Dynamic Factor Demands and the Effects
of Energy Price Shocks

By ROBERT S. PINDYCK AND JULIO J. ROTEMBERG*

Most models of the industrial demand for energy, capital, and other factor inputs can be grouped into two categories. The first is static models that consistently account for substitution among several factors (i.e., expenditure shares sum to unity), without imposing strong *a priori* restrictions on the production structure. Such models typically utilize a generalized functional form (for example, the translog) to represent the underlying production or cost function.[1] The second consists of dynamic models that incorporate costs of capital stock adjustments. But such models often apply to a single factor, and typically impose *ad hoc* restrictive assumptions.[2]

To understand how sharp changes in energy prices affect investment behavior, employment, and energy use itself, a dynamic model is essential. To begin with, recent studies of industrial energy demand based on static models have produced conflicting results, including widely differing estimates of own-price elasticities, and much of the discussion over these results has focused on whether the elasticities in question are short run or long run.[3] Also, projections and policy analyses related to energy prices and energy use require an understanding of the pattern of demand response over time. Finally, understanding the macroeconomic impact of changing energy prices requires an understanding of the response of investment, which in turn requires a dynamic model of factor demands.

Ideally, a dynamic factor-demand model should retain the generality of functional form that has characterized much of the recent static modeling work. But it should also embody rational expectations and dynamic optimization in the presence of adjustment costs. In other words, it should describe producers that use all available information to choose both flexible and quasi-fixed factor inputs over time so as to maximize the expected present discounted value of the flow of net revenue, given possible uncertainty over the evolution of factor and output prices.

Some recent work has begun to move in this direction. Randall Brown and Laurits Christensen (1981) and Nalin Kulatilaka (1980), for example, have used static translog restricted cost functions to compare factor use under partial vs. full adjustment. However, while such models provide a comparison of short-run vs. long-run elasticities, they do not describe the *path* to the long run, or the pattern of investment over time. Berndt, Melvyn Fuss, and Leonard Waverman (1980), on the other hand, develop a fully dynamic model in which capital is quasi fixed and subject to quadratic adjustment costs. But their approach utilizes an explicit solution to the optimal investment problem, and in so doing, imposes the assumption that

[1] Ernst Berndt and David Wood (1975) and James Griffin and Paul Gregory (1976) are good examples.

[2] As with the flexible accelerator model of investment demand.

[3] Some studies (for example, Berndt and Wood, 1975), show energy-capital complementarity, while others (for example, Griffin and Gregory, 1976, and Pindyck, 1979a,b), show energy-capital substitutability. For attempts to reconcile these differences, see Berndt and Wood (1979, 1981) and Griffin (1981). A survey of industrial energy demand elasticity estimates is given in Pindyck (1979b).

producers have static expectations regarding the evolution of factor and output prices, and requires that the underlying cost function be quadratic. Richard Meese (1980) estimated a dynamic factor-demand model in which producers have rational expectations, but also imposed the restriction of a quadratic production structure.[4] Finally, we should mention models based on the dynamic q theory of investment, as in the work of Fumio Hayashi (1982) and Lawrence Summers (1981). Here Tobin's q theory provides an estimate of the shadow price of capital. However, this approach requires the assumptions of constant returns to scale, a stock market that is "strong form efficient," and that there is only one quasi-fixed input.

In this paper we take a different approach. In a stochastic environment, firms that have rational expectations and maximize the expected sum of discounted profits also minimize the expected sum of discounted costs. Given the restricted cost function, we derive the stochastic Euler equations (i.e., first-order conditions) for this latter dynamic optimization problem. Although these Euler equations do not provide a complete solution to the optimization problem, they can be estimated directly for any parametric specification of the technology. We specify a translog restricted cost function, and then estimate the Euler equations, together with the cost function itself and the share equations for the flexible factors, using three-stage least squares. This permits us to test restrictions such as constant returns, or zero-adjustment costs, for particular factors.

The estimated equations provide a complete empirical description of the production technology, including both short-run (only flexible factors adjust) and long-run (all factors fully adjust) elasticities of demand. The parameter estimates are fully consistent with rational expectations, and in particular with

firm behavior that utilizes the solution to the stochastic control problem. But since we do not actually solve the stochastic control problem (beyond writing the first-order conditions), we cannot calculate optimal factor-demand trajectories corresponding to particular stochastic processes for prices.[5]

However, we can calculate deterministically optimal factor-demand trajectories, that is, those consistent with producers choosing input levels that are solutions to the corresponding deterministic control problem, and adapting to stochastic shocks by repeatedly resolving that deterministic problem. We calculate such trajectories as a way of simulating the effects over time of changes in energy prices on the use of energy, capital, labor, and materials inputs.

The theory underlying our approach is presented in the next section, where we derive the Euler equations and static share equations for an arbitrary restricted cost function. In Section II, estimating equations are presented for a translog restricted cost function, and the requisite concavity conditions are discussed. In Section III, we discuss the data and issues relating to the estimation of the model. The estimated parameters and corresponding elasticities are presented in Section IV. The method of simulating the effects of changes in factor prices is explained in Section V, where we use the model to illustrate the effects of changing energy prices and a changing output level on factor demands. Section VI contains a summary of our results, and some concluding remarks.

I. Theory

In this model, firms choose the optimal levels of four inputs: capital K, labor L, energy E, and materials M. We denote the real rental price of capital by v, the real wage rate by w, and the real prices of energy and materials by e and m, respectively. These prices can evolve stochastically over time.

---

[4]Related to this is John Kennan's (1979) dynamic model of the demand for capital. Here, too, restrictive assumptions are imposed on the structure of production; in this case the production function is linear. For a survey of recent work in dynamic factor-demand modelling, see Berndt, Catherine Morrison, and G. Campbell Watkins (1981).

[5]Stochastic control problems of this sort are generally difficult, if not impossible, to solve. This, of course, raises the question of whether rational expectations provides a realistic behavioral foundation for studying investment behavior and factor demands in general.

We treat energy and materials as flexible factors, but capital and labor as quasi fixed. The production technology is represented by a restricted cost function; conditional at time t on K_t, L_t, and output Q_t, the minimum real expenditure on the two flexible inputs energy and materials is given by

$$(1) \qquad C(e_t, m_t, K_t, L_t, Q_t, t),$$

where C is increasing and concave in the two prices, but decreasing and convex in K and L. Finally, we assume that changes in K and L result in costs of adjustment, represented by the convex functions $c_1(T_t)$ and $c_2(H_t)$. Here I is that part of investment subject to adjustment costs, and H is net hirings, that is,[6]

$$(2) \qquad I_t = K_t - (1 - \delta) K_{t-1},$$

$$(3) \qquad H_t = L_t - L_{t-1},$$

where δ measures the extent to which investment for replacement purposes incurs adjustment costs.[7]

We apply our model to the aggregate U.S. manufacturing sector, treating it as competitive in factor markets, that is, firms take input prices as given. We can therefore view the sector as consisting of a single firm that has the technology of (1), or equivalently as consisting of many firms whose *aggregate* technology is represented by our model.[8]

[6] Note that L_t is the number of hours worked, so that H includes changes in hours as well as changes in the number of workers. We make adjustment costs a function of H for analytical convenience, and also because it is reasonable to expect that such costs are incurred when there are changes in overtime, etc.

[7] Capital adjustment costs are a function of *gross* investment if δ is equal to the depreciation rate, and a function of *net* investment if δ is equal to zero. In general δ can be anywhere between these values. Also, we model adjustment costs as external to the firm. An alternative approach is to make adjustment costs internal by writing the cost function as $C(e_t, m_t, K_t, L_t, Q_t, I_t, H_t, t)$, with $C_I, C_{II}, C_{II}, C_{III} > 0$, but unless restrictions are placed on C a priori, this introduces too many parameters.

[8] But this is *not* equivalent to many firms, *each* of which has the technology described by (1). There are clearly potential aggregation problems here, as is often

Firms are assumed to minimize the expected sum of discounted costs; factor demands are therefore given by the solution to

$$(4) \quad \min_{\{K, L\}} \mathscr{E}_t \sum_{\tau = t}^{\infty} R_{t,\tau} [C(e_\tau, m_\tau, K_\tau, L_\tau, Q_\tau)$$
$$+ v_\tau K_\tau + w_\tau L_\tau + c_1(I_\tau) + c_2(H_\tau)],$$

subject to equations (2) and (3). Here $\mathscr{E}_t$ denotes the expectation conditional on information available at t, and $R_{t,\tau}$ is the discount factor applied at t for revenues accruing at τ.[9] The expectation in (4) is taken over all future values of e, m, v, w, and Q, which are treated as random. But note that this does not mean that output Q must be viewed as "exogenous" or "predetermined." The path of Q depends on the realization of e, m, v, and w as firms maximize profits. However, as long as revenues depend on output alone and not on the choice of inputs, the maximization of profits implies the minimization of costs.[10]

---

the case in work of this sort. If in addition firms are competitive, our approach can be justified by a result of Robert Lucas and Edward Prescott (1971). They show that when competitive firms maximize profits, they act as if a central planner maximized aggregate welfare. This latter maximization requires the minimization of the discounted values of aggregate costs.

[9] That is, $R_{t,\tau} = 1/(1 + \bar{r}_{t,\tau})$ where $\bar{r}_{t,\tau}$ is the real interest rate from t to τ. Note that $R_{t,\tau}$ can also be written as $R_{t,\tau} = P_\tau / (1 + r_{t,\tau}) P_t$, where P_τ is the price of output at τ, and $r_{t,\tau}$ is the nominal interest rate between t and τ.

[10] To see this, suppose firms maximize expected profits. This leads to a contingency plan for K_t, L_t, E_t and M_t that maximizes

$$(a) \quad \mathscr{E}_t \sum_{\tau = t}^{\infty} R_{t,\tau} [f(Q_\tau) - C(e_\tau, m_\tau, K_\tau, L_\tau, Q_\tau)$$
$$- v_\tau K_\tau - w_\tau L_\tau - c_1(I_\tau) - c_2(H_\tau)],$$

subject to (2) and (3). But this maximization implies a contingency plan for Q. Therefore, treating Q as a random variable, (4) must be minimized by the choice of inputs that maximizes (a). If this were not the case, (a) could be made even larger by using the contingency plan for Q, together with the associated contingency plan for K, L, E, and M that minimizes (4).

The minimization of (4) yields the following first-order conditions:

(5) $E_t = \partial C / \partial e_t;$

(6) $M_t = \partial C / \partial m_t;$

(7) $\partial C / \partial K_t + v_t$

$$+ \partial c_1 [K_t - (1 - \delta) K_{t-1}] / \partial K_t$$

$$+ \mathscr{E}_t \{ R_t \partial c_1 [K_{t+1} - (1 - \delta) K_t] / \partial K_t \} = 0;$$

(8) $\partial C / \partial L_t + w_t + \partial c_2 [L_t - L_{t-1}] / \partial L_t$

$$+ \mathscr{E}_t \{ R_t \partial c_2 [L_{t+1} - L_t] / \partial L_t \} = 0.$$

For notational simplicity, we have denoted the one-period discount factor $R_{t,t+1}$ by R_t. Equations (5) and (6) are consequences of Shepherd's Lemma, and the fact that C gives the minimum variable cost $e_t E_t + m_t M_t$. Equations (7) and (8) are the Euler equations, and describe the (expected) evolution of the quasi-fixed factors. For example, equation (7) says that the net effect on expected profits from the last unit of capital is just zero. That net effect consists of the variable cost savings $\partial C / \partial K_t$, a rental cost v_t, a current adjustment cost $\partial c_1 / \partial K_t$, and an expected (discounted) savings in future adjustment costs (by installing the capital now rather than in the future) of $R_t \partial c_1 [K_{t+1} - (1 - \delta) K_t] / \partial K_t$.

The following transversality conditions must also hold:

(9) $\lim_{\tau \to \infty} \mathscr{E}_t R_{t,\tau} \{ \partial C / \partial K_\tau + v_\tau$

$$+ \partial c_1 [K_\tau - (1 - \delta) K_{\tau-1}] / \partial K_\tau \} = 0;$$

(10) $\lim_{\tau \to \infty} \mathscr{E}_t R_{t,\tau} \{ \partial C / \partial L_\tau + w_\tau$

$$+ \partial c_2 [L_\tau - L_{\tau-1}] / \partial L_\tau \} = 0.$$

In other words, as the firm looks farther into the future, the quantities of K and L that it expects to hold should not differ very much from the quantities it would hold in the absence of adjustment costs.

A full solution of equations (5)–(10) is a path for K, L, E, and M that depends on the current states L_t, K_t, e_t, and m_t, as well as the expected future values of prices and output. In general, solving for such a path is extremely difficult unless C, c_1, and c_2 are all quadratic functions. We discuss an approach to simulating the path in Section V of this paper. Here we simply point out that equations (5)–(8) are in effect regression equations, and can be used to estimate the parameters of C, c_1, and c_2, *whatever the forms of those functions*.

Note that firms also choose output Q to solve their intertemporal profit-maximization problem. This implies an additional first-order condition, namely that variations in Q around the optimal level will not make the firm better off. This optimal Q depends on such things as monopoly power in output markets, the presence of costs of price adjustment, etc. Given assumptions about the market environment, one could derive this first-order condition, and its inclusion in the estimation might yield more efficient parameter estimates. However, if these auxiliary assumptions are incorrect, all the parameter estimates would be inconsistent. We therefore limit ourselves to the dynamic cost-minimization problem; in this way we can obtain consistent parameter estimates without relying on dubious assumptions about market structure.

II. Model Specification

We now specify functional forms for C, c_1, and c_2, To limit the number of parameters to be estimated, we make the adjustment cost functions quadratic, that is,[11]

(11) $c_1 (I_t) = \beta_1 I_t^2 / 2;$

(12) $c_2 (H_t) = \beta_2 H_t^2 / 2.$

We use a translog form for the restricted cost function, and impose parameter restrictions to make that function symmetric and

[11] This is in keeping with tradition. See, for example, Kennan and Meese.

homogeneous of degree 1 in prices:

$$(13) \quad \log C_t = \alpha_0 + \log m_t + \alpha_1 \log(e_t/m_t)$$
$$+ \alpha_3 \log K_t + \alpha_4 \log L_t + \alpha_5 \log Q_t + \lambda t$$
$$+ \tfrac{1}{2}\gamma_{11}\left[\log(e_t/m_t)\right]^2 + \gamma_{13}\log(e_t/m_t)\log K_t$$
$$+ \gamma_{14}\log(e_t/m_t)\log L_t$$
$$+ \gamma_{15}\log(e_t/m_t)\log Q_t$$
$$+ \tfrac{1}{2}\gamma_{33}(\log K_t)^2 + \gamma_{34}\log K_t\log L_t$$
$$+ \gamma_{35}\log K_t\log Q_t + \tfrac{1}{2}\gamma_{44}(\log L_t)^2$$
$$+ \gamma_{45}\log L_t\log Q_t + \tfrac{1}{2}\gamma_{55}(\log Q_t)^2,$$

where λ represents the rate of neutral disembodied technical progress.[12]

With this specification, the first-order conditions (5)–(8) become[13]

$$(14)$$
$$S_{et} \equiv \frac{e_t E_t}{e_t E_t + m_t M_t} = \alpha_1 + \gamma_{11}\log(e_t/m_t)$$
$$+ \gamma_{13}\log K_t + \gamma_{14}\log L_t + \gamma_{15}\log Q_t,$$

$$(15) \quad S_{mt} \equiv \frac{m_t M_t}{e_t E_t + m_t M_t} = 1 - S_{et},$$

$$(16) \quad C_t S_{Kt}/K_t + v_t + \beta_1\left[K_t - (1-\delta)K_{t-1}\right]$$
$$- \mathscr{E}_t\left\{R_t(1-\delta)\beta_1\left[K_{t+1} - (1-\delta)K_t\right]\right\} = 0,$$

$$(17) \quad C_t S_{Lt}/L_t + w_t + \beta_2(L_t - L_{t-1})$$
$$- \mathscr{E}_t\left\{R_t\beta_2(L_{t+1} - L_t)\right\} = 0,$$

where S_{Kt} and S_{Lt} are defined as

$$(18)$$
$$S_{Kt} = \partial\log C/\partial\log K_t = \alpha_3 + \gamma_{13}\log(e_t/m_t)$$
$$+ \gamma_{33}\log K_t + \gamma_{34}\log L_t + \gamma_{35}\log Q_t,$$

$$(19)$$
$$S_{Lt} = \partial\log C/\partial\log L_t = \alpha_4 + \gamma_{14}\log(e_t/m_t)$$
$$+ \gamma_{34}\log K_t + \gamma_{44}\log L_t + \gamma_{45}\log Q_t.$$

We discuss the estimation of these equations in the next section. First, however, note that the inputs and parameter estimates must be such that the function C satisfies monotonicity and curvature conditions at all sample points. Since C should be monotonically increasing in e and m and decreasing in K and L, estimated values of S_e, S_K and S_L must satisfy $0 \le S_e \le 1$ and $S_K, S_L \le 0$. Since C should be concave in the prices e and m, the matrix of second partials of C with respect to e and m must be negative semidefinite. Since C should be convex in the inputs K and L, the matrix of second partials of C with respect to K and L must be positive semidefinite. We check whether these curvature conditions hold at each observation.

Once estimated, the model can be used to calculate demand elasticities, but a caveat is needed regarding their interpretation. Short-run elasticities, that is, only flexible factors change, have a straightforward meaning, and can be interpreted as usual. However intermediate- and long-run elasticities, that is, those that apply when quasi-fixed factors have partially or fully adjusted, must be interpreted with caution. The reason is that if prices evolve stochastically, the adjustment path for any particular discrete change in a price, as well as the long-run expected equilibrium, are solutions to a stochastic control problem.[14] Since such solutions are typically infeasible, we must utilize the solution to the

---

[12] Note that λ applies to *variable*, not total cost. Also, our specification of the cost function implies that both new investment and new workers become productive in the same year as their installation or hiring. An alternative view is that a year or more is required for these factor additions to become productive.

[13] Note that equation (15) is an identity, reflecting the fact that for given K and L, once E has been chosen optimally using (14), there is a unique M such that the firm can produce Q and no more.

[14] Furthermore, as Andrew Abel (1983) has shown for a Cobb-Douglas production function and an output price that follows a geometric random walk, a long-run expected equilibrium may not exist. (In Abel's example, investment and capital stock are expected to grow without bound.)

corresponding deterministic control problem to compute these elasticities, that is, we implicitly assume that firms ignore the variance of future prices in responding to price changes. Intermediate- and long-run elasticities can therefore be best viewed as a description of the technology. Formulas for the calculation of elasticities are available from the authors upon request.

III. Estimation Method and Data

To obtain parameter values, we simultaneously estimate the cost function (13), the energy cost share equation (14), and the Euler equations for capital and labor (16) and (17), using three-stage least squares. This method deserves some comment. The Euler equations state that the expected values, conditional on information available at t, of one additional unit of capital or labor at t are zero. Therefore, as Lars Peter Hansen (1982) and Hansen and Kenneth Singleton (1982) have shown in another context, a natural estimator of these equations is an instrumental variables procedure which minimizes the correlation between any variable known at t and the residuals of (16) and (17). These residuals, which can be interpreted as expectational errors, are computed using the actual values of K_{t+1} and L_{t+1} on the left-hand side of (16) and (17). Also, the minimized value of the objective function of this procedure provides a statistic J, which is distributed as χ^2 with degrees of freedom equal to the number of instruments times the number of equations minus the number of parameters. This statistic can be used to test the overidentifying restrictions of the model.

Using any variable known at t as an instrument when estimating (16) and (17) is only appropriate when the cost function and energy share equation hold without error, as they theoretically should. In practice, those equations will also contain errors, which must be interpreted properly. Additive errors in the cost function and share equation could reasonably arise from three sources: measurement error, optimization error (i.e., firms operate suboptimally), or technological shocks (i.e., randomness in the technology itself). Errors like these are likely to be corre-

lated with variables in the cost function, the share equation, *and* the Euler equations.

It is plausible, however, to view the cost function, share, and Euler equations as holding *in expectation* with respect to some conditioning set. That conditioning set would be a proper subset of the set that would apply to the Euler equations if the cost function and share equation fit exactly. We therefore use a conditioning set (i.e., set of instrumental variables) that does not include any current variables appearing in the cost function, share equation, or Euler equations.

The procedure proposed by Hansen and Hansen and Singleton allows for errors that are conditionally heteroscedastic. As can be seen in Hansen, this procedure reduces to three-stage least squares when the errors are conditionally homoscedastic. We make this additional assumption for simplicity, although the resulting estimates will be consistent even if the assumption is wrong.[15]

We test structural restrictions in two ways. For restrictions involving a single parameter —whether labor is subject to adjustment costs ($\beta_2 = 0$), or whether adjustment costs are incurred only from net investment ($\delta = 0$)—the asymptotic standard error is used directly. To test for constant returns, we use the procedure analogous to the likelihood ratio test suggested by Ronald Gallant and Dale Jorgenson (1979). This involves reestimating the model with the restrictions imposed, and comparing the minimized values of J.[16]

We estimate our model using the data developed by Berndt and Wood (1975). The data include the total use of capital, labor, energy, and materials for U.S. manufacturing, the corresponding prices of output, energy, materials, and labor, and an up-

---

[15]However the standard errors, and the J statistic of Hansen, which we use to test the overidentifying restrictions, are valid only if the errors are in fact conditionally homoscedastic.

[16]Under constant returns, multiplying K, L, and Q by e^μ multiplies costs by e^μ, and this requires $\alpha_3 + \alpha_4 + \alpha_5 = 1$. Also, the sum of the coefficients of $\mu \log(e_t/m_t)$, $\mu \log K_t$, $\mu \log L_t$, $\mu \log Q_t$, and μ^2 must be zero. This implies the additional parameter restrictions $\gamma_{13} + \gamma_{14} + \gamma_{15} = 0$, $\gamma_{33} + \gamma_{34} + \gamma_{35} = 0$, $\gamma_{34} + \gamma_{44} + \gamma_{45} = 0$, and $\gamma_{35} + \gamma_{45} + \gamma_{55} = 0$.

dated series for the rental price of capital computed a la Christensen and Jorgenson (1969).[17] The data are annual, covering the years 1948–71. Despite the fact that recent years are not covered, use of this data is desirable for two reasons. First, the series are well-constructed and have been carefully checked by a number of researchers. Second, their use by others provides a basis for comparison of our method and results.

We estimated the model using two alternative sets of instruments. Theoretically, the parameter estimates should not depend on the subset of the information set used, that is, on the choice of instruments. This suggests testing the overidentifying restrictions by using a large set of instruments, but that is not feasible given the limited number of observations. Instead we check the robustness of the model by using two alternative sets of instruments and comparing the resulting parameter estimates. The first set is that used by Berndt and Wood (1975) which we denote "B-W."[18] The second, "P-R," includes the lagged values of output, the capital stock, the real rental rate of capital, the quantities and real prices of labor, energy, and materials, as well as the rate on commercial paper.

IV. Estimation Results

Parameter estimates, using the two alternative sets of instruments, are shown in columns 1 and 2 of Table 1. We can observe from these estimates that the parameter $1 - \delta$ is very close to 1 in magnitude, and not statistically significantly different from 1. In addition, β_2, the adjustment cost parameter

for labor, is very small. (Using the estimated values for β_1 and β_2, the average value of $\beta_1 I^2$ is about 15 times as large as the average value of $\beta_2 H^2$.) These results hold for both sets of instruments, and suggest that adjustment costs for labor are much less important than those for capital, and that the latter depend on net rather than gross investment.[19]

Columns 3 and 4 apply to the same model, but with adjustment costs for capital a function of *net* investment, that is, δ is set equal to 0 in equation (16). Note that all of the parameter estimates are substantially the same whether or not we impose the restriction $\delta = 0$. In particular, β_2 is very close to 0, again indicating that labor might be better viewed as a flexible factor.

In column 5 we impose constant returns, and estimate the model using the B-W instruments. To test constant returns, we use the difference in the values of J with and without the parameter restrictions imposed. That difference is $105.1 - 35.55 = 69.55$. With five restrictions (see fn. 16), the critical 5 percent *chi*-square value is 11.07, so that constant returns is rejected.

Curvature and monotonicity conditions on the cost function are satisfied for all of the data points in all of the models reported in Table 1. However, the values for J indicate rejection of the overidentifying restrictions at the 5 percent level, throwing some doubt on the validity of the estimated standard errors.[20] On the other hand, the use of different sets of instruments results in little change in the values of most of the parameter estimates, supporting the validity of these estimates for the computation of elasticities.

Demand elasticities are reported in Table 2 for the parameter estimates presented in column 3 of Table 1, that is, for the model in

---

[17]This rental rate of capital is unfortunately not fully consistent with all of the other assumptions of the model. In particular, it embodies static expectations of the expected return on capital. Also, it is only consistent with either constant returns to scale (which we test and reject), or zero profits under monopolistic competition.

[18]It includes total U.S. population, working age population, the net sales tax rate, the effective personal income tax rate, purchases of durable goods by government and foreigners, inventory investment in durables and in consumer goods, the value of labor input hired by government and the foreign sector, the net private claims on government by U.S. households, and the tangible capital stock at the end of the previous year.

[19]These results are similar to those in our earlier 1983 article, where energy and materials are neglected, but labor and capital are disaggregated. It appears that adjustment costs are an important consideration mainly when firms are planning new ventures and changing the size of their capital stock, but not when they are replacing old equipment.

[20]The failure of the overidentifying restrictions can be attributed to either a failure of our specification of the cost function, or to the absence of optimization with rational expectations on the part of firms.

TABLE 1—PARAMETER ESTIMATES: CAPITAL AND LABOR QUASI FIXED[a]

Parameter	B-W (1)	P-R (2)	B-W (3)	P-R (4)	B-W[b] (5)
α_0	5.2323	5.8018	5.2275	5.8068	7.6525
	(1.2143)	(.9132)	(1.2141)	(.90973)	(.14769)
α_1	.20606	.21593	.20536	.21708	.02590
	(.07765)	(.07862)	(.07755)	(.07851)	(.017852)
α_3	.2666	.301245	.2764	.29414	−.11571
	(.2584)	(.23267)	(.2500)	(.22095)	(.04247)
α_4	.0342	−.28221	.03041	−.27991	−.97018
	(.5949)	(.43994)	(.5951)	(.43968)	(.07414)
α_5	1.1336	1.2559	1.1279	1.2593	
	(.39129)	(.30893)	(.3843)	(.30823)	
γ_{11}	.03759	.014411	.03759	.01479	.03624
	(.01156)	(.01390)	(.01155)	(.013742)	(.011242)
γ_{13}	.03690	.01871	.03715	.01837	.02944
	(.00884)	(.01006)	(.00862)	(.009984)	(.00620)
γ_{14}	−.0608	−.05078	−.06078	−.050872	−.00862
	(.01906)	(.02008)	(.01899)	(.020038)	(.00585)
γ_{15}	−.00257	.00257	−.00276	.002884	
	(.01301)	(.01326)	(.01296)	(.013235)	
γ_{33}	−.07227	−.02609	−.07388	−.023959	.01222
	(.03667)	(.04837)	(.03665)	(.04011)	(.02839)
γ_{34}	−.04598	−.08471	−.04751	−.084234	−.00044
	(.05655)	(.04525)	(.05340)	(.04524)	(.01566)
γ_{35}	.07536	.05466	.07751	.05325	
	(.04904)	(.04445)	(.04526)	(.043664)	
γ_{44}	−.07507	.03274	−.07306	.031776	.15761
	(.15527)	(.11506)	(.15449)	(.115089)	(.02015)
γ_{45}	−.02243	−.03762	−.02228	−.037146	
	(.08699)	(.06611)	(.08678)	(.066015)	
γ_{55}	−.03296	.05745	−.03729	.05771	
	(.09874)	(.09244)	(.09361)	(.09212)	
λ	.00289	−.00465	−.00286	−.004711	−.01285
	(.00231)	(.00209)	(.00230)	(.00199)	(.00044)
β_1	.21194	.192022	.21366	.18799	.106175
	(.090745)	(.09575)	(.05100)	(.04435)	(.03856)
β_2	.00807	−.000321	.00809	−.000326	−.008199
	(.00435)	(.00229)	(.00434)	(.00229)	(.002951)
$(1-\delta)$	.999798	.98082			
	(.51543)	(.53520)			
J	35.613	40.497	35.55	40.4367	105.1

Note: Adjustment costs for cols. 1 and 2 are based on gross investment, and cols. 3, 4, and 5 are based on net investment.
[a]Asymptotic standard errors are shown in parentheses.
[b]Constant returns to scale are imposed.

which adjustment costs for capital are a function of net investment. Observe that all of the own-price elasticities are negative. The own-price elasticity of energy is −.36 in the short run and −.99 in the long run. The short-run number is close to the value obtained by Berndt and Wood (1975), while the long-run number is close to those obtained by Griffin and Paul Gregory (1976) and by Pindyck (1979a, b), and is thus consistent with the view that the Berndt-Wood elasticities are short run, while the latter are long run.

Also note from Table 2 that both capital and energy, and capital and labor are complements in the long run. However, this finding of capital-energy and capital-labor complementarity should be viewed as purely a characteristic of the production structure. To determine whether a higher price of energy

TABLE 2—ELASTICITIES FOR MODEL WITH CAPITAL AND LABOR QUASI FIXED

	Elasticity of Demand for			
	E	M	L	K
A. Short-Run Elasticities **(E and M adjust)**				
e	−.3616	.0254		
m	.3616	−.0254		
Q	1.2106	1.2556		
L	−1.3685	−.3774		
K	.4683	−.1375		
B. Intermediate-Run Elasticities (L also adjusts)				
e	−.5791	−.0345	.1589	
m	.1347	−.2614	.6239	
Q	−.1854	.87063	1.0201	
w	1.0713	.2760	−.7892	
K	.4579	−.1403	.0075	
C. Long-Run Elasticities (L and K also adjust)				
e	−.9914	.0914	.1521	−.9001
m	1.3064	−.8121	.6535	3.9272
Q	.4905	.6634	1.0312	1.4758
w	1.0256	.3094	−.7836	−.0998
v	−1.3406	.4108	−.0220	−2.9271

results in less demand for capital, one must also consider the general equilibrium response of output and the real wage to changes in the price of energy, and any subsequent effects on the demand for capital.

The elasticities of demand for capital, especially the own-price elasticity, are large in magnitude, so that investment will respond strongly to changes in real prices. Also, capital is the input most responsive to output changes. This is consistent with the high cyclical variability of investment, and helps explain it.

Since adjustment costs for labor appear to be negligible, we estimate an alternative model in which labor is flexible and only capital is quasi fixed. That alternative model could simply have been equations (13), (14), (16), and (17), but with β_2 constrained to be zero. Instead we use a new translog restricted cost function which depend upon the prices of energy, materials, and labor, and the quantities of capital and output. This has two advantages—it simplifies the simulations, and it provides a check on the robustness of our whole approach, as well as our elasticity estimates. The cost function we now

use is

$$(20) \quad \log C_t = \phi_0 + \log m_t + \phi_1 \log(e_t/m_t)$$
$$+ \phi_2 \log(w_t/m_t) + \phi_3 \log K_t + \phi_4 \log Q_t$$
$$+ \tfrac{1}{2}\psi_{11}\left[\log(e_t/m_t)\right]^2$$
$$+ \psi_{12}\log(e_t/m_t)\log(w_t/m_t)$$
$$+ \psi_{13}\log(e_t/m_t)\log K_t$$
$$+ \psi_{14}\log(e_t/m_t)\log Q_t$$
$$+ \tfrac{1}{2}\psi_{22}\left[\log(w_t/m_t)\right]^2$$
$$+ \psi_{23}\log(w_t/m_t)\log K_t$$
$$+ \psi_{24}\log(w_t/m_t)\log Q_t$$
$$+ \tfrac{1}{2}\psi_{33}(\log K_t)^2 + \psi_{34}\log K_t\log Q_t$$
$$+ \tfrac{1}{2}\psi_{44}(\log Q_t)^2 + \lambda t,$$

where C_t is the minimum value of $e_t E_t + m_t M_t + w_t L_t$. This cost function implies the

TABLE 3 — ELASTICITIES AND PARAMETER ESTIMATES FOR MODEL WITH CAPITAL QUASI FIXED

	Elasticity of Demand for			
	E	M	L	K
A. Short-Run Elasticities				
(E, M, and L adjust)				
e	−.6567	−.0157	.1313	
m	−.2200	−.1821	.4145	
Q	.0847	.9706	.8841	
w	.8768	.1978	−.5458	
K	.3513	−.1382	.0207	
B. Long-Run Elasticities				
(K also adjusts)				
e	−.9348	.0937	.1149	−.7916
m	1.1549	−.7229	.4955	3.9138
Q	.6340	.7545	.9165	1.5638
w	.6981	.2681	−.5663	−.5086
v	−1.0115	.3979	−.0596	−2.8793

C. Parameter Estimates

ϕ_0	5.137	ϕ_4	.8438	ψ_{14}	−.0378	ψ_{33}	−.0501	λ	−.00186
	(.0985)		(.0731)		(.0036)		(.0323)		(.00158)
ϕ_1	.00069	ψ_{11}	.0137	ψ_{22}	.0450	ψ_{34}	.0235	β_1	.2245
	(.0111)		(.0071)		(.0410)		(.0193)		(.0512)
ϕ_2	.2016	ψ_{12}	.0262	ψ_{23}	.0269	ψ_{44}	−.0179	J	39.72
	(.0722)		(.0056)		(.0294)		(.0122)		
ϕ_3	.0599	ψ_{13}	.0193	ψ_{24}	−.0058				
	(.0794)		(.0044)		(.0189)				

following equations for the shares of energy and labor expenditures:

$$(21) \quad e_t E_t / (e_t E_t + m_t M_t + w_t L_t)$$
$$= \phi_1 + \psi_{11} \log(e_t/m_t) + \psi_{12} \log(w_t/m_t)$$
$$+ \psi_{13} \log K_t + \psi_{14} \log Q_t;$$

$$(22) \quad w_t L_t / (e_t E_t + m_t M_t + w_t L_t)$$
$$= \phi_2 + \psi_{12} \log(e_t/m_t) + \psi_{22} \log(w_t/m_t)$$
$$+ \psi_{23} \log K_t + \psi_{24} \log Q_t.$$

Finally, it implies the following Euler equation for capital accumulation:

$$(23)$$

$$\frac{ e_t E_t + m_t M_t + w_t L_t }{ K_t } \left[\phi_3 + \psi_{13} \log(e_t/m_t) \right.$$
$$\left. + \psi_{23} \log(w_t/m_t) + \psi_{33} \log K_t + \psi_{34} \log Q_t \right]$$
$$+ v_t + \beta_1 \left[K_t - (1-\delta) K_{t-1} \right]$$
$$- \mathscr{E}_t \beta_1 (1-\delta) R_t \left[K_{t+1} - (1-\delta) K_t \right] = 0.$$

The results of estimating equations (20)–(23) with the B-W instruments and the assumption $\delta = 0$ are reported in Table 3. The results, as well as those (not reported) which correspond to the other columns of Table 1, are very similar to those obtained for the model of equations (13), (14), (16), and (17). Constant returns are rejected, $(1 - \delta)$ is close to 1, β_1 is near the values reported in Table 1, and the estimates satisfy the monotonicity and curvature restrictions at all sample points. Finally, the elasticities are quite close to those in Table 2, even though the cost function specifications are quite different, suggesting that the elasticity estimates are robust.

V. Simulation of the Model

As explained earlier, we can only carry out simulations in a deterministic context. If the stochastic elements in the evolution of factor prices and the real interest rate are small, then the results should provide a close approximation to actual behavior. Even if these stochastic elements are large, the simulations
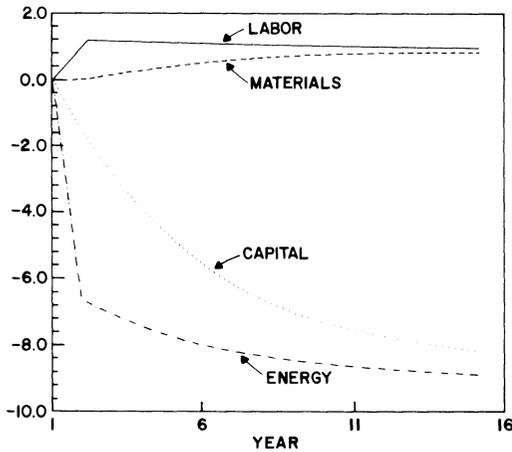
FIGURE 1. TEN PERCENT UNEXPECTED INCREASE
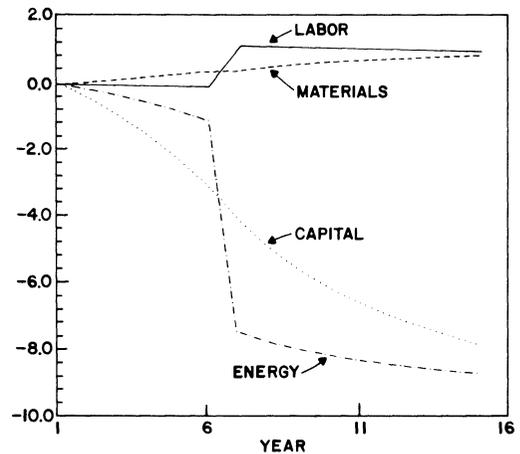IN ENERGY PRICES IN YEAR 2

FIGURE 2. TEN PERCENT INCREASE IN ENERGY PRICES
BEGINNING IN YEAR 7, ANTICIPATED AS OF YEAR 2

provide an illustration of the importance and role of adjustment costs.

We examine the ways in which factor inputs respond over time to anticipated and unanticipated changes in the price of energy and the level of output. To do this, we utilize the model of equations (20)–(23) to simulate the effects of the following "events": (i) the price of energy unexpectedly increases by 10 percent, and is then expected to remain at this higher level; (ii) the same 10 percent increase in the price of energy is anticipated by firms five years before it occurs; (iii) an unanticipated recession occurs, but has a two-year duration that is anticipated.

Carrying out the simulations involves the solution of a deterministic optimal control problem. That solution is obtained by searching for an initial condition for investment which yields a steady-state capital stock (i.e., satisfies the transversality condition) when the Euler equation and cost function are solved together recursively through time. This provides the path for the capital stock, and the share equations together with the cost function then determine the paths for the other inputs.

Simulation results are shown graphically in Figures 1–3. Each figure shows percentage changes in capital, labor, energy, and materials inputs over time. These percentage changes are relative to the base year 1971, at

which time we assume that all factor inputs are in steady-state equilibrium.[21]

Figure 1 shows the effect of an unanticipated 10 percent increase in the price of energy. The major impact is a significant drop in the use of both capital and energy (which are complements). Because of adjustment costs, capital falls gradually, while energy, a flexible factor, falls by a significant amount in the first period, and continues to fall in subsequent periods in conjunction with the drop in the use of capital. The adjustment is fairly rapid; three-fourths of the total drop in capital occurs in seven years, so that substantial net disinvestment occurs during the first two or three years.

Figure 2 shows the effect of an *anticipated* 10 percent increase in the price of energy. As expected, the price increase eventually leads to the same steady-state equilibrium values as in Figure 1, but the dynamics of adjustment are quite different. The demand for capital now begins dropping immediately because of the presence of adjustment costs, but the major changes in the use of energy

[21] For computational simplicity we assumed that there is no technological change in steady-state equilibrium, i.e., $\lambda = 0$. To compute base-year equilibrium values, v was first chosen to make desired net investment zero. Then, values of E, M, and L were found which made the cost and share equations hold without error.
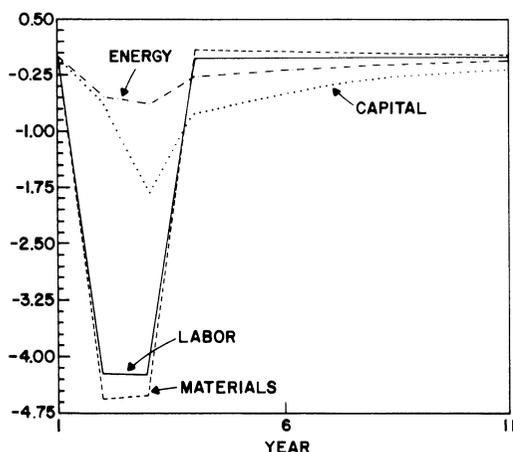
FIGURE 3. FIVE PERCENT UNEXPECTED DECLINE
IN OUTPUT BEGINNING IN YEAR 2 UNTIL YEAR 4,
WHEN OUTPUT RETURNS TO ITS INITIAL LEVEL

and labor occur after the price increase is realized. There is still a noticeable decline in the use of energy prior to the price increase, however, because of the complementarity of energy and capital.

Figure 3 illustrates the effect of an *unanticipated* recession whose duration— *both anticipated and actual* —is limited to two years. Specifically, output is assumed to decline by 5 percent, remain at the lower level for two years, and then return to its initial level.

Because of adjustment costs and the limited expected duration of the recession, the capital stock decline is smaller than it would be otherwise. Investment, however, is driven strongly by the stock adjustment effect; during the two years of the recession net investment is sharply negative, but during the recovery year, it is large and positive. Also, note that there is little change in energy use, in part because of the limited decline in the capital stock. Finally, observe that the relatively large declines in materials and labor are roughly consistent with the experience of recent business cycles.

VI. Conclusions

This paper has shown how a general non-linear model of dynamic factor demands that is consistent with rational expectations can

be estimated and used to study the effects over time of unexpected changes in factor prices, of a changing output level, and of policies in which future price changes are anticipated (such as the Natural Gas Policy Act of 1978). Also, our empirical results provide some insight into the structure of aggregate production, the importance of adjustment costs, and the role of energy as a factor input. We would stress the following results.

First, the data strongly reject the hypothesis of constant returns to scale within our specification of aggregate production. This puts into question earlier studies that impose constant returns, as well as empirical q theory models which equate marginal and average q. On the other hand, the use of time-series data makes it difficult to disentangle the extent of returns to scale from various types of technical progress. Our rejection of constant returns may be dependent on our assumption of Hicks-neutral technical change.

Second, the data indicate that any adjustment costs on labor are small. This is significant because it runs counter to the widely held view that adjustment costs are a major cause of the procyclical movement of productivity.[22] Of course our data aggregate blue- and white-collar workers, and disaggregated data might yield different results.

Third, our results help reconcile some of the conflicting estimates of energy demand elasticities that have appeared in the literature in recent years. In particular, our results support the argument that time-series studies such as Berndt and Wood (1975) have yielded shorter-run elasticities, while cross-section and pooled cross-section time-series studies such as Griffin and Gregory and Pindyck (1979a,b) have yielded longer-run elasticities. Also, our results support the energy-capital complementarity finding of Berndt and Wood (1975).

Of course, these findings are subject to some caveats. Our approach involves several

---

[22] These results also seem to shed some doubt on the approach used recently by Thomas Sargent (1978) in which adjustment costs on capital are ignored while those for labor are included.

forms of aggregation, each of which might be
questioned. In particular, we aggregate out-
puts and inputs across firms and industries,
we aggregate a variety of diverse inputs un-
der the headings of energy, materials, capital,
and labor, and we take averages of prices
and quantities over each year as the objects
of analysis. Moreover, we ignore certain fea-
tures of investment which might be im-
portant, including the lumpiness of capital
goods, investment decision lags, and the time
intervals required for capital construction.
However, even as a "first cut," our approach
helps to illustrate the importance of using a
dynamic model to analyze factor demands.

REFERENCES

Abel, Andrew B., "Optimal Investment under
Uncertainty," *American Economic Review*,
March 1983, *83*, 228–33.

Berndt, Ernst R., Fuss, Melvyn A. and Waverman,
Leonard, "Dynamic Adjustment Models of
Industrial Energy Demand: Empirical
Analysis for U.S. Manufacturing, 1947–
74," Research Project No. 683–1, Final
Report, Electric Power Research Institute,
Palo Alto, November 1980.

_____, Morrison, Catherine and Watkins,
G. Campbell, "Dynamic Models of Energy
Demand: An Assessment and Compari-
son," in E. Berndt and B. Field, eds.,
*Measuring and Modelling Natural Resource
Substitution*, Cambridge: MIT Press, 1981.

_____ and Wood, David O., "Technology,
Prices and the Derived Demand for En-
ergy," *Review of Economics and Statistics*,
August 1975, *57*, 259–68.

_____ and _____, "Engineering and Econ-
ometric Approaches to Industrial Energy
Demand: A Reconciliation," *American
Economic Review*, June 1979, *69*, 342–54.

_____ and _____, "Engineering and
Econometric Approaches to Industrial En-
ergy Demand: Reply," *American Economic
Review*, December 1981, *71*, 1105–110.

Brown, Randall and Christensen, Laurits, "Esti-
mating Elasticities of Substitution in a
Model of Partial Static Equilibrium: An
Application to U.S. Agriculture, 1947 to
1974," in E. Berndt and B. Field, eds.,

*Measuring and Modelling Natural Resource
Substitution*, Cambridge: MIT Press, 1981.

Christensen, Laurits R. and Jorgenson, Dale W.,
"The Measurement of U.S. Real Capital
Input, 1929–1967," *Review of Income and
Wealth*, December 1969, *15*, 293–320.

Gallant, Ronald A. and Jorgenson, Dale W.,
"Statistical Inference for a System of
Simultaneous, Non-Linear, Implicit Equa-
tions in the Context of Instrumental Vari-
able Estimation," *Journal of Econometrics*,
October/December 1979, *11*, 275–302.

Griffin, James M., "Engineering and Econo-
metric Approaches to Industrial Energy
Demand: Comment," *American Economic
Review*, December 1981, *71*, 1100–04.

_____ and Gregory, Paul R., "An Intercoun-
try Translog Model of Energy Substitution
Responses," *American Economic Review*,
December 1976, *66*, 845–57.

Hansen, Lars Peter, "Large-Sample Proper-
ties of Method of Moments Estimators,"
Econometrica, July 1982, *50*, 1029–54.

_____ and Singleton, Kenneth, "Generalized
Instrumental Variables Estimation of
Nonlinear Rational Expectations Models,"
Econometrica, September 1982, *50*, 1269–
86.

Hayashi, Fumio, "Tobin's Marginal q and
Average q: A Neoclassical Interpretation,"
Econometrica, January 1982, *50*, 213–24.

Kennan, John, "The Estimation of Partial Ad-
justment Models with Rational Expecta-
tions," *Econometrica*, November 1979, *47*,
1441–456.

Kulatilaka, Nalin, "A Partial Equilibrium
Model of Derived Demand for Production
Factor Inputs," Working Paper No. 1176
–80, Sloan School, MIT, October 1980.

Lucas, Robert E. and Prescott, Edward C., "In-
vestment Under Uncertainty," *Econ-
ometrica*, September 1971, *39*, 659–81.

Meese, Richard, "Dynamic Factor Demand
Schedules for Labor and Capital Under
Rational Expectations," *Journal of Econo-
metrics*, Summer 1980, *14*, 141–58.

Pindyck, Robert S., (1979a) "Interfuel Sub-
stitution and the Industrial Demand for
Energy: An International Comparison,"
Review of Economics and Statistics, May
1979, *61*, 169–79.

_____, (1979b) *The Structure of World En-*

ergy Demand, Cambridge: MIT Press, 1979.

_____ and Rotemberg, Julio J., "Dynamic Factor Demand Under Rational Expectations," Scandinavian Journal of Economics, July 1983, 85.

Sargent, Thomas J., "Estimation of Dynamic Labor Demand Schedules Under Rational Expectations," Journal of Political Economy, December 1978, 86, 1009–044.

Summers, Lawrence, "Taxation and Corporate Investment: A q Theory Approach," Brookings Papers on Economic Activity, 1:1981, 67–140.