the fact that the guidance can be over time or at a point in time. (All major results seem to go through with minor differences for a model with sequential choices that are publicly observed, though that fits less for large organizations.) In stage 2c, payoffs are realized.

Each participant P_k tries to maximize the payoff from her task Z_k, $\Pi_k = \alpha_k I_k + \sum_{T_{kl} \in Z_k} \gamma_{kl} I_{kl}$, which is equivalent to assuming that P_k's utility is a strictly increasing function of Π_k and that players are risk neutral. (Van den Steen (2012a) and Van den Steen (2012b) analyze instead a team theory version of the model where all players, including S, share the same objective.) The strategist's objective is to maximize overall project payoff $\tilde{\Pi} = \Pi - c_I(I)$. To break indifference – which considerably simplifies the statements of proofs and propositions without affecting the essential results and which obviously only matters in a set of measure zero – I will assume that upon indifference, any player prefers an α-payoff over a γ-payoff; when still indifferent prefers the payoff with the lowest index or sum of indices. Moreover, when indifferent, players also have a strict (lexicographic) preference relationship over the alternatives in $\mathcal{C}_k$ (for any decision C_k) and over the interactions for any T_{kl}, which players learn when they learn the local payoffs or when they investigate states. I will use $\succ$ and $\prec$ to indicate the preferences.

I will focus in the analysis on pure-strategy equilibria that are locally symmetric: when permutating the (arbitrary) labels on a choice (and on all its interactions), the labels for that choice

[12] All results hold if there was a cost of announcing choices. I state the results for the cost being zero to make clear that the results are not driven (here) by the cost of announcing.

are also permutated in the equilibrium. The local symmetry condition ensures that the equilibrium does not depend on a particular labeling and is robust to arbitrary labels. This property could be endogenized as part of the game but at the cost of considerable additional notation and complexity. I will discuss its effect below. Note that this does *not* affect the equilibrium itself: it is a traditional Bayesian-Nash equilibrium.

To ensure the existence of a pure-strategy equilibrium in this model – that has many decisions and investigations, that has agency incentives but no commitment, and that has a general interaction structure – I need to avoid a matching pennies or rock-papers-scissors equilibrium issue by imposing an extra parametric condition. (Alternatively, using a simpler model or a team theory model, as in (2012b) and Van den Steen (2012c), or allowing the strategist to commit eliminates the need for this assumption (but creates other trade-offs). Hence, this parametric assumption is not what drives the results.)[13] I will, in particular, impose an assumption to exclude (for this version of the model) '*strong* loops': there is no closed loop sequence of choices such that *each* decision *strictly* prefers to align with the next over making its optimal standalone choice. Formally, there does not exist a sequence of distinct choices $(C_k, \dots, C_l)$ such that simultaneously $\gamma_{kl} p_{kl}(1 - \Delta_l) > \alpha_k p_k$ and $\gamma_{mn} p_{mn}(1 - \Delta_m) > \alpha_n p_n$ for each C_m and C_n where C_n follows C_m in the sequence. This does not exclude loops per se, only *strong* loops where the strict inequality holds for *each* step. If such strong loop exists, it may lead to a matching pennies issue where each tries to outguess the other and thus no pure strategy exists. Again, the assumption is not necessary in a simpler model or a team theory model or when the strategist can commit.

I will finally use some recurring notation throughout the paper. Let $\beta_k = \alpha_k p_k$ and $\eta_{kl} = \gamma_{kl} p_{kl}$ combine, for respectively the decision and the interaction states, the importance with the eventual confidence. Let t_k^k denote the piecemeal choice for C_k, i.e., the choice that maximizes I_k according to P_k's beliefs. Define the 'piecemeal outcome' or 'trivial outcome' as the outcome where each player chooses t_k^k. Let $\Gamma_k = \{C_l : T_{kl} \in Z_l\}$ denote the set of all interactions guided by C_k, which I will call the 'inbound' interactions for C_k.

Variations In the analysis, I will sometimes consider variations on the basic model to study specific effects, such as the role of specific investments, of commitment, or of irreversibility. While each variation will be discussed at the time of analysis, it is useful to preview them here already.

- To study the role of specific investments, I will allow in some of the analysis that some (additional) participants can make investments that pay off only if the firm follows a specific

[13] I conjecture that all results also hold in this model without the condition, but mixed equilibria in a game of almost unlimited complexity make things intractable. All essential results have been proved without any conditions and with $M = 2$ both for $K = 2$ and for $I = 1$.

course of action. For example, cost-reduction know-how only fully pays off if the company follows a cost-focused strategy.

- To study the role of commitment, I will allow in some of the analysis that S can fix, at the end of stage 1, some of the choices announced in 1c at a cost of c_C per commitment.
- To study the effect of irreversibility, I will allow in some of the analysis that some participants can reverse their respective choice C_k after observing others' choices.
- To study the effect of uncertainty, I will assume in some of the analysis that there is a public signal about a decision – which thus reduces the ex-ante uncertainty about that decision.
- To study the role of competition, I will consider a setting where some of the choices are not part of the project, but part of a competing firm with, obviously, a different objective.

Moreover, as mentioned before, the results will be derived for a slightly more general payoff structure, to capture the fact that a high payoff from a good choice may materialize only if other choices align on it. To that purpose, let for $T_{kl} \in Z_k$, $\gamma_{kl} = \tilde{\gamma}_{kl} + \check{\gamma}_{kl}\Pi_l$ with both $\tilde{\gamma}_{kl}, \check{\gamma}_{kl} \geq 0$ exogenous parameters. In this case, some part of the payoff from C_l (namely $\check{\gamma}_{kl}\Pi_l$) gets realized only when C_k aligns correctly on C_l. The size of $\check{\gamma}_{kl}$ then captures the degree to which the full realization of Π_l depends on C_k also aligning correctly on it.

Interpretation, Limitations, and Boundaries While the model is in principle very general, the C_k are best interpreted as functional choices (in the context of business strategy): one C_k could then be the choice of production technology, another the choice of product, or the type of advertising, or the marketing channels, etc. Subsection 3.3 will show that the strategic choices will typically be central and high-level choices, because such decisions are most effective at giving guidance. The model then assumes that the strategist knows, for this particular firm, how important the choice of production technology is, how important the choice of product range is, and how important it is to align the production technology with the product range (or the other way around). But the strategist does *not* know – without further investigation – what the optimal production technology is, what the optimal products are, and which technologies fits best with which products. But the strategist can investigate all these things. A participant also knows the importance of these choices (at least for her own task) and learns about the optimal choices and alignments for her own task. When the strategist and the participants learn about the optimal choices and interactions, they do so imperfectly. And the participants are more up to date than the strategist, at least by the time of making the decision.

This model has – due to its simplifying assumptions – some important limitations or 'boundary conditions' (as they are sometimes called). The most important one is that, while the model includes some dynamic elements to study the basics of irreversibility and commitment, it is not a full dynamic model where the strategy can be revised over time. Subsection 3.7 considers such dynamic, but the analysis is – in this first iteration – very limited. Similarly, while Subsection 3.6 illustrates the model's use for competitive strategy and derives some useful insights, the focus of the current model is on strategy as an internal tool for setting direction. The payoff function is also relatively specific, with, for example, separable internal and external consistency. The local symmetry condition implicitly assumes that all choices are ex-ante equivalent in the participants' eyes. When some choices – such as the status quo – are focal or salient, some of the results may be affected. But in all these cases, it seems that the qualitative results would carry over. Furthermore, all these assumptions can be relaxed or generalized, but such analysis goes beyond the current paper.

One thing that is important to note, though, is that strategy – as defined here – is *not* about planning out each and every detail, in at least 4 ways. First, as explained in Subsection 3.8, an optimal strategy for a project with 1000 choices may consist of investigating just 6 or 7 states (or even one single state) and announcing 6 or 7 choices as the strategy, and then letting the participants choose based on their local information. In such case, less than 1% of choices is planned ahead. Second, even for a given strategy, the guided choices are not fixed as the participant still has a choice among N optimal alternatives. Third, given a cost of investigation, not all choices will be guided. Fourth and most importantly, Subsection 3.7 will show that optimal strategies – as defined here – will often be dynamic once learning and experimentation are introduced.

3 The Nature of Strategic Decisions and the Value of Strategy

I now turn to the main focus of this paper, the nature and value of strategy: what makes decisions strategic and what makes strategy important? These questions are of obvious importance: How do you find a strategy if you don't know what to look at? And why would you look for a strategy if you don't know why it matters?

To relate these questions to the model, Section 4 observes – and will show formally – that the announcements in stage 1c coincide with 'a strategy' in the sense of the 'smallest set of (core) choices to optimally guide the other choices.' I will therefore define a decision to be 'strategic' to the degree that the decision was either investigated or announced as part of the strategy, where I will say that a decision was investigated if the information acquired by the strategist affects the (conditionally) optimal choice for that decision.

Definition 1 *The degree to which a decision is 'strategic' equals the probability that it is, in equilibrium, investigated or announced as part of the optimal strategy.*

Let π_k denote the probability that C_k is investigated or announced as part of the equilibrium strategy. The probabilistic nature of the definition reflects the fact that the set of strategic decisions may depend on the state realization.

Before discussing how specific characteristics make decisions strategic or strategy important, let me first discuss the equilibrium outcome of the game of Section 2, as that drives the results. The equilibrium strategy announcement in step 1c creates effectively a hierarchy of decisions, consisting of a set of trees of decisions, with higher level decisions guiding lower level ones (Figure 2). Some of the strategic choices announced in 1c become the apexes (or roots) of the set of hierarchical trees. (C_1 and C_2 in Fig. 2) These root choices are chosen standalone optimal ($C_l = t_l^l$) and thus not guided by any other choices. The other strategic choices announced in 1c align with, and are thus guided by, one of these root choices (C_3 and C_4 in Fig. 2) or with other strategic choices that are already part of a tree. The non-strategic choices, finally, either align with one of the strategic choices (as the end-branch of such a tree, such as C_5) or do not align (and are then simply standalone optimal but without guiding any other decisions, such as C_6). Every strategic choice has at least one other choice aligning with it, and is thus either a root or an intermediate branch of a tree. Obviously, every strategic choice can have many other choices aligning with it. In fact, there can be one strategic choice which all others choices align on. *The strategy thus guides towards a pattern of choices, though that does not mean that the strategy is itself a pattern.* Note, though, that while such a tree may in theory contain a very long path, in practice many 'trees' in a strategy are more like a brush: a set of roots on which a set of non-strategic decisions align. The formal equilibrium is stated and derived in Lemma 2 of Appendix A (as it requires a lot of notation and definitions for the tree structure).

An important observation on this equilibrium (and thus on the model) is that, with probability one, no decision aligns with multiple choices at once. The most important example of (potentially) aligning with multiple decisions at once is when some decision, say C_n, aligns with C_k unless C_l and C_m both guide to the same choice, in which case C_n aligns with C_l and C_m. The reason why this doesn't happen (with probability one) under the assumptions of Section 2 is that each choice has infinite alternatives, hence two choices guiding to the same alternative is a probability zero event. If there were a *finite* number of alternatives then this *does* happen, as in Van den Steen (2012c) which studies this explicitly and shows that the gain from strategy is larger in a supermodular case than in a non-supermodular case. But introducing that possibility in a model with many (K) choices, with potentially many (I) investigations, and with the possibility of reversion ($\Delta_k > 0$), makes the

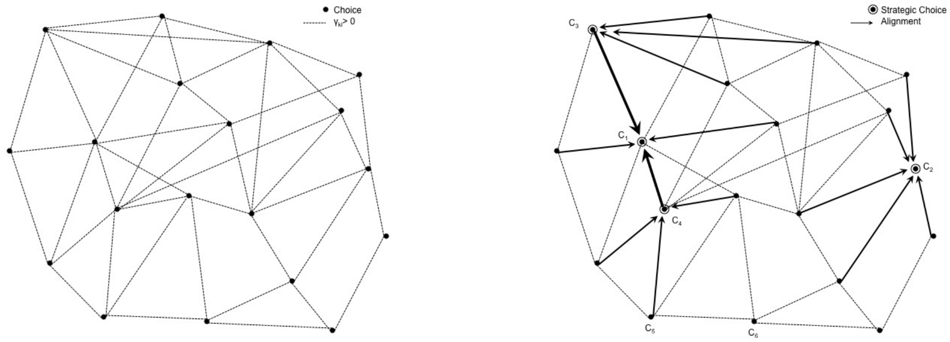

Figure 2: Strategy creates a hierarchy of decision

model intractable and thus beyond the scope of this paper. While it is unclear what insights it would deliver beyond Van den Steen (2012c), it may be an important topic for further research. I now turn to the results themselves.

3.1 Persistence, Commitment, and Irreversibility

A first set of results is about the role of commitment, irreversibility, and persistence. These results are particularly important for a number of reasons. First, irreversibility and commitment are the only generic characteristic that the literature has explicitly identified as making a decision strategic, in particular in Ghemawat's (1991) seminal work, making them obvious characteristics of interest. It also means that comparing this paper's results to Ghemawat (1991) may give insight into both. Second, the analysis has some practical implications for optimal strategies. And, finally, these three characteristics have implications for the trade-off between commitment and flexibility, which is one of the most challenging issues in strategy.

In the analysis here, I will use the following definitions (in the context of 'strategy') to distinguish among irreversibility, persistence, and commitment:

- A decision is *irreversible* to the degree that it is hard to change or cannot be changed.
- A decision is *persistent* if it tends to remain unchanged over time. A decision's persistence can be due not only to irreversibility but also to the fact that there is no reason for changing it, for example, because the environment is very stable so that the optimal decision today is also the optimal decision tomorrow, or because the decision results from in-depth research or because it is founded on strong beliefs of the manager. I will refer to the choices that are persistent despite being reversible as 'stable' decisions. (The degree of stability is partially endogenous.)

- As a *commitment* is a pledge to do something, a choice results in a commitment if it is an 'intentionally irreversible and visible choice', i.e., a choice that is *intended* to be irreversible and observable (to the right party). A commitment can be made in two ways: intentionally making an irreversible choice (e.g., a sunk cost) or intentionally making a choice irreversible (e.g., deciding to burn money rather than putting it into a safe). The latter type of commitments imply that the set of committed decisions is not a subset of the decisions that are inherently irreversible. Commitment requires an ability to make the cost of reversion high; irreversibility is the inability to make the cost of reversion low.

Whereas I thus define commitment in the sense of 'the act of committing' – with its forward-looking, voluntary, and visible nature – 'commitment' can also mean 'the state of being committed' and then has the nature of a constraint and is a source of 'irreversibility'. This distinction is critical because Ghemawat (1991) focused on the second meaning: he defines commitment as 'the difficulty to flip-flop,' argues that one can identify the commitment-intensive choices (in this sense) by looking for a specific set of adjustment costs (lock-in, lock-out, lags, and inertia) that make a choice difficult to reverse, but he does not mention intentionality or visibility (and hence the ability to influence others) as an essential part of commitment.[14] Ghemawat then argued 1) that such irreversibility is the essence of strategy, 2) that it makes strategy valuable, and 3) that irreversible choices 'are the ones that should be treated as strategic.' I will discuss later how the results of this paper partially confirm and partially differ from these results.

An important implication of stability is the fact that the optimal decision does not change between the strategist investigating it and the participant making the decision based on her local information at that time (i.e., low Δ_k). In other words, stability leads to *consistency* between the announced strategic decisions and the eventual choices for these decisions.

The main results of this paper are as follows:

- Persistence, and especially consistency, makes a decision more strategic as it increases (ex-post) the value of aligning with the decision and hence also (ex-ante) the incentives to align with the decision, and thus its effectiveness in guiding other decisions.

- The *possibility* of commitment may make a decision more strategic (when its natural persistence is insufficient). However, commitment will *not* always be used as it comes at a cost of potential ex-post misalignment. In fact, automatic commitment may make a decision *less* strategic. Persistence through stability makes a decision more strategic than persistence through commitment.

[14] Ghemawat's focus was on the reasons why differences in performance persist, which leads naturally to the issue of irreversibility rather than to (intentional) commitment (as defined here).

- Irreversibility does not necessarily make a decision more strategic and may even make it *less* strategic. It does, however, always make strategy more valuable and also makes decisions with which it interacts more strategic.

The following proposition captures the effect of *persistence* measured by the likelihood Δ_k that the optimal choice has changed by the time the participant makes the decision (which results in inconsistency between announcement and choice).

Proposition 1a *A decision C_k is more strategic when it is more persistent, i.e., when Δ_k decreases. The value of strategy increases in the persistence of the decisions.*

While the above result captures the effect of persistence between the strategist's investigation and the participant's decision, a simple variation on the model shows that the result also holds for persistence in the participant's choice *over time*. Consider, in particular, a variation on the model of Section 2 where stages 2a and 2b are repeated twice, but the set of signals of each participant P_k changes with some probability $\check{\Delta}_k$ between the first and second repetition. The payoffs, which are the sum of the payoffs over the two repetitions, are realized at the end of the game. To simplify the analysis, assume that $c_I(2) = \infty$ so that S can investigate only one choice. The following proposition then confirms the result for persistence in the participant's choices.

Proposition 1b *A decision C_k is more strategic when it is more persistent across the two periods, i.e., when $\check{\Delta}_k$ decreases. The value of strategy increases in the persistence of the decisions.*

Persistent decisions – decisions that are unlikely to change – are more strategic for two reasons. First, a change in the decision will undo all the internal consistency that it was meant to generate. Hence, a more persistent decision will generate more internal consistency (ex-post). Second, anticipating this exact issue, other decisions will be less inclined to let themselves be guided by a non-persistent decision, making such decisions less effective as a guide.

These two propositions have important implications for the *content* of strategy. First, they favor strategies that are built around more stable factors, such as strategies that focus on consumer needs that are unlikely to change. Second, in a volatile environment, strategy should be built more around internal factors, such as capabilities and resources, that are under the control of the organization and can thus be kept stable, rather than around products or needs that may change quickly. This is clear in high tech industries where firms often build their strategy around their capabilities and broad market needs, rather than around specific products or solutions. A third implication is that longer-term strategies, strategies in more volatile environments, and strategies for start-ups will be simpler or more concise because there are fewer persistent decisions. This effect will typically also make decisions about which the strategist is confident more strategic, as the strategist will expect less potential reversion (as $\Delta_k < q_k$).

The first proposition is somewhat reminiscent of Van den Steen (2005) where the employees' choice of project (A vs B) is driven by their belief about the manager's future decision, with strategy as one way to communicate the manager's beliefs (Van den Steen 2001). But the focus of that work is on the role of the manager's beliefs rather than on the role of strategy (which is reflected in the differing priors assumption in that previous work, versus the common prior assumption here).

The results seem to suggest that *commitment* may be important to strategy as a commitment makes a choice persistent. The following proposition partially confirms that intuition, but also points out important caveats. To state the results, consider a variation on the main model of Section 2 in which the strategist S can publicly fix, at the end of stage 1, some of the choices announced in 1c at a cost of c_C per commitment. The choices that can be committed are a subset $\mathcal{D}_c$ of all decisions.

Proposition 2 *1) The option to commit makes a decision more strategic.*

2) The strategist may not *commit to a strategic decision even when it is possible. In fact,* automatic *commitment upon a strategy announcement may make a decision* less *strategic.*

3) Persistence through stability makes a decision more strategic than persistence through commitment. Formally: if two decisions, C_k and C_l, are identical (incl. $q_k = q_l$) except that $\Delta_k = 0 < \Delta_l$, but $C_l \in \mathcal{D}_c$, then decision C_k is more strategic than C_l (even when $c_C = 0$).

Let me start with the third result. The two decisions, C_k and C_l, have the same persistence *if* the strategist commits to C_l. Hence, the two decisions differ only in the source of their persistence: commitment (C_l) versus stability (C_k). And the result then says that persistence through stability makes a decision more strategic than persistence through commitment. The reason is that commitment comes at a cost: there may be an ex-post misalignment that could have been avoided absent commitment. Persistence through stability does not have that issue as there will be no reason to change. It is for this same reason – losing the option to resolve ex-post misalignment – that the strategist may not use her ability to commit and that automatic commitment may make a choice *less* strategic: if ex-post optimality is more important than the alignment that would come from commitment, then it is optimal not to commit, even if that means (in the case of automatic commitment) completely leaving the decision out of the strategy. Such automatic commitment may come, for example, from reputation concerns or from reactions by others.

Whereas a full analysis of *irreversibility* goes beyond the scope of this paper, a simple model can already give important insight in the effect of irreversibility and in Ghemawat's arguments on this issue. In particular, I will consider a setting where all decisions are reversible and then investigate the effect of making one decision *ir*reversible. Formally, consider a variation on the basic model with a third stage where all participants observe all choices and signals and then in random but pre-determined order publicly choose their (new) action. The following result then says that a decision does not become more strategic when it becomes irreversible but an increase in irreversibility makes strategy more important.

Proposition 3a *Irreversibility of C_k increases the value of strategy but does not make C_k more strategic.*

In fact, irreversibility can make a decision *less* strategic (in the presence of a public signal).[15] The first part of the proposition confirms Ghemawat's (1991, p29-31) argument that irreversibility makes strategy more important: since you can't align ex post, alignment has to come ex-ante through strategy. The second part, however, and especially the observation that irreversibility can make a decision less strategic, goes against his argument that irreversibility would make a decision more strategic. Ghemawat's intuition was that irreversible decisions constrain other decisions and should therefore be chosen with great care. But the fact that an irreversible decision constrains other decisions does not imply that it *should* guide other decisions. The purpose of strategy is in part to let the *optimal* decisions guide rather than the *default* – i.e., irreversible – ones. This is best illustrated with an example from Ghemawat (1987). Around 1985, Coors needed to decide on the construction of a large brewery on the East Coast, which only made sense if Coors pursued a national (versus regional) strategy. Whereas the construction decision was the irreversible decision, the choice of geographic scope should be the strategic decision, guiding decisions such as the brewery construction. Letting the brewery construction – the irreversible decision – drive the choice of geography would put the cart before the horse. In conclusion, an irreversible decision makes it vital to develop a strategy but is not necessarily part of the strategy – at least not in the current setting – because irreversibility does not directly affect how useful the decision is as a guide for other decisions. We can in fact be more specific: irreversibility makes the decisions that interact with it more strategic because they can can guide the irreversible decision.

Proposition 3b *Irreversibility of C_k makes decisions that interact strongly with C_k more strategic. (Formally: a choice C_m is more strategic when γ_{km} is larger.)*

This fits nicely with the Coors example: the irreversibility of the brewery decision made it important to guide that decision, which then made decisions that are effective guides for the brewery decision –

[15] For an example, consider a setting with 3 choices with $Z_1 = \{C_2, C_3\}$, $Z_2 = \{C_3\}$, and $c_I(I) = -.1 + .2I$. All $p_k = p_{kl} = 1$, while $\alpha_1 = 0$, $\gamma_{13} = 0.85$, $\gamma_{12} = 1 = \alpha_2$, and $\gamma_{23} = 100 = \alpha_3$. There is a signal about C_3 that is correct with probability .9. Decision C_1 is always irreversible. Consider the effect of making C_2 irreversible. With C_2 reversible, C_2 is strategic, but not when it becomes irreversible: in that case C_3 becomes strategic and with the guidance that already provides for C_1, the added value from also making C_2 strategic becomes too small.

in this case the choice of geographic scope – more strategic. Irreversibility makes strategy important; persistence makes it feasible.

In a multi-period model, the result of Proposition 3b will be strengthened because the irreversibility of C_k will make the decision C_m that interacts strongly with C_k more persistent and thus more strategic. Irreversibility will also turn C_k into a constraint that determines future strategies.

3.2 Uncertainty, Clarity, and Strategic Bets

Informally, it seems obvious that uncertainty must play an important role in strategy. But the exact effect of uncertainty on strategy is not so obvious. On the one hand, uncertainty makes it hard to develop a strategy, leading some to conclude that uncertainty makes strategy useless since tomorrow will look different (Martin 2013). On the other hand, uncertainty seems essential to strategy: without uncertainty, everyone knows what to do and where to go and there is no role for strategy. Uncertainty thus seems to make strategy both more and less valuable.

The analysis in this paper shows that the effect depends on the type of uncertainty: prior uncertainty (i.e., uncertainty that exists before investigating states) makes strategy more valuable and makes decisions more strategic while residual uncertainty (i.e., uncertainty that remains after investigating) makes strategy less valuable and makes decisions less strategic. A critical role of strategy is to reduce uncertainty *about what actions others will take*, to make it possible to align.

To formally study the effect of *ex-ante uncertainty*, consider a variation on the basic model in which all players observe a public signal $\tilde{\tau}_k$ about decision C_k. The signal $\tilde{\tau}_k$ will be a garbling of the strategist's (potential) signal τ_k that is correct with probability $\rho_k < q_k$. The signal thus reduces the ex-ante uncertainty about decision C_k. The question is how such reduction in uncertainty affects whether decision C_k is strategic and the value of strategy. The following proposition – which uses $\Delta_{k'}$ for the probability that $\tilde{\tau}_k \neq \theta_k$ – then says that uncertainty makes a decision more strategic and increases the value of strategy:

Proposition 4 *Decision C_k is more strategic and the value of strategy is higher when there is more uncertainty, i.e., when the signal $\tilde{\tau}_k$ is* less *informative ($\Delta_{k'}$ increases). Moreover, uncertainty and interactions η_{kl} are complements with respect to the value of strategy.*

The complementarity result gives some important intuition: the fact that uncertainty makes strategy valuable *only when* combined with a high level of interaction shows that ex-ante uncertainty matters *not* because it makes it hard to find the correct decision but because it makes it hard to predict what others will do and thus to align with them. The effect of strategy is indeed to reduce uncertainty about what others will do.

This result implies that high-level generic choices, such as 'be the preferred service provider' or 'maximize shareholder value,' are typically not strategic: as there is not much uncertainty about such choices, making them explicit as part of the strategy does not provide additional guidance to people's decision making. In the other direction, this also explains why a choice *not* to do something that seems at first attractive – such as a choice *not* to be the preferred vendor in a particular segment or to *not* have high quality – is often strategic, as such choices go against general expectations and are thus very informative.

The fact that *residual uncertainty* makes strategy less valuable and makes the decision less strategic was already captured in the results on persistence. But there is a further result, though that is easiest to see from the opposite direction: decisions with clear implications for other decisions are more strategic. As clarity of implications is captured by p_{kl}, the formal result is:

Proposition 5 *A decision C_k is more strategic, and the value of strategy increases, when the confidence p_{kl} in its inbound interactions (with $T_{kl} \in \Gamma_k$) increases.*

Prior uncertainty makes strategy more important; residual uncertainty makes it less effective.

If instead of an objective up-front signal, people held differing priors, then a similar result could be derived that choices that are (ex-ante) more ambiguous are more strategic. The intuition is that such ambiguity makes it hard to predict what others will do and thus hard to align.

Strategy Bets When organizations face large uncertainty, it is sometimes said that it is more important for them to choose *some* direction than to delay in order to find the *optimal* direction. This is related to the observation that managers of high-tech start-ups often talk in terms of 'bets' rather than strategy, reflecting a sense that they are forced to make important and far-reaching choices without having much information to base these choices on. Does it make sense to make such 'bets'? In other words, what is the gain from *some* strategy, even when it may be uninformed and thus potentially suboptimal.

To analyze this formally, consider a variation on the basic model where the strategist cannot investigate any states at all but can still announce (and can now commit to) a strategy, i.e., can announce and fix decisions in stage 1c. What is the value from such a strategy 'bet' and what would such strategy look like? The following proposition shows that strategy can add value without information about the optimal decisions and even without knowing the interactions among them. Moreover, the best decisions to build a strategy bet on are decisions with limited standalone importance (α_k) or eventual confidence (p_k) but strong and clear interactions.

Proposition 6 *There is value from a* strategy bet *that contains C_k when the strength of inbound interactions γ_{kl} (with $T_{kl} \in \Gamma_k$) and confidence in the interactions $(1 - p_{kl})$ are large and when the* *importance of, and confidence in, standalone optimality are small. The strength of interaction* γ_{kl} *and confidence in the interaction* $(1 - p_{kl})$ *are complements with respect to the value of a* strategy bet.

A first obvious question is how strategy can add value without the strategist even knowing how the decisions interact. The reason why strategy 'works' here is because people *want* to align their decisions with others when η_{kl} is large, but they can only do so if they know what others will do. When η_{kl} is sufficiently large relative to β_k, it becomes optimal to blindly commit one or more decisions, in order to allow others to align with these. But since the strategy is uninformed, the internal alignment comes at the cost of a loss of external alignment: under the optimal strategy bet, the external alignment is no better than random. The optimal strategy bet is therefore most valuable at high η_{kl} but at low β_k.

This benchmark clarifies the role of strategy from a different angle: Without *any* strategy, the organization does relatively well on external alignment, but no better than random on internal alignment. With the optimal *strategy bet*, things switch to the other extreme: the organization does well on internal alignment, but no better than random on external alignment. The optimal *informed* strategy, finally, optimally trades off internal and external alignment.

An important challenge for 'strategy as a bet' is implementation: employees may doubt that managers will follow-through on the announced strategy. In fact, the optimal strategy bet is not an equilibrium if strategy is just cheap talk. A strategy bet thus requires a commitment device, which can be managerial reputation, a strategist-leader with strong views (Van den Steen 2013b), or a committed decision.

3.3 Interactions, Level, Centrality, and Overlap

Another important determinant of whether a decision is strategic is the degree to which it has many and strong interactions, especially when the decision's interactions don't overlap with those of other strategic decisions. The importance of this result derives from its implications:

- The fact that many and strong interactions make a decision more strategic implies, as illustrated in Figure 3, that both more *central* choices and *higher-level* choices are more strategic because they have more interactions. This suggests that when there is – for technological or organizational reasons – some exogenous hierarchy in decisions, then the strategic structure of the decisions will tend to mirror that exogenous hierarchical structure. In this sense, more important decisions will thus be more strategic.

- The role of overlap implies that effective strategies will often have one choice per function, such as a marketing choice, a production choice, etc.

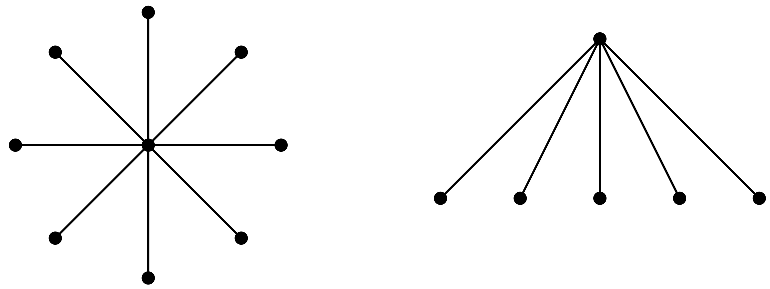

Figure 3: Interactions from centrality and from hierarchy

This result – that many strong interactions make a choice more strategic – is intuitive (and not so surprising, once one works from the definition of this paper): decisions with many interactions can guide many decisions at once and, hence, should be more strategic. Overlap, on the other hand, leads to both duplication and conflict in guidance, making decisions less strategic.

The result also fits Merriam-Webster Online's definition of 'strategic' as being 'of importance within an integrated whole', which indeed requires interaction as a necessary condition for being strategic. Whereas Merriam-Webster takes it as part of the definition, I derive it here endogenously as an implication of this paper's definition.

Given the important implications, it is useful to start with a fairly black-and-white formal result.

Proposition 7 *A decision with no interactions is never strategic. Absent interactions, a company should have no strategy. (Formally: For any $k \in K$, if $\gamma_{kl} = 0, \forall C_l \in \Gamma_k$, then $\pi_k = 0$. If $\gamma_{kl} = 0, \forall k, l$, then the optimal strategy is empty and strategy has no value.)*

A university department, for example, does not gain from a department-wide research strategy unless there are important interactions among the faculty members' research agendas, for example for hiring, joint projects, or external reputation. In the extreme, a firm with no interactions among its decisions (and with no specific investments and no strategic interactions with competitors) has no use for strategy, and may hurt its performance by trying to follow one.

While this extreme result puts the mechanism in perspective, a more general result is useful for practical purposes. The following proposition says that decisions are more strategic when they interact more – and more strongly – with other decisions, in particular for inbound interactions, and that strategy is more valuable when there are more and stronger interactions:

Proposition 8 *A decision C_k is more strategic when the number and strength of its (inbound) interactions γ_{kl} increase. The value of strategy increases in the number and strength of interactions γ_{kl}, $\forall k, l \in K$. (Formally: The probability π_k and the value of strategy both increase in γ_{kl}, and in* *particular when going from $\gamma_{kl} = 0$ to $\gamma_{kl} > 0$ ('one more interaction') for $T_{kl} \in \Gamma_k$. The value of strategy increases in γ_{kl}, $\forall k, l \in K$.)*

One important example of interactions through *centrality* is the choice of scope, i.e., the choice of which customers to serve with which products. Andrews (1971), Bower et al. (1995), Saloner et al. (2001), and Collis and Rukstad (2008) all identified, as an experience-based rule-of-thumb, the choice of scope as an essential part of a business strategy. A small change in scope can reverberate throughout the whole business system. An example of *higher-level* decisions being more strategic is that a decision to be 'low-cost' is more strategic than a decision to 'use CFL lamps in the store' because 'being low cost' has more implications for other decisions.

The formal result that a decision is more strategic when its interactions overlap less with those of other strategic decisions requires some of the formalism of Lemma 2. Let $B = (\tilde{C}_l)_{l \in B^i}$ and $B' = B \cup \{\tilde{C}_k\}$ be two candidate strategies in the sense of two IC sets of disjoint trees. The following proposition says that the added value of including C_k into the (candidate) strategy decreases with the overlap in interactions that C_k has with the other (candidate) strategic decisions.

Proposition 9 *The added value from investigating and announcing B' instead of B decreases in γ_{lm} for $l \in B^i$ and $m \in \Gamma_k \cap \Gamma_l$.*

3.4 Standalone Importance

'Being strategic' is often associated (or even equated) with 'being important'. But in which direction does this relationship run: is a strategic decision more important or is an important decision more strategic? The effect runs in fact both ways. In one way, strategic decisions are important because they influence or guide a lot of other decisions. In the other direction, last subsection showed that important decisions in the sense of hierarchically higher-level decisions, are more strategic.

But there is another sense in which important decisions are more strategic: standalone importance (α_k) and confidence (p_k) make a decision more likely to be a root choice of strategy.

Proposition 10 *A decision C_k is more likely to be a root choice of strategy when its standalone importance α_k and eventual confidence p_k increase. (Formally, the set of parameters (conditional on α_k and p_k) for which $C_k \in B^r$ increases in α_k and p_k.)*

There is also a *negative* result that gives important intuition:

Proposition 11 *No matter how standalone important, a decision is not strategic unless it has sufficient interactions.*

This shows that standalone important decisions are more strategic, *not* because they have more impact on the project payoff, but because they will be made on their own terms, i.e., without regard to what is optimal for other decisions. The other decisions thus have to adapt to them and be guided by them if there is sufficient interaction. A practical example of an important decision that is typically not strategic is an airline's decision to hedge currency or fuel contracts: whereas such decisions have a tremendous impact on the bottom line, they usually do not guide other decisions – such as which customers to target – but are themselves guided by the cash flow needs implied by other decisions, and are therefore not strategic. Similarly, a technological choice buried deeply in a product design may critically affect a company's success or failure, but that does not – by itself – make that decision strategic.

3.5 Specific Investments and Capabilities

A clear strategy not only facilitates alignment (or coordination) but also encourages investment in resources, skills, and capabilities, in particular when these resources and capabilities are specific to the firm's course of action. The intuition is simple: with a clear strategy, employees know better which investments will pay off. For example, IKEA's know how in low-cost flat-pack furniture design and Walmart's know how in super-efficient logistics both seem to result, at least in part, from a persistent and clear strategy of cost minimization along specific dimensions. This intuition suggests that a choice will be more strategic, and strategy more important, when action-specific investments (in skills and capabilities) depend on it.

To derive this formally, consider a variation on the basic model in which – apart from the K participants P_k making decisions C_k– there are also L participants $\tilde{P}_l$ that can each develop a capability K_l, such as know-how or a skill at performing a task. Each such capability K_l is related to a specific decision C_k. Moreover, each capability is choice-specific: when building a skill or capability K_l, $\tilde{P}_l$ has to specialize this skill towards some specific choice $c_k^f \in \mathcal{C}_k$ for that decision C_k. Capability K_l pays off only if $C_k = c_k^f$. Design skills, for example, only pay off if the firm pursues design-sensitive segments while cost-reduction skills only fully pay off if the firm pursues mass markets. (Almost all capabilities are to some degree choice-specific in the sense that the returns from the capability depend on the firm's choices.) To develop the capability, participant $\tilde{P}_l$ thus picks a specific choice alternative $\tilde{c}_k \in \mathcal{C}_k$ and then invests effort $e_l \in \mathbb{R}^+$, at a cost to the firm of $e_l^2/2$, which then generates an additional payoff $\lambda_l e_l$ if and only if P_k chooses $\tilde{c}_k$. I will assume that this investment is a separate task for which $\tilde{P}_l$ is responsible, so that $\tilde{P}_l$ maximizes $\lambda_l e_l I_{C_k=\tilde{c}_k} - e_l^2/2$ while the objective of P_k remains unchanged. The $\tilde{P}_l$ develop their know-how or capabilities in period 2a, simultaneous with the choice of decisions by the P_k. Let $\mathcal{K}_k$ denote the set of all capabilities that are specific to the choice of C_k. The following proposition then says that choice-specific capabilities make a decision more strategic. Moreover, choice-specific capabilities and persistence are complements.

Proposition 12 *A decision C_k is more strategic and the value of strategy increases when more, and more important, choice-specific capabilities depend on C_k. Moreover, choice-specific capabilities and persistence are complements in making a choice strategic. [Formally, π_k increases when for some $K_l \in \mathcal{K}_k$, λ_l increases. Moreover, λ_l and $1 - \Delta_k$ are complements w.r.t. π_k.]*

Another nice example of such choice-specific capabilities is Akamai (Van den Steen 2013a). Due to the fact that Akamai's technology is very different from its competitors', Akamai's employees have to make large investments in specific skills. As implied by the model, Akamai made its technology choice the core of its strategy and then committed to that strategy through an almost 'religious belief' by its leadership in the technology.

This result builds on Zemsky (1994) who showed that commitment to a strategy (interpreted as a choice of project) can create incentives for investments in choice-specific skills. This paper differs and contributes in two ways. First, the result here does not require commitment: a cheap-talk strategy is sufficient. Second, as Zemsky (1994) had only one choice and hence no way to investigate which choices are more or less strategic, that paper was about the effect of committing, while this result focuses on the structure of strategy.

3.6 Competition

Up to this point, the analysis focused on one firm where the strategist tried to maximize the firm's payoff by announcing a strategy to guide internal decisions. But the paper's model and logic can also be used to analyze competitive strategy, where the firm tries to influence competitors, complementers, and other external organizations. While the 'guiding' will then sometimes be more like 'influencing' or 'forcing,' the same logic applies. For example, a firm may expand capacity to force others to delay expansion. In that case, it tries to influence or guide others, and this paper's definition of strategy as the 'smallest set of (core) choices to optimally guide the other choices' works. The purpose of this section is to derive some high-level results and, in the process, show more concretely how the model can work in a competitive strategy setting.

To study competitive strategy in a very simple context, consider a setting with a focal firm F with K choices, as before, and a second firm G that has one choice, C_g, with the two firms' choices interacting and each firm having its own objective. The second firm can be any firm that interacts with the focal firm, such as a competitor, a complementer, or a supplier. To fix ideas, I will interpret, and refer to, firm G as the 'competitor'. Formally, let there be one ('as if') project with $K+1$ choices C_k that are partitioned into $K+1$ tasks Z_k of which one task – consisting of C_g and the interactions in Z_g– forms the competitor and the K other tasks form the focal firm. Interactions are simple, i.e., $\gamma_{kl} = \tilde{\gamma}_{kl}$. The payoff of the competitor is exactly like before: $\Pi_g = \alpha_g I_g + \sum_{T_{kg} \in Z_g} \gamma_{kg} I_{kg}$ where $I_k = I_{C_k = T_k}$ and $I_{kl} = I_{(C_k, C_l) \in T_{kl}}$. The focal firm F's payoff, however, is now also *directly* affected by G's choices: F's profits have an additional firm-wide payoff term $\breve{\alpha}_g I_g + \sum_{T_{kg} \in Z_g} \breve{\gamma}_k I_{kg}$, where the competitive effects ($\breve{\gamma}_k$ and $\breve{\alpha}_g$), which capture the effects of G's choices on F's profits, can be either positive *or negative*. The participants P_k still care (only) about the payoff from their task Z_k and thus, in this simple case, don't consider these competitive effects. Each firm's strategist can (as a starting point) only investigate the states of its own choices. The firms make their strategy announcements sequentially in random order.

I want to consider two important questions within the context of this very simple model: what makes a decision strategic in such a competitive setting and what makes a competitor a 'strategic rival' in the sense that it has to be endogenized as part of the strategic analysis, i.e., in the sense that firm F has to consider G's reaction when designing its strategy.

With respect to decisions being strategic, the following proposition says that decisions can be both more and less strategic when they can influence the competitor, depending on whether they influence in the right direction or not.

Proposition 13 *A decision C_k is more (resp. less) strategic when it influences a competitor's or complementer's choices in a sufficiently favorable (resp. unfavorable) direction to the focal firm. [Formally: π_k increases either if γ_{kg} increases at sufficiently high $\breve{\gamma}_k$ or if $\breve{\gamma}_k$ increases.]*

It follows that, in a one-shot static situation, capacities (strategic substitutes) are more strategic than prices (strategic complements). The result that competitive interactions can make a decision *less* strategic, however, depends on the competitor observing the firm's strategy. An interesting question for further research is what happens if the firm can make a cheap talk strategy announcement that is different for internal and external parties.

For the question on 'strategic rivals', I will formally define a firm G to be 'a strategic rival of a focal firm F' if F's optimal strategy depends on whether G has already made all its choices. Strategic rivals require extra attention when developing strategy since the focal firm will have to consider how strategic rivals will react to the firm's own choices. In other words, strategic rivals need to be endogenized when analyzing a competitive situation or developing a strategy, while other competitors can be taken as exogenous. For example, something like a demand curve for a set of firms is only well defined if and only if all strategic rivals are excluded when determining the demand curve. Similarly, strategic rivals have to be modeled explicitly when determining 'added value' in the sense of Brandenburger and Stuart (1996). Hence, it would be useful to have a simple criterium to identify strategic rivals from the primitives of the setting. Consider, for example, the Nissan Leaf and Chevrolet Volt. Nissan's CEO had explicitly stated that he did not consider the Volt to be competition for the Leaf because of their different technological characteristics. The Volt, on the other hand, did react immediately to the Leaf's price changes in mid-2013, which suggests that they *did* consider the Leaf to be a competitor. What does this imply for the Volt and Leaf being potentially strategic rivals or not (from each firm's perspective)?

The following proposition establishes that a necessary and sufficient condition for being a strategic rival is the following two directional effect: the focal firm's choices must influence the competitor's *actions* – by affecting its payoff – and these competitor's actions must affect the focal firm's *payoffs*.

Proposition 14 *Firm G is a strategic rival to F if and only if* both $\check{\gamma}_k$ *(or* $\check{\alpha}_g$*) and* γ_{kg} *are sufficiently different from 0.*

The answer is thus that, if we take Nissan at its word, neither the Volt nor the Leaf are a strategic rival to each other. From Nissan's perspective, the Volt is not a strategic rival because, according to Nissan's beliefs, the Volt's actions do not affect Nissan's profits. From the Volt's perspective, the Leaf is not a strategic rival because it does not react to Volt's actions. Given their beliefs, neither firm needs to consider the other when developing its strategy. This may seem surprising: as the Volt reacted to the Leaf's price change, shouldn't the Leaf consider that when developing its strategy? The answer is negative *in as far as* Nissan maintains its original beliefs: as it does not consider the Volt to be a competitor, it does not matter to Nissan that the Volt's price changed, so Nissan should not consider the potential Volt change when making its own choices.

3.7 Dynamics

The dynamics of strategy is one of its most challenging but also one of its most important aspects, as it drives the trade-offs between flexibility and persistence and between exploration and exploitation. Even though the basic model already considered some dynamics via the change in signals (Δ_k), it is for the most part static. A complete dynamic analysis would require learning and the possibility to change the strategy over time. While a full analysis is beyond the scope of this paper, this section presents a very simple example of a dynamic analysis in order to illustrate some basic points about the dynamics of strategy and to show how the model lends itself quite naturally to such analysis.

For the example, consider the model of section 2 with $K = 3$ choices and with $\gamma_{12} = 0$, $\gamma_{13} = \gamma_{23} = \alpha_k = 1$, $\Gamma_1 = \Gamma_2 = \{C_3\}$, all $\Delta_k = 0$, and all $p_k = p_{kl} = .8$. A concrete setting would be that C_1 is a marketing choice, C_2 is a production choice, and C_3 is a product design choice.

Product design can either be marketing-focused or production-focused. The timing repeats the basic model twice but with some interim feedback at the end of the first repetition, where everyone learns whether the payoff of $\alpha_1 + \gamma_{13} I_{1,3} = 2$ or not, and all payoffs come at the end of period 4. In other words, everyone learns after the first repetition whether the marketing choice is a success but *only if* the company went 'all the way' on marketing.

The optimal dynamic strategy in this example is as follows: 1) In period 1, the strategist investigates and announces C_1 as the strategy. In period 2, C_3 aligns with C_1. 2) If the marketing approach turns out to be a success, i.e., $\alpha_1 + \gamma_{13} I_{1,3} = 2$, then there is no more investigation or announcement and all players keep doing what they did before. 3) If, on the other hand, the marketing approach fails, then the strategist investigates and announces C_2 in period 3 and C_3 then aligns with C_2 in period 4. However, if there was a need for an important strategy-specific investment in resources or capabilities or an important irreversible decision, then the optimal strategy may sometimes be to investigate and announce C_2 and stick to that for the rest of the game.

This illustrates a few simple but important points. First, the optimal strategy is now dynamic: it changes over time depending on the events. Second, the strategy explicitly considers learning and such learning may make a choice both more strategic but also less strategic. Third, whether learning makes a choice more or less strategic depends on the importance of external alignment versus the need to make sunk investments, either as irreversible decisions or as action-specific investments in resources and capabilities. I conjecture that in a more complex setting, there would be persistence along some strategic dimensions and adaptation/flexibility along other dimensions, with learning and sunk investments being key drivers.

3.8 Strategy Process

The focus of this paper has been on the role of strategy as an *organizational tool* to give clear direction to an organization in order to improve alignment and specific investments. But strategy is also a useful *decision-making tool* to determine that direction. In particular, the paper (implicitly) shows how understanding the structure of strategy may enable a strategist to develop the optimal strategy in a very parsimonious way. First, the strategist needs to investigate and announce only the strategic decisions. Second, in many settings, the optimal number of strategic decisions is small so that the strategist needs to investigate and announce few decisions.

It is useful to make the latter point somewhat more explicit with an example. For the example, consider settings with $K = 100$ and $K = 1000$ choices with, for simplicity, all $p_k = p_{kl} = 1$, all $\alpha_k = 1$, and all $\gamma_{kl} = 5$. In a team-theory version of the model with no tasks, the optimal strategy will consist of exactly *one* choice for both $K = 100$ and $K = 1000$. Moreover, that optimal strategy would give perfect guidance and achieve the first best. In the agency version of the model, the number of choices to investigate is random and depends on the cost of investigation and announcement. However, most often a strategy of 4 or 5 choices for $K = 100$ and 6 or 7 for $K = 1000$ can again achieve first-best if investigation were free. The strategies will be even more concise once a cost of investigation or announcement is considered but then, obviously, won't achieve the first-best any more. This example makes a few important points. First, a strategy can be very sparse relative to the set of decisions it is trying to guide. Second, this is particularly true in team-theory settings where employees care about the overall outcome. Third, and maybe somewhat surprisingly at first, a denser network of interactions seems to make strategy *more* effective, though more research is needed to confirm this.

There are two mechanisms outside the model that could further strengthen this parsimonious nature of strategy. First, Simon (1962) pointed out that systems in nature tend to have a hierarchical structure. Such hierarchical structure will tend to make strategy more effective. Second, employees typically know about more interactions than only their own. Such additional knowledge would also increase the effectiveness of strategy as a decision can be guided through a chain of links. Especially when combining the latter with the hierarchical structure, it is clear that strategy can be a very effective tool to guide organizations in real-world settings.

4 Strategy: Definition and Equilibrium Outcome

As this paper's definition of strategy is central to the analysis, it is useful to motivate and clarify it in some more detail. The purpose of this section is therefore fourfold: 1) clarify important aspects of the definition, 2) concisely relate this paper's definition to the existing strategy literature, 3) formalize the definition in the context of the game of Section 2, and 4) show that such a strategy is indeed the equilibrium outcome of the game. (The main body of the paper – outside the proofs – does not need the level of detail and formalism that is developed in the latter 2 parts. The informal definition of strategy and the statement of Observation 1 suffice.)

Let me thus start with some important observations and clarifications. First, the high-level choices in strategy – such as 'being low-cost' – often function as objectives for lower levels of the organization (Simon 1947), which helps to relate this definition to the literature. A second and closely related observation is that the players in this model can also be interpreted as parts of the firm, such as 'production' or 'marketing'. Each function, such as marketing or production, may further translate the overall strategy to a functional strategy. A third and final observation is that this definition implicitly assumes both a set of target outcomes towards which the strategy guides and an organizational context within which the strategy operates. With regard to the first, while a company can thus have a clear strategy that guides it towards a disastrous outcome, an 'optimal strategy' guides towards the constrained optimal outcome. With respect to the latter, a very important part of the organizational context is the 'audience', i.e., the people towards who the strategy is targeted, and what that audience knows, because that may determine what a strategy needs to specify. This was partially formalized in Subsection 3.2 on uncertainty. Other important aspects of the organizational context include the identity of the strategist and the ability of the strategist to collect information. Some organizational choices will then precede strategy, some will be strategic, and some will be guided by the strategy. Strategy itself is essentially an organizational tool.

The introduction already related the definition to the literature. To do this in somewhat more detail, it is useful to relate the different elements of the definition to the existing management literature. The fact that the strategy is expressed in terms of a 'set of choices' is consistent with much of the management literature. Andrews (1971), for example, defines strategy as a 'pattern of *decisions* [...]'; Porter (1996) describes it as '*choosing* [...] activities.'[16] The idea that the choices and decisions (that make up the strategy) 'guide' (towards an objective) is obviously implicit in the idea of 'strategy as plan' and is explicit in Minztberg's (1987) reference to 'guidelines.' An important difference with existing definitions is that this perspective on strategy sees it as a means to guide towards a pattern of choices, though the strategy itself is not necessarily a pattern (though it can be a pattern if one of the trees has more than one node).

It is also instructive to relate this paper's definition to the practice-oriented list-based definitions of strategy by Andrews (1971), Bower et. al. (1995), Saloner et al. (2001) or Collis and Rukstad (2008). Collis and Rukstad (2008), for example, describe strategy – based on their experience – as specifying a choice of objective, a choice of scope, and a choice of advantage. This list of choices or decisions can be interpreted as an average experience-based 'smallest set of choices to optimally guide the other choices' for the most common situations. These thus give very concrete form to this paper's definition. In the other direction, this paper provides a rationale and criterium for such a list and provides a logic for adjusting it to specific settings.

I now turn to the formalization of the definition in the context of the model of Section 2. (What follows is more abstract and detailed than the rest of the paper.) The definition of strategy as the 'smallest set of choices to optimally guide the other choices' can be reformulated as follows: a

[16] Simon (1947) refers explicitly to game theory when defining strategy as 'a series of such *decisions* which determine behavior'. Drucker (1973) defines strategic planning in part as 'the continuous process of making (...) *decisions* (...).' Barney (2011), despite defining strategy formally as a 'theory', informally describes it as a '*actions* [that] firms take' (p10). Note that Barney's (2011) 'choice of theory' can be interpreted in the context of Section 2 as the strategist announcing her beliefs about state variables, thus explaining the *logic* of the strategy. Doing so often makes sense, though giving *only* the logic – without the actual choices and decisions – might not be sufficient to get alignment.

strategy – given the set of target outcomes and given what the participants know and observe – is the smallest set of choices C_k to announce so that the equilibrium of the subgame starting in stage 2a implements one of the target outcomes. There is, however, an issue if we want to define 'strategy' for a potentially suboptimal outcome: the participants responsible for the strategic decisions may prefer not to implement them if the target outcome is suboptimal, even if the decisions would in fact guide to the target outcome. To sidestep this implementation issue, I will – for purposes of the definition of a general strategy (only) – condition on each decision C_k in the strategy being implemented when $\theta_k = \tau_k$, i.e., on all decisions announced in 1c automatically being fixed in 2b when the participant gets the strategist's signal.

To now completely formalize this definition, I need to introduce some notations and terminology. Let a target outcome, which may depend on the vector of signals θ, be denoted $\breve{C}(\theta) = (\breve{C}_1(\theta), \ldots, \breve{C}_K(\theta))$. Let the set of target outcomes be denoted $\breve{\mathbf{C}}(\theta)$. Let a 'pattern of investigation' be a complete contingent plan for the strategist with regard to which states to investigate in stage 1. (A 'pattern of investigation' is thus a game-theoretic-strategy for the strategist for stages 1a and 1b.) Let an 'investigation outcome' $\tilde{\tau}$ be the set of realized signals that the strategist has observed by the start of stage 1c. Let $\theta_{\tilde{\tau}}, \theta_{-\tilde{\tau}} \subset \theta$ be the subvectors of signals that the strategist has respectively investigated and not investigated by the start of stage 1c and $\tilde{\theta}_{-\tilde{\tau}}$ a particular realization of $\theta_{-\tilde{\tau}}$. Note that – because the strategist's choice of signals to investigate may depend on the realization of earlier investigated signals – $\theta_{-\tilde{\tau}}$ may depend on that particular realization of signals $\tilde{\tau}$ and not just on the investigation pattern. Denote the set of all possible realizations of $\theta_{-\tilde{\tau}}$, i.e., the set of all $\tilde{\theta}_{-\tilde{\tau}}$, as $\Theta_{-\tilde{\tau}}$. Let, finally, $K_{\mathcal{S}} \subset K$ denote the indices of the subset of decisions that are part of the strategy $\mathcal{S}$.

Definition 2 *A* strategy $\mathcal{S}$ *(for a set of target outcomes $\breve{\mathbf{C}}(\theta)$, for a commonly known pattern of investigation, and for an investigation outcome $\tilde{\tau}$) is a set of choices $(\tilde{c}_k)_{k \in K_{\mathcal{S}}}$ for a subset of decisions $K_{\mathcal{S}} \subset K$ such that for some particular target outcome $\breve{C}(\theta) \in \breve{\mathbf{C}}(\theta)$*

1. *$\tilde{c}_k = \breve{C}_k(\tilde{\tau}, \theta_{-\tilde{\tau}})$ for all $k \in K_{\mathcal{S}}$ and for all $\theta_{-\tilde{\tau}} \in \Theta_{-\tilde{\tau}}$,*

2. *for any $\tilde{\theta}_{-\tilde{\tau}} \in \Theta_{-\tilde{\tau}}$, the outcome $\breve{C}(\tilde{\tau}, \tilde{\theta}_{-\tilde{\tau}})$ is an equilibrium outcome of the subgame starting in stage 2a for $\theta_{\tilde{\tau}} = \tilde{\tau}$ and $\theta_{-\tilde{\tau}} = \tilde{\theta}_{-\tilde{\tau}}$ when $\tilde{c}_k$ was announced in stage 1c and fixed in stage 2b for all $k \in K_{\mathcal{S}}$ and when the players update their beliefs given the pattern of investigation and the announcement in 1c,*

3. *there does not exist a set of decision choices $\tilde{\tilde{c}}_k$ for a subset of decisions $K_{\tilde{\tilde{\mathcal{S}}}} \subset K$ such that the two previous conditions are satisfied and $\#K_{\tilde{\tilde{\mathcal{S}}}} < \#K_{\mathcal{S}}$.*

An optimal strategy formulation *is a pattern of investigation and a set of strategies, one for each possible investigation outcome, that maximizes the overall payoff* ($\Pi - c_I(I)$) *and where the announced choices are also part of the subgame equilibrium.*

An optimal strategy *for $\tilde{\tau}$ is a strategy for $\tilde{\tau}$ that is part of an optimal strategy formulation.*

A strategy does not necessarily exist for every pattern of investigation and $\breve{\mathbf{C}}(\theta)$, however. For example, if the pattern of investigation is empty and the desired outcome $\breve{\mathbf{C}}$ is neither the trivial outcome nor a constant, then no strategy exists. But when the pattern of investigation investigates all signals, then a strategy exists for any $\tilde{\tau}$ (that then includes a realization for each signal) and $\breve{C}$: one candidate strategy that satisfies the first two conditions of the definition is $C_k = \tilde{c}_k = \breve{C}(\tilde{\tau}), \forall k$, so that condition 3 then minimizes over a finite non-empty set and a strategy always exists. This further ensures that the overall problem of finding an optimal strategy is well behaved (as there is only a finite number of possible investigation patterns).

The following observation (with proof in Appendix) then captures the fact that the strategist will in equilibrium announce exactly an optimal strategy.

Observation 1 *In equilibrium, the announcement in stage 1c is an optimal strategy.*

While this result follows directly from the setup, it is important because it explicitly and formally connects this paper's definition of strategy with the process of 'looking ahead to formulate an overall plan before making any particular decision'. This provides a clear rationale for the use of strategy in practice and a reference point to think about the concept.

5 Conclusion

This paper developed a formal theory of strategy – starting from a very simple but concrete formalization of strategy as '*the smallest set of (core) choices to optimally guide the other choices*' – and studied which decisions are strategic and what makes strategy important, considering factors such as persistence, commitment, centrality, level, uncertainty, standalone importance, and more. In the process, it also showed that strategy – as defined here – can be a very effective organizational and decision tool, giving effective guidance with a limited number of investigations and announcements. Strategy, so defined, guides towards a pattern of choices but is itself not necessarily a pattern of choices as it can, for example, consist of a single choice.

An important insight of the paper is how precisely the many things that we intuitively associate with strategy fit together: strategy as committing to one path, strategy as being decided by the CEO or general manager, strategy as coordination device, strategy as looking ahead, strategy as broad direction, etc. Such conceptual understanding of how these ideas hang together is helpful for thinking about and developing good strategies.

While the paper gives many insights, it also raises many questions. The study of dynamics and competitive strategy, for example, derived some high-level insights and showed how this theory can be used in these settings, but fell clearly short of a complete and in-depth analysis. Especially the question of dynamics is an important one, as balancing flexibility and persistence and, closely related, dealing with high uncertainty and volatility are some of the most important challenges in practice. Beyond these, relaxing or modifying some of the assumptions could generate important insights. Examples are considering different payoff structures, endogenizing the precision of the signals, and allowing participants to learn more than just their immediate interactions.

This paper hopefully contributes to a broader study of the structure of strategy.

A Proofs of Propositions

A.1 Notation and Lemmas

The ordering $\succ$ over payoffs is defined as follows (with f and g strictly increasing functions): If $X > Y$, then $X \succ Y$; If $f(\alpha_m) = g(\gamma_{kl})$ then $f(\alpha_m) \succ g(\gamma_{kl})$; If $f(\alpha_m) = g(\alpha_k)$ then $f(\alpha_m) \succ g(\alpha_k)$ iff $m < k$; If $f(\gamma_{kl}) = g(\gamma_{mn})$ then $f(\gamma_{kl}) \succ g(\gamma_{mn})$ iff $k + l < m + n$ or, when $k + l = m + n$, $\min(k,l) < \min(m,n)$; $\succ$ follows the lexicographic ordering among choice or interactions alternatives. Let $\text{ch}(X)$ denote the choice according to $\succ$ from a set X.

Let $\mathcal{I}$ be the set of states that is investigated, O the set of observed signals, $\mathcal{M}$ be the set of messages, $Z_m^c = \{C_k : T_{km} \in Z_m\}$, and $\mathcal{M}^i = \{i : m_i \in \mathcal{M}\}$. Define $\tilde{a}_k^\tau$ and a_k^τ as follows: $c_k^f \in \tilde{a}_k^\tau(c_l^g)$ if $(c_k^f, c_l^g) \in \tau_{kl}$ and $a_k^\tau(c_l^g) = \text{ch}(\tilde{a}_k^\tau(c_l^g))$. Define $\tilde{a}_k^\theta$ and a_k^θ analogously. Define a as $a(\tau_k) = \text{ch}(\tau_k)$ and similarly for $a(\theta_k)$. Let an (alignment) path H_{kl} be a finite sequence of $i_{H_{kl}}$ elements drawn from $\{C_1, \ldots, C_K\}$ with all elements distinct, with i'th element denoted $H_{kl}^{(i)}$, with $H_{kl}^{(1)} = C_l$ as the 'root,' and with $H_{kl}^{(i_{H_{kl}})} = C_k$ as the last. For some H_{kl}, let H_{kl}^T be a sequence with $i_{H_{kl}}$ elements, with $H_{kl}^{T(1)} = H_{kl}^{(1)}$ and for $i > 1$ $H_{kl}^{T(i)} = T_{mn}$ for $C_m = H_{kl}^{(i-1)}$ and $C_n = H_{kl}^{(i)}$. Say that H_{kl} is directed if $H_{kl}^{(i)} \in Z_{H_{kl}^{(i+1)}}, \forall i$. (The direction of alignments runs towards the root.) Define an implied action path $\tilde{h}_{kl}$ and h_{kl} that corresponds to H_{kl} recursively as follows: 1) $\tilde{h}_{kl}^{(1)} = \tau_l$ and $h_{kl}^{(1)} = a(\tau_l)$; 2) for $i > 1$, if $H_{kl}^{(i)} = C_m$ then $h_{kl}^{(i)} = a_m^\tau(h_{kl}^{(i-1)})$ and $\tilde{h}_{kl}^{(i)} = \bigcup_{c \in \tilde{h}_{kl}^{(i-1)}} \tilde{a}_m^\tau(c)$. Define $h_{kl}(\tau) = h_{kl}^{(i_{H_{kl}})}(\tau_l)$ and analogously for $\tilde{h}_{kl}$, and $h_{kl}^\theta = a_k^\theta(h_{kl}^{(i_{H_{kl}}-1)})$ and $\tilde{h}_{kl}^\theta = \bigcup_{c \in \tilde{h}_{kl}^{(i_{H_{kl}}-1)}} \tilde{a}_k^\theta(c)$. Let $\mathcal{H}$ be the set of all (alignment) paths H_{kl}. Let $\underline{h}_{kl} = \tilde{h}_{kl} \cup \tilde{h}_{kl}^\theta$. Say that H_{kl} and H_{mn} are disjoint if they have no elements in common. A tree b_l is a set of directed paths with common root C_l and such that if $C_o = H_{kl}^{(i)}$ for $H_{kl} \in b_l$ and $C_o = H_{k'l}^{(i')}$ for $H_{k'l} \in b_l$ then $i' = i$ and $\forall j < i$, $H_{kl}^{(j)} = H_{k'l}^{(j)}$.

Let $B = \{b_1, \ldots, b_v\}$ be a set of disjoint trees, $B^c = \{C_m : \exists H_{kl} \in b_u \in B \text{ s.t. } C_m = H_{kl}^{(i)}\}$ the set of included decisions, $B^r = \{C_l : \exists H_{kl} \in b_u \in B\} \subset B^c$ the set of roots of these trees, $B^T = \{T_{mo} : \exists H_{kl} \in b_u \in B \text{ s.t. } C_m = H_{kl}^{(i)} \text{ and } C_o = H_{kl}^{(i-1)}\}$ the branches of these trees. Let, for $C_m \in B^c$, $\text{Sc}(C_m) = \{C_n \in B^c : \exists H_{kl} \in b_u \in B, \text{ s.t. } C_m = H_{kl}^{(i)}, C_n = H_{kl}^{(i+1)}\}$ denote the set of successors to C_m and $\text{Pr}(C_m) = \{C_n \in B^c : \exists H_{kl} \in b_u \in B, \text{ s.t. } C_m = H_{kl}^{(i)}, C_n = H_{kl}^{(i-1)}\}$ the predecessor to C_m.

Define $\tilde{N}_k$ to be the set of choices that are not part of B but that align with $C_k \in B^c$ (in some equilibrium with B as announcement):

$$\tilde{N}_k = \{C_m \in K \setminus B^c : C_k \in Z_m^c \text{ and } \eta_{km}(1 - \Delta_k) \succ \beta_m \text{ and } \eta_{km}(1 - \Delta_k) \succ \max_{C_l \in B^c \cap Z_m^c} \eta_{lm}(1 - \Delta_l)\},$$

$N_k = \tilde{N}_k \cup \text{Sc}(C_k)$, and $M = K \setminus \left(B^c \cup \left(\bigcup_{k \in B^c} \tilde{N}_k\right)\right)$.

Say that B is incentive compatible iff for $C_k \in B^r$, $\beta_k \succ \max_{C_n \in (B^c \cap Z_k^c)} \eta_{kn}(1 - \Delta_n)$ and for $C_k \in B^c \setminus B^r$ and $C_l \in \text{Pr}(C_k)$, $\eta_{kl}(1 - \Delta_l) \succ \max(\beta_k, \max_{C_n \in (B^c \cap Z_k^c)} \eta_{kn}(1 - \Delta_n))$. Define Π_B for such an IC set of disjoint trees B, as follows:

$$\Pi_B = \sum_{C_k \in B^c} \sum_{C_n \in N_k} \eta_{kn}(1 - \Delta_k) + \sum_{C_k \in B^r \cup M} \beta_k$$

Let $\mathbf{\Pi}$ be the max over all IC sets of disjoint trees and $\mathbf{B}$ the argmax set of trees.

Let the event $\text{ND}_1 = \{\tau : \forall H_{kl} \neq H_{km} \in \mathcal{H}, \tilde{h}_{kl} \cap \tilde{h}_{km} = \emptyset \text{ and } \#\tilde{h}_{kl} = N^{i_{H_{kl}}}\}$, $\text{ND}_2 = \{\tau, \theta : \forall H_{kl} \neq H_{km} \in \mathcal{H}, \underline{h}_{kl} \cap \underline{h}_{km} = \emptyset \text{ and } \#\tilde{h}_{kl} = N^{i_{H_{kl}}}\}$, and $\text{ND}_\tau = \{(\tau, \tau') : \forall k, \tau_k \cap \tau'_k = \emptyset\}$.

Lemma 1 *Both the events ND_τ and ND_2 have probability 1.*

Proof : For the event ND_τ, consider first the case where each choice has M, rather than an infinite number of, alternatives. For randomly drawn (τ, τ'), the probability that $\tau_k \cap \tau'_k = \emptyset$ converges to $((M-N)/M)^N$ for large M. It follows that ND_τ is true with probability $((M-N)/M)^{NK}$, which converges to 1 as $M \to \infty$. For the event ND_2, we need to show that for any $H_{kl}, H_{km} \in \mathcal{H}$ (with $H_{kl} \neq H_{km}$) the probability that $\underline{h}_{kl} \cap \underline{h}_{km} = \emptyset$ and $\#\tilde{h}_{kl} = N^{i_{H_{kl}}}$ is 1. Consider again the case where each choice has M, rather than an infinite number of, alternatives. For any $H_{kl} \neq H_{km} \in \mathcal{H}$ the probability that $\tilde{h}_{kl} \cap \tilde{h}_{km} = \emptyset$ conditional on $\#\tilde{h}_{kl} = N^{i_{H_{kl}}}$ converges to $\left((M - N^{i_{H_{kl}}})/M\right)^{i_{H_{km}}}$ for large M, which converges to 1 as $M \to \infty$. The argument for the rest of ND_2 is analogous. ■

Let $\mathcal{H}(\mathcal{I}) = \{H_{kl} : H_{kl}^T \subset \mathcal{I}\}$ be the set of all (alignment) paths that can be constructed from $\mathcal{I}$. Let $\tilde{h}_\mathcal{I}$ be the full set of (all possible) implied decisions: $\tilde{h}_\mathcal{I} = \bigcup_{H_{kl} \in \mathcal{H}(\mathcal{I})} \tilde{h}_{kl}$, which has a finite number of elements, and $h_\mathcal{I} = \{h_{kl} : H_{kl} \in \mathcal{H}(\mathcal{I})\}$. Let $\mathcal{I}_B = \{T_k : C_k \in B^r\} \cup \{T_{kl} \in B^T\}$. Let $Z_k^i = \{i : T_{ki} \in Z_k\}$ denote the set of indices with which P_k has an interaction in his task. Let $\mathcal{T}_k$ denote the set of actual strategist signals τ_k and τ_{kl} for Z_k, which may potentially be empty, and Θ_k the set of local signals θ_k and θ_{kl} for $l \in Z_k^i$. I will use $\tilde{C}_k$ for a particular choice for C_k.

Lemma 2 *The unique locally-symmetric pure-strategy equilibrium outcome is for S to investigate all states in $\mathcal{I}_B$ for $B = \mathbf{B}$ and then, with probability 1 (as conditional on the ND_2 event), to announce the set of implied choices as messages $\mathcal{M} = h_{\mathcal{I}_B}$, and for the participants, also with probability 1 (as conditional on the ND_2 event) to then choose $\tilde{C}_l = a_l(m_k)$ for any $C_l \in N_k$ (for $C_k \in B^c$), and choose $\tilde{C}_l = a(\theta_l)$ otherwise, with expected payoff $\Pi_\mathbf{B} = \sum_{C_k \in \mathbf{B}} \sum_{C_n \in N_k} \eta_{kn}(1 - \Delta_k) + \sum_{C_k \in \mathbf{B}^r \cup M} \beta_k$.*

Proof : Throughout the proof, I will condition on the (probability 1) event ND_2 unless otherwise noted. (As payoffs are bounded and the complement is a probability zero event, it will affect neither optimal actions prior to the signals nor the expected payoff.)

Consider first the (iterative) investigation in steps 1a and 1b. In any locally-symmetric (LS) pure-strategy (PS) equilibrium (henceforth LSPSEq), LS implies that, conditional on ND_2, the set $\mathcal{I}$ does not depend on the outcome of earlier investigations and is thus common knowledge. If it did depend on the outcome of earlier investigations, then switching the signals would affect which states get investigated, contradicting LS. (This does not hold outside ND_2. In particular, when implied choices may coincide, then the optimal investigation will sometimes depend on observations, as analyzed in Van den Steen (2012c).)

Messages Consider next the messages in stage 1c. First, in any pure-strategy equilibrium (henceforth PSEq), $\mathcal{M}$ can depend only on O.

Second, for any LSPSEq, there exists a subset $\check{h} \subset \tilde{h}_\mathcal{I}$ such that, with probability 1, $\mathcal{M} = \{m_k = \check{h}_{kl} : \check{h}_{kl} \in \check{h}\}$ for any realization of signals. To see this, fix first a τ. Assume (by contradiction) that $\exists m_k \in \mathcal{M}$ with no such $\check{h}_{kl} = m_k$. Pick some other $c_k^x \notin \tilde{h}_{\mathcal{I}}$. Relabeling C_k such that $c_k^f = m_k$ and c_k^x are switched, changes $\mathcal{M}$ even though no element of O changed, contradicting the fact that $\mathcal{M}$ can only depend on O. Assume next (by contradiction) that $\exists \tau, \tau' \in \text{ND}_1$ with $(\tau, \tau') \in \text{ND}_\tau$ such that $\check{h}_{kl}(\tau) \in \mathcal{M}(\tau)$ but $\check{h}_{kl}(\tau') \notin \mathcal{M}(\tau')$. Relabeling choice alternatives to switch every $\tau_k \in \tau$ to $\tau'_k \in \tau'$ while preserving the preference $\succ$ (but nothing more) should relabel all actions and choices in the equilibrium accordingly. But that implies that $\check{h}_{kl}(\tau') \in \mathcal{M}(\tau')$, which leads to a contradiction. This further implies that $\mathcal{M}^i$ is fixed for any equilibrium.

Action restrictions from local symmetry Consider now participant P_k. In any PSEq, P_k's choice C_k must be a deterministic function of his information $\Theta_k \cup \mathcal{T}_k \cup \mathcal{M}$. LS further restricts this to $A_k = \{\theta_k, \tau_k, m_k, \tilde{a}_k^\theta(\mathcal{M} \mid Z_k), \tilde{a}_k^\tau(\mathcal{M} \mid Z_k)\}$, where m_k is only included if $m_k \in \mathcal{M}$ and where $\tilde{a}_k^\theta(\mathcal{M} \mid Z_k) = \{\tilde{a}_k^\theta(m_l) : l \in \mathcal{M}^i \cap Z_k^i\}$ and analogous for $\tilde{a}_k^\tau$.

Second, in any equilibrium, P_k's strategy is – potentially conditional on $\tau_k = \theta_k$ – 'always choose X' for some fixed $X \in A_k$ (where 'fixed' includes rank-order according to $\succ$ within a set). To see this, I will first show that $C_k \in A_k$ for any θ and τ. To that purpose, fix some equilibrium and some θ and τ. Assume (by contradiction) that $\exists C_k$ with $\tilde{C}_k = c_k^x \notin A_k$. Pick some other $c_k^y \notin A_k$. Relabeling $\mathcal{C}_k$ to switch c_k^x and c_k^y (and nothing more) changes C_k even though no element of A_k changed, contradicting the fact that $\tilde{C}_k$ can only depend on A_k. Pick next some C_k and some $(\tau, \theta), (\tau', \theta') \in \text{ND}_2$ with $\tau'_k = \theta'_k$ iff $\tau_k = \theta_k$. Let $\tilde{C}_k = X$ for some $X \in A_k$ at (θ, τ). Now permutate all signals from (τ, θ) to (τ', θ'). Following LS, it remains true that $C_k = X$ (even though the actual choice for C_k may change).

Beliefs Consider now the beliefs (at the start of 2b) of some participant P_n about C_k. Let this belief, which is a distribution over $\mathcal{C}_k$, be denoted μ_k^n. P_n's beliefs can only depend on her information $\Theta_n \cup \mathcal{T}_n \cup \mathcal{M}$. The choice C_k, on the other hand, can only depend on $A_k = \{\theta_k, \tau_k, m_k, \tilde{a}_k^\theta(\mathcal{M} \mid Z_k), \tilde{a}_k^\tau(\mathcal{M} \mid Z_k)\}$ (where m_k is only included if $m_k \in \mathcal{M}$). Combining these two implies that μ_k^n can only depend on m_k or, if $k \in Z_n^c$, on $\tilde{a}_k^\tau(m_n)$. Any potential equilibrium with beliefs based on the latter is dominated when messages are costless.[17] It follows that μ_k^n will depend only on m_k and be the same for all participants. Let $\mu_k = \mu_k^n$. It further follows that when $C_k \notin \mathcal{M}^c$, μ_k is the ignorance belief and puts equal (zero) probability on all alternatives. When $C_k \in \mathcal{M}^c$, μ_k puts some probability $\check{\mu}_k$ on $C_k = m_k$ and puts equal probability on all other alternatives for C_k (with overall the complementary probability $1 - \check{\mu}_k$ but with the probability of any particular alternative equal to 0).

Action choices Consider now the action choice of some player P_k in stage 2b. Participant P_k solves

$$\max_{C_k} \beta_k I_{C_k \in T_k} + \sum_{l \in Z_k^c} \eta_{kl} E[I_{(C_k, C_l) \in T_{kl}}] = \max_{C_k} \beta_k I_{C_k = t_k^k} + \sum_{l \in \mathcal{M}^i \cap Z_k^c} \eta_{kl} \check{\mu}_l I_{C_k = a_k^\theta(m_l)}$$

with $t_k^k = a(\theta_k)$. Since $\beta_k > 0$ and all $\eta_{kl}, \check{\mu}_l \geq 0$, the payoff increases in both $I_{C_k = t_k^k}$ and $I_{C_k = a_k^\theta(m_l)}$. Note, moreover, that ND_2 implies that t_k^k and all $a_k^\theta(m_l)$ are distinct. It then follows that in a LSPSEq, C_k either always chooses t_k^k or for some $l \in \mathcal{M}^i \cap Z_k^c$ always chooses $a_k^\theta(m_l)$. Let the set of participants (excluding P_l) who choose $a_k^\theta(m_l)$ be denoted N_l. So the set of P_n is partitioned into a set who choose t_n^n (and do not align

[17] When there is a real cost from messages, such beliefs can be part of the equilibrium. This may affect the message and action choices but does not affect the fundamental structure of equilibria.

with any decision) and a number of sets N_k who align with $C_k \in \mathcal{M}$.

The set of equilibrium actions is thus completely determined by the set $\mathcal{M}$. Furthermore, in any equilibrium, if $m_k \in \mathcal{M}$, then $N_k \neq \emptyset$ since otherwise m_k does not affect the outcome so that lexicographic preferences for less announcements imply that such announcements cannot be optimal. But that further implies that if $m_k \in \mathcal{M}$ then $C_k = m_k$ with strictly positive probability (since otherwise $\breve{\mu}_k = 0$ and $N_k = \emptyset$).

Consider now some equilibrium and some C_k with $m_k \in \mathcal{M}$. If $C_k = t_k^k$, then it must be that $m_k = a(\tau_k)$ (since this is the only $m_k = h_{kl}$ for some $H_{kl} \in \mathcal{H}(\mathcal{I})$ such that $P(C_k = m_k) > 0$). If $C_k = a_k^\theta(m_l)$ and thus aligns on m_l, then it must be that $m_k = a_k^\tau(m_l)$ (since this is again the only $m_k = h_{kl}$ for some $H_{kl} \in \mathcal{H}(\mathcal{I})$ for which $P(C_k = m_k) > 0$). But the fact that either $m_k = a(\tau_k)$ or $m_k = a_k^\tau(m_l)$ (combined with the facts that there can be only one m_k for each C_k and that there can't be strong loops) directly implies that for every $m_k \in \mathcal{M}$, there exists exactly one directed H_{kl} such that $m_k = h_{kl}$ and such that for each $C_m \in H_{kl}$, $m_m \in \mathcal{M}$ and $m_m = h_{ml}$ for $H_{ml} \subset H_{kl}$. Combined with the fact that each choice C_k is optimal in equilibrium, this finally implies that the m_k form an IC set of disjoint trees B. Furthermore, since $m_k = h_{kl}$ for some $H_{kl} \in \mathcal{H}(\mathcal{I})$, it follows that S then must have investigated at least $\mathcal{I}_B$. Finally, as investigation is costly and no investigation beyond B^r and T_B affect the outcome, S will investigate exactly $\mathcal{I}_B$. The payoff in this case is Π_B. Since S tries to maximize the payoff Π_B, it finally follows that the equilibrium outcome is for S to investigate all states in $\mathcal{I}_B$ for $B = \mathbf{B}$ and then to announce the set of implied choices as messages $\mathcal{M} = h_{\mathcal{I}_B}$, and for the participants (conditional on the probability 1 ND_2 event) to choose $\tilde{C}_l = a_l(m_k)$ for any $C_l \in N_k$ (for $C_k \in B^c$), and choose $\tilde{C}_l = a(\theta_l)$ otherwise, with expected payoff $\Pi_{\mathbf{B}} = \sum_{C_k \in \mathbf{B}} \sum_{C_n \in N_k} \eta_{kn}(1 - \Delta_k) + \sum_{C_k \in \mathbf{B}^r \cup M} \beta_k$. This completes the proof. ■

Let $\tilde{\Pi}_B$ be the payoff from B including the cost of investigating B. Let $\zeta_{kl} = \eta_{kl}(1-\Delta_k) = \gamma_{kl} p_{kl}(1-\Delta_k)$.

Lemma 3 *A decision C_k is more strategic and the value of strategy increases when ζ_{kl} increases for some $l \in \Gamma_k$. The value of strategy is supermodular in η_{kl} and $(1 - \Delta_k)$. [Formally: The probability π_k that $C_k \in \mathcal{S}$ increases in ζ_{kl}. The value of strategy increases in ζ_{kl}, $\forall k, l \in K$ and is supermodular in η_{kl} and $(1 - \Delta_k)$.]*

Proof : Pick any set of parameters and any two decisions C_k and C_l with $k \in Z_l^i$. I will consider the effect of an increase in the value of ζ_{kl} to ζ'_{kl}, keeping all other parameters fixed. Let $\mathbf{B}$ and $\mathbf{B}'$ be the optimal IC sets of disjoint trees for, respectively, ζ_{kl} and ζ'_{kl}, and let $\mathbf{\Pi}$ and $\mathbf{\Pi}'$ be the respective payoffs. Note that for any B with $C_k \notin B$, the payoff under B does not depend on ζ_{kl} and whether B is IC or not also does not change with ζ_{kl}.

I will first show that when C_k is strategic at ζ_{kl}, it will remain strategic at ζ'_{kl} and the payoff increases and is supermodular in η_{kl} and $(1 - \Delta_k)$. So assume that $C_k \in \mathbf{B}$. This implies that $\mathbf{\Pi} \succ \Pi_B$ for any B that does not contain C_k and that is IC at ζ_{kl}. That further implies – since IC does not change with ζ_{kl}– that $\mathbf{\Pi} \succ \Pi_B$ for any B that does not contain C_k (i.e., with C_k non-strategic) and that is IC at ζ'_{kl}. It thus suffices to show that either $\mathbf{B}$ is IC at ζ'_{kl} or that some other B that contains C_k is IC at ζ'_{kl} and has $\Pi_B \succ \Pi_{\mathbf{B}}$. To that purpose, I will now argue that if some set of disjoint trees B that contains C_k is IC at ζ_{kl} but not at ζ'_{kl}, then there exists a $\tilde{B}$ that contains C_k that is IC at ζ'_{kl} and that has a strictly preferred payoff. Consider thus any such set of disjoint trees B, i.e., with $C_k \in H_{no} \in b_u \in B$ and that is IC at ζ_{kl}. Let x denote the payoff from C_l under B, i.e., $x = \beta_l$ if $l \in M$ and $x = \zeta_{lr}$ if $l \in N_r$. Note first that B remains IC at ζ'_{kl} if $C_l \notin B$ or if $C_l \in B$ and $l \in N_k$. So consider now $C_l \in B$ but $l \notin N_k$. If $\zeta'_{kl} \prec x$ then B is still IC at ζ'_{kl}. If, on the other hand, $\zeta'_{kl} \succ x$, then consider $\tilde{B}$ constructed from B as follows. Replace any $H_{pq} \in B$ that contains C_l, i.e., $H_{pq} = (C_q, \ldots, C_l, \ldots C_p)$ (and appropriately adapted when $p = l$ and/or $q = l$), with $H_{po} = (C_o, \ldots, C_k, C_l, \ldots, C_p)$. Notice that B and $\tilde{B}$ contain exactly the same choices variables. Moreover, all choices are the same except for C_l. $\tilde{B}$ is IC at ζ'_{kl} (given $\zeta'_{kl} \succ x$) and $\Pi_{\tilde{B}} = \Pi_B + (\eta'_{kl}(1-\Delta_k) - x) \succ \Pi_B$. This concludes the proof of the first part, i.e., that if C_k is strategic at ζ_{kl}, then it will remain strategic at ζ'_{kl}. To see that the value of strategy increases and is supermodular in η_{kl} and $(1-\Delta_k)$, it suffices to show that the optimal payoff increases and is supermodular in η_{kl} and $(1-\Delta_k)$. This follows immediately from above for the case that C_k is strategic at ζ_{kl}. The argument for when C_k is not strategic at ζ_{kl} is straightforward. In that case, $\mathbf{B}$ does not contain C_k and neither its payoff nor whether it is IC depends on the value of ζ'_{kl}. Since $\mathbf{B}$ remains feasible at ζ'_{kl}, the optimal payoff must be weakly higher. For supermodularity, the argument follows from the observation that the above implies that there exists a critical ζ_{kl} below which the payoff is independent of ζ_{kl} and above which the payoff contains the term ζ_{kl} (and is thus supermodular in η_{kl} and $(1-\Delta_k)$).

To see that the increases are sometimes strict, consider a setting with two choices, C_k and C_l, and let $\zeta_{kl} = 0$ but $\zeta'_{kl}(1-\Delta_k) > \beta_l + c(I)(1)$. This proves the proposition. ■

Proof of Proposition 1a: The payoff depends on Δ_k only through the interactions between C_k and $C_l \in \Gamma_k$. Applying Lemma 3 to these interactions proves the result. ■

Proof of Proposition 1b: Let C_l be the decision that S investigates and announces. Following a proof analogous to that of Lemma 2, any participant $P_k \neq P_l$ will do one of 3 things: choose t_k^k each time; align with C_l in the first repetition and choose t_l^l in the second; or align twice with C_l. Define $\check{N}_l$ to be the set of choices that align with C_l in the first repetition only $\check{N}_l = \{C_m \in \Gamma_l : \eta_{lm}(1-\Delta_l) \succ \beta_m \succ \eta_{lm}(1-\Delta_l)(1-\check{\Delta}_l)\}$ and $\check{N}_l^{'}$ as the set of choices that align with C_l in both repetitions $\check{N}_l^{'} = \{C_m \in \Gamma_l : \eta_{lm}(1-\Delta_l)(1-\check{\Delta}_l) \succ \beta_m\}$. The expected payoff from investigating and announcing C_l is then $\Pi = \sum_{C_m \in \check{N}_l'} [\eta_{lm}(1-\Delta_l)(2-\check{\Delta}_l)] + \sum_{C_m \in \check{N}_l} [\eta_{lm}(1-\Delta_l) + \beta_m] + \sum_{C_m \in K \setminus (\check{N}_l \cup \check{N}_l')} 2\beta_k$. The result then follows directly. ■

Proof of Proposition 2: The option to commit cannot make a decision less strategic: the option only matters if it is used and it can be used only if the decision is part of the strategy. For a setting where such commitment makes a choice more strategic, consider $K = 2$, $N = 1$, $T_{12} \in Z_2$, $p_1 = p_2 = p_{12} = 1$, $\Delta_1 = 1$, $\Delta_2 = 0$, $\beta_1 = \beta_2 = .1$, $\gamma_{12} = 1$. Without commitment, no choice is strategic and $\Pi = .2$. With commitment an option for C_1, C_1 becomes strategic and the payoff increases to $\Pi = 1$

For the second part of the proposition, consider the setting above but now with $\beta_1 = 4$ and $\Delta_1 = .5$. The payoff without strategy equals $\beta_1 + \beta_2 = 4.1$. The optimal strategy absent commitment is $\mathcal{M} = \{\tau_1\}$ with payoff $\beta_1 + \gamma_{12}(1-\Delta_1) = 4.5$. If S were to commit to $C_1 = \tau_1$, the payoff would become $\beta_1(1-\Delta_1) + \gamma_{12} = 3$

and thus drops below the trivial payoff. It follows that S will not commit and C_1 would cease to be strategic if commitment to the announced strategy would be automatic.

For the last part of the proposition, note the following. First, $q_k = q_l$ and $\Delta_k = 0$ imply $p_k = q_k = q_l = (1-\Delta_l)p_l)$. Next, pick a potential equilibrium with $m_l \in \mathcal{M}$ but $m_k \notin \mathcal{M}$ (and with equilibrium choices $\tilde{C}_k$ and $\tilde{C}_l$). I will argue that replacing m_l with $m_k = \tilde{C}_k$ will weakly and sometimes strictly increase the payoff, which then completes the proof. Consider first the case that C_l were committed. Replacing m_l with $m_k = \tilde{C}_k$ keeps all payoffs from $C_m \neq C_l$ identical – as any choice that aligned on C_l will now align on C_k with the identical same payoff – while the payoff from C_l will weakly or strictly increase because C_l can now be chosen based on more informative signals. The payoff will thus improve weakly and sometimes strictly. Consider next the case that C_l were not committed but chosen optimally based on P_l's local information. Replacing m_l with $m_k = \tilde{C}_k$ keeps the payoff from C_l identical but will weakly or strictly improve the payoffs from $C_m \neq C_l$ as it corresponds to an increase in persistence (from $(1-\Delta_l) < 1$ to $(1-\Delta_k) = 1$), so that the result follows from Proposition 1a. This concludes the proof. ■

Proof of Proposition 3a: Let $\hat{C}_k$ for $k \in K$ denote the choices that maximize the project payoff (including lexicographic preferences but not considering the cost of investigation) for a given set of signals θ. Let $\hat{C}_{k|X}$ denote the choices that maximize Π (for a given set of signals) conditional on a set of choices X (eg. $X = \{\tilde{C}_l = c_l^g\}$). Let $\hat{\Pi}$ and $\hat{\Pi}_|$ denote the respective optimal project payoffs.

With full reversibility, there is no value from strategy: under ND_2 (and no strong loops), the choice adjustments will lead to the $\hat{C}_k$ and do so in a cheaper way than strategy – as investigations are costly but reversions are not – making this optimal.

Consider now the case that one choice C_k is irreversible. If $\hat{C}_k = t_k^k$, then there is no need for strategy as P_k will choose $\hat{C}_k$ to start with. Consider then the case that $\hat{C}_k \neq t_k^k$. The outcome *without* strategy will then be $\hat{C}_{l|C_k=t_k^k}$ with payoff $\hat{\Pi}_{|C_k=t_k^k}$. (With a strategy, however, the optimal outcome is not necessarily simply the unconditional optimum $\hat{C}_l$ as that optimum may require a lot of investigations while one with a slightly lower payoff may require a lot less investigations.) To determine now the optimum for such cases *with* a strategy, let $\mathcal{H}_{k-i}$ be the set of all directed paths with $i \geq 1$ (distinct) elements that end in C_k such that for each $H_{kl} \in \mathcal{H}_{k-i}$ and for each $j \leq i$: if $C_m = H_{kl}^{(j)}$ then $h_{kl}^{(j)} = \hat{C}_{m|C_k=h_{kl}^\theta}$. (Note that $\mathcal{H}_{k-1} = \{H_{kk}\}$.) In other words, these are the directed paths such that if C_k is fixed according to that path, the path is indeed part of the optimal outcome.

For each $i < K$, let H_{k-i} be the element H_{kl} of $\mathcal{H}_{k-i}$ that results in the highest overall payoff $\hat{\Pi}_{|C_k=h_{kl}^\theta}$. Let finally H_k be the H_{k-i} that maximizes the payoff subject to the cost of investigation, i.e., that maximizes $\Pi - c_I(i-1)$. The following is then the equilibrium: for $H_{kl} = H_k$ investigate in stages 1a and 1b all but the last element of H_{kl}^T. If $i_{H_{kl}} \geq 2$ and $C_m = H_{kl}^{(i_{H_{kl}}-1)}$, announce $m_m = h_{kl}^{(i_{H_{kl}}-1)}$. (If $i_{H_{kl}} = 1$, investigate nothing and announce nothing. This is thus the outcome with no strategy.) In the subgame, P_k will choose $a_k^\theta(m_m)$ (as he knows that that is, in equilibrium, his payoff-maximizing choice) while all others will choose, ultimately, $\hat{C}_{n|C_k=a_k^\theta(m_m)}$. (Note that all others will be able to adjust after observing P_k's choice.) It follows that the only strategic choice is (at most) a choice interacting with C_k and a choice C_m is more likely to be strategic when η_{km} is larger. C_k itself is never strategic, and the irreversible choice is in this case thus less strategic than the reversible ones. Moreover, whenever this is optimal, strategy has value and the value of strategy thus increased relative to the case without irreversibility. This proves the proposition. ■

Proof of Proposition 3b: This follows from the proof of Proposition 3a ■

Proof of Proposition 4: Let C_k be the decision about which there is a public signal. A variation on the proof of Lemma 2 implies that the equilibrium takes one of the following two forms:

1. For some IC set of disjoint trees, B, with $C_k \in B^c$, S makes the investigation and announcement as if there were no public signal. The actions are also as if there were no public signal. In other words, the signal gets overruled by an investigation.
2. For some set of disjoint trees, B', with $C_k \in B'^r$ and that is IC when replacing Δ_k with $\Delta_{k'}$, S makes the investigation and announcement as if no public signal exists with the following exceptions: S does not investigate T_k but uses $\tilde{\tau}_k$ as its signal; S does not announce any m_k. In other words, the signal gets used as a message. In this case, C_k is thus not strategic.

For the first case, the IC condition and the implied actions imply that conditional on m_k, the public signal is uninformative about C_k and the outcome is thus independent of the signal. In this case, Π_B will thus be independent of the precision of the signal (as the IC conditions will never be affected by the presence of a signal that is a garbling of τ_k). It follows that the payoff of any potential equilibrium strategy with $m_k \in \mathcal{M}$ is unaffected by the precision of the public signal.

For the second case, Proposition 1a implies that the payoff increases in the precision of the public signal. It then follows that, as the precision of the signal increases, if the equilibrium is of type 1, then it will either not change (and the payoff remains the same) or change to an equilibrium of type 2 (with a higher payoff), making C_k non-strategic. If the equilibrium is of type 2, it will stay of type 2, keeping C_k non-strategic. This proves the first part of the proposition.

For the second part of the proposition, note that the payoff absent any strategy has increased since the public signal about C_k allows other choices to align with C_k. In particular, let $\tilde{N}_k = \{C_m \in \Gamma_k : \eta_{km}(1-\Delta_{k'}) \succ \beta_m\}$ for the subgame equilibrium without strategy, then $\Pi = \sum_{C_l \in \tilde{N}_k} \eta_{kl}(1-\Delta_{k'}) + \sum_{C_l \notin \tilde{N}_k} \beta_l$ which thus decreases in $\Delta_{k'}$ with derivative $-\sum_{C_l \in \tilde{N}_k} \eta_{kl} < 0$. If the optimal strategy is of type 1, then the payoff is independent of $(1-\Delta_{k'})$ so that the gain from strategy indeed increases in $\Delta_{k'}$. If the optimal strategy is of type 2, then the payoff contains the term $\sum_{C_l \in \tilde{\tilde{N}}_k} \eta_{kl}(1-\Delta_{k'})$ for $\tilde{\tilde{N}}_k = \{C_m \in \Gamma_k : \eta_{km}(1-\Delta_{k'}) \succ \max(\beta_m, \max_{l \in B^c \cap Z_m^c} \eta_{lm}(1-\Delta_l))\} \subset \tilde{N}_k$. The derivative (for Δ_k) then equals $-\sum_{C_l \in \tilde{\tilde{N}}_k} \eta_{kl}$ so that the derivative of the gain from strategy equals $-\sum_{C_l \in \tilde{\tilde{N}}_k} \eta_{kl} - (-\sum_{C_l \in \tilde{N}_k}) = \sum_{C_l \in \tilde{N}_k \setminus \tilde{\tilde{N}}_k} \eta_{kl} \geq 0$ and the cross partial is also positive. This completes the proof. ■

Proof of Proposition 5: This follows directly from Lemma 3. ■

Proof of Proposition 6: Let $\mathcal{S}$ denote the strategy bet. Following an argument analogous to the proof of Lemma 2, the payoff will be $\sum_{C_k \in \mathcal{S}} \sum_{C_l \in N_k} \eta_{kl} + \sum_{C_k \in M} \beta_k$ whereas the payoff without $\mathcal{S}$ is $\sum_{C_k} \beta_k$, so that the gain in payoff is $\sum_{C_k \in \mathcal{S}} \left[-\beta_k + \sum_{C_l \in N_k} \eta_{kl}\right]$, which implies the proposition. ■

Proof of Proposition 7: Following Lemma 2, if $\eta_{kl} = 0 \; \forall l \in \Gamma_k$, then $N_k = \emptyset$, so that Π_B does not depend on whether C_k is investigated or announced, so that S will prefer not to investigate or announce C_k, so that C_k will never be strategic. The second part of the proof follows directly from the expression of $\Pi_{\mathbf{B}}$. ■

Proof of Proposition 8: This follows directly from Lemma 3. ■

Proof of Proposition 9: Let $\check{C}_k$ be the optimal choice for C_k given B: $\check{C}_k = t_k^k$ if for all $n \in B^i \cap Z_k^i$, $\beta_k \succ \eta_{kn}(1-\Delta_n)$ while $\check{C}_k = a_k^\theta(C_n)$ for some $n \in B^i$ if $\eta_{kn}(1-\Delta_n) \succ \max(\beta_k, \max_{l \in B^i \cap Z_k^i} \eta_{kl}(1-\Delta_l))$. Let $V_{l|B}$ denote the value generated by decision C_l under B: $V_{l|B} = \beta_l$ if $C_l \in B^r \cup M$ and $V_{l|B} = \eta_{ml}(1-\Delta_m)$ if $C_l \in N_m$.

Let B' then be the IC set of disjoint trees generated by adding $\check{C}_k$ to B. To determine now the value from investigating and announcing B', note that the payoff from C_k itself will not change as $C_k = \check{C}_k$ both when B and when B' are investigated and announced. The difference is in the choices that align on C_k, which add a term $\sum_{C_l \in N'_k} \eta_{kl}(1-\Delta_k)$ with $N'_k = \{C_m \in \Gamma_k : \eta_{km}(1-\Delta_k) \succ \max(\beta_m, \max_{l \in B'^i \cap Z_m^i}(\eta_{lm}(1-\Delta_l))\}$ and analogous for the N'_l in B' for $l \in B^i$. The added value from announcing B' instead of B can now be written as $\sum_{C_m \in N'_k}[\eta_{km}(1-\Delta_k) - V_{m|B}]$. An increase in γ_{lm} for some $l \in B^i$ and $m \in \Gamma_k \cap \Gamma_l$ has two effects that both reduce the added value from announcing B' instead of B. First, it may reduce N'_k if $(\eta_{lm}(1-\Delta_l) \succ \eta_{km}(1-\Delta_k)$ due to the increase in γ_{lm}. Second, it will increase $V_{m|B}$ if $m \in N_l$. This proves the proposition. ■

Proof of Proposition 10: The payoff from a strategy increases in β_k iff $C_k \in B^r \cup M$, and in that case the derivative for β_k always equals 1. It follows that if at α_k, $C_k \in B^r$, then $C_k \in B^r$ at $\alpha'_k > \alpha_k$. And similarly for $p'_k > p_k$. This proves the proposition. ■

Proof of Proposition 11: This follows immediately from Proposition 7. ■

Proof of Proposition 12: With investments in specific capabilities, Lemma 2 and its proof change on only one point. Consider a capability K_l that depends on choice C_k. In stage 2a, $\tilde{P}_l$ will invest only if $k \in \mathcal{M}^i$ and then make the capability specific to m_k and choose $e_l = (1-\Delta_k)\lambda_l$. Let $\tilde{\Pi}_B$ denote the payoff from an IC set of disjoint trees B so that the equilibrium investigation and announcements will be $\tilde{\mathbf{B}} = \mathrm{argmax}_B \tilde{\Pi}_B$. An IC set of disjoint trees B with $C_k \notin B$ is independent of λ_l for $K_l \in \mathcal{K}_k$. For an IC set of disjoint trees B with $C_k \in B$, on the other hand, $\frac{d\tilde{\Pi}_B}{d\lambda_l} = (1-\Delta_k)^2\lambda_l$ for $K_l \in \mathcal{K}_k$. Note that this is the same for *all* B with $C_k \in B$. It follows that if C_k is strategic, it will remain strategic when λ_l increases. When C_k is not strategic, it may become strategic. Finally, $\frac{\partial^2 \Pi_B}{\partial \lambda_l \partial(1-\Delta_k)} = 2(1-\Delta_k)\lambda_l > 0$. This completes the proof. ■

Proof of Proposition 13: The competitor G will not announce a strategy (as announcing a strategy can only affect F's payoff and G prefers less announcement) and F will thus never align on C_g. Let $\hat{k} = \text{argmax}_{k \in \mathcal{M}^i \cap Z_g^i} \eta_{kg}(1 - \Delta_k)$. An argument completely analogous to the proof of Lemma 2 then implies that any potential equilibrium outcome takes the following form: For some IC set B, S investigates all states in $\mathcal{I}_B$ and announces the set of implied choices $\mathcal{M} = h_{\mathcal{I}_B}$; the participants (conditional on the probability 1 ND$_2$ event) choose $\tilde{C}_l = a_l^\theta(m_k)$ for any $C_l \in N_k$ (for $C_k \in B^c$), and choose $\tilde{C}_l = a(\theta_l)$ otherwise; if $\eta_{\hat{k}g}(1 - \Delta_{\hat{k}}) \prec \beta_g$ then $\tilde{C}_g = a(\theta_g)$ else $\tilde{C}_g = (a_g^\theta(m_{\hat{k}})$. This gives F a payoff

$$\Pi_B = \sum_{C_k \in B} \sum_{C_l \in N_k} \eta_{kl}(1 - \Delta_k) + \sum_{C_k \in \mathbf{B}^r \cup M} \beta_k + \check{\alpha}_g I_{\eta_{\hat{k}g}(1-\Delta_{\hat{k}}) \prec \beta_g} + \check{\eta}_{\hat{k}}(1 - \Delta_{\hat{k}}) I_{\eta_{\hat{k}g}(1-\Delta_{\hat{k}}) \succ \beta_g}$$

and it gives G a payoff that is simply $\beta_g I_{\eta_{\hat{k}g}(1-\Delta_{\hat{k}}) \prec \beta_g} + \eta_{\hat{k}g}(1 - \Delta_{\hat{k}}) I_{\eta_{\hat{k}g}(1-\Delta_{\hat{k}}) \succ \beta_g}$.

It now suffices to show that in any equilibrium with C_k strategic, C_k remains strategic when either γ_{kg} increases at sufficiently high $\check{\gamma}_k$ or when $\check{\gamma}_k$ increases. Note first that the payoff from any potential equilibrium with C_k non-strategic is independent of $\check{\gamma}_k$ or γ_{kg}. So it suffices to show that the payoff from any potential equilibrium with C_k strategic increases in $\check{\gamma}_k$ and increases in γ_{kg} for sufficiently large $\check{\gamma}_k$. Pick then any equilibrium with C_k strategic. Consider first the case that $\hat{k} = k$ and $\eta_{\hat{k}g}(1 - \Delta_{\hat{k}}) \succ \beta_g$ so that $\tilde{C}_g = a_g^\theta(C_k)$. An increase in $\check{\gamma}_k$ increases the payoff while an increase in γ_{kg} ensures that $\hat{k} = k$ remains.

Consider next the case that either $\hat{k} \neq k$ and/or $\eta_{\hat{k}g}(1 - \Delta_{\hat{k}}) \prec \beta_g$ so that $\tilde{C}_g \neq a_g^\theta(C_k)$. In that case, an increase in $\check{\gamma}_k$ does not affect the payoff. An increase in γ_{kg}, on the other hand may make $\hat{k} = k$ and $\eta_{\hat{k}g}(1 - \Delta_{\hat{k}}) \succ \beta_g$, switching C_g to $\tilde{C}_g = a_g^\theta(C_k)$. This increases the payoff iff $\check{\eta}_k(1 - \Delta_k)$ exceeds the payoff that F received before, which will be the case if $\check{\eta}_k$ is sufficiently high. This proves the proposition. ■

Proof of Proposition 14: That follows from the proof of proposition 13 ■

Proof of Observation 1: Consider Lemma 2. I need to show that the announcements in 1c are an optimal strategy. Let me first show that the equilibrium announcement is a strategy for any investigation outcome. Following the lemma, the pattern of investigation is common knowledge and the set of S's signals is the investigation outcome $\tilde{\tau}$. Take as target outcome simply the equilibrium outcome of Lemma 2 (which is defined for all possible investigation outcomes and states). Note that if S announces an action for C_k in period 1c (as part of the supposed strategy) then that action will in equilibrium indeed be chosen as long as $\tau_k = \theta_k$. This implies the first condition for a strategy. The second condition follows by construction from the definition of the target outcome. The third condition follows from the fact that in equilibrium every announced decision is also investigated and investigations are costly. Hence if a smaller announcement existed, S would have chosen it.

It also follows from Lemma 2 that the equilibrium investigations and announcement form an optimal strategy formulation. In particular, S chooses the pattern of investigation and – conditional on the investigation outcome – the announcements to maximize the overall payoff (including the cost of investigation). Furthermore, the announced choices are part of the subgame equilibrium as long as $\tau_k = \theta_k$. It follows that it is an optimal strategy formulation and the set of choices announced by the strategist in stage 1c is an optimal strategy. This completes the proof. ■

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