Future Schiff-Moment Calculations

in Radioactive Nuclei

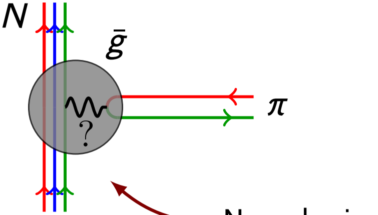
J. Engel

June 29, 2021

How Diamagnetic Atoms Get EDMs, Roughly

Because SM *CP* violation is so weak, an additional undiscovered source is required to explain why there is so much more matter than antimatter.

The source can work its way into nuclei through CP-violating πNN vertices (in chiral EFT)...



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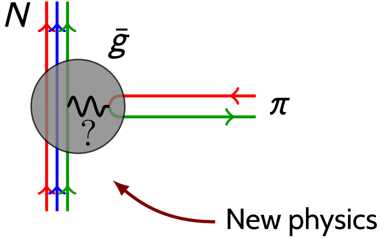
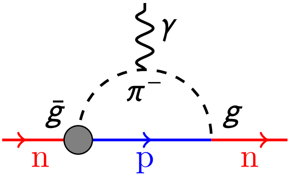


leading, e.g. to a neutron EDM…



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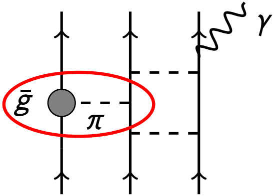

…and to a nuclear EDM from the nucleon EDM or a *T*-violating *NN* interaction:

$$V_{PT} \propto \bar{g}\, (\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2) \cdot (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)\, \frac{\exp\left(-m_\pi |\boldsymbol{r}_1 - \boldsymbol{r}_2|\right)}{m_\pi |\boldsymbol{r}_1 - \boldsymbol{r}_2|} + \text{contact terms/etc.}$$

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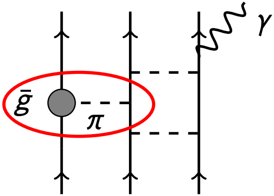

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Atoms get EDMs from nuclei. But electronic shielding replaces nuclear dipole operator with "Schiff operator,"

$$S \propto \sum_p \left(r_p^2 - \frac{5}{3}R_{\text{ch}}^2\right) z_p + \ldots\,,$$

making relevant nuclear quantity the Schiff moment:

$$\langle S \rangle = \sum_m \frac{\langle 0|\, S\, |m\rangle \langle m|\, V_{PT}\, |0\rangle}{E_0 - E_m} + c.c.$$

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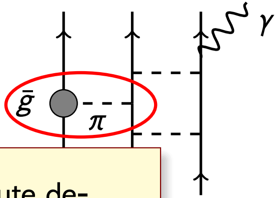

$V_{PT} \propto$ … ns/etc.

Job of nuclear-structure theory: compute dependence of $\langle S \rangle$ on the three $\bar{g}$'s (and on the contact-term coefficients and nucleon EDM).

It's up to QCD/EFT to compute the dependence of the $\bar{g}$ vertices on fundamental sources of *CP* violation.

See talks by Vincenzo, Jordy, Michael.

Atom… s

nuclea…

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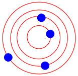
Nuclear Models in One Slide



Starting point is always a mean field

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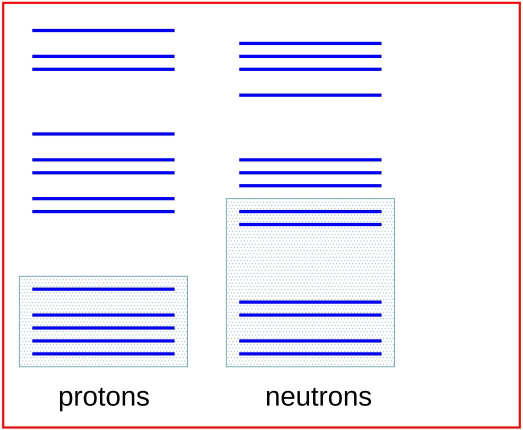

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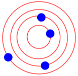

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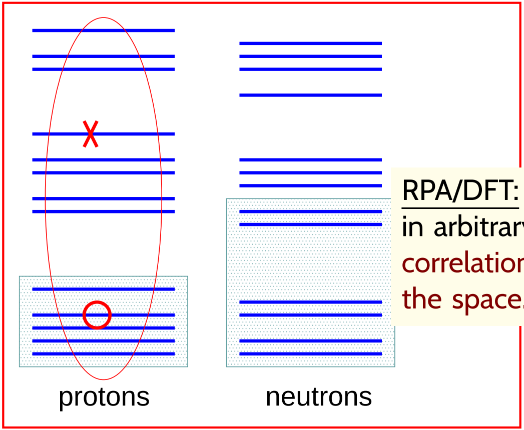

RPA/DFT: Large single-particle spaces in arbitrary single mean field; simple correlations and excitations within the space.

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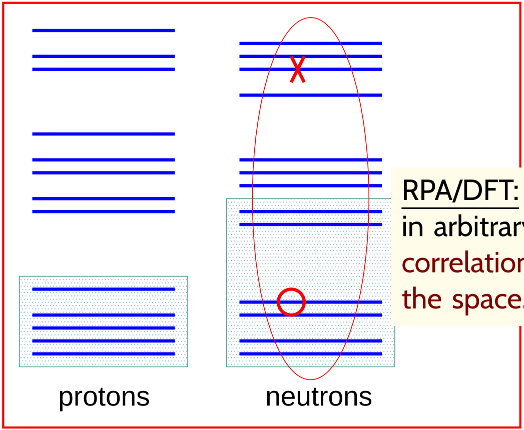

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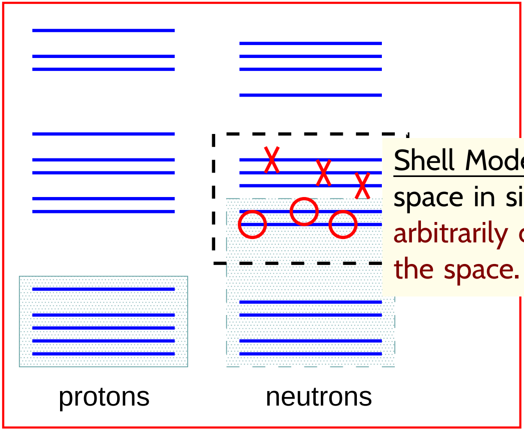

Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

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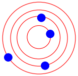

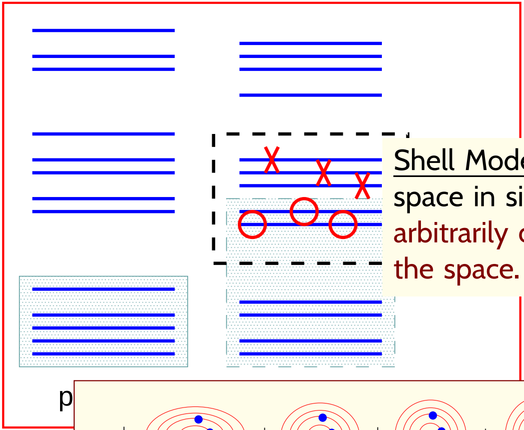

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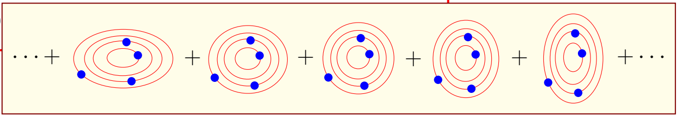

Generator-Coordinate Method (GCM): extension of DFT that mixes many mean-field states with different collective properties.

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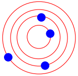

All such models require phenomenological Hamiltonian/operators, with coefficients fit to energies/transitions in heavy nuclei.

This is a problem if you're looking at operators such as V_{PT}, for which there are no data.

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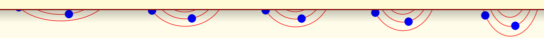

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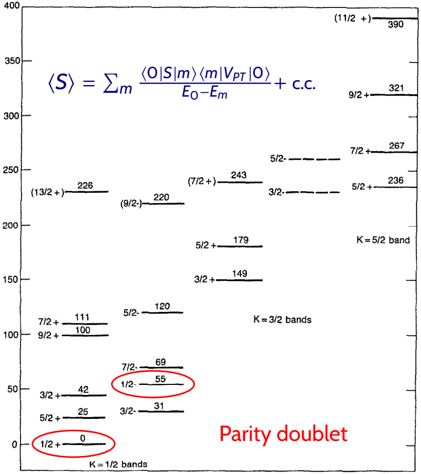
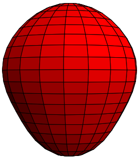
^{225}Ra and Other Light Actinides

Octupole Physics and DFT



Unlike in other nuclei, these two states are the whole story.



Deformed density

Two members of the parity doublet correspond to the same intrinsic mean-field state:

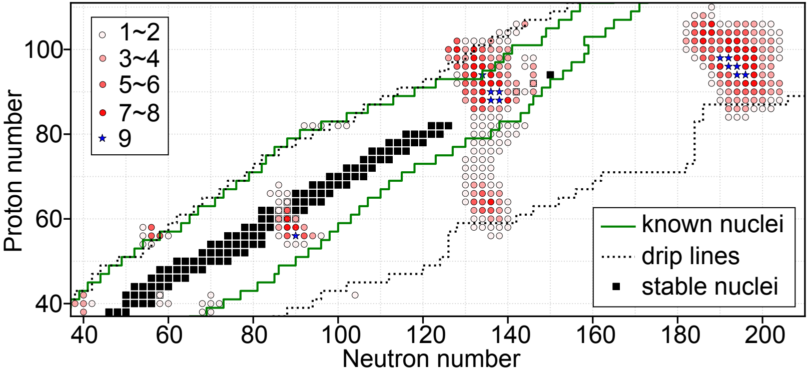
$$\left|\frac{1}{2}^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle \pm |\bigcirc\rangle\right)$$

and, to good approximation,

$$\langle S\rangle \approx \frac{2}{3}\frac{\overbrace{\langle\bigcirc|\,S_z\,|\bigcirc\rangle}^{\langle S\rangle_{\text{intr.}}}\langle\bigcirc|\,V_{PT}\,|\bigcirc\rangle}{E_+ - E_-}$$

Octupole Systematics

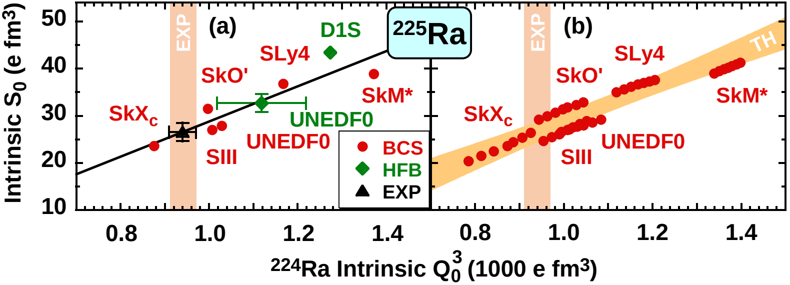
More DFT



From Cao et al., arXiv:2004.01319

See talk by W. Nazarewicz?

Correlation of ^{225}Ra $\langle S \rangle_{\mathrm{intr.}}$ with ^{224}Ra Octupole Defm.



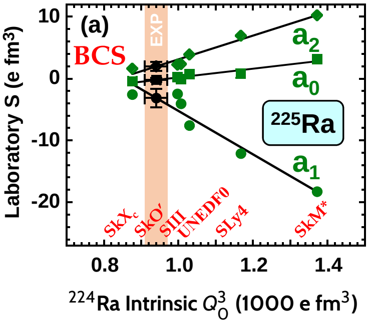
J. Dobaczewski, JE, M. Kortelainen, P. Becker

Correlation with octupole moment of ^{225}Ra even better.

That moment will be determined at ANL.

Observables also correlated in other octupole-deformed actinides.

Implications of Correlation for Lab Schiff Moment



Looks good, but situation is more complicated when we include octupole moments in other nuclei. The resulting a_i for ^{225}Ra:

isoscalar	isovector	isotensor
-0.4 – 0.8	-2 – -8	2 – 5

Range doesn't include systematic uncertainty.

Reducing Uncertainty of Lab Moments

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Can we find measurements that will constrain its matrix element?

- In one-body approximation $V_{PT} \approx \vec{\sigma} \cdot \vec{\nabla}\rho$. The closest simple one body operator is $\vec{\sigma} \cdot \vec{r}$.

  Can we measure $\langle 1/2^- | \vec{\sigma} \cdot \vec{r} \, | 1/2^+ \rangle$ or something like it, in some octupole-deformed nucleus?

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- What about charge-changing transition strength to isobar analog of $|1/2^-\rangle$ in ^{225}Fr? Axial-charge β decays in other nuclei?

  V_{PT} is similar to two-body current operator in axial-charge channel.

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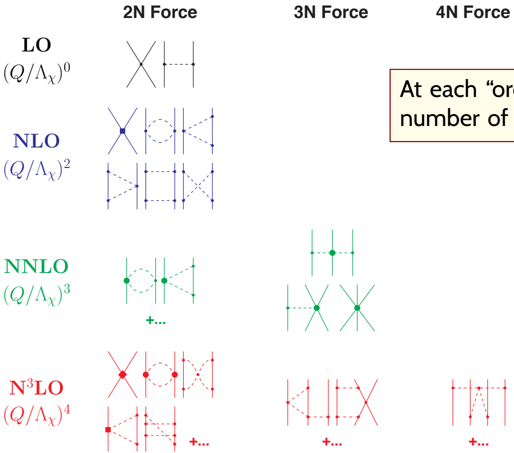
The other option is an ab-initio calculation with controlled errors!

Ab Initio Nuclear Structure

Starts with chiral effective-field theory

Nucleons, pions sufficient below chiral-symmetry breaking scale.
Expansion of operators in powers of Q/Λ_χ.

$Q = m_\pi$ or typical nucleon momentum.



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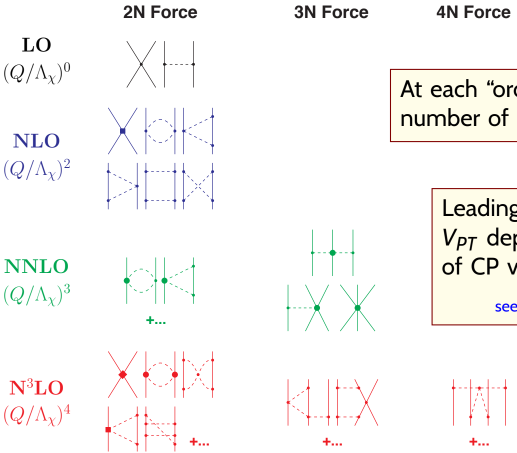

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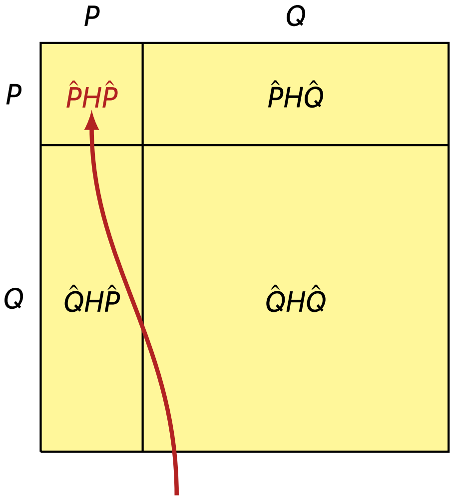
Leading-order terms in V_{PT} depend on source of CP violation.

see Jordy's talk.

Ab Initio Many-Body Methods

Partition of Full Hilbert Space



Simpler calculation done here.

P = "reference" space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing most important eigenvalues.

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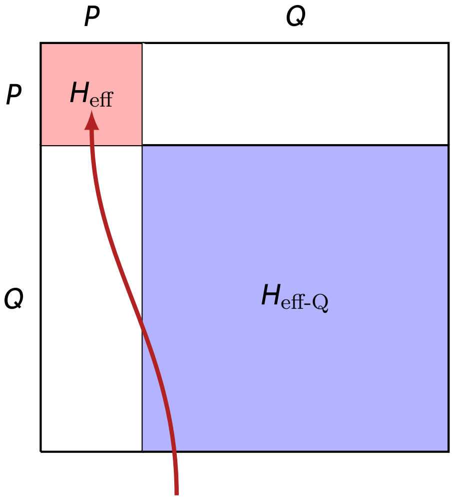

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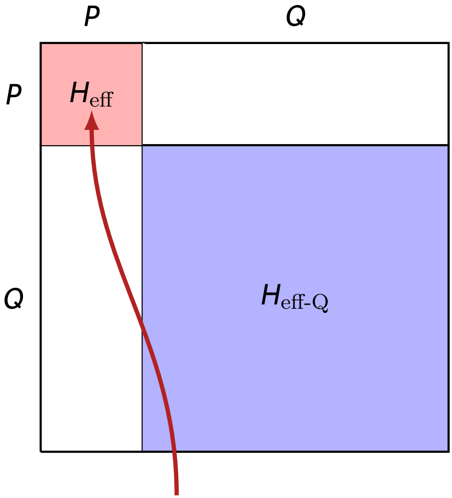

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Must must apply same unitary transformation to transition operator.

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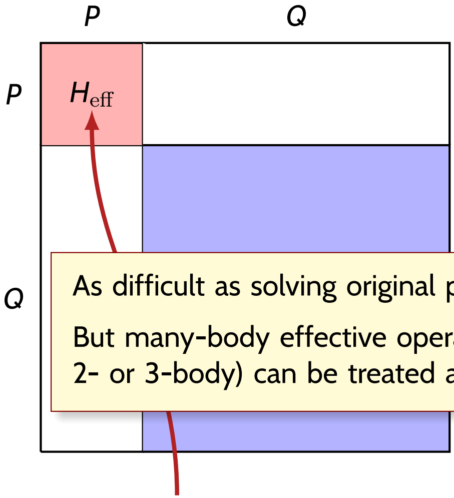

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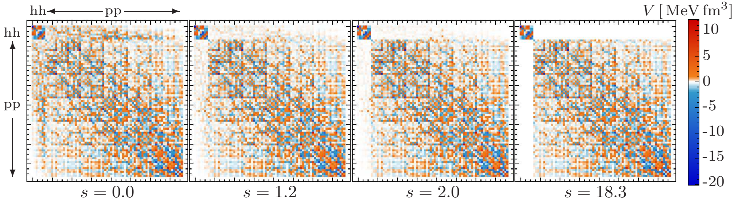
As difficult as solving original problem.

But many-body effective operators (beyond 2- or 3-body) can be treated approximately.

In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian.
Gradually decouples reference space.



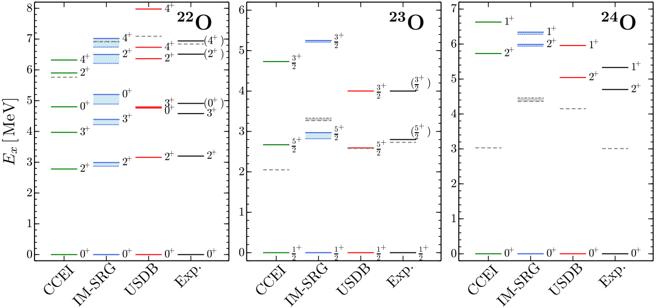
from H. Hergert

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]\,, \qquad \eta(s) = [H_d(s), H_{od}(s)]\,, \qquad H(\infty) = H_{\text{eff}}$$

Trick is to keep all 1- and 2-body terms in H at each step *after normal ordering* (**IMSRG-2**, includes most important parts of 3, 4-body …terms).

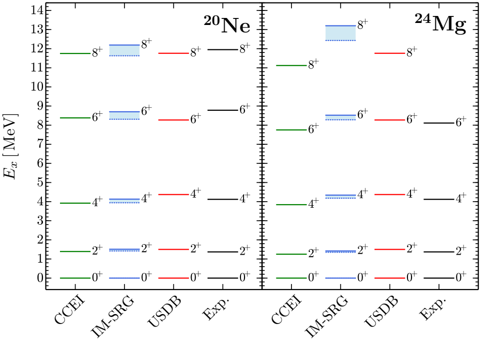
If reference space is a single state, end up with g.s. energy. If, e.g., it is a valence space, get effective shell-model interaction and operators.

Ab Initio Calculations of Spectra



Neutron-rich oxygen isotopes

Deformed nuclei



Reference State with Collective Correlations

Background: Generator Coordinate Method

Construct set of mean fields by constraining coordinate(s), e.g. quadrupole moment $\langle Q_0 \rangle$. Then diagonalize H in space of symmetry-restored quasiparticle vacua with different $\langle Q_0 \rangle$.

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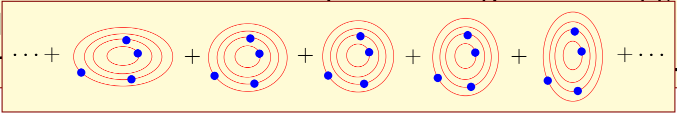
quad
symm .



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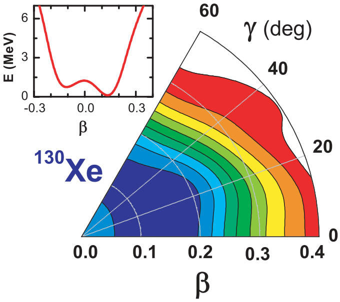
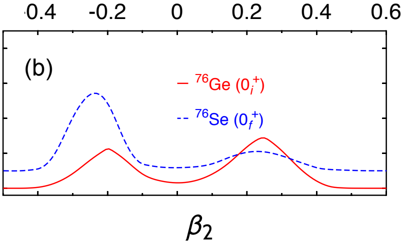
Potential energy surface



Li et al.: Potential energy surface for ^{130}Xe

Collective squared wave functions



Rodríguez and Martínez-Pinedo: Wave functions in ^{76}Ge,Se peaked at two different deformed shapes.

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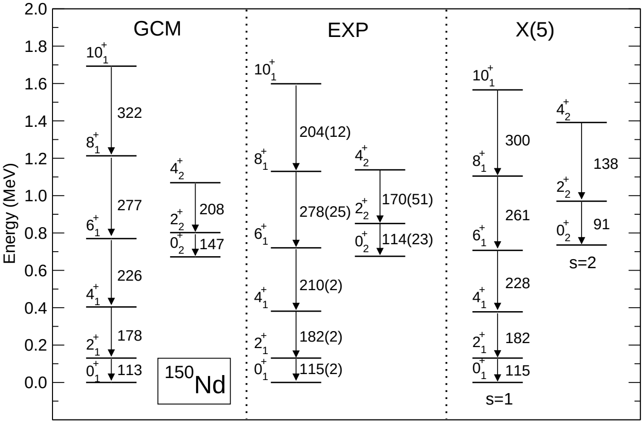
GCM in DFT



P. Ring et al, 2011

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J.M. Yao, B. Bally, JE, R. Wirth, T.R. Rodríguez, H. Hergert

Uses GCM reference state to capture collective multi-particle multi-hole correlations, include the rest through IMSRG.

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Procedure for Schiff moment, which involves two states:

1. Use bare Hamiltonian to construct approximate GCM states for ground state and its partner, and create the statistical ensemble:

$$\rho = a \left|1/2^+\right\rangle\left\langle 1/2^+\right| + b \left|1/2^-\right\rangle\left\langle 1/2^-\right|$$

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Procedure for Schiff moment, which involves two states:

We have already done this for the $\beta\beta$ decay of ^{48}Ca to ^{48}Ti (also involving two states).

Now doing it for the decay of ^{76}Ge.

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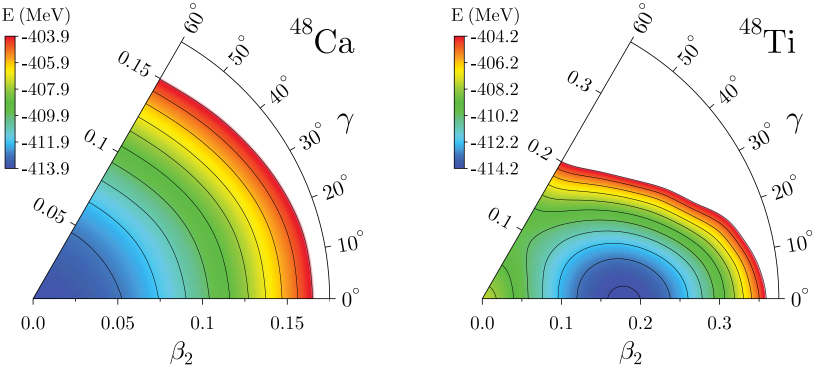
In-Medium GCM for Decay of ^{48}Ca

GCM Reference States for IMSRG

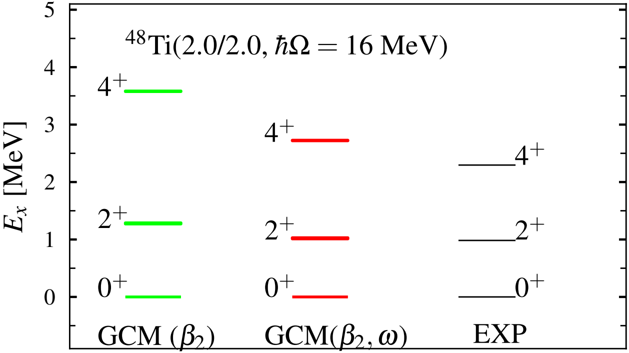
Potential Energy Surfaces



^{48}Ca is spherical and ^{48}Ti is weakly deformed.

Spectrum in ^{48}Ti

In 9 shells



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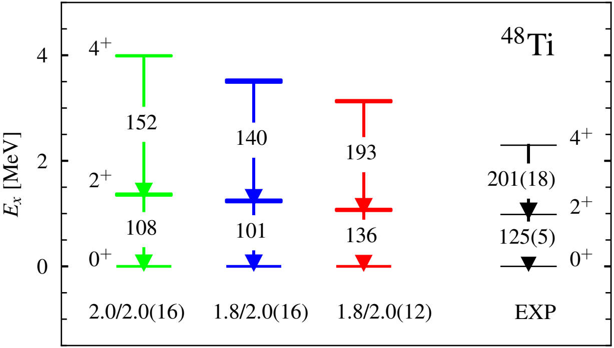

β_2 only

We get large E2's.

Application to Schiff Moment of, e.g., ^{225}Ra

Required Improvements to Methods

- More nucleons will require larger spaces = more memory, processors = supercomputer use (maybe).

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That's about it!

How much smaller will the uncertainty be?

Not clear, but it will be more believable.

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That's all; thanks. 

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