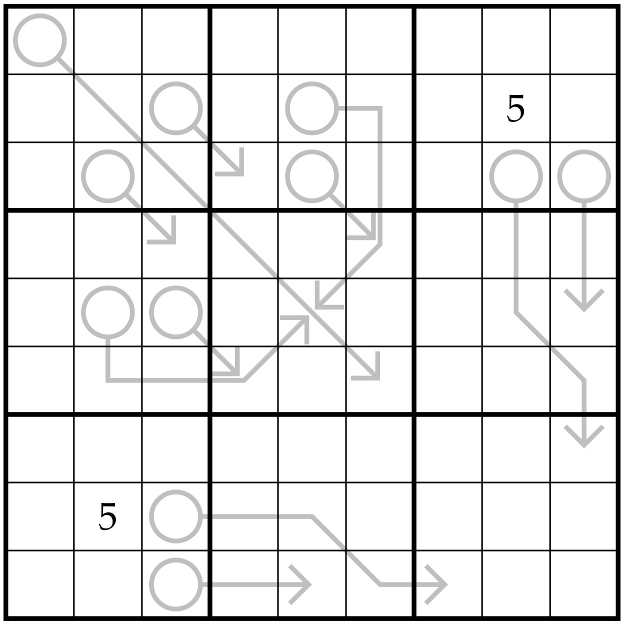
The solver is presented with a collection of sudoku puzzles. Each one is a standard sudoku variant whose rules can easily be found on many puzzle blogs and contests. In this document, the phrasing of the rules will be based on the phrasing from the WPF Sudoku GP.

Aside from the Little Killer Sudoku, which is used to extract the answer to the puzzle, none of the sudoku puzzles has a unique solution. The first step of the puzzle is to find the exact number of solutions to each of the sudoku puzzles.

Arrow Sudoku



Rules

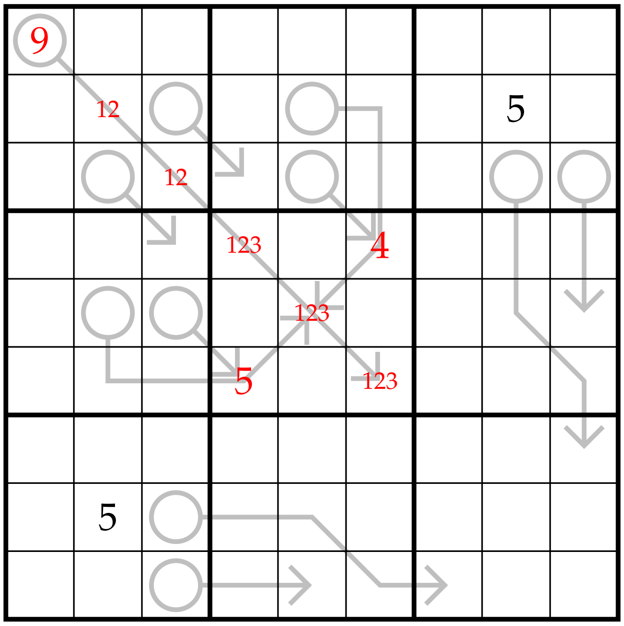
Apply classic sudoku rules. Each digit placed in a cell with a circle must be the sum of the digits placed in the cells that the adjoining arrow passes through. Digits may repeat on arrows.

Counting the solutions

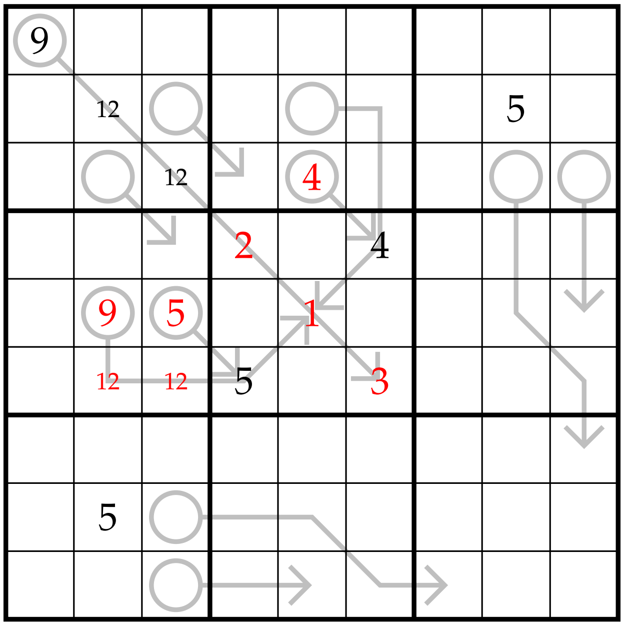
We start by applying some usual Arrow Sudoku deductions. The digit in R1C1 must be 9, with a 12-pair in {R2C2, R3C3} and a 123-triple in {R4C4, R5C5, R6C6}.

Because of the arrows passing through the cells R4C6 and R6C4, neither of them can contain a digit greater than 5. Since there is already a 123-triple in the box, there must be a 45-pair in {R4C6, R6C4}.

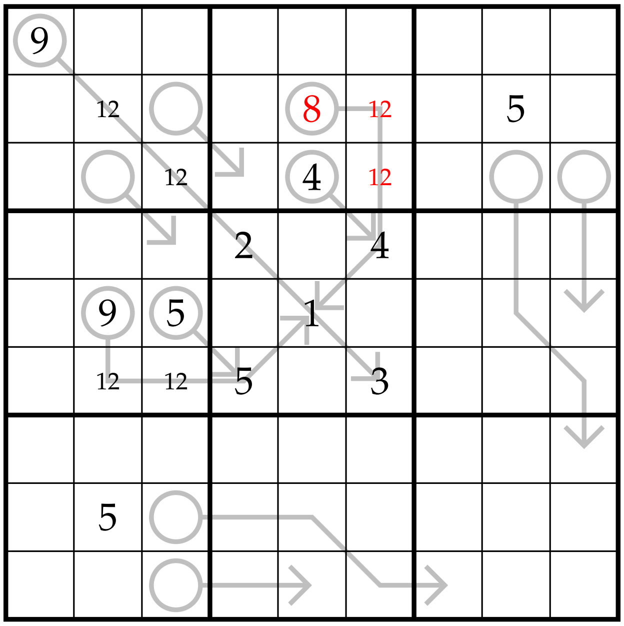
Because the puzzle is reflectionally symmetric in the main diagonal, exactly half of its solutions have 4 in R4C6 and 5 in R6C4, and the other half have 5 in R4C6 and 4 in R6C4. Let us write 4 in R4C6 and 5 in R6C4, count the solutions to the resulting puzzle, and then multiply the result by 2 to find the number of solutions to the original puzzle.



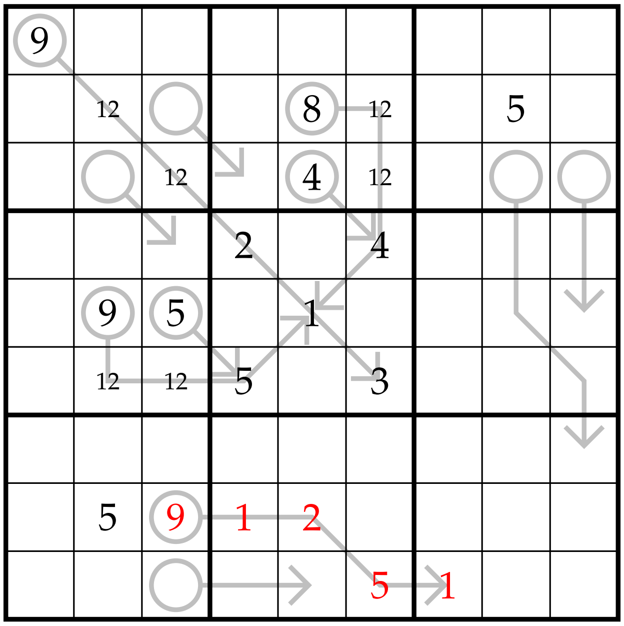
With a 5 in R6C4, the remainder of the arrow must be filled with its minimum possible values to make a sum of 9. We may also use the arrows of length 1 to complete R5C3 and R3C5.



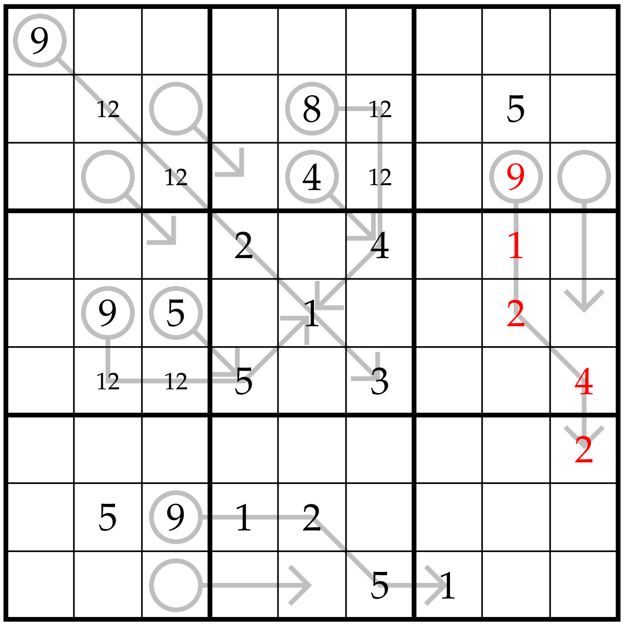
It is impossible to place a digit which is 5 or greater in either of R2C6 or R3C6 because of the arrow passing through those cells. Along with the 3 and the 4 already placed in column 6, this gives a 12-pair in {R2C6, R3C6} and places 8 in R2C5.



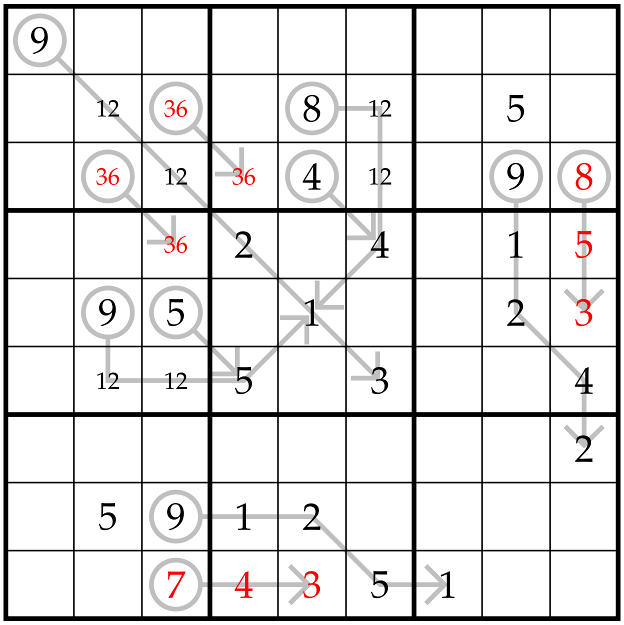
Now, consider the cell R9C6. It cannot be 1, 2, 3, or 4 because those digits already appear in the column. Hence, it must be at least 5, and we can complete the arrow.



Now, consider the cell R6C9. It cannot be 1, 2, or 3 because those digits already appear in the row. Hence, it must be at least 4, and we can complete the arrow, taking into account that R7C9 can no longer be 1.

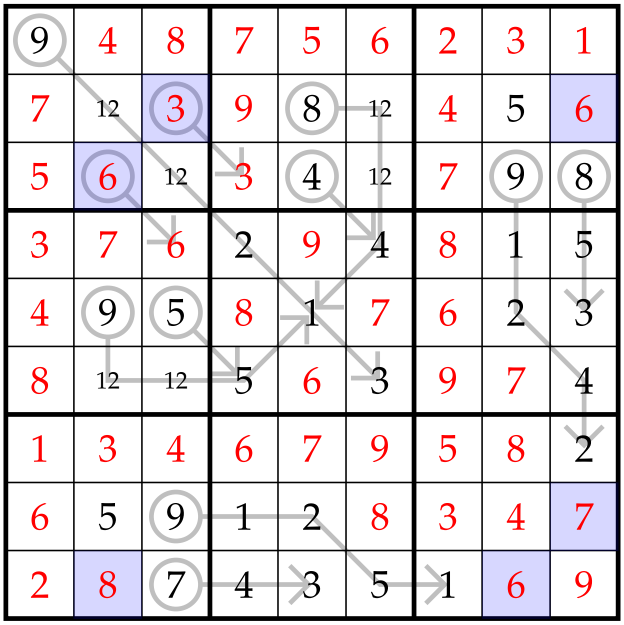


There is now only one way to complete the remaining arrows of length 2. Then, 3 and 6 are the only digits that can appear on the remaining arrows of length 1.



9								
	12	36		8	12		5	
	36	12	36	4	12		9	8
		36	2		4		1	5
	9	5		1			2	3
	12	12	5		3			4
								2
	5	9	1	2				
		7	4	3	5	1		

After applying classic sudoku strategies, including an X-Cycle on 6's in the highlighted cells and some XY-Chains, we arrive at the following grid.



9	4	8	7	5	6	2	3	1
7	12	3	9	8	12	4	5	6
5	6	12	3	4	12	7	9	8
3	7	6	2	9	4	8	1	5
4	9	5	8	1	7	6	2	3
8	12	12	5	6	3	9	7	4
1	3	4	6	7	9	5	8	2
6	5	9	1	2	8	3	4	7
2	8	7	4	3	5	1	6	9

There are two ways to resolve the remaining 12-pairs. Hence the number of solutions to the original puzzle is twice 2, which is $\boxed{4}$.

Alternatively, we could have used the f-puzzles tool to count the solutions to the original puzzle and see that there are exactly 4.

Classic Sudoku

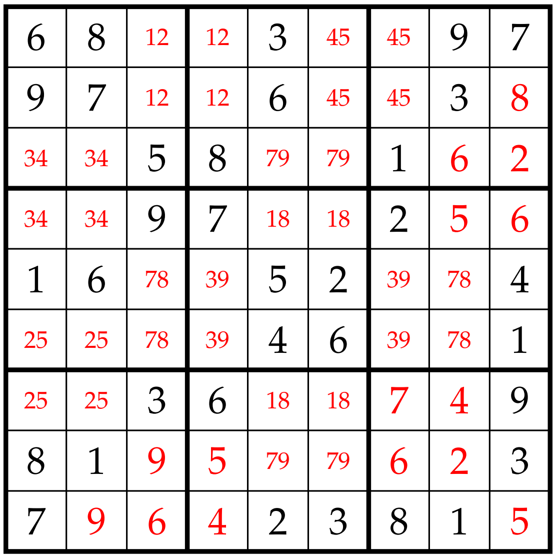
6	8			3			9	7
9	7			6			3	
		5	8			1		
		9	7			2		
1	6			5	2			4
				4	6			1
		3	6					9
8	1							3
7				2	3	8	1	

Rules

Place a digit from 1–9 in each empty cell in the grid such that each row, column, and marked 3×3 box contains each digit exactly once.

Counting the solutions

Applying classic sudoku strategies yields the following grid.



6	8	12	12	3	45	45	9	7
9	7	12	12	6	45	45	3	8
34	34	5	8	79	79	1	6	2
34	34	9	7	18	18	2	5	6
1	6	78	39	5	2	39	78	4
25	25	78	39	4	6	39	78	1
25	25	3	6	18	18	7	4	9
8	1	9	5	79	79	6	2	3
7	9	6	4	2	3	8	1	5

There are eight unique rectangles in this grid, each of which can be resolved in two ways, so the number of solutions is 2^8, which is $\boxed{256}$.

Even Sudoku

■	■		■	■			1	
■	■			1		■	■	
	1		■	■		■	■	
■		■	■		■			
■		■				■		■
			■		■	■		■
	■	■		■	■			1
	■	■			1		■	■
		1		■	■		■	■

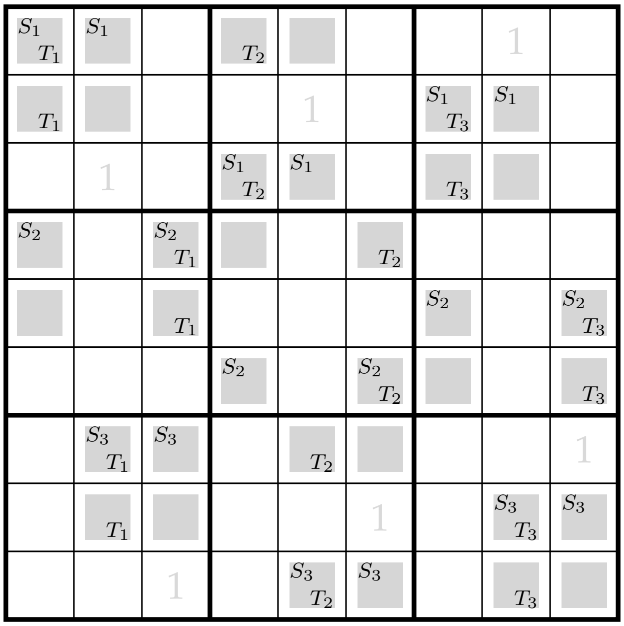
Rules

Apply classic sudoku rules. Digits in squares must be even.

Counting the solutions

We will first find the number A of ways to fill the cells with squares with the digits 2, 4, 6, and 8 in such a way that each of the four digits appears once in each row, column, and box. Then, we will find the number B of ways to fill the cells without squares with the digits 1, 3, 5, 7, and 9 in such a way that each of the five digits appears once in each row, column, and box. The number of solutions to the puzzle is the product $A \times B$.

Let $S_1 \subset \{2, 4, 6, 8\}$ be the set of digits that appear in the cells marked S_1 in the diagram below. Similarly define S_2, S_3, T_1, T_2, and T_3.



▨ S_1 T_1	▨ S_1		▨ T_2	▨			1	
▨ T_1	▨			1		▨ S_1 T_3	▨ S_1	
	1		▨ S_1 T_2	▨ S_1		▨ T_3	▨	
▨ S_2		▨ S_2 T_1	▨		▨ T_2			
▨		▨ T_1				▨ S_2		▨ S_2 T_3
			▨ S_2		▨ S_2 T_2	▨		▨ T_3
	▨ S_3 T_1	▨ S_3		▨ T_2	▨			1
	▨ T_1	▨			1		▨ S_3 T_3	▨ S_3
		1		▨ S_3 T_2	▨ S_3		▨ T_3	▨

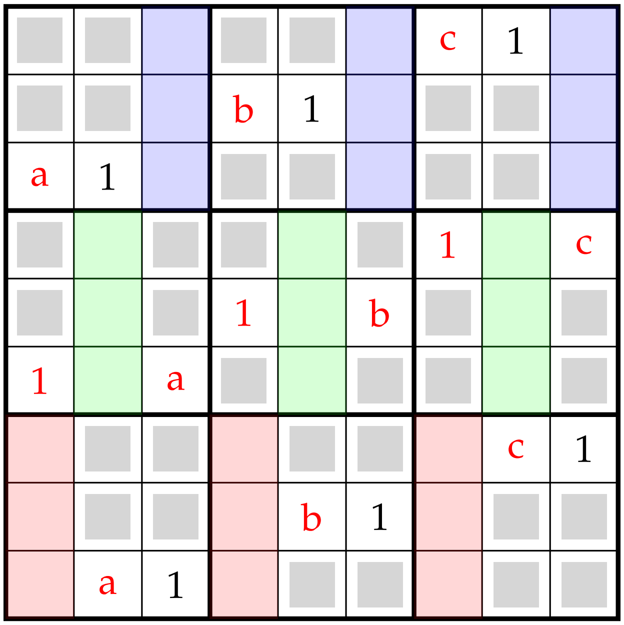
By classic sudoku rules, each of the sets $S_1, S_2, S_3, T_1, T_2, T_3$ contains exactly two elements. By looking at the contents of each of the 3×3 boxes, we see that $|S_i \cap T_j| = 1$ for $1 \leq i, j \leq 3$.

The sets S_1, S_2, S_3, T_1, T_2, T_3 can be any sets satisfying the above conditions, and the digits that appear in the cells with squares are completely determined by S_1, S_2, S_3, T_1, T_2, and T_3. Hence A is equal to the number of choices of two-element subsets $S_1, S_2, S_3, T_1, T_2, T_3 \subset \{2, 4, 6, 8\}$ such that $|S_i \cap T_j| = 1$ for $1 \leq i, j \leq 3$.

Let us call $\{2, 4\}$ and $\{6, 8\}$ the *type-1* subsets of $\{2, 4, 6, 8\}$. In a similar fashion, call $\{2, 6\}$ and $\{4, 8\}$ the *type-2* subsets of $\{2, 4, 6, 8\}$ and call $\{2, 8\}$ and $\{4, 6\}$ the *type-3* subsets of $\{2, 4, 6, 8\}$. The condition that $|S_i \cap T_j| = 1$ for $1 \leq i, j \leq 3$ is equivalent to the condition that none of the sets S_1, S_2, S_3 have the same type as any of the sets T_1, T_2, T_3.

There are 42 ways to choose the types of the sets S_1, S_2, S_3, T_1, T_2, and T_3 in such a way that none of the sets S_1, S_2, S_3 have the same type as any of the sets T_1, T_2, T_3. (This number can easily be found by doing casework on the number of types that appear among S_1, S_2, and S_3.) Given a choice of types, there are 2^6 ways to choose the sets S_1, S_2, S_3, T_1, T_2, and T_3. Hence $A = 42 \times 2^6 = 2688$.

To compute B, we first apply classic sudoku deductions to reach the following grid. Here, the letters a, b, and c represent three digits, not necessarily distinct, from the set $\{3, 5, 7, 9\}$.



■	■	🟦	■	■	🟦	c	1	🟦
■	■	🟦	b	1	🟦	■	■	🟦
a	1	🟦	■	■	🟦	■	■	🟦
■	🟩	■	■	🟩	■	1	🟩	c
■	🟩	■	1	🟩	b	■	🟩	■
1	🟩	a	■	🟩	■	■	🟩	■
🟥	■	■	🟥	■	■	🟥	c	1
🟥	■	■	🟥	b	1	🟥	■	■
🟥	a	1	🟥	■	■	🟥	■	■

Now, we count the ways to fill the cells without squares by casework on the values of a, b, and c. In the below, the term “floor” refers to three boxes in the same three rows of the grid.

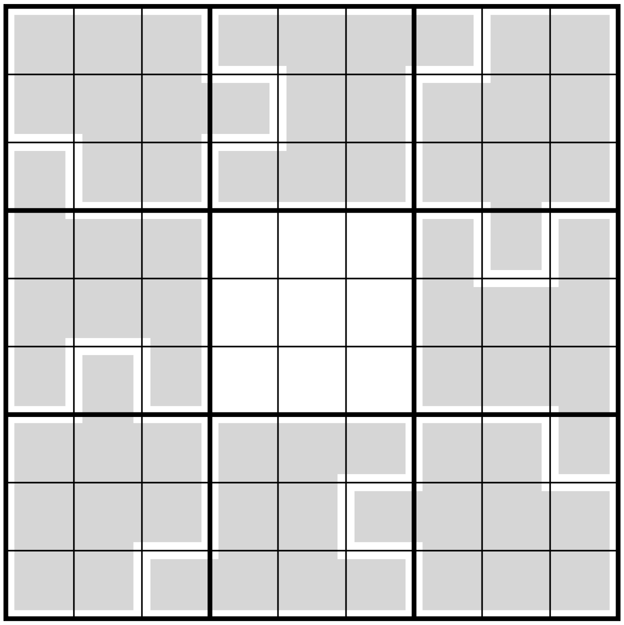
Case 1: In the case that a $=$ b $=$ c, there are four ways to choose the values of a, b, and c. Given a choice of a, b, and c, it is easy to verify that there are 12 ways to fill the highlighted cells in each of the three floors of the grid. So, the number of ways to fill the cells in this case is 4×12^3.

Case 2: In the case that two of a, b, and c are equal and the third is different, there are 36 ways to choose the values of a, b, and c. Given a choice of a, b, and c, it is easy to verify that there are 4 ways to fill the highlighted cells in each of the three floors of the grid. So, the number of ways to fill the cells in this case is 36×4^3.

Case 3: In the case that a, b, and c are distinct, there are 24 ways to choose the values of a, b, and c. Given a choice of a, b, and c, it is easy to verify that there are 4 ways to fill the highlighted cells in each of the three floors of the grid. So, the number of ways to fill the cells in this case is 24×4^3.

Hence $B = 4 \times 12^3 + 36 \times 4^3 + 24 \times 4^3 = 10{,}752$, and the number of solutions to the original puzzle is $A \times B = 2688 \times 10{,}752 = \boxed{28{,}901{,}376}$.

Extra Region Sudoku

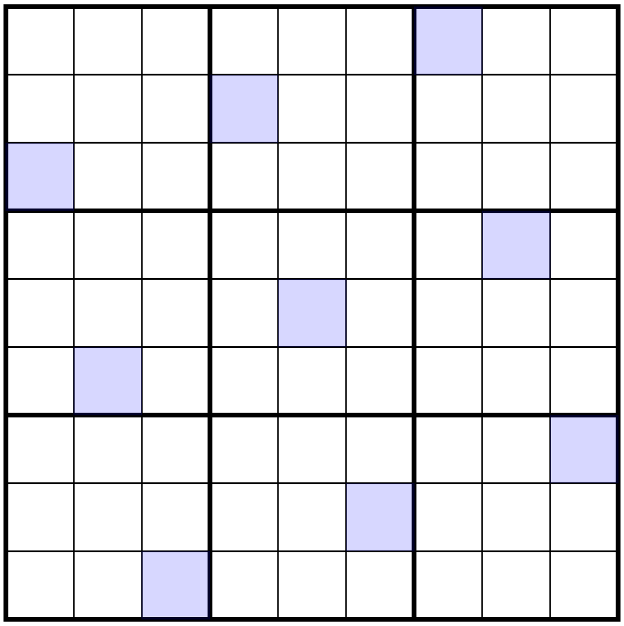


Rules

Apply classic sudoku rules. Each of the shaded regions must also contain each digit from 1–9 exactly once.

Counting the solutions

A completed sudoku grid is a solution to the above puzzle if and only if the highlighted cells in the diagram below all contain the same digit.



It is well-known that there are 6,670,903,752,021,072,936,960 sudoku grids. There are various symmetries that act on those grids. For example, given a sudoku grid, permuting the rows of the grid yields another sudoku grid, as long as the rows stay within the same floor. Similarly, permuting the columns of the grid yields another sudoku grid, as long as the columns stay in the same tower. (Here, the term “floor” refers to three boxes in the same three rows of the grid and the term ”tower” refers to three boxes in the same three columns of the grid.)

There are 6^6 ways to write the digit 1 into some cells of a sudoku grid in such a way that it appears exactly once in each row, column, and box. Each of those 6^6 ways can be obtained from a single one by permuting the rows within a floor or permuting the columns within a tower. By symmetry, each of those 6^6 ways occurs equally often among the 6,670,903,752,021,072,936,960 sudoku grids.

Therefore, in a sudoku grid chosen uniformly at random, the probability that the digit 1 appears in each of the highlighted cells in the above diagram is $\frac{1}{6^6}$. Hence, the probability that each of the highlighted cells in the above diagram contain the same digit is $\frac{9}{6^6}$. In conclusion, the number of solutions to the puzzle is

$$6{,}670{,}903{,}752{,}021{,}072{,}936{,}960 \times \frac{9}{6^6} = \boxed{1{,}286{,}825{,}569{,}448{,}509{,}440}.$$

Frame Sudoku

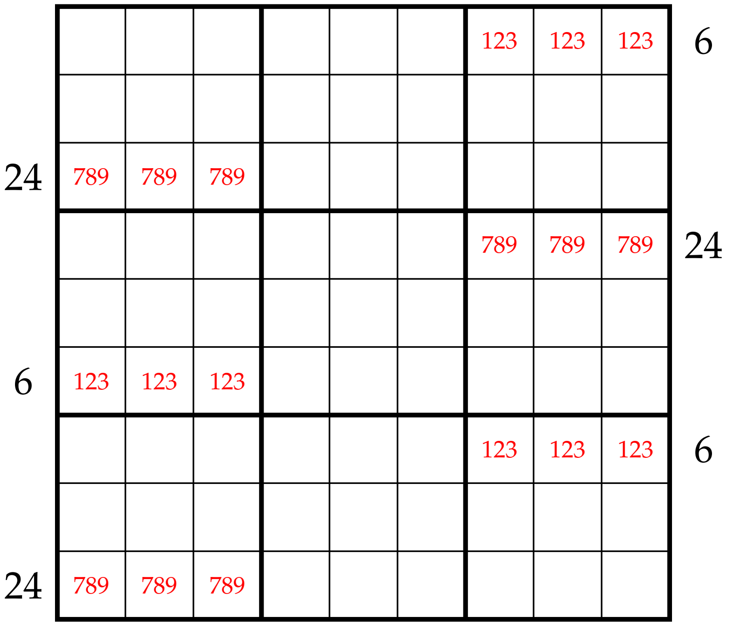
										6
24										
										24
6										
										6
24										

Rules

Apply classic sudoku rules. Numbers outside the grid are equal to the sum of the first 3 digits appearing in the corresponding row or column.

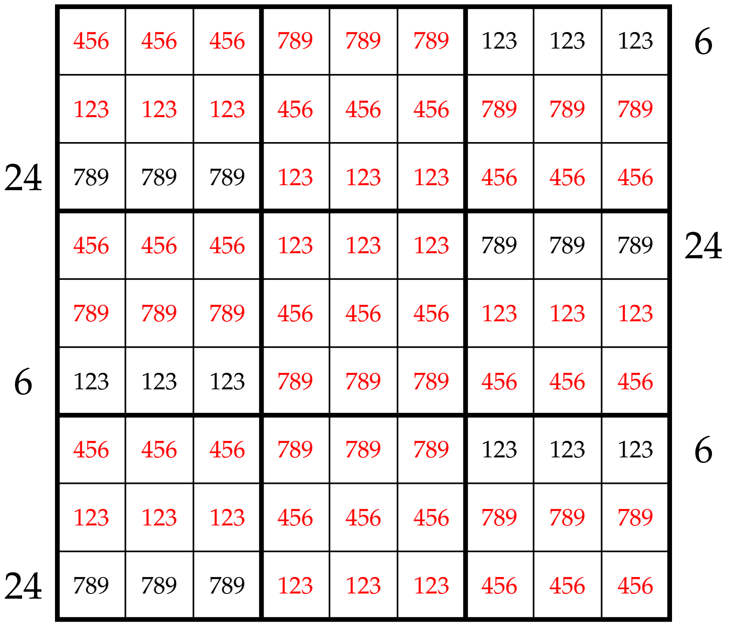
Counting the solutions

To start, the only three digits which sum to 6 are 1, 2, and 3, and the only three digits which sum to 24 are 7, 8, and 9, so we can pencil in some triples.



							123	123	123	6
24	789	789	789							
							789	789	789	24
6	123	123	123							
							123	123	123	6
24	789	789	789							

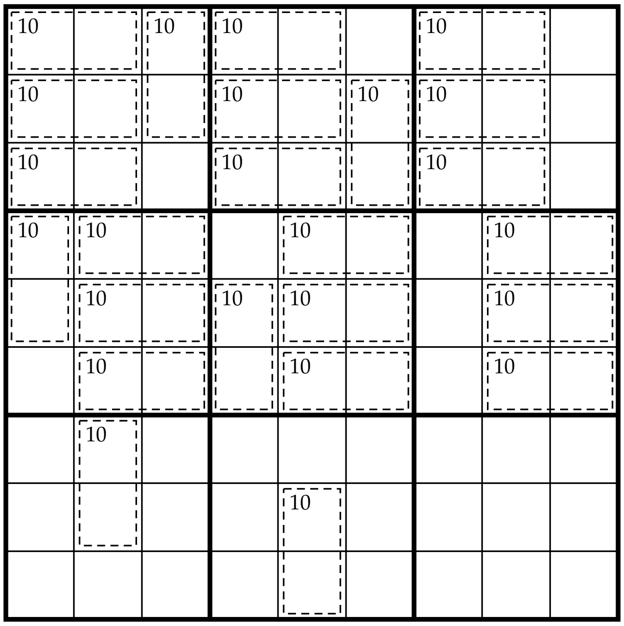
We can now deduce whether each cell has to be from among $\{1,2,3\}$, $\{4,5,6\}$, or $\{7,8,9\}$. For instance, in box 1, we see that $\{1,2,3\}$ have to be in row 2, since row 3 is already taken by $\{7,8,9\}$, and row 1 is ruled out by the $\{1,2,3\}$ in box 3, and then it follows that $\{4,5,6\}$ have to be in row 1 in box 1. We can apply similar logic to fill the whole grid with triples.



	456	456	456	789	789	789	123	123	123	6
	123	123	123	456	456	456	789	789	789	
24	789	789	789	123	123	123	456	456	456	
	456	456	456	123	123	123	789	789	789	24
	789	789	789	456	456	456	123	123	123	
6	123	123	123	789	789	789	456	456	456	
	456	456	456	789	789	789	123	123	123	6
	123	123	123	456	456	456	789	789	789	
24	789	789	789	123	123	123	456	456	456	

From here, we notice that many of the different choices left to complete the grid are independent of each other. First, resolving each of the three triples, $\{1, 2, 3\}$, $\{4, 5, 6\}$, and $\{7, 8, 9\}$, is independent of the other two. Let's focus on $\{1, 2, 3\}$, but the other two triples are nearly identical. We can see that resolving $\{1, 2, 3\}$ in boxes 1, 4, and 7 is independent of resolving them in the rest of the grid. The number of ways to place them in those three boxes is simply the number of 3×3 Latin squares, which is $3! \times 2 = 12$. There are similarly 12 ways to place $\{1, 2, 3\}$ in boxes 2,5, and 8, and 12 ways to place $\{1, 2, 3\}$ in boxes 3,6, and 9, for a total of $12^3 = 1728$ ways to place $\{1, 2, 3\}$. The counts for $\{4, 5, 6\}$, and $\{7, 8, 9\}$ are the same giving a final total of $1728^3 = \boxed{5{,}159{,}780{,}352}$.

Killer Sudoku

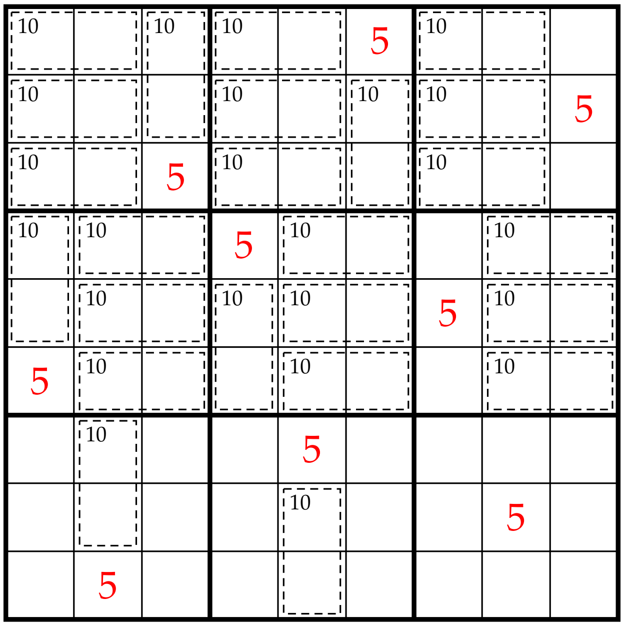


Rules

Apply classic sudoku rules. The digits placed in each marked cage must sum to the total given in its top-left. Digits must not repeat in cages.

Counting the solutions

To begin, we notice that a 5 cannot go in any of the given cages, since otherwise we would have to put a second 5 in the same cage for the two digits to sum to 10. We can thus place all the 5s as follows.

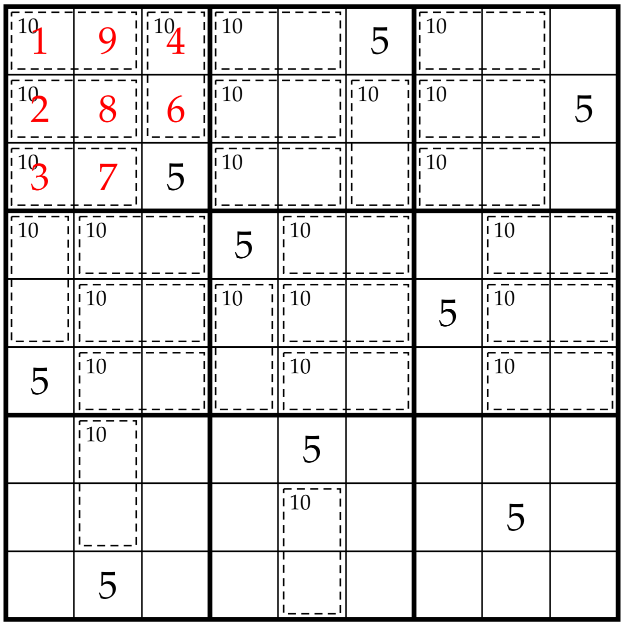


10		10	10		5	10		
10			10		10	10		5
10		5	10			10		
10	10		5	10			10	
	10		10	10		5	10	
5	10			10			10	
	10			5				
				10			5	
	5							

In each cage which sums to 10, we need to place 1 and 9, 2 and 8, 3 and 7, or 4 and 6. Since these cages are the only given constraints, we could permute the digits in any valid solution in the following ways to produce another valid solution:

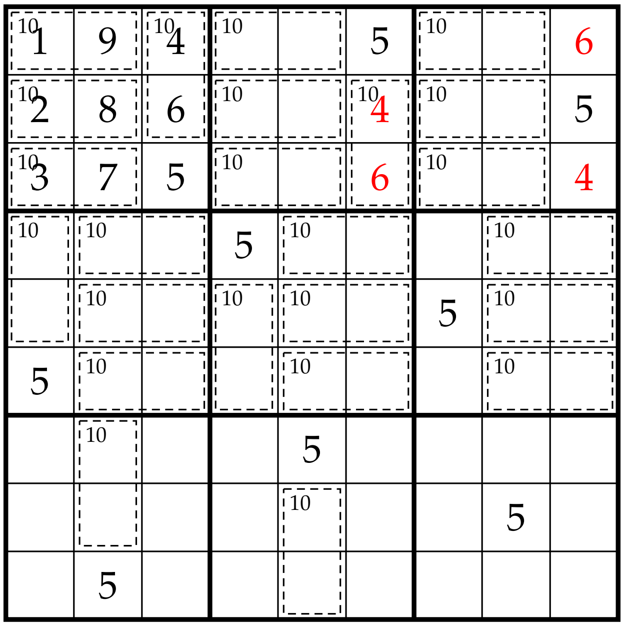
- Swap the two digits in any of the four pairs of digits which sum to 10, e.g. swap all 1s with 9s.
- Permute the four pairs. E.g. we could swap the (1,9) pair with the (2,8) pair by swapping all 1s with 2s and all 9s with 8s.

The number of such permutations is $2^4 \times 4! = 384$. We can thus arbitrarily fix the permutation, e.g. by placing the digits in the top-left box, and then make sure to multiply our final answer by 384.



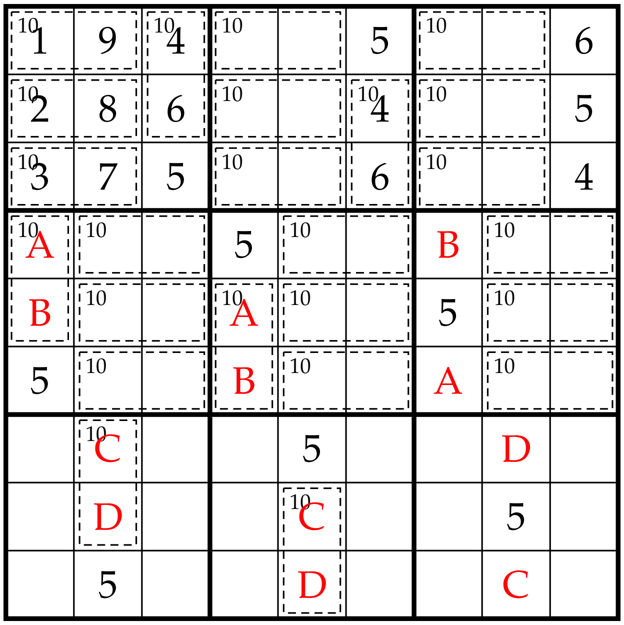
1	9	4			5			
2	8	6						5
3	7	5						
			5					
						5		
5								
				5				
							5	
	5							

From here we can place the digits 4 and 6 in more cells. In box 2, the 4 and 6 we have already placed rule out putting a 4 and 6 in the cages containing R1C4 and R2C4, and we can see that putting them in the cage containing R3C4 would leave no options for placing the 4 and 6 in box 3. We must therefore put them in the cage containing R2C6. We can then place them in box 3 as well.



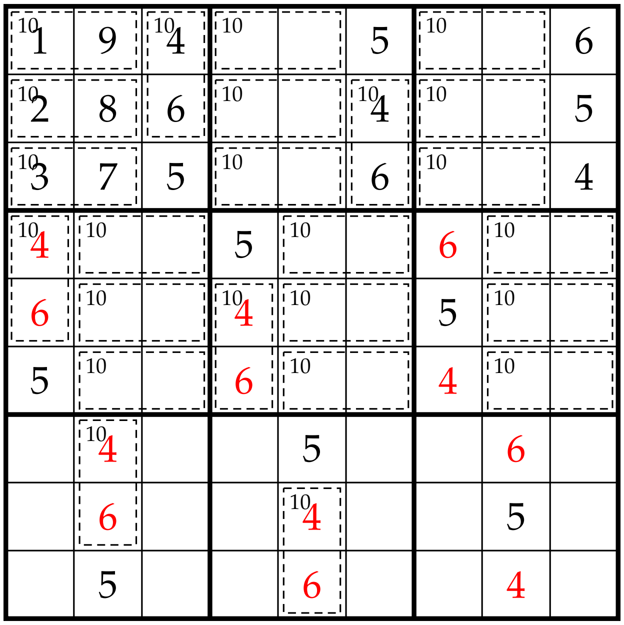
1	9	4			5			6
2	8	6			4			5
3	7	5			6			4
			5					
						5		
5								
				5				
							5	
	5							

From here, we can deduce which cells in boxes 4–9 contain a 4 or 6, but there is some ambiguity about which cells contain 4 and which contain 6. E.g. the 4 and 6 we have placed in box 1 tell us that, in box 4, only the cage containing R4C1 can contain the 4 and 6. Using A, B, C, and D to represent digits that may be a 4 or a 6, we can place them as follows.



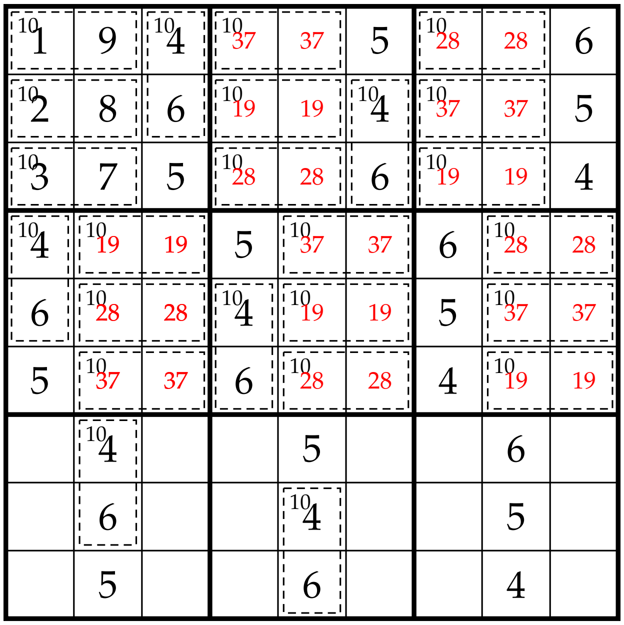
1	9	4			5			6
2	8	6			4			5
3	7	5			6			4
A			5			B		
B			A			5		
5			B			A		
	C			5			D	
	D			C			5	
	5			D			C	

From here, we see that we can freely assign (A,B) to either (4,6) or (6,4), and assign (C,D) to either (4,6) or (6,4), and the solution remains valid, and these decisions do not affect anything else in the grid since the same cells will be occupied by 4s and 6s, and all 4s and 6s will be placed. We may thus arbitrarily assign (A,B) and (C,D) to (4,6), and make sure to multiply our final answer by $2^2 = 4$.



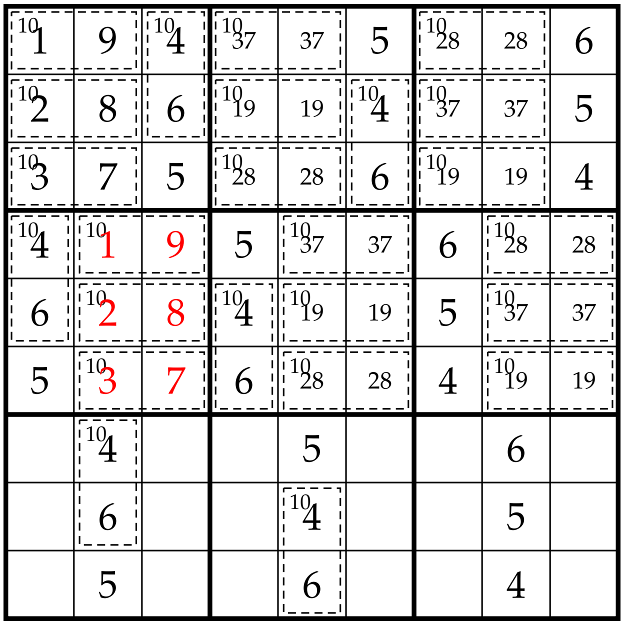
1	9	4			5			6
2	8	6			4			5
3	7	5			6			4
4			5			6		
6			4			5		
5			6			4		
	4			5			6	
	6			4			5	
	5			6			4	

Next, we will consider which pair of digits is contained in each of the cages in boxes 1–6. First, in boxes 2 and 3, we can see that there are two choices, depending on whether 1 and 9 go in the cage containing R2C4 or the cage containing R3C4. We can see that this decision does not impact the rest of the puzzle, the set of digits which appear in each column is unaffected by this decision, so we will arbitrarily pick one and make sure to multiply our final answer by 2. Nearly identically, in boxes 4–6, there are $3! = 6$ choices for which pair of digits goes in each cage in box 4, then 2 more choices for which pair of digits goes in each cage in box 5, and these decisions do not impact the rest of the puzzle, so we pick one decision arbitrarily and make sure to multiply our final answer by 12.



1	9	4	37	37	5	28	28	6
2	8	6	19	19	4	37	37	5
3	7	5	28	28	6	19	19	4
4	19	19	5	37	37	6	28	28
6	28	28	4	19	19	5	37	37
5	37	37	6	28	28	4	19	19
	4			5			6	
	6			4			5	
	5			6			4	

Using column 2, we can resolve the digits in box 4.



10 1	9	10 4	10 37	37	5	10 28	28	6
10 2	8	6	10 19	19	10 4	10 37	37	5
10 3	7	5	10 28	28	6	10 19	19	4
10 4	10 1	9	5	10 37	37	6	10 28	28
6	10 2	8	10 4	10 19	19	5	10 37	37
5	10 3	7	6	10 28	28	4	10 19	19
	10 4			5			6	
	6			10 4			5	
	5			6			4	

From here, some casework seems necessary. Although it is possible to proceed by hand (and indeed, some of our testsolvers did so), we imagine most solvers will use a computer-aided approach, so we want to outline one such approach.

Fortunately, with our given pencil marks, all the killer cages are now satisfied, and so any software which counts the number of solutions to a Classic Sudoku (taking pencil marks into account) is sufficient. There are many options for what program to use, and what casework to do. We will outline one which uses `https://www.f-puzzles.com/`. When using this site, make sure to

- Use “Mode: Setting” rather than “Mode: Solving” in the top-right corner.
- In the Settings (second button in the top-left; if you don’t see it, first click on “≪” next to “Solve”),
  - Make sure “Treat Pencil Marks as Given” is checked, and
  - Slide the “Solution Count Limit” slider all the way to 1000.

With these settings, the site will count the number of solutions for you when you click “Solution Count” (the fourth button on the far right; if you don’t see it, first click on “≫” next to “Solve”), but only if the number of solutions is at most 1000.

The current state of our puzzle is here: `https://f-puzzles.com/?id=y9cyaqq4`. Unfortunately, the solution count here gives us the unhelpful message “This puzzle has at least 1000 solutions.” One way to deal with this is to manually try the different cases for box 2, and then get a solution count for each one. Here they are, along with their solution counts from the site:

- `https://f-puzzles.com/?id=y9rcove3`, 576 solutions.

- `https://f-puzzles.com/?id=ybwotlt6`, 480 solutions.
- `https://f-puzzles.com/?id=y8vzuflt`, 480 solutions.
- `https://f-puzzles.com/?id=y97tkfqe`, 480 solutions.
- `https://f-puzzles.com/?id=ycecelfw`, 480 solutions.
- `https://f-puzzles.com/?id=y88xgzmg`, 480 solutions.
- `https://f-puzzles.com/?id=y77ob92s`, 480 solutions.
- `https://f-puzzles.com/?id=yb7wmoae`, 576 solutions.

(The fact that the first and last cases have the same answer, and the other 6 cases have the same answer, is not too hard to see by a symmetry argument. Noticing this reduces the number of cases we have to solve from the eight above to only two.)

We can now compute our final answer, remembering to multiply by all the factors from earlier:

$$384 \times 4 \times 2 \times 12 \times (2 \times 576 + 6 \times 480) = \boxed{148{,}635{,}648}.$$

Search 9 Sudoku

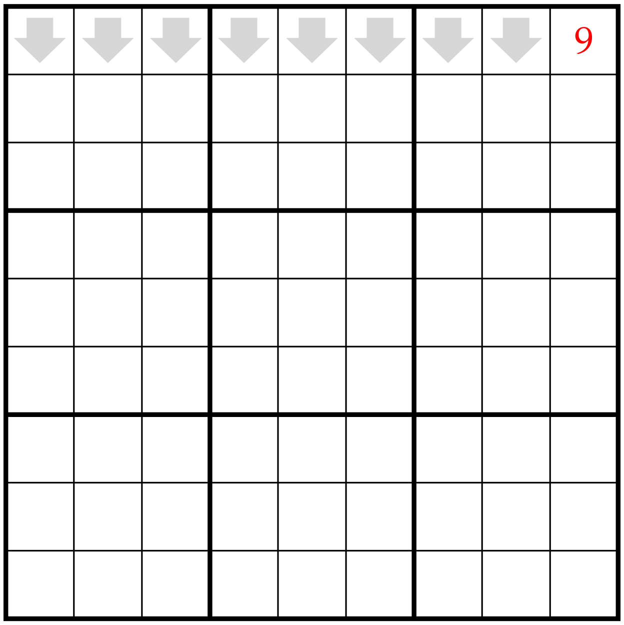
↓	↓	↓	↓	↓	↓	↓	↓	

Rules

Apply classic sudoku rules. Some arrows are marked in the grid. Each arrow points to the digit 9 in the respective row or column. The digit in the cell with the arrow is the distance from this cell to the corresponding digit 9.

Counting the solutions

First, there is only one place to write the digit 9 in row 1.



Choose a sudoku grid uniformly at random. The probability that there is a 9 in R1C9 is $\frac{1}{9}$. Given that there is a 9 in R1C9, the probability that the digits in R1C1–8 match the distance to the 9 is $\frac{1}{8!}$.

Hence, the probability that a sudoku grid chosen uniformly at random is a solution to the given puzzle is $\frac{1}{9} \times \frac{1}{8!}$. The number of sudoku grids is well-known to be $6{,}670{,}903{,}752{,}021{,}072{,}936{,}960$. So the number of solutions to the given puzzle is

$$6{,}670{,}903{,}752{,}021{,}072{,}936{,}960 \times \frac{1}{9} \times \frac{1}{8!} = \boxed{18{,}383{,}222{,}420{,}692{,}992}.$$

Skyscraper Sudoku

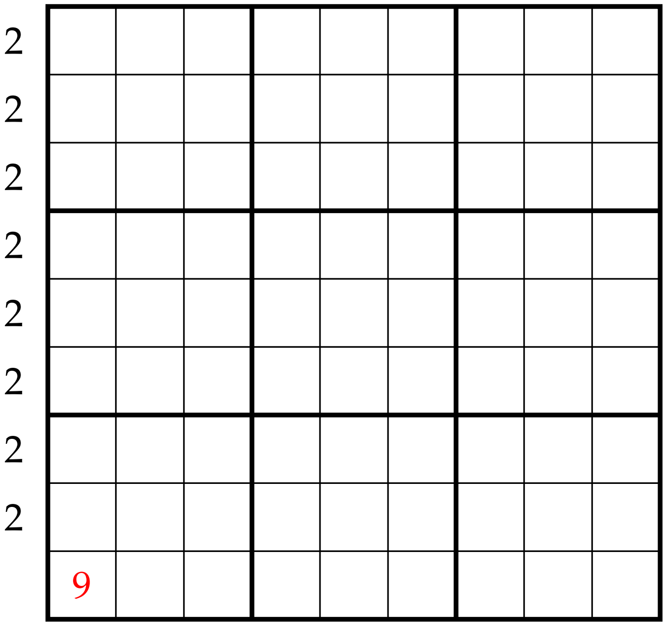
2									
2									
2									
2									
2									
2									
2									
2									

Rules

Apply classic sudoku rules. Each digit represents the height of a building. The clues outside the grid indicate the number of buildings in the corresponding direction. A taller building will hide any shorter buildings behind it.

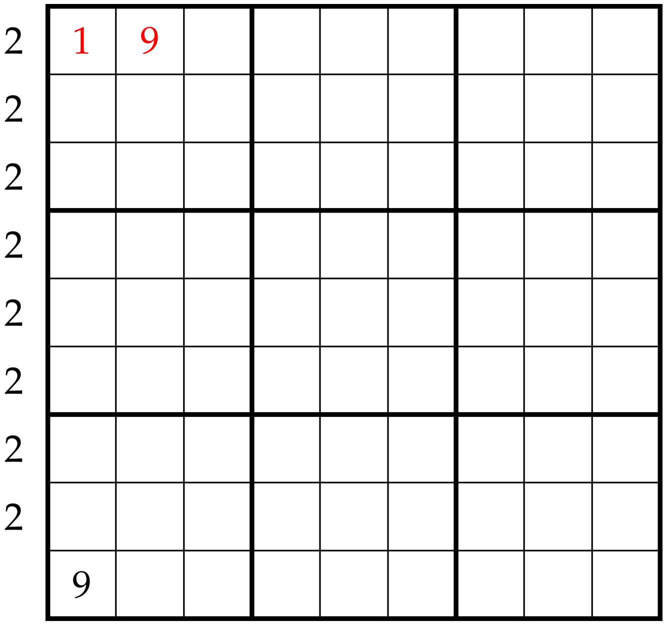
Counting the solutions

First, in column 1, the 9 must be in R9C1, since otherwise it would violate one of the skyscraper clues.



2									
2									
2									
2									
2									
2									
2									
2									
	9								

Next, let's consider the location of the 1 in column 1. Its row will have a '2' skyscraper clue, and so we must put the 9 in its row immediately adjacent to it in row 2. Since box 7 already has a 9 in it, we know this must be in one of the first six rows. We can see by symmetry that each of the six rows results in the same final number of solutions, so let's place this in row 1, and remember to multiply our final answer by 6.



2	1	9							
2									
2									
2									
2									
2									
2									
2									
	9								

We next consider the location of the 2 in column 1. Because of the ‘2’ skyscraper clue, the digit in column 2 in that row must be a 1 or a 9, but since we’ve already placed a 9 in column 2, it must be a 1. Then, again because of the ‘2’ skyscraper clue, the digit in column 3 in that row must be a 9. This 9 rules out boxes 1 and 7, so we know this must be in row 4, 5, or 6. Again by symmetry, the final solution count will be the same in the three cases, so let’s place this in row 4 and remember to multiply our final answer by 3.

2	1	9							
2									
2									
2	2	1	9						
2									
2									
2									
2									
	9								

We continue with similar logic. Whichever row has a 3 in column 1 must then have 2, 1, 9 in columns 2, 3, 4, respectively (since the digits we’ve already placed prevent a 1 or 9 from going in column 2, and prevent a 9 from going in column 3). The digits already in boxes 1 and 4 mean this has to be in row 7 or 8. We will pick row 7, and remember to multiply our final answer by 2.

2	1	9							
2									
2									
2	2	1	9						
2									
2									
2	3	2	1	9					
2									
	9								

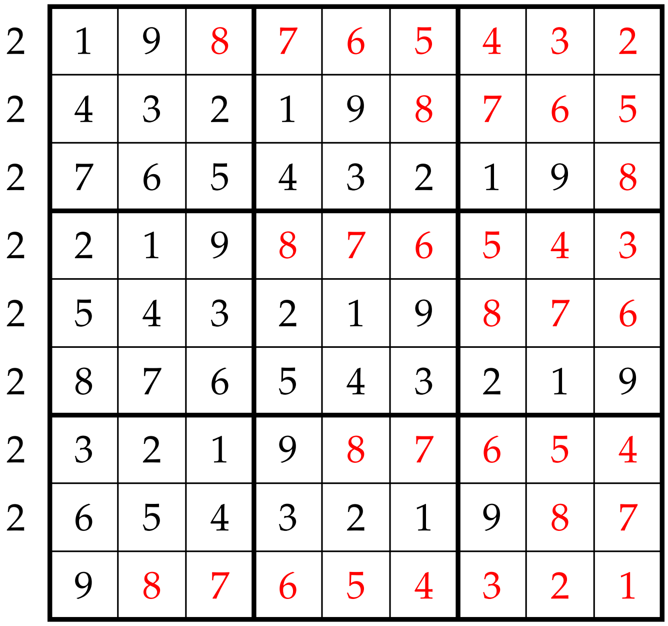
There are similarly two choices for the row with a 4 in column 1, and two choices for the row with a 5 in column 1. We place them in rows 2 and 5, respectively, and remember to multiply our final answer by $2 \times 2 = 4$.

2	1	9							
2	4	3	2	1	9				
2									
2	2	1	9						
2	5	4	3	2	1	9			
2									
2	3	2	1	9					
2									
	9								

Now there is only one choice each for which row will have a 6, 7, and 8 in column 1.

2	1	9							
2	4	3	2	1	9				
2	7	6	5	4	3	2	1	9	
2	2	1	9						
2	5	4	3	2	1	9			
2	8	7	6	5	4	3	2	1	9
2	3	2	1	9					
2	6	5	4	3	2	1	9		
	9								

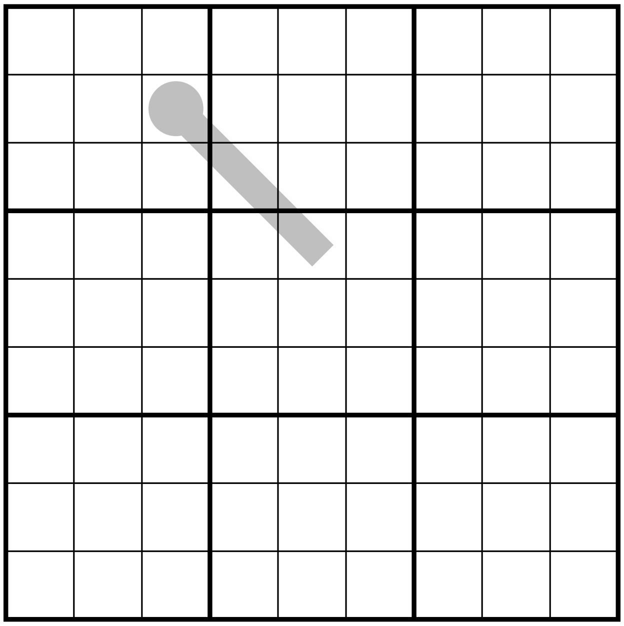
From here, we can fill in the rest of the grid using classic Sudoku logic. In fact, we'll find that filling in columns from 2 to 9 is not too hard since the digits we just placed in each column result in naked singles in the next column.



2	1	9	8	7	6	5	4	3	2
2	4	3	2	1	9	8	7	6	5
2	7	6	5	4	3	2	1	9	8
2	2	1	9	8	7	6	5	4	3
2	5	4	3	2	1	9	8	7	6
2	8	7	6	5	4	3	2	1	9
2	3	2	1	9	8	7	6	5	4
2	6	5	4	3	2	1	9	8	7
	9	8	7	6	5	4	3	2	1

We've completed the grid! Interestingly, each row is a cyclic permutation of 9, 8, 7, 6, 5, 4, 3, 2, 1. We can multiply the number of choices from previous steps to get our final answer: $6\times3\times2\times4 = \boxed{144}$.

Thermo Sudoku



Rules

Apply classic sudoku rules. Starting at the “bulb”, digits placed along each marked thermometer must form a strictly increasing sequence.

Counting the solutions

Choose a sudoku grid uniformly at random. Let A, B, and C be the digits in R2C3, R3C4, and R4C5, respectively. The sudoku grid is a solution to the above puzzle if and only if $A < B < C$.

		A						
			B					
				C				

It is well-known that there are 6,670,903,752,021,072,936,960 completed sudoku grids, so the number of solutions to the above puzzle is

$$6{,}670{,}903{,}752{,}021{,}072{,}936{,}960 \times \Pr[A < B < C].$$

It remains to compute $\Pr[A < B < C]$.

Using symmetry (in particular, the symmetries of permuting the rows within a floor and permuting the columns within a tower), we may compute:

$$\begin{aligned} \Pr[A = B] &= \frac{1}{6} \\ \Pr[B = C] &= \frac{1}{6} \\ \Pr[A = C] &= \frac{1}{9} \\ \Pr[A = B = C] &= \frac{1}{36} \end{aligned}$$

Then

$$\begin{aligned} \Pr[A < B < C] &= \frac{1}{3!} \Pr[A, B, \text{and } C \text{ are distinct}] \\ &= \frac{1}{3!} \left(1 - \Pr[A = B] - \Pr[B = C] - \Pr[A = C] + 2 \Pr[A = B = C]\right) \\ &= \frac{1}{3!} \left(1 - \frac{1}{6} - \frac{1}{6} - \frac{1}{9} + 2 \times \frac{1}{36}\right) \\ &= \frac{11}{108}. \end{aligned}$$

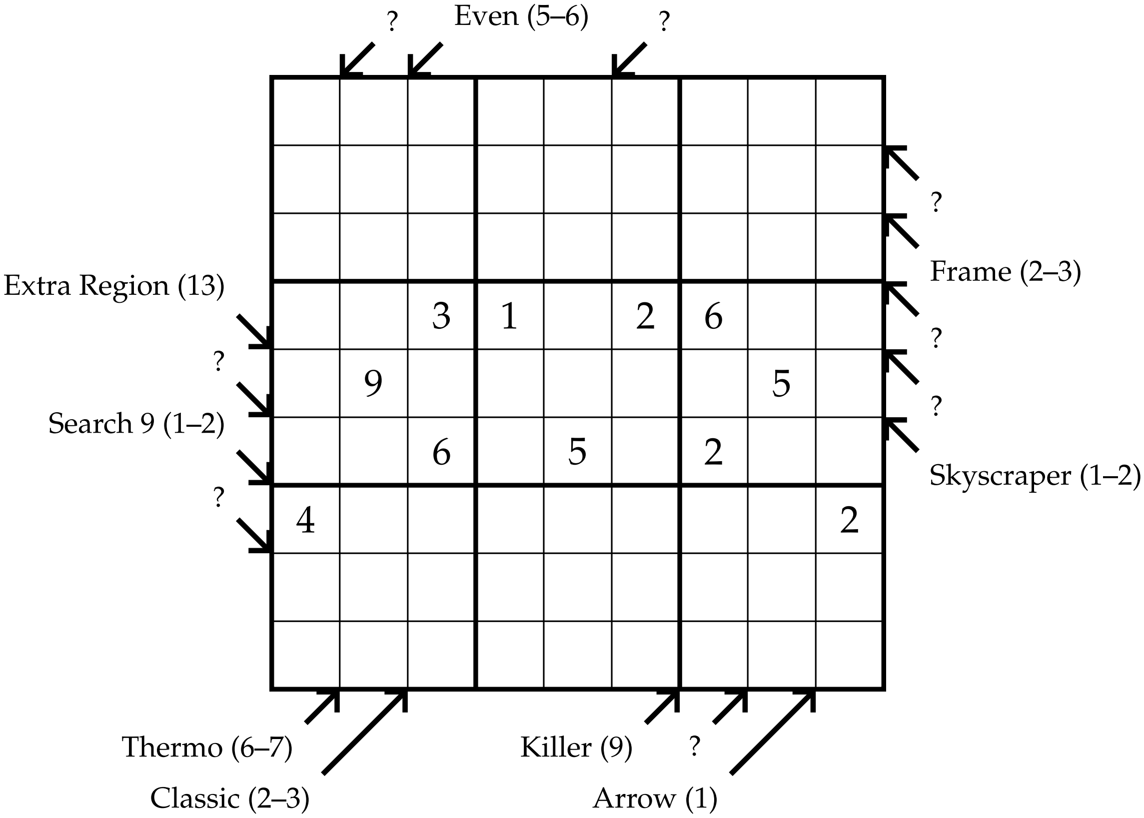
So the number of solutions to the above puzzle is

$$6{,}670{,}903{,}752{,}021{,}072{,}936{,}960 \times \frac{11}{108} = \boxed{679{,}443{,}900{,}668{,}812{,}984{,}320}.$$

Little Killer Sudoku

(Digits may repeat along any diagonal.)

This Little Killer Sudoku is used to extract the final answer to the puzzle.



		3	1		2	6		
	9						5	
		6		5		2		
4								2

Rules

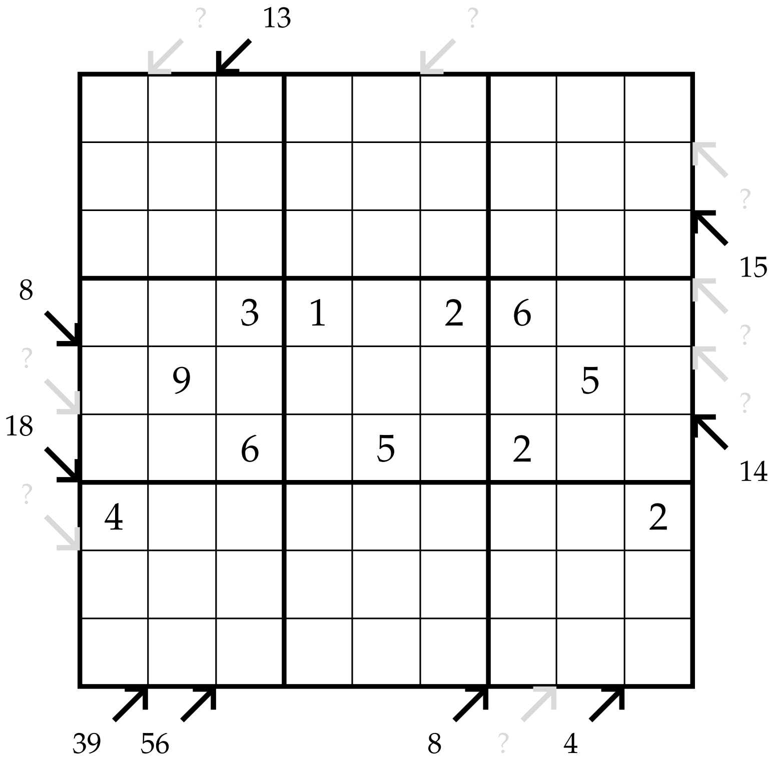
Apply classic sudoku rules. The clues outside the grid indicate the sum of the digits contained in the cells in the direction of the corresponding arrow.

Completing the puzzle

First, recall the number of solutions to the previous parts of the puzzle.

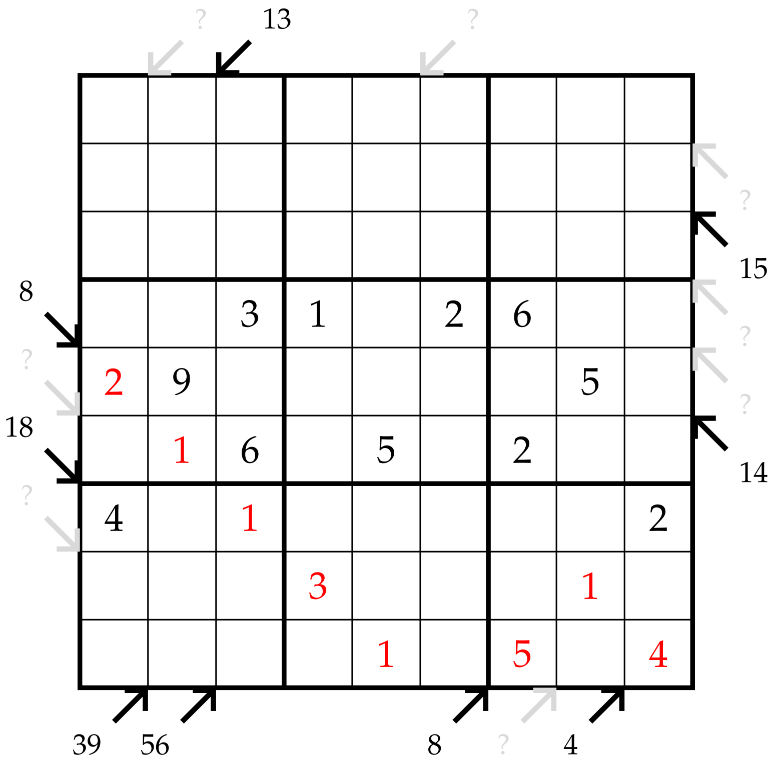
Arrow Sudoku	4
Classic Sudoku	256
Even Sudoku	28,901,376
Extra Region Sudoku	1,286,825,569,448,509,440
Frame Sudoku	5,159,780,352
Killer Sudoku	148,635,648
Search 9 Sudoku	18,383,222,420,692,992
Skyscraper Sudoku	144
Thermo Sudoku	679,443,900,668,812,984,320

Other than the clues marked "?", which we ignore for now, we may replace each clue outside the grid with a numerical clue by using the provided numbers and ranges as indices into the numbers in the above table.



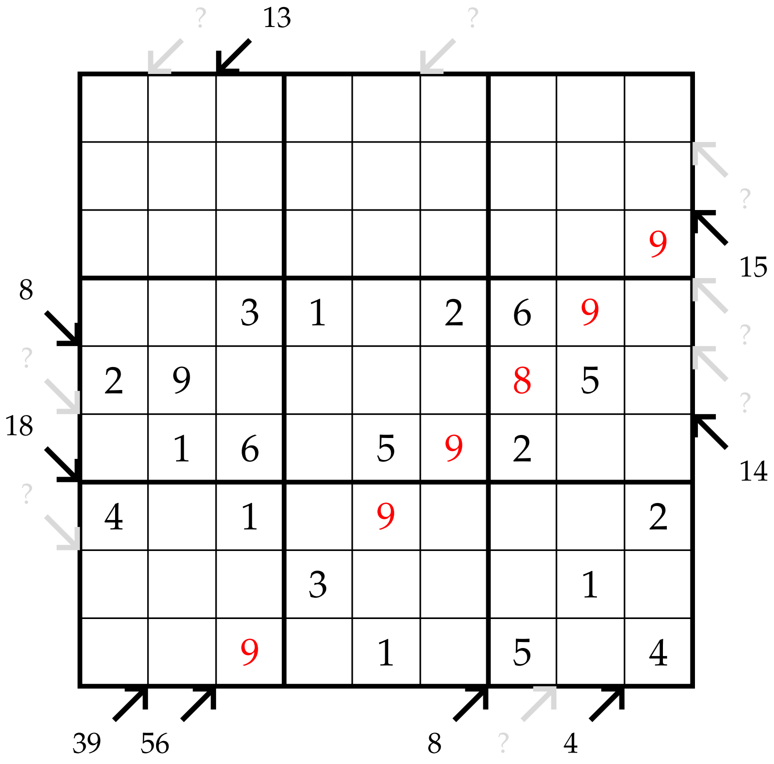
		3	1		2	6		
	9						5	
		6		5		2		
4								2

Now, we solve the resulting puzzle as a Little Killer Sudoku. Using the given digits, there is only one way to complete the two diagonals with an 8 clue and the diagonal with a 4 clue.



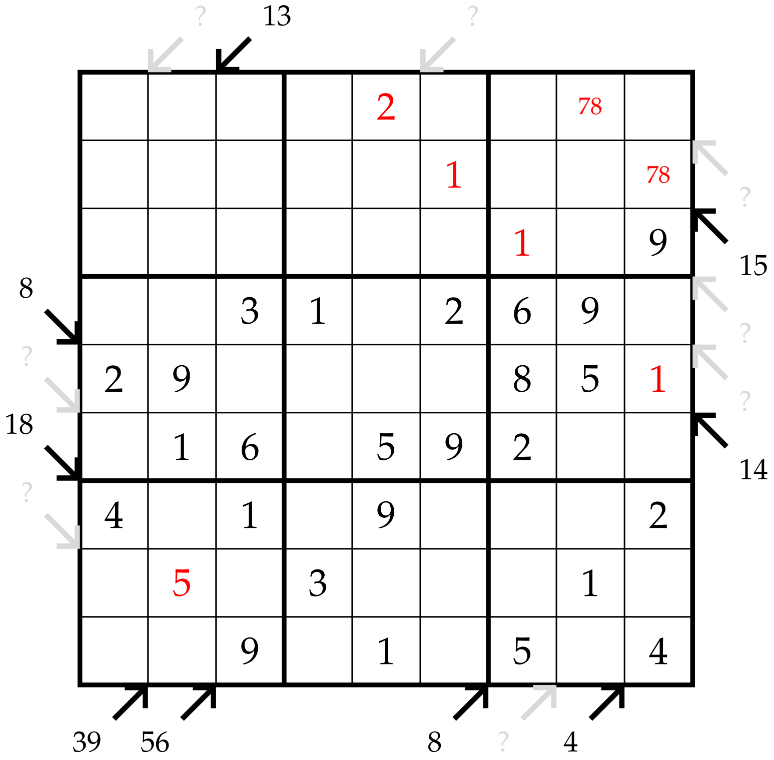
		3	1		2	6		
2	9						5	
	1	6		5		2		
4		1						2
			3				1	
				1		5		4

To complete the diagonal with a 56 clue, we must fill the empty cells along that diagonal with the maximum possible digits.



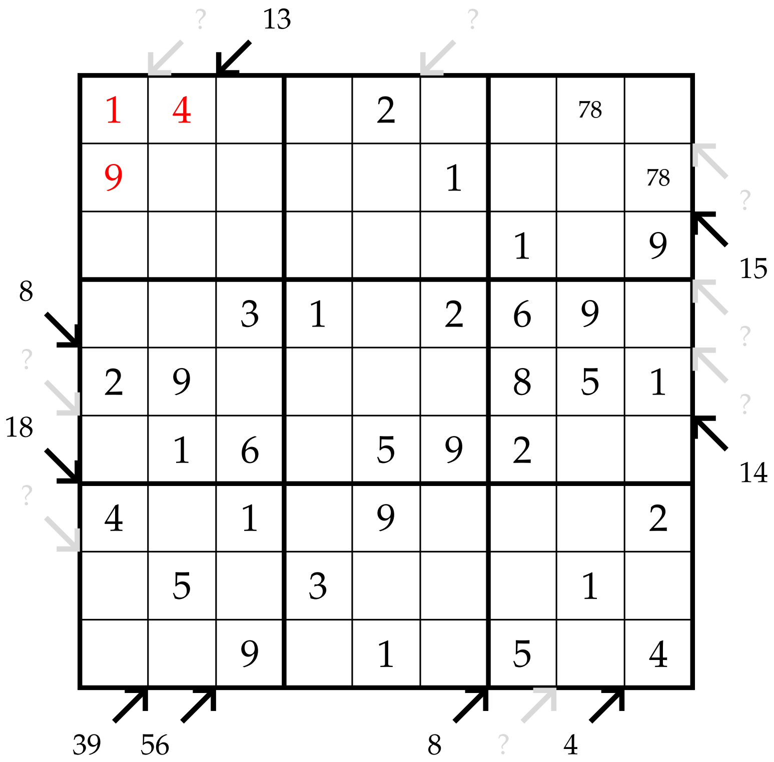
								9
		3	1		2	6	9	
2	9					8	5	
	1	6		5	9	2		
4		1		9				2
			3				1	
		9		1		5		4

We may now complete the diagonal with an 18 clue. To complete the diagonal with a 14 clue, we must fill the empty cells along that diagonal with the minimum possible digits. To complete the diagonal with a 15 clue, we must fill the cells along that diagonal with the maximum possible digits.



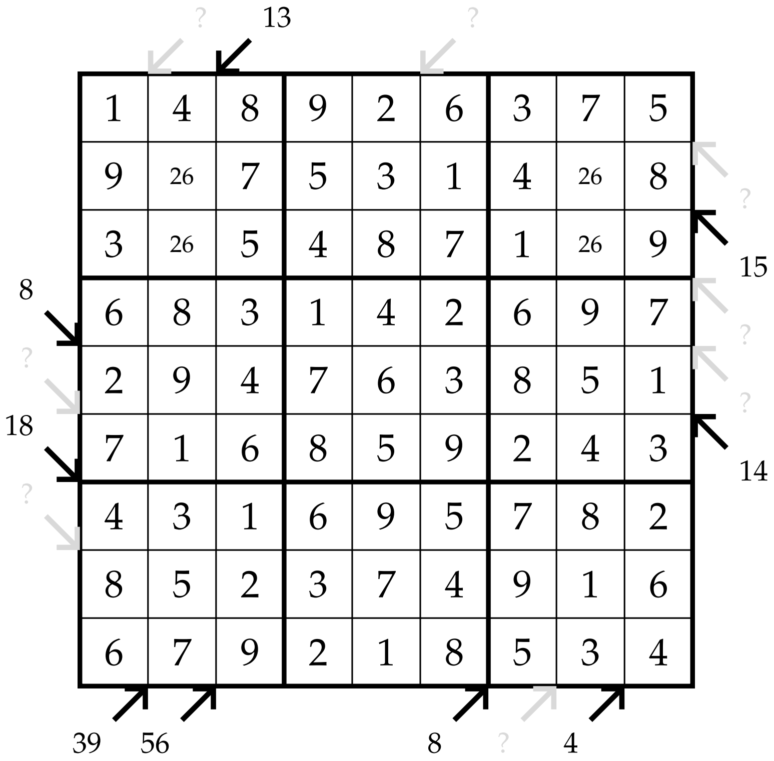
				2			78	
					1			78
						1		9
		3	1		2	6	9	
2	9					8	5	1
	1	6		5	9	2		
4		1		9				2
	5		3				1	
		9		1		5		4

Now R1C1 is the only possible location for 1 in box 1. Then R2C1 is the only possible location for 9 in box 1. We may then complete the diagonal with a 13 clue.



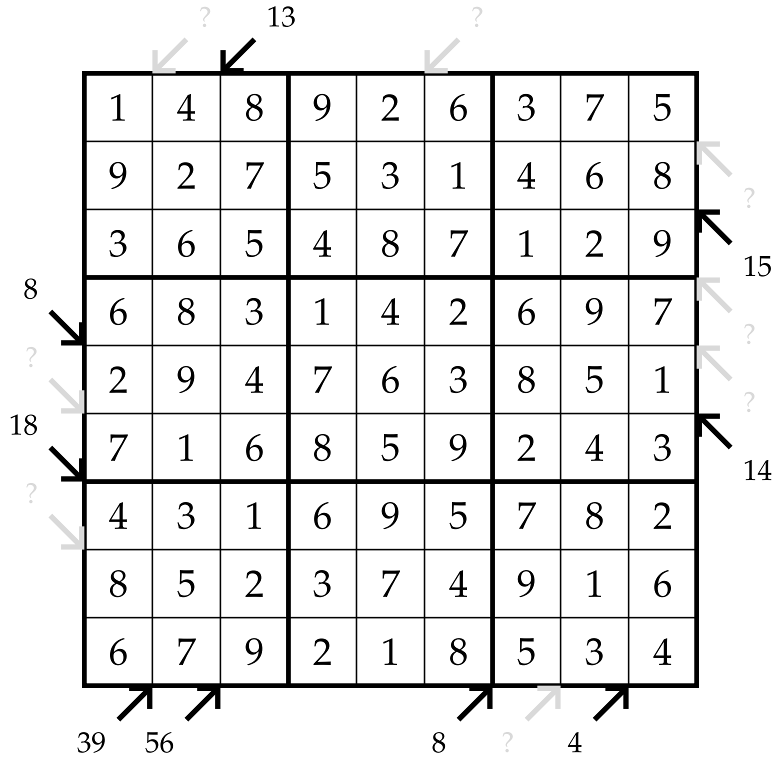
1	4			2			78	
9					1			78
						1		9
		3	1		2	6	9	
2	9					8	5	1
	1	6		5	9	2		
4		1		9				2
	5		3				1	
		9		1		5		4

Using normal sudoku rules, we arrive at the following grid.



1	4	8	9	2	6	3	7	5
9	26	7	5	3	1	4	26	8
3	26	5	4	8	7	1	26	9
6	8	3	1	4	2	6	9	7
2	9	4	7	6	3	8	5	1
7	1	6	8	5	9	2	4	3
4	3	1	6	9	5	7	8	2
8	5	2	3	7	4	9	1	6
6	7	9	2	1	8	5	3	4

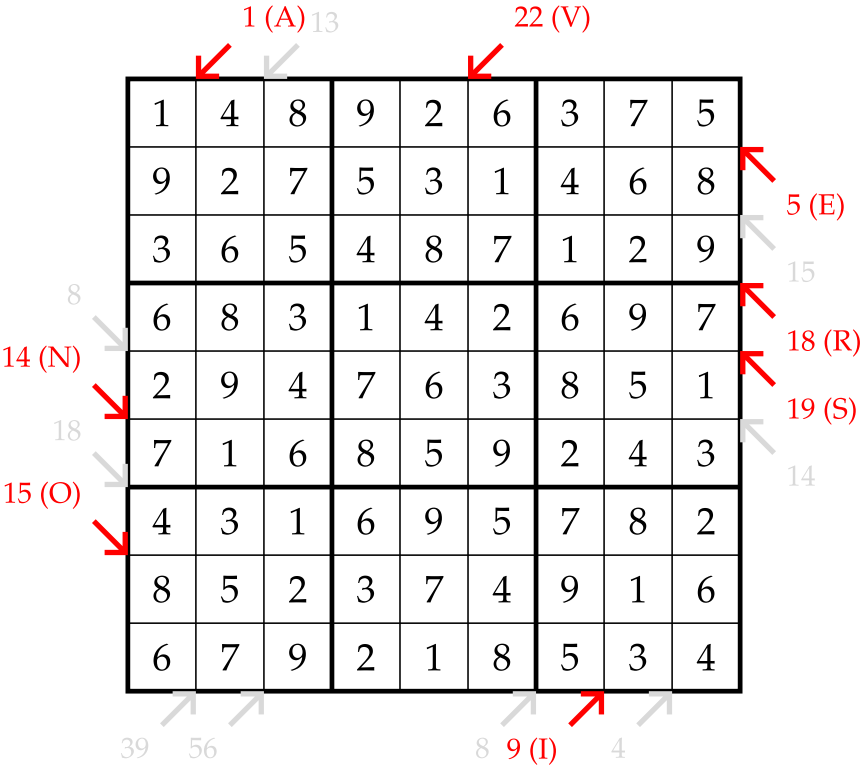
Finally, we use the remaining clue to resolve the rectangle in rows 2 and 3 and columns 2 and 8.



1	4	8	9	2	6	3	7	5
9	2	7	5	3	1	4	6	8
3	6	5	4	8	7	1	2	9
6	8	3	1	4	2	6	9	7
2	9	4	7	6	3	8	5	1
7	1	6	8	5	9	2	4	3
4	3	1	6	9	5	7	8	2
8	5	2	3	7	4	9	1	6
6	7	9	2	1	8	5	3	4

All of the clues outside the grid are necessary to reach the unique solution to the Little Killer Sudoku, but it is still possible to narrow the solution grid to a few possibilities if one or two of the clues are missing.

Now, we turn to the diagonals marked “?”. We compute the sum of the digits along each of those diagonals. Converting those sums to letters and reading them in clockwise order yields the answer to the puzzle: **AVERSION**.



1	4	8	9	2	6	3	7	5
9	2	7	5	3	1	4	6	8
3	6	5	4	8	7	1	2	9
6	8	3	1	4	2	6	9	7
2	9	4	7	6	3	8	5	1
7	1	6	8	5	9	2	4	3
4	3	1	6	9	5	7	8	2
8	5	2	3	7	4	9	1	6
6	7	9	2	1	8	5	3	4