OPTIMAL CONTROL OF AFFINE HEREDITARY DIFFERENTIAL SYSTEMS WITH A QUADRATIC COST .*

by

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April 1971

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* This research was supported by NSF Grant GK-25781 and by AFOSR Grant AFOSR-70-1941, both at the Electronic Systems Laboratory, M.I.T., Cambridge, Mass.

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If Π^{00}, Π^{10}, Π^{01} and Π^{11} are known, the feedback law and the optimal cost can be computed for any initial datum h in $M^2(-b,0;H)$. However our task is considerably reduced by Corollary 5.3 where it was established that it is sufficient to know $P^0(s,\eta)$, $s \in [0,T]$, $\eta \in I(-b,0)$, $s-\eta \leq T$, in order to compute Π^{00}, Π^{10} and Π^{01}; as for Π^{11} it can be determined from $P^1(s,\eta)$, $s \in [0,T]$, $\eta \in I(-b,0)$, $s-\eta \leq T$. In general the map $s \to A_{00}(s)$ will not be absolutely continuous and hence the map $\alpha \to \Pi^{10}(s,\alpha)$ will also not be absolutely continuous (equation (5.16)). However the map $\eta \to P(s,\eta)$ is absolutely continuous (by definition).

D.W. Ross and I. Flügge-Lotz [3] formally derived equations for Π^{00}, Π^{01} and Π^{11} which involve differentiation of $\Pi^{01}(s,\alpha)$ with respect to α . This was possible since the maps $s \to A_i(s)$ they consider are constant. Their results were generalized by H.J. Kushner and D.I. Barnea [5] where the maps $s \to A_i(s)$ were assumed to be absolutely continuous. Here our system is not subject to the above restrictions and as pointed out earlier we cannot perform certain operations on the operator Π .

Thus the next step is the study of the operator P . There are several technical difficulties involved and it is not sufficient to formally differentiate equation (4.26) ,

$$p(s-\eta) = P(s,\eta)\ (h\circ x)_s + r(s,\eta)$$

in order to obtain results analogous to the Riccati equation results for control of linear ordinary differential equations. We plan to do this in a subsequent paper.

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