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Population, Productivity, and Sustainable Consumption†

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How does sustainable consumption depend on productivity growth, the size and growth rate of the population, and uncertainty over these growth rates? I address these questions using a model in which productivity and population growth are stochastic and human lives can have (positive or negative) intrinsic social value. I show how sustainable consumption depends on expected rates of productivity and population growth, the volatility of those rates, and the dependence of welfare on population. For plausible parameter values, sustainable consumption is well below the optimal welfare-maximizing level. This raises a question: given its cost, should sustainability be a social objective? (*JEL* E21, E22, E23, J11, Q01)

Given current and projected future levels and growth rates of aggregate production and wealth, what level of consumption is sustainable? How does the sustainable level of consumption depend on the size and growth rate of the population? How does it compare to the optimal level of consumption that maximizes welfare? And how are the answers to these questions affected by uncertainty, over both the growth and productivity of the capital stock and the growth of population?

These questions presume a definition of "sustainable." A common definition is that future generations should be at least as well off as we are. But does "as well off" mean that there is no reduction in per capita consumption or no reduction in the utility from consumption? And do we care about the number of people who are well off?

Much of the economics literature that addresses these questions defines a sustainable path for consumption as one for which social welfare is nondeclining throughout the future. In turn, social welfare is usually defined as the present value of a flow of utility generated from consumption—i.e.,

$$V_t = \int_t^{\infty} U(C_\tau)\, e^{-\rho(\tau-t)}\, d\tau, \tag{1}$$

where ρ is the social rate of time prefence (and, hence, discount rate) and sustainability requires that $dV/dt \geq 0$ for all t.

A variety of studies have used this framework to examine how sustainable consumption can depend on such things as the level and growth rate of the capital stock, depletion of natural resources, and technological change. For example, Dasgupta

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(2009) and Arrow et al. (2012, 2013) show that sustainability implies that a properly defined comprehensive measure of productive wealth—which includes stocks of physical and human capital, natural resources, and the technological knowledge base—must never decline. Others have used this framework to assess whether current consumption exceeds the sustainable level.[1]

These studies have two important limitations. First, they are inherently deterministic in nature. They typically examine how sustainable trajectories for consumption depend on the (deterministic) growth rates of the capital stock, productivity, natural resources, and other factors that affect output and welfare. These studies yield insights into the relative importance of different factors that can limit future consumption, but they ignore the fact that the economy evolves stochastically, so it is impossible to ensure that welfare will never decline. An important exception to this literature is the fully stochastic model developed by Campbell and Martin (2021), discussed below.

A second limitation is that population is usually taken as incidental. Equation (1) might be modified by replacing C with per capita consumption C/N or by expressing (instantaneous) welfare as $NU(C/N)$. But apart from their consumption, there is no social utility (or disutility) from the very existence of people in these welfare functions.[2] This is at odds with a growing literature that examines how life itself might be valued. There are good reasons to believe that population growth will affect social welfare (apart from its impact on per capita consumption), and we will see that this can have profound implications for sustainable consumption.

If the determinants of welfare—and, thus, welfare itself—evolve stochastically, sustainability can be defined in expected value terms. A natural definition is that the expected value of social welfare (which itself evolves stochastically) is not expected to decline at any point in the future. That is the definition of sustainability used in the recent paper by Campbell and Martin (2021)—henceforth, CM—on which this paper builds, and it is used here as well. With this definition of sustainability, equation (1) becomes

$$V_t = \mathcal{E}_t \int_t^{\infty} U(C_\tau)\, e^{-\rho(\tau - t)}\, d\tau, \tag{2}$$

and sustainability requires $(1/dt)\mathcal{E}_t dV \geq 0$ for all t.[3]

[1] For an overview of studies of sustainable consumption, see Arrow et al. (2004). As they point out, a sustainable trajectory for consumption may not exist, and if it does, it may not be unique and it need not be optimal (in the sense of maximizing V_t). Are we consuming too much? Arrow et al. (2004) answer this by measuring the growth rates of per capita "genuine wealth" (i.e., comprehensive productive wealth) for different countries. They find that some (mostly poor) countries are on unsustainable trajectories because their investments in physical and human capital do not offset their depletion of natural capital.

[2] Expressing welfare as $U(C/N)$ is often referred to as average or classic utilitarianism and $NU(C/N)$ as total utilitarianism. The latter values the number of people, but only in terms of utility from consumption.

[3] There are other ways to define sustainability. The objective could be to maintain (expected) per-period utility as opposed to the entire forward-looking discounted utility of equation (2). Maintaining per-period utility corresponds to the max-min approach introduced by Solow (1974) and Hartwick (1977) in the context of exhaustible resources. Another objective, which lends itself to including uncertainty, is to give future generations the ability to sustain certain targets, as in Fleurbaey (2015). Agliardi (2011) shows how the stochastic evolution of the capital stock and productivity can affect $\mathcal{E}_t dV$ in (2) but does not solve for V_t.

In their paper, CM assume that social welfare derives from wealth-generated consumption via a constant relative risk aversion (CRRA) utility function, that wealth can be invested in risk-free and risky capital, and that the value of risky capital is driven by both Brownian motion and Poisson jumps of random size. They show how these two types of stochastic shocks affect portfolio choice—i.e., the fraction of wealth that society optimally invests in risky capital and the sustainable consumption-wealth ratio. They find that the sustainability constraint does not affect portfolio choice, and the sustainable consumption-wealth ratio lies between the risk-free interest rate and the expected return on optimally invested wealth.

A definition of sustainable consumption based on the expected rate of change of expected future social welfare should not be controversial. But how should we define social welfare? CM define it as the expected discounted flow of CRRA utility from consumption. I use a more general definition of social welfare that includes population, and I allow both population and productive wealth to evolve stochastically.

How might population affect social welfare? I consider the following:

(i) Total productive wealth generates total consumption, but what matters for individual welfare is per capita consumption. Holding total consumption fixed, when population increases, per capita consumption falls. If the population is homogeneous, the welfare of each individual falls, and thus so does social welfare. (CM also allow for this.)

(ii) People consume, but they also utilize capital to produce, and they raise productivity by generating new ideas. Thus, total consumption should depend on the total population. If the population increases, does the productive capacity of the economy—and, hence, total output and consumption—increase more or less than proportionally in response? Although the subject of considerable research, to my knowledge there is no clear answer to that question. Thus, I examine how sustainable consumption depends on how population growth affects aggregate production.

(iii) Both economists and philosophers have long argued that society values the very existence of people—alive now and potentially in the future—and not just the consumption-based utility enjoyed by those people and/or their contribution to aggregate production. Then, population can enter the social welfare function in a more complex way, so that in principle more people can raise—or lower—welfare.

I start with a simplified version of the CM model: (i) I assume that all individuals are the same—i.e., there is no heterogeneity within the population. (ii) I assume that production and, hence, consumption require productive wealth, which includes physical and human capital, as well as the technological know-how to make that capital productive. (iii) I measure sustainable consumption in terms of its relationship to wealth; i.e., I calculate a sustainable consumption-wealth ratio and compare it to the optimal (unconstrained) ratio that maximizes welfare. But unlike CM, I assume that all productive wealth is risky, so I ignore portfolio choice. I also assume that all fluctuations in wealth are continuous (there are no jumps). Eliminating portfolio choice and Poisson jumps simplifies the analysis and facilitates the introduction of population as a second stochastic state variable.[4]

My model diverges from CM in other important respects. First, I assume that production (and, hence, consumption) requires labor in addition to capital, and I take the labor force to be proportional to population. Likewise, productivity growth depends on the generation of new ideas, which also requires people.[5] I assume that the relationship between population and output is isoelastic, and I examine how the elasticity affects the sustainable and optimal consumption-wealth ratios. Second, I introduce a more general social welfare function that explicitly includes population. I use this to explore how sustainable consumption depends on how we value the existence of people (whether positively or negatively), apart from their consumption and their contribution to aggregate output. As for population itself, I allow it to evolve as a continuous stochastic process.

Because productive wealth and population follow geometric Brownian motions (GBMs) in my model, social welfare also follows a GBM, and its drift depends on the consumption-wealth ratio. I define a sustainable consumption-wealth ratio as one for which the drift of the stochastic process for social welfare is nonnegative. (Hence, the expected value of social welfare is not expected to decline in the future.) This condition yields a constraint on the maximum consumption-wealth ratio. Also, the social welfare function can be determined analytically and maximized to yield the optimal (unconstrained) consumption-wealth ratio.

The model yields several insights: (i) As in earlier deterministic models, if the return on capital is low and/or population growth is high, a positive sustainable consumption-wealth ratio may not exist. (ii) An increase in the volatility of the return on capital always reduces the sustainable consumption-wealth ratio, but an increase in the volatility of population growth can increase or decrease the ratio, depending on the parameters of the model. (iii) Sustainable consumption depends critically on how society values human lives, now and in the future. A positive (negative) intrinsic social value of lives raises (reduces) the sustainable consumption-wealth ratio. The reason is that consumption and population become substitutes in terms of their contributions to social welfare. (iv) For plausible parameter values, the sustainable consumption-wealth ratio is well below the optimal ratio that maximizes social welfare. Thus, achieving sustainability can come at the cost of a substantial welfare loss.

[4] CM also show how their results change when there is no risk-free asset and there are no jumps in wealth.

[5] Arrow, Dasgupta, and Mäler (2003b) examine sustainable consumption when welfare depends on per capita consumption and population (labor) enters the production function, but in a deterministic setting.

I. The Model

Most studies of optimal population, income growth, and sustainability describe welfare in terms of average or total utilitarianism—i.e., $U(C/N)$ or $NU(C/N)$, respectively.[6] I characterize social welfare in terms of a more general utility function:

$$U(C,N) = \frac{1}{1-\gamma}\left[(C/N)^{1-\phi}N^{\phi}\right]^{1-\gamma}. \tag{3}$$

This is a CRRA function of a composite that combines per capita consumption (C/N) and population N, with weights $1-\phi$ and ϕ, respectively, and $\phi < 1$. If $\phi = 0$, social welfare depends only on per capita consumption, as in most models of sustainability. But if $\phi \neq 0$, equation (3) says that we care not only about our individual consumption but also about the existence of other people. Depending on the value of ϕ, holding per capita consumption fixed, we might prefer the existence of more people to fewer people, or the reverse.

Utility functions related to (3) have been used by others in different contexts. Ashraf and Galor (2011) developed a Malthusian growth model with technological change in which individual utility depends on consumption and also on the number of surviving children, with weights $1-\phi$ and ϕ, respectively, as in (3). And Razin and Yuen (1995) and Boucekkine and Fabbri (2013) use a utility function of the form $N^{\eta}(C/N)^{1-\gamma}/(1-\gamma)$, where the "altruism" parameter η captures the welfare contribution of surviving children. These other studies are focused on growth, while the emphasis here is on the welfare effects of changes in population and productive wealth and on the implications for sustainable consumption.

In what follows, I assume $\gamma > 1$ for simplicity. If $\phi = 0$, γ is a coefficient of relative risk aversion, and $\gamma > 1$ would be consistent with economic and financial data. More generally, γ just defines the marginal social utility of an increase in the $[(C/N), N]$ composite, whether resulting from a change in C, N, or both.

For now, I assume that C is independent of N. (This assumption will be relaxed shortly.) With $\phi < 1$, the utility function (3) has the usual dependence on consumption—i.e., $U_C > 0$, $U_{CC} < 0$, and $U_C \to \infty$ as $C \to 0$. However, U_N can be positive or negative, depending on ϕ. If $\phi = 0$, a catastrophic event that causes a drop in N is a pure blessing for those that remain alive, because per capita consumption rises and N itself is not directly valued.[7] In general, $U_N > (<)\ 0$ if $\phi > (<)\ 1/2$, and $U_N = 0$ if $\phi = 1/2$.

What can we say about the value of ϕ? We expect social welfare to be an increasing function of per capita consumption, but should it also be increasing (or decreasing) in population? More generally, why might we care at all about the existence

[6] Several authors explored the implications of average versus total utilitarianism (in a deterministic context). Parfit (1984) argued that welfare maximization under total utilitarianism implies a growing population with per capita consumption approaching zero (the "Repugnant Conclusion"). But Boucekkine and Fabbri (2013); Palivos and Yip (1993); and Razin and Yuen (1995) showed that the "Conclusion" is avoided if population is linked to growth or parents value their offspring. Bonneuil and Boucekkine (2014) show how optimal and sustainable consumption differ in the context of a Ramsey growth model and total utilitarian welfare.

[7] This is similar to the "benefits" of wars and major pandemics in the articles by Young (2005) and Voigtländer and Voth (2009, 2013).

of other people, and why should a larger population be more or less preferable to a smaller one? These questions have been addressed from several angles. Studies of the optimal population size or growth rate have used alternative social welfare functions to evaluate outcomes, in the context of growth models with population exogenous or endogenous.[8] The social choice literature instead uses an axiomatic approach that derives social orderings for which population is one of the choice variables (see, e.g., Broome 2004 and Blackorby, Bossert, and Donaldson 2005).

In practice, society values changes in population asymmetrically. Based on the social norms of most countries, mortality-based *decreases* in population are almost always treated as "bad," in that societies go to great lengths to save lives and prevent life-threatening disasters (but not always for decreases dues to low birthrates.) *Increases* in population, however, are seen by some as "good" (on purely ethical grounds, but also by contributing to technological change, and ensuring that the old are cared for) and by some as "bad" (because of crowding and environmental stress). Thus, social welfare should depend at least in part on population, even though there may be no consensus regarding the form of that dependence.

Why might an increase in N be bad? A common argument is that more people cause unwanted "congestion," in the broad sense of the term. This argument is usually environmental in nature; e.g., more people will crowd our national parks, accelerate climate change and pollution generally, and more rapidly deplete our natural resources.[9] Even if resources and externalities are priced properly, welfare can fall. For example, if carbon emissions are taxed at their social cost, an increase in N that increases emissions will still reduce total and per capita consumption. And if the externalities are not priced properly (as is usually the case), the negative impact of an increase in N would be even greater. Thus, environmentalists (and others) might argue for a low or even negative value of ϕ, so that $U_N < 0$.

One argument for a social preference for a *higher N* is that more people are needed to drive technological progress and economic growth, as in Kremer (1993); Jones (1999); Desmet, Nagy, and Rossi-Hansberg (2018); and related models.[10] Population growth is also needed to provide workers to generate output, especially as the pool of older retirees expands. Many societies exhibit a preference for a higher N, insofar as they try to prevent or limit the deaths of their citizens and subsidize the

[8] Welfare functions include the utility of a representative agent, per capita utility, and the sum of utilities over the existing population. Recent work has made population growth endogenous and allowed a parent's utility to depend on both her own consumption and the utility and number of her children; see, e.g., Harford (1998); Razin and Yuen (1995); Razin and Sadka (1995); and Ashraf and Galor (2011). When welfare depends on the utility of those who exist as well as those yet to be born, Pareto efficiency is not well defined, unless it is based on a representative agent whose preferences apply across generations.

[9] The environmental argument for $\phi < 0$ would be weakened if we include environmental amenities as part of consumption. See Harford (1998) and Bohn and Stuart (2015). Based on climate change impacts, Casey and Galor (2016) and Scovronick et al. (2017) estimate a social benefit from slower population growth. Acemoglu, Fergusson, and Johnson (2020) show that increases in population from improvements in health led to more civil wars and other violent conflicts. What is the limit to how many people can live (tolerably) on Earth? We don't know, but Cohen (1995) surveys estimates of Earth's human carrying capacity.

[10] Jones (2021, 2022) shows how in semiendogenous growth models, research-generated growth is proportional to the rate of population growth, so declines in population growth could reduce GDP growth. Peters (2022) provides empirical evidence on the positive relationship between population growth and GDP growth.

bearing and rearing of children.[11] Finally, many will argue that lives simply have intrinsic social value, so that two million people each enjoying utility U_0 is preferred to one million people enjoying that same utility, and *may* be preferred to one million people each enjoying utility $1.5U_0$. This argument implies a social preference for a higher N and, thus, a higher value of ϕ, so $U_N > 0$ and $U_{NN} < 0$.

We will explore the implications for sustainable consumption of a social preference for more or fewer people. But first we need to lay out the full model.

A. *Consumption, Wealth, and Population*

Consumption in this model is generated by productive wealth, W_t:

$$C_t = \theta W_t,$$

where θ is the consumption-wealth ratio, which, as we will see, is constant as long as the parameters of the model are constant. Productive wealth is a function of physical capital K_t and a stock of know-how and ideas A_t that contributes to productivity. But production also depends on population N_t. People provide labor to operate capital, so output is proportional to $K_t N_t^{\beta_1}$, with $\beta_1 > 0$. People also drive the production of new ideas, which I model as $A_t = AN_t^{\beta_2}$, where A is a constant and $\beta_2 > 0$. Thus, productive wealth can be written as

$$W_t = AK_t N_t^{\beta}, \tag{4}$$

where $\beta = \beta_1 + \beta_2 > 0$. With no loss of generality, I set $A = 1$. A growing population yields consumption growth, and if $\beta > 1$, there are increasing returns to population growth.

How does W_t change over time? It will increase if K_t increases (as part of output is invested in additional capital) or if N_t increases. It will decrease if K_t or N_t decrease, but also as a fraction of wealth, θ, is consumed. Consumption is expressed as a fraction of wealth, but it comes out of physical capital K_t, which is proportional to wealth, so $C_t = \theta K_t N_t^{\beta} = \theta W_t$, as above.[12]

Substituting $C_t = \theta K_t N_t^{\beta}$ into (3), the utility function becomes

$$U(K,N) = \frac{1}{1-\gamma}\left[\left(\theta K_t N_t^{\beta-1}\right)^{1-\phi} N_t^{\phi}\right]^{1-\gamma}. \tag{5}$$

If $\beta = 0$, C is independent of N; if $\beta = 1$, an increase in N leaves $C/N = \theta K N^{\beta-1} = \theta K$ unchanged, and if $\beta > 1$, an increase in N raises C/N. What is a reasonable value for β? The share of GDP paid to labor is around two-thirds, which

[11] Societies often weigh lives against consumption, and a large literature addresses the economic and ethical aspects of those trade-offs. See, e.g., Broome (2004) and Blackorby, Bossert, and Donaldson (2005). Millner (2013) provides an argument for including population as a determinant of social welfare, and de la Croix and Doepke (2021) show how we might value changes in population.

[12] This is a one-sector economy with output that can be consumed or utilized as capital. Note that $A_t = AN_t^{\beta_2}$ can fall (if N_t falls). The stock of ideas can decline if they become less useful—e.g., if there are fewer people who want the output generated by those ideas. In principle, A_t could vary stochastically and independently of N_t, but introducing another state variable will complicate the analysis without providing additional insight.

suggests a value around 0.6 or 0.7. This would imply that increasing N alone would reduce C/N (but might increase welfare, depending on ϕ). On the other hand, some studies suggest that β may exceed 1.[13]

Note from equation (5) that $U_K > 0$ and $U_{KK} < 0$ for all $\phi < 1$ and all β. However, the signs of U_N and U_{NN} are determined by β and ϕ together. To have $U_N > 0$ (so that adding another person to the population increases welfare), we need $(\beta - 1)(1 - \phi) + \phi > 0$. If $\phi \geq 1/2$, this holds for any $\beta > 0$, but if $\phi < 1/2$, then $U_N > 0$ only if $\beta > (1 - 2\phi)/(1 - \phi)$.

Given equation (5) for $U(K,N)$, we can modify equation (2) to write welfare at time 0 as

$$V_0 = \frac{1}{1-\gamma}\mathcal{E}_0 \int_0^\infty \left[g(K_t,N_t)\right]^{1-\gamma} e^{-\rho t}\,dt, \tag{6}$$

where

$$g(K_t,N_t) = (\theta K_t)^{1-\phi} N_t^{\omega} \tag{7}$$

and $\omega \equiv \beta(1 - \phi) + 2\phi - 1$.

To complete the model, we need to describe the evolution of K_t and N_t. I assume that both follow independent GBMs:

$$dK/K = (r - \theta)dt + \sigma_K dz_K, \tag{8}$$

$$dN/N = ndt + \sigma_N dz_N. \tag{9}$$

Here, r is the expected return on capital, and n is the expected rate of population growth. Because consumption draws down the capital stock, $(r - \theta)$ is the drift of dK/K.

B. *Sustainable and Optimal Consumption*

We want the maximum value of θ consistent with the sustainability constraint—i.e., the value θ_{max} that just satisfies the constraint that welfare (6) is not expected to decline. Thus, we must find the drift (expected rate of change) of this integral. For comparison, we also want the optimal value of θ—i.e., the value θ_{opt} that maximizes welfare (and might exceed θ_{max}).

Given the form of $g(K_t,N_t)$ in equation (7) and the assumption that K_t and N_t follow GBMs, V_t follows a GBM, so I can apply the approach used by CM to find θ_{max} and θ_{opt}. To find θ_{max}, note that V_t is proportional to $\frac{1}{1-\gamma}\left[g(K_t,N_t)\right]^{1-\gamma}$, which is negative because $\gamma > 1$.Thus, the sustainability constraint implies that $X_t \equiv \left[g(K_t,N_t)\right]^{1-\gamma}$ is not expected to increase—i.e., the drift of dX_t/X_t is nonpositive. To find θ_{opt}, I solve for V_0 and maximize with respect to θ.

[13] See, e.g., Kuznets (1960) and Kremer (1993). The value of β is likely to change over time and vary with per capita income; see, e.g., Kelley (1988); Kelley and Schmidt (1994); and Robinson and Srinivasan (1997).

Sustainable Consumption.—To find the maximum consumption-wealth ratio consistent with sustainability, we need the drift of dX_t/X_t. Substituting (7) for $g(K_t, N_t)$ yields the following expression for X_t:

$$X_t = (\theta K_t)^{(1-\phi)(1-\gamma)} N_t^{\omega(1-\gamma)}. \tag{10}$$

(Recall that $\omega \equiv \beta(1-\phi) + 2\phi - 1$.) Then, using Ito's lemma,

$$\begin{aligned} dX_t/X_t = (1-\gamma)\Big\{ &(1-\phi)(r-\theta) + \omega n \\ &+ \frac{1}{2}(1-\phi)\big[(1-\phi)(1-\gamma) - 1\big]\sigma_K^2 \\ &+ \frac{1}{2}\omega\big[\omega(1-\gamma) - 1\big]\sigma_N^2\Big\} dt \\ &+ (1-\gamma)(1-\phi)\sigma_K dz_K + \omega(1-\gamma)\sigma_N dz_N. \end{aligned} \tag{11}$$

Since $(1-\gamma) < 0$, the constraint implies that the bracketed part of the drift of dX_t/X_t must be nonnegative—i.e.,

$$\begin{aligned} &(1-\phi)(r-\theta) + \omega n + \frac{1}{2}(1-\phi)\big[(1-\phi)(1-\gamma) - 1\big]\sigma_K^2 \\ &+ \frac{1}{2}\omega\big[\omega(1-\gamma) - 1\big]\sigma_N^2 \geq 0. \end{aligned} \tag{12}$$

This can be rewritten as

$$\begin{aligned} \theta_{max} = r + \left(\frac{\omega}{1-\phi}\right)n &+ \frac{1}{2}\big[(1-\phi)(1-\gamma) - 1\big]\sigma_K^2 \\ &+ \frac{1}{2}\left(\frac{\omega}{1-\phi}\right)\big[\omega(1-\gamma) - 1\big]\sigma_N^2. \end{aligned} \tag{13}$$

Note that as long as the parameters r, ϕ, n, etc. remain constant, θ_{max} will be constant. Also, θ_{max} does not depend on the discount rate ρ, a social preference parameter that values utility in the future versus today. As shown below, the unconstrained optimal consumption-wealth ratio θ_{opt} does depend on ρ, but θ_{max} is a limit that ensures consumption is sustainable, irrespective of society's time preference. However, θ_{max} depends on γ, which also reflects social preferences, via σ_K and σ_N. As can be seen from equation (10), welfare is a nonlinear function of K and N and thus is impacted by stochastic fluctuations in K and N. An increase in γ increases the curvature of the welfare function and, thus, the sizes of those impacts.

Optimal Consumption.—To obtain θ_{opt}, start with dg_t/g_t. From (7), (8), and (9),

$$\begin{aligned} dg_t/g_t = &\left[(1-\phi)(r-\theta) + \omega n - \frac{1}{2}\phi(1-\phi)\sigma_K^2 + \frac{1}{2}\omega(\omega-1)\sigma_N^2\right]dt \\ &+ (1-\phi)\sigma_K dz_K + \omega\sigma_n dz_N. \end{aligned} \tag{14}$$

By Ito's lemma, $d\log g_t = dg_t/g_t - \frac{1}{2}(dg_t/g_t)^2$, so

$$d\log g_t = \Big[(1-\phi)(r-\theta) + \omega n - \frac{1}{2}(1-\phi)\sigma_K^2 - \frac{1}{2}\omega\sigma_N^2\Big]dt + (1-\phi)\sigma_K dz_K + \omega\sigma_n dz_N. \tag{15}$$

Integrate this forward and exponentiate to get g_t:

$$g_t = g_0 \exp\Big\{\Big[(1-\phi)(r-\theta) + \omega n - \frac{1}{2}(1-\phi)\sigma_K^2 - \frac{1}{2}\omega\sigma_N^2\Big]t + (1-\phi)\sigma_K z_K + \omega\sigma_N z_N\Big\}, \tag{16}$$

where $g_0 = (\theta K_0)^{1-\phi} N_0^{\omega}$. Now raise g_t to the power $(1-\gamma)$, substitute into equation (6), and integrate to get the following expression for V_0:

$$V_0 = \frac{(\theta K_0)^{(1-\phi)(1-\gamma)} N_0^{\omega(1-\gamma)}}{(1-\gamma)\rho - (1-\gamma)^2\Big[(1-\phi)(r-\theta) + \omega n - \frac{1}{2}(1-\phi)\sigma_K^2 - \frac{1}{2}\omega\sigma_N^2\Big]}. \tag{17}$$

Maximizing V_0 with respect to θ yields θ_{opt}:

$$\theta_{opt} = \frac{\rho + (\gamma-1)\Big[(1-\phi)r + \omega n - \frac{1}{2}(1-\phi)\sigma_K^2 - \frac{1}{2}\omega\sigma_N^2\Big]}{\gamma(1-\phi)+\phi}. \tag{18}$$

The sustainability constraint is binding when $\theta_{max}/\theta_{opt} < 1$. As one would expect, θ_{opt} is increasing in ρ, because a higher ρ means that society wants more utility today versus in the future, regardless of the impact on sustainability. Thus, the constraint is more likely to be binding if ρ is large. If $\sigma_N = 0$, the constraint is binding if $\rho > \theta_{max}$ (as in the CM model). Later, we will see how $\theta_{max}/\theta_{opt}$ depends on σ_N, r, n, and the other parameters.

C. *Sustainable Consumption When Lives Have No Intrinsic Value*

I introduced population N into the social welfare function for three reasons: (i) Welfare should depend on per capita consumption C/N. (ii) People are needed to produce and innovate, so C is an increasing function of N; here, $C_t = \theta K_t N_t^{\beta}$, with $\beta > 0$. (iii) Society might value the existence of people apart from their consumption and contribution to production.

Letting welfare depend on per capita consumption and letting consumption be an increasing function of population should not be controversial. But letting population affect welfare directly might appear strange to some readers. Therefore, I begin by assuming that lives have no intrinsic social value—i.e., $\phi = 0$. This provides a link to the existing literature and a reference case for comparison when lives do have intrinsic value.

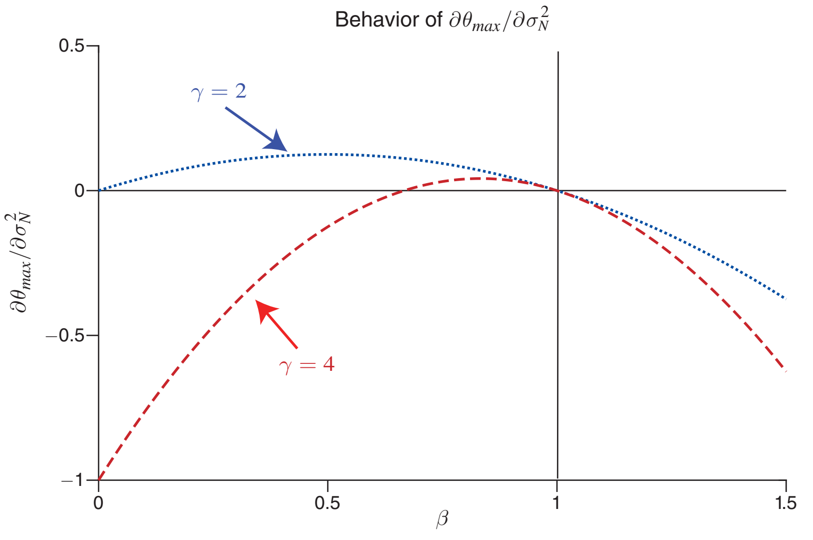

Figure 1. Dependence of $\partial\theta_{max}/\partial\sigma_N^2$ on β and γ When $\phi = 0$

Notes: If $\gamma = 2$, $\partial\theta_{max}/\partial\sigma_N^2 \geq 0$ for all values of β between 0 and 1. Increasing γ increases the range over which $\partial\theta_{max}/\partial\sigma_N^2 < 0$.

If $\phi = 0$, then $\omega \equiv \beta(1-\phi)+2\phi-1 = \beta-1$, and equation (13) for θ_{max} becomes

$$(19)\quad \theta_{max} = r + (\beta - 1)n - \frac{1}{2}\gamma\sigma_K^2 - \frac{1}{2}(\beta - 1)\big[(\beta - 1)(\gamma - 1) + 1\big]\sigma_N^2.$$

With no uncertainty, $\theta_{max} = r + (\beta - 1)n$. If $\beta = 1$, population growth does not affect C/N, so we have the standard sustainability constraint $\theta_{max} = r$; i.e., if $r - \theta \geq 0$, consumption will not draw down productive wealth.[14] If $\beta < (>)\ 1$, then $\theta_{max} = r + (\beta - 1)n < (>) r$; i.e., a lower (higher) consumption-wealth ratio ensures that total productive wealth does not decline.

Suppose just the return on capital is uncertain ($\sigma_K > 0$, $\sigma_N = 0$). Then $\theta_{max} = r + (\beta - 1)n - \frac{1}{2}\gamma\sigma_K^2$, so σ_K^2 reduces θ_{max}, relative to the expected return on capital, r.[15] As we will see, this reduction in θ_{max} can be substantial.

The dependence of θ_{max} on $\sigma_N > 0$ is more complex. First, uncertainty over population growth matters only if $\beta \neq 1$. If $\beta = 1$, an increase in N increases C commensurately, leaving welfare unchanged. If $\beta \neq 1$, an increase in N alters per capita consumption. In general, the sign of $\partial\theta_{max}/\partial\sigma_N^2$ depends on β and γ. With $\phi = 0$,

$$\frac{\partial\theta_{max}}{\partial\sigma_N^2} = \begin{cases} > 0 & \text{if } (\gamma - 2)/(\gamma - 1) < \beta < 1 \\ 0 & \text{if } \beta = (\gamma - 2)/(\gamma - 1) \quad \text{or} \quad \beta = 1 \\ < 0 & \text{if } \beta < (\gamma - 2)/(\gamma - 1) \quad \text{or} \quad \beta > 1 \end{cases}.$$

[14] This is equivalent to the constraint in Dasgupta (2009) and Arrow et al. (2012) that there is no reduction in “comprehensive wealth.” If $\beta = 1$, the sustainability constraint is binding (i.e., $\theta_{max} < \theta_{opt}$) if $\rho > r$. If $\beta = 0$, the constraint is binding if $\rho > (r - n)$.

[15] This differs from CM, who find that σ_K^2 increases θ_{max}, but relative to the risk-free rate r_f. In CM, $r = r_f + \mu$, where μ is the expected excess return, and $\mu = \gamma\sigma_K^2$, so $r = r_f + \gamma\sigma_K^2$. In my model, r is independent of σ_K.

This dependence of $\partial\theta_{max}/\partial\sigma_N^2$ on β and γ is illustrated in Figure 1 and is driven by the fact that in equations (6) and (7) for welfare, when $\phi = 0$, the exponent on N is $(\beta - 1)(1 - \gamma)$, which is $<$ $(>)$ 0 if $\beta > (<)(\gamma - 2)/(\gamma - 1)$, so stochastic fluctuations in N_t increase (reduce) welfare and, thus, θ_{max}.

If $\phi = 0$ and there is no uncertainty, θ_{max} depends only on the expected return on capital r, the expected population growth rate n, and the elasticity of consumption with respect to population β. Apart from this last term, it follows the earlier literature, and the parameter γ plays no role. If $\sigma_K > 0$, θ_{max} is reduced by $\frac{1}{2}\gamma\sigma_K^2$. A reasonable value for σ_K is 0.20, which is roughly the annual standard deviation of returns on the S&P 500. Even if $\beta = 1$, a value of γ above 2 can drive θ_{max} below zero, so that there is *no* sustainable level of consumption. Fluctuations in population growth can raise or lower θ_{max} depending on β and γ, but the impact is small. Historical values of σ_N vary across countries but are usually around 0.01 or 0.02. Then, if $\gamma = 4$ and $\beta = 0$, θ_{max} would fall by 0.0004 or less.

In the next section, I explore how θ_{max} changes when lives have intrinsic social value, but I ignore uncertainty until the following section.

II. Sustainability in a Deterministic World

In most models, population growth reduces sustainable consumption, which must be spread among more people. An exception is when an increase in population proportionally or more than proportionally increases total consumption. But if lives have intrinsic social value, population growth can affect sustainable consumption via a different route.

A. *Lives versus Consumption*

If β is sufficiently large and $\phi > (<) 0$, θ_{max} increases (decreases) when n increases, because population growth and consumption growth become substitutes in terms of their contributions to welfare. With $\phi > 0$, the welfare gain from a growing population can partially offset the loss from reduced future consumption, so productive wealth can be drawn down faster by increasing current consumption (raising θ). Note from (6) and (7) that $g(K_t, N_t) = (\theta K_t N_t^{\beta-1})^{1-\phi} N_t^{\phi} = (C_t/N_t)^{1-\phi} N_t^{\phi}$, so changes in welfare can result from changes in N_t and/or changes in (C_t/N_t). Holding N_t fixed, future consumption can increase via growth in K_t or via growth in wealth from reducing current consumption (reducing θ).

Figure 2 shows $\partial\theta_{max}/\partial n$, i.e., how θ_{max} changes in response to a change in n for several values of β. If $\beta = 1$ (the solid black curve), population growth leaves C/N unchanged. Then, if $\phi = 0$ so only C/N affects welfare, changing n doesn't affect θ_{max}. But if $\phi > 0$, there is a welfare gain from faster population growth, allowing current consumption to be sustainably increased, so $\partial\theta_{max}/\partial n > 0$. The increase in θ_{max} means wealth will be drawn down, reducing future consumption, but the resulting welfare loss is offset by the gain from a growing population.

As Figure 2 shows, this effect holds for every value of β. If $\beta = 0$ so that growth in N_t does not increase consumption, $\partial\theta_{max}/\partial n = -1$ when $\phi = 0$, but increases with ϕ and is positive when $\phi > \frac{1}{2}$. Again, a higher θ_{max} means that wealth is

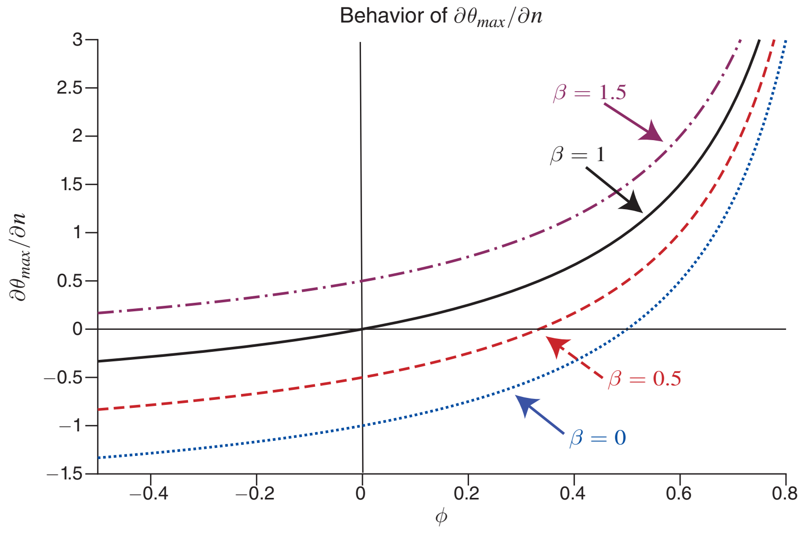

Figure 2. Dependence of $\partial\theta_{max}/\partial n$ on ϕ for Different Values of β

Notes: An increase in β implies an increase in the contribution of population to total consumption, raising $\partial\theta_{max}/\partial n$. For any value of β, $\partial\theta_{max}/\partial n$ is increasing in ϕ. If $\phi > 0$, growth in N_t adds to welfare, raising θ_{max} so wealth can be drawn down faster. This reduces future consumption and, thus, welfare, but is sustainable because it is offset by the increase in welfare from a larger future population.

drawn down faster, but this is sustainable because of the welfare gain from a larger population.

What if lives have a negative social value, so $\phi < 0$? Now if $\beta = 1$, increasing n reduces θ_{max}, because the welfare loss from faster population growth must be offset by a gain from greater future consumption. But faster consumption growth requires growth in productive wealth and, thus, reduced current consumption, so θ_{max} is lower.

B. *Bounds on Population Growth*

Figure 2 shows how a change in the mean rate of population growth n affects θ_{max}, reducing it if $\phi \leq 0$ and $\beta < 1$ and possibly increasing it if $\phi > 0$ or $\beta > 1$. But depending on ϕ and β, a sufficiently high—or low—value of n could result in $\theta_{max} < 0$, so that there is *no* sustainable level of consumption. I define the *critical* values of n as the upper and lower bounds at which $\theta_{max} = 0$. Figure 3 shows these bounds, n_c, as functions of ϕ for $\beta = 0$ (blue dotted line), $\beta = 0.5$ (red dashed line), and $\beta = 1.0$ (green dot-dash line). The vertical lines are at values of ϕ for which θ_{max} is independent of n.

Suppose $\beta = 1$, so $\theta_{max} = r + \phi n/(1 - \phi)$ is independent of n when $\phi = 0$. If $\phi < 0$, any population growth reduces welfare, and if population growth is large enough, it can drive θ_{max} below zero. In Figure 3, $\theta_{max} > 0$ only if $n < n_c = -r(1 - \phi)/\phi$, i.e., n is below the green dashed line at the top left corner of the figure.

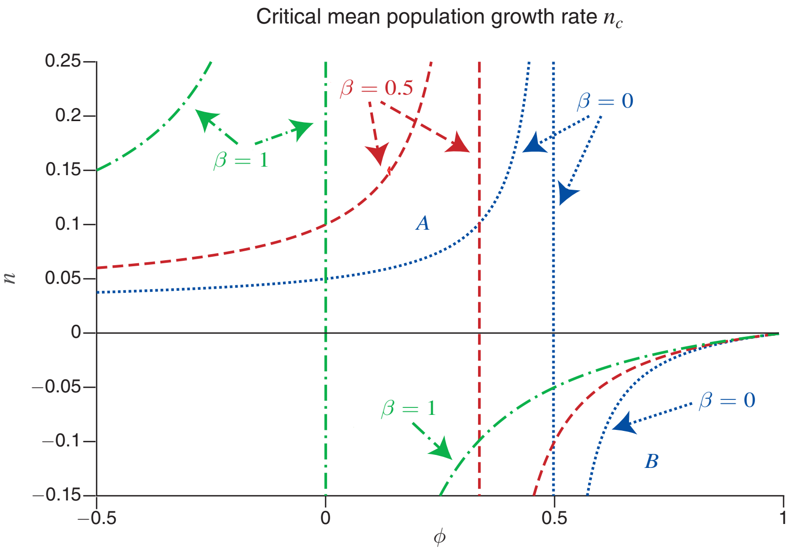

FIGURE 3. CRITICAL POPULATION GROWTH RATES n_c AT WHICH $\theta_{max} = 0$, FOR $r = 0.06$ AND $\sigma_A = \sigma_N = 0$

Notes: If $\beta = 1$ (green dot-dash line), $\theta_{max} = r + \phi n/(1 - \phi)$ is independent of n when $\phi = 0$. If $\phi \neq 0$, n must be between the green lines to have $\theta_{max} \geq 0$. If $\beta = 0$ (blue dotted line), $\theta_{max} = r + (2\phi - 1)n/(1 - \phi)$, so if $\phi = 0.2$ and $n = 0.09$ as at point A, $\theta_{max} = -0.01$, and no level of consumption is sustainable. For large ϕ, n must not be too low. If $\phi = 0.7$, $n_c = -0.045$, so if $n = -0.10$ (point B), $\theta_{max} = -0.073$; now no level of consumption yields enough welfare to offset the loss from a rapidly declining population.

If $n > n_c$, then even reducing current consumption to zero to build up wealth and increase future consumption, the resulting welfare gain will be smaller than the loss from a growing population. But if $\phi > 0$, the constraint is turned around. Now $n_c < 0$, and we need $n > n_c$ to have $\theta_{max} > 0$. If $n < n_c$, the welfare loss from a falling population cannot be offset by the gain from increasing future consumption, even by consuming nothing today.

If $\beta = 0$, doubling the population cuts C/N in half. Now $\theta_{max} = r + (2\phi - 1)n/(1 - \phi)$, which is independent of n if $\phi = 0.5$, and $n_c = r(1 - \phi)/(1 - 2\phi)$, shown as the blue dotted line in Figure 3. If $\phi = 0$, sustainable consumption requires $n < n_c = r$. If $\phi = 0.2$, $n_c = 1.33r = 0.08$ with $r = 0.06$. So if $n = 0.09$ as at point A, then $\theta_{max} = -0.01$, and no positive consumption is sustainable. But if n is only 0.02, then $\theta_{max} = 0.045$. If society consumes at this level, wealth is drawn down, but the welfare loss from reduced consumption growth is offset by the gain from population growth. For large ϕ, population growth must not be too low. If $\phi = 0.7$, $n_c = -0.045$, so if $n = -0.10$ (as at point B), $\theta_{max} = -0.073$, so no level of consumption provides enough welfare to offset the loss from a rapidly declining population. Welfare is unsustainable, not because of too much consumption but because of too few people.

As Figure 3 shows, depending on the value that society places on lives, population growth can be too high—or too low—to allow for any sustainable level of consumption.

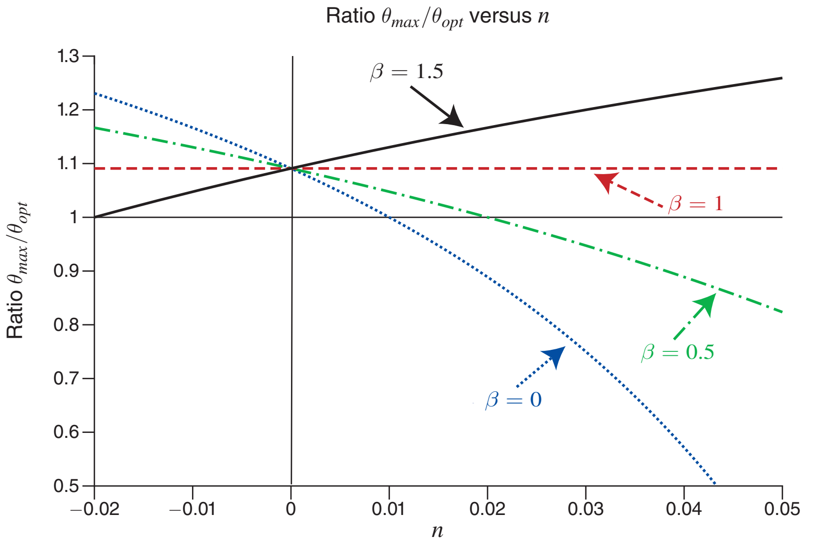

Figure 4. The Ratio $\theta_{max}/\theta_{opt}$ versus the Mean Population Growth Rate n, for $r = 0.06$, $\rho = 0.05$, $\gamma = 2$, $\sigma_A = \sigma_N = 0$, and $\phi = 0$

Notes: If $\beta = 0$ or 0.5, the ratio declines as n increases because population growth contributes little or nothing to output and has no intrinsic value ($\phi = 0$). If $\beta = 1.5$, population growth raises per capita consumption, so the ratio increases with n.

C. *Population Growth and the Cost of Sustainability*

Population growth affects the sustainable and optimal consumption-wealth ratios differently, thereby affecting the social cost of sustainability. Here, we show how the ratio varies with n.

Figure 4 shows $\theta_{max}/\theta_{opt}$ versus n for $r = 0.06$, $\rho = 0.05$, $\gamma = 2$, $\phi = 0$, and, again, $\sigma_K = \sigma_N = 0$. If $n = 0$ (no population growth), the ratio is independent of β. If $\beta = 0$ (the blue dotted line in the figure), population growth is a pure burden on welfare because it contributes nothing to output and has no intrinsic value ($\phi = 0$), so the ratio declines with n, falling below 1 at $n = 0.01$. The decline is less steep if $\beta = 0.5$, and if $\beta = 1$, population growth leaves C/N unchanged, so the ratio is independent of n. If $\beta = 1.5$, population growth raises C/N, so the ratio increases with n. The relationship between population growth and total consumption is symmetric; if $\beta = 1.5$ ($\beta = 0$), negative population growth reduces (increases) C/N. So when $n < 0$, $\theta_{max}/\theta_{opt}$ is larger for $\beta = 0$ than for $\beta > 0$.

Figure 5 also shows $\theta_{max}/\theta_{opt}$ versus n, but now with $\phi = 0.3$, so population growth has positive social value. Now population growth affects welfare through its intrinsic value and (as before) its effect on per capita consumption. When $\beta = 0.5$, the two effects nearly offset each other; a higher n reduces welfare by reducing C/N but increases welfare via the intrinsic value of more people. If $\beta = 0$, the ratio declines with n, but less rapidly than in Figure 4 because a larger population increases welfare. If $\beta = 1$ (so population growth leaves C/N unchanged), the ratio

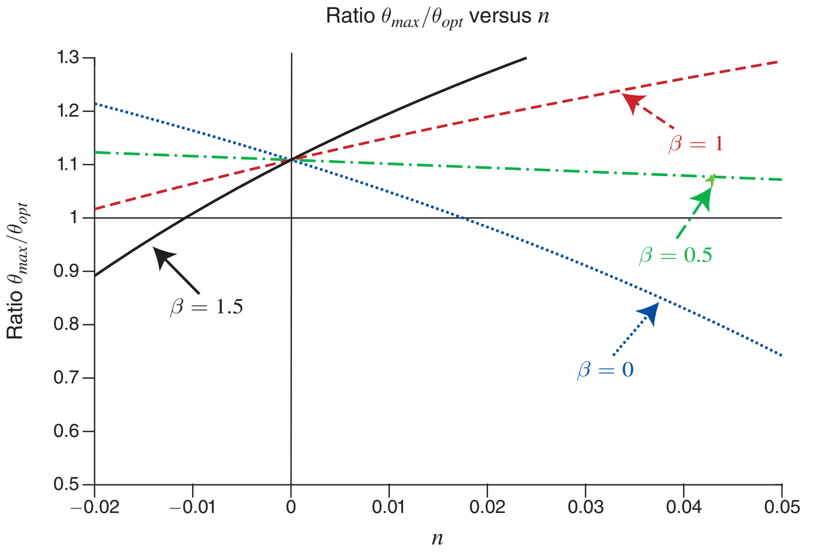

Figure 5. The Ratio $\theta_{max}/\theta_{opt}$ versus the Mean Population Growth Rate n, for $r = 0.06$, $\rho = 0.05$, $\gamma = 2$, $\sigma_A = \sigma_N = 0$, and $\phi = 0.3$, So Population Growth Has Positive Social Value

Note: Now if $\beta = 1$, the ratio increases with n (and increases rapidly if $\beta = 1.5$).

increases with n and increases rapidly if $\beta = 1.5$. If $\beta = 1.5$, a *negative* value of n reduces social welfare in two ways, by reducing C/N and by reducing the intrinsic value of the (shrinking) population.

III. Risk and the Sustainability Constraint

Stochastic fluctuations in K_t reduce θ_{max}, and depending on β and γ, fluctuations in N_t also affect θ_{max}. A reasonable value for σ_K is 0.2, which significantly reduces $\theta_{max}/\theta_{opt}$ for every value of ϕ. Depending on β, if $\sigma_K = 0.2$, $\theta_{max}/\theta_{opt}$ can be well below 1, unless ϕ is large.

Figure 6 shows $\theta_{max}/\theta_{opt}$ versus ϕ, for $n = 0.02$, $r = 0.06$, $\rho = 0.05$, $\gamma = 2$, $\sigma_N = 0.02$, $\beta = 0$ and 1, and $\sigma_K = 0$ and 0.2. Start with $\sigma_K = 0$, shown by the two lines starting close to $\theta_{max}/\theta_{opt} = 1$. If $\beta = 0$ (green dotted line), C/N is falling (because $n = 0.02$), so $\theta_{max}/\theta_{opt} < 1$ when $\phi = 0$. But $\theta_{max}/\theta_{opt}$ increases with ϕ, and the gain in value from a growing population eventually outweighs the loss from the lower per capita consumption. If $\beta = 1$ (solid black line), population growth leaves C/N unchanged, and $\theta_{max}/\theta_{opt}$ is always above 1.

For the two curves at the bottom of Figure 6, $\sigma_K = 0.2$. Now, unless ϕ is very large, $\theta_{max}/\theta_{opt} < 1$ for both $\beta = 0$ and 1. If $\beta = 0$ (blue dotted line), $\theta_{max}/\theta_{opt} < 0$ if $\phi < 0$, so no positive level of consumption is sustainable. If $\beta = 1$ (red dash-dot line), $\theta_{max}/\theta_{opt} \approx 0.4$ if $\phi = 0$ and exceeds 1 only if $\phi > 0.5$ In the figure, $\gamma = 2$; raising γ will push the ratio down.

The two curves at the bottom show a sustainable consumption-wealth ratio well below the optimal, so sustainability would impose a welfare cost on society. But

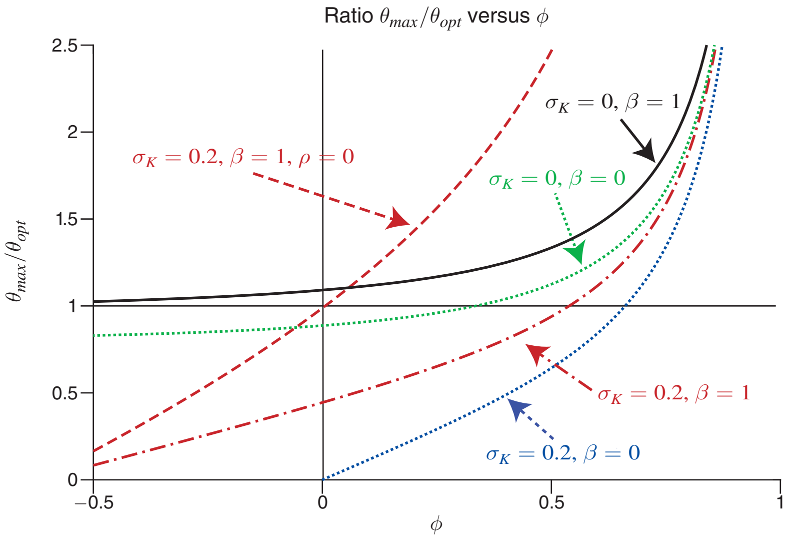

Figure 6. The Ratio $\theta_{max}/\theta_{opt}$ versus ϕ for $n = 0.02$, $r = 0.06$, $\rho = 0.05$, $\gamma = 2$, $\sigma_N = 0.02$, $\beta = 0$ and 1, and $\sigma_K = 0$ and 0.2

Notes: For the two lines starting close to $\theta_{max}/\theta_{opt} = 1$, $\sigma_K = 0$. If $\beta = 0$ (green dotted line), C/N is falling (because $n = 0.02$), so $\theta_{max}/\theta_{opt} < 1$ if $\phi = 0$. But the ratio increases with ϕ because of the gain in welfare from a growing population. If $\beta = 1$ (solid black line), population growth leaves C/N unchanged, and $\theta_{max}/\theta_{opt} > 1$ always. When $\sigma_K = 0.2$, both curves are lower, and the ratio is well below 1 unless ϕ is large. (If $\beta = 0$, as in the blue dotted line, there is no sustainable consumption if $\phi \leq 0$.) But the ratio rises sharply if $\rho = 0$ (the red dashed line for $\beta = 1$) and is above 1 if $\phi > 0$.

there is one parameter to consider in more detail—namely, the discount rate ρ. I set $\rho = 0.05$ to be consistent with numbers in the macroeconomics and finance literature and with experimental evidence on people's time preferences. One could argue, however, that ρ is a normative number that should be based on how society trades off the welfare of future versus current generations. As in the debate over the "correct" discount rate for climate change policy, one could argue that ρ should be much lower than a "market" value, perhaps even 0.

Reducing ρ to zero substantially reduces θ_{opt} and thereby raises $\theta_{max}/\theta_{opt}$, as shown by the red dashed line in Figure 6, for which $\sigma_K = 0.2$ and $\beta = 1$ but $\rho = 0$ instead of 0.05. With this change, $\theta_{max}/\theta_{opt}$ is well above 1 for all $\phi > 0$. This suggests that support for government policies focused on sustainability may have to rely on "ethical" arguments for parameters such as ρ. In this sense, the discount rate plays the same role in "sustainability policy" as it does in climate change policy.

IV. The Social Cost of Sustainability

When the sustainable consumption-wealth ratio is below the optimal ratio, sustainability imposes a welfare cost on society. How large is that cost? To find out, I use equation (6) to compare welfare at time $t = 0$ when $\theta = \theta_{opt}$ (denoted by V_{opt}) to welfare when $\theta = \theta_{max}$ (denoted by V_{max}). Because $\gamma > 1$, V_{opt} and V_{max} are

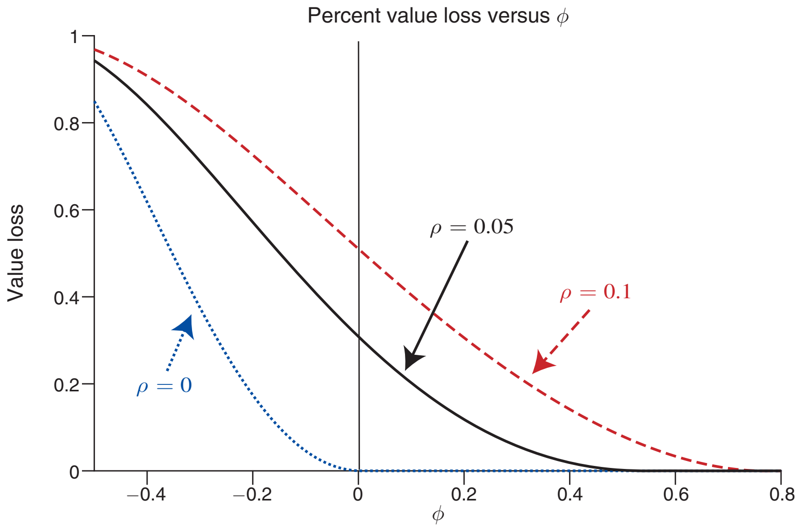

Figure 7. Percentage Loss of Value from Sustainability Constraint

Note: Loss is given by equation (20).

both negative, so this comparison is expressed in terms of the following percentage welfare loss:

$$\text{Loss} = (V_{max} - V_{opt})/V_{max}. \tag{20}$$

Figure 7 shows this percentage loss for three values of ρ and for $n = 0.02$, $r = 0.06$, $\beta = 1$, $\gamma = 2$, $\sigma_N = 0.02$, and $\sigma_K = 0.2$. For $\rho = 0.05$, the welfare loss is considerable unless ϕ is large; if $\phi = 0$, the loss is over 30 percent and becomes 0 only if $\phi > 0.4$. Doubling ρ to 0.10 makes the loss much larger. Also, for any ρ, the percentage loss is large for negative values of ϕ. The reason is that if a growing population has negative social value, sustainability requires it to be offset by the positive value of greater consumption growth. But greater consumption growth requires reducing current consumption so that productive wealth can increase.

Even restricting the analysis to $\phi \geq 0$, the welfare loss is substantial if ρ is 0.05 or greater. But there is no loss if $\rho = 0$. (Reducing ρ reduces θ_{opt} relative to θ_{max}.) This brings us back to the question of whether ρ should reflect people's actual time preferences or instead be treated as a normative number based on the (government's?) ethical view of how society should trade off the welfare of future versus current generations. This is another way of stating the more general question of whether sustainability, as opposed to the maximization of social welfare, should be the objective of government policy.[16]

[16] An alternative measure of the social cost of sustainability is in terms of lost consumption—i.e., Loss $= (\theta_{opt} - \theta_{max})/\theta_{max} = (C_{opt} - C_{max})/C_{max}$.

V. Conclusions

Much of the literature on sustainability focuses on consumption and defines a sustainable consumption path as one for which social welfare never declines. But that literature is largely deterministic and treats population growth as incidental. In this paper, social welfare depends on consumption and population, both of which evolve stochastically. Population affects social welfare through its contribution to total consumption, but also through its intrinsic social value. That social value, which could be positive or negative, can have a substantial effect on the sustainable and optimal levels of consumption.

There are some caveats to keep in mind. Most importantly, the results in this paper follow from the utility function (3) and the sustainability criterion $\mathcal{E}_t dV \geq 0$ for all t. A related family of social utility functions has the form $U(C/N) = F(N)G(C/N)$, where we might have $F(N) = N^{\eta}$ and $G(C/N) = \left[(C/N)^{1-\gamma} - c_0\right]/(1-\gamma)$, and c_0 is a minimally tolerable level of per capita consumption. Likewise, the sustainability criterion might be a per-period constraint, such as $\mathcal{E}_t dU(C_t/N_t) \geq 0$ for all t.

A limitation of the model is that population-induced environmental damage affects welfare only via the parameter ϕ. Ideally, the production of environmental amenities would be modeled separately from the production of other goods and services that are part of consumption, and the production of those amenities would be reduced by an increase in N. Then, the welfare function could include E/N as well as C/N, where E is the production of environmental amenities (and could be negative). But this would complicate the model, which I have tried to keep as simple as possible. So $\phi < 0$ is a proxy for bad behavior (crowding, burning carbon, etc.) on the part of a growing population.

The model in this paper is simple, with few parameters. But determining the values of those parameters is not straightforward and raises questions about sustainability as a social objective. Notably, a high value of the discount rate ρ means that society wants utility—and, hence, consumption—now rather than later, possibly in conflict with the sustainability constraint. Should society be bound by that constraint and reduce its current consumption to benefit future generations rather than consuming at the higher level that maximizes social welfare? Should ρ be set close to zero on "ethical" grounds, which could put the sustainable level of consumption above the optimal level? This dilemma is analogous to the discount rate problem in climate change policy: a high market-based discount rate will imply an optimal policy of limited carbon dioxide emission abatement, at a cost to future generations, leading some to argue for a low ethics-based discount rate instead.

Another difficult problem is how to determine a value for ϕ. How should we decide whether human lives have intrinsic value and what that value is relative to the value of consumption? We have shown that sustainable consumption depends critically on the value society places on lives. There may be no "correct" value for ϕ; instead, this parameter might be viewed as a vehicle for exploring how an intrinsic value of lives affects sustainability.

More generally, plausible parameter values put the sustainable consumption-wealth ratio below the optimal ratio that maximizes welfare. This result is reversed if society places a large positive value on lives, the elasticity of output with respect to population is above 1, and the volatility of the return on capital is low. But without these conditions, the goal of sustainability creates a policy dilemma. Should we reduce consumption to a sustainable level, even if this pushes social welfare below what it could be otherwise? There are some who would argue that sustainability is more important than maximizing welfare. But in that case we should be aware of the costs of sustainability, which we have seen can be substantial.

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