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Lecture 21:
Polarization & Scattering

21.1 Summary: radiation so far

In the last few lectures, we examined solutions of the source free Maxwell equations:

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}\ .$$

With a little massaging, we discovered that these equations can be rewritten as *wave equations* for $\vec{E}$ and $\vec{B}$:

$$\frac{\partial^2 \vec{E}}{\partial t^2} - c^2 \nabla^2 \vec{E} = 0$$
$$\frac{\partial^2 \vec{B}}{\partial t^2} - c^2 \nabla^2 \vec{B} = 0\ .$$

A particularly instructive solution to the wave equations are the *plane wave* forms:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \sin(\vec{k} \cdot \vec{r} - \omega t)$$
$$\vec{B}(\vec{r}, t) = \vec{B}_0 \sin(\vec{k} \cdot \vec{r} - \omega t)\ .$$

This solution represents an electromagnetic wave propagating in the $\hat{k} = \vec{k}/k$ direction (where $k = \sqrt{\vec{k} \cdot \vec{k}} = \sqrt{k_x^2 + k_y^2 + k_z^2}$).

By considering how the wave behaves at some fixed time, we learned that k is simply related to the wavelength λ:

$$k = 2\pi/\lambda\ .$$

The requirement that this solution satisfy the wave equation tells us that

$$\omega = ck\ .$$

From the definition $\omega = 2\pi\nu$ (angular frequency is 2π radians times “regular” frequency), we then obtain

$$\lambda\nu = c\ .$$

Finally, requiring that the plane wave solution satisfy all of Maxwell’s equations leads to some important constraints on the vector amplitudes $\vec{E}_0$ and $\vec{B}_0$. These constraints are:

- The amplitudes are orthogonal to the propagation direction: $\hat{k} \cdot \vec{E}_0 = 0$, $\hat{k} \cdot \vec{B}_0 = 0$.
- The amplitudes are orthogonal to each otherL $\vec{E}_0 \cdot \vec{B}_0 = 0$.
- The amplitudes have the same magnitude: $|\vec{E}_0| = |\vec{B}_0|$.
- The propagation direction is parallel to $\vec{E} \times \vec{B}$.

These are important and rather constraining conditions. Nonetheless, they leave us with a great deal of freedom in the amplitudes. This freedom is described in terms of the radiation's *polarization state*.

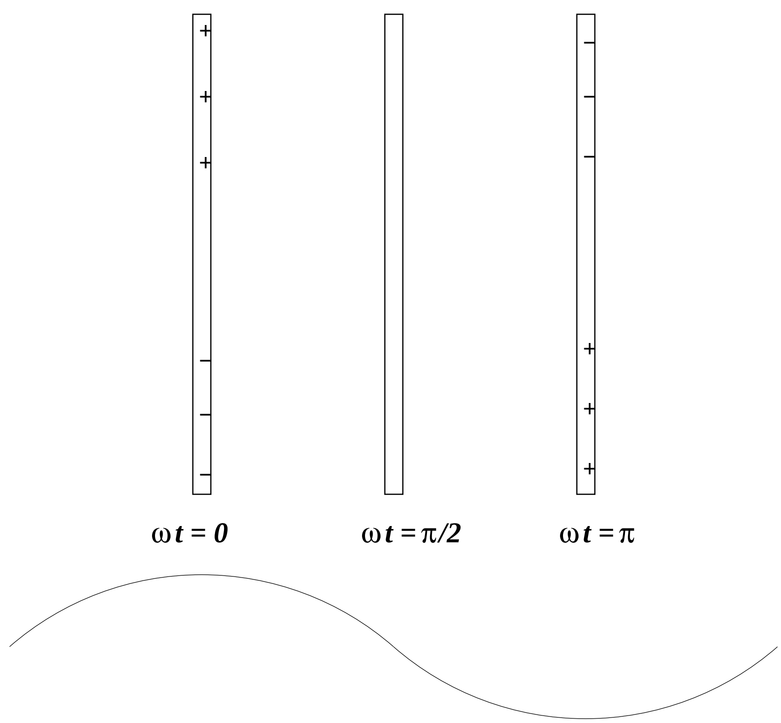
21.2 Linear polarization

Since plane waves propagate in a straight line, we might as well just define their propagation direction as something simple and be done with it. In what follows, we will take $\hat{k} = \hat{z}$, so our wave is of the form

$$\begin{aligned} \vec{E} &= \vec{E}_0 \sin(kz - \omega t) \\ \vec{B} &= \vec{B}_0 \sin(kz - \omega t) \ . \end{aligned}$$

Suppose that the wave is arranged (somehow) so that the electric field is aligned with the x axis: $\vec{E}_0 = E_0\hat{x}$. Then, our requirement that $\vec{E} \times \vec{B}$ be parallel to $\hat{k}$ tells us that $\vec{B}_0 = E_0\hat{y}$. (We're also using $|\vec{E}_0| = |\vec{B}_0|$.)

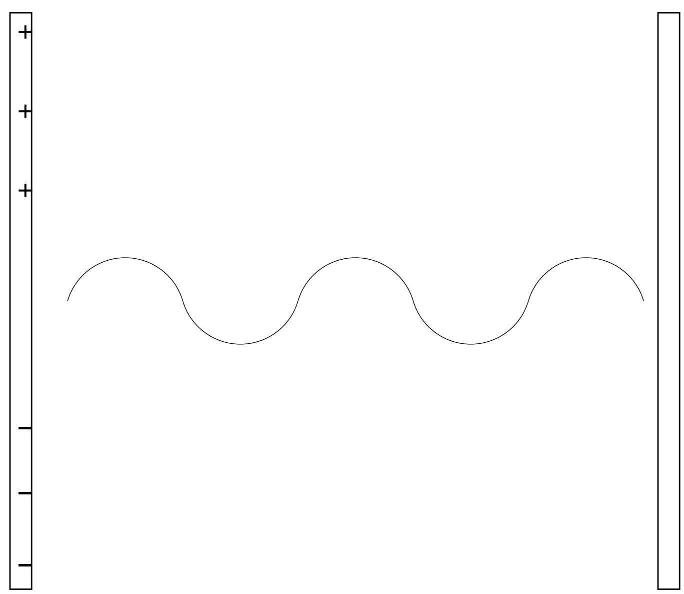
This configuration is known as a *linearly polarized* wave, as the electric (and magnetic) fields at all points align parallel to a line. Such an electromagnetic wave is quite simple to produce: we just need to take a conductor and arrange things so that it has an oscillating charge distribution. Here's an example:



The top part of this figure shows an *antenna*: a long conductor in which we drive a very rapidly oscillating current, so that the conductor has a charge *distribution*. As we move to the right, time increases.

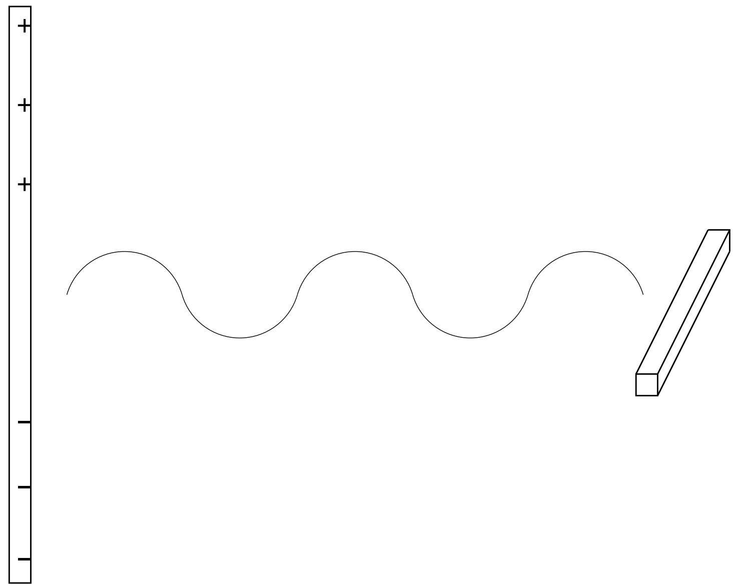
At any given instant, one end is likely to have a net positive charge, the other end a net negative charge; the whole thing is neutral overall. This setup creates an electric field which oscillates in phase with the current's oscillation; this is sketched by the sinusoid below the antenna.

The antenna we have described here is more properly a *generating* or *broadcast* antenna. What if we want to measure the electromagnetic wave that this antenna is broadcasting? Quite simple: we just need to setup a second antenna that will "catch" the oscillating electromagnetic field of the broadcaster. If we put a second antenna down near it, the oscillating electric field will shake charges in this *receiving* antenna:



The oscillating charges themselves produce a current, which we can measure and (possibly) extract information from. This is how a radio works!

When the generated electromagnetic wave is linearly polarized, it is (hopefully) intuitive obvious that the receiving antenna should be oriented parallel to the polarization vector. Suppose we get it wrong by 90°:



For this orientation, the receiving antenna measures doodly squat: there's not enough "room" for a significant charge oscillation to occur.

21.3 Polaroids

Certain substances act like antennas for radiation in the visual band — light that we can see. They typically consist of organic molecules that are fairly extended in one direction. These molecules can carry a current along that direction; but, no current can flow in the direction orthogonal to the molecule's orientation.

A *polaroid* is a sheet of plastic embedded with molecules of this type all lined up along some axis. Suppose linearly polarized light impinges on these molecules. Suppose further that the light is polarized such that $\vec{E}$ is aligned with its organic molecules. Then, the light shakes charges in the polaroid; it generates a current, which generates heat in the plastic sheet. The light is absorbed — nothing gets through.

Suppose instead that light is linearly polarized in a direction *perpendicular* to the sheet's organic molecules. This light will not be able to shake the charges in the sheet — they are not free to move in that direction. Rather than being absorbed, the light passes right through the sheet.

The direction that *allows* light through is, by convention, the "preferred" direction of the polaroid. The punchline of this discussion is that

Polaroids are transparent to light with polarization parallel to their preferred direction. They are opaque to light with polarization perpendicular to their preferred direction.

21.3.1 Changing the polarization direction

What if the light is polarized in a direction somewhere between parallel and perpendicular to the polaroid's preferred orientation? For concreteness, suppose the light's electric field is oriented along the x axis:

$$\vec{E}_{\text{incoming}} = E_0 \hat{x} \cos(kz - \omega t) \ .$$

Suppose the polaroid's orientation $\hat{p}$ is at some angle θ between the x and y axes:

$$\hat{p} = \hat{x} \cos\theta + \hat{y} \sin\theta \ .$$

Since the polaroid has a component of its orientation parallel to the light's electric field, the light will *partially* be able to go through the polaroid. The amount of electric field coming out of the polaroid will be given by the overlap between the incoming electric field and the polaroid's orientation:

$$|\vec{E}_{\text{outgoing}}| = \vec{E}_{\text{incoming}} \cdot \hat{p} = E_0 \cos\theta \cos(kz - \omega t) \ .$$

This outgoing light must be parallel to the polaroid's orientation — the polaroid defines the new polarization direction. Thus, the full solution for the outgoing light's $\vec{E}$ field is

$$\vec{E}_{\text{outgoing}} = \left(\vec{E}_{\text{incoming}} \cdot \hat{p}\right) \hat{p} \ .$$

A polaroid reduces the amplitude of linearly polarization light by $\cos\theta$, where θ is the angle between the polaroid's orientation and the light's electric field. It rotates the orientation of that field by θ.

21.3.2 Random light

In many situations, light is not linearly polarized — the light generated by a bulb, or coming from the surface of the sun, or from a fire, is best described as a superposition of many plane waves, each with their own polarization[1]:

$$\vec{E}_{\text{random}} = \sum_{j=1}^{\text{huge}} E_0 \left(\cos\theta_j \hat{x} + \sin\theta_j \hat{y}\right) \cos(kz - \omega t) \ .$$

We can make this light linearly polarized by passing in through a polaroid. For concreteness, imagine the polaroid is aligned along the x axis, so $\hat{p} = \hat{x}$. The light that comes out of the polaroid is no longer randomly oriented:

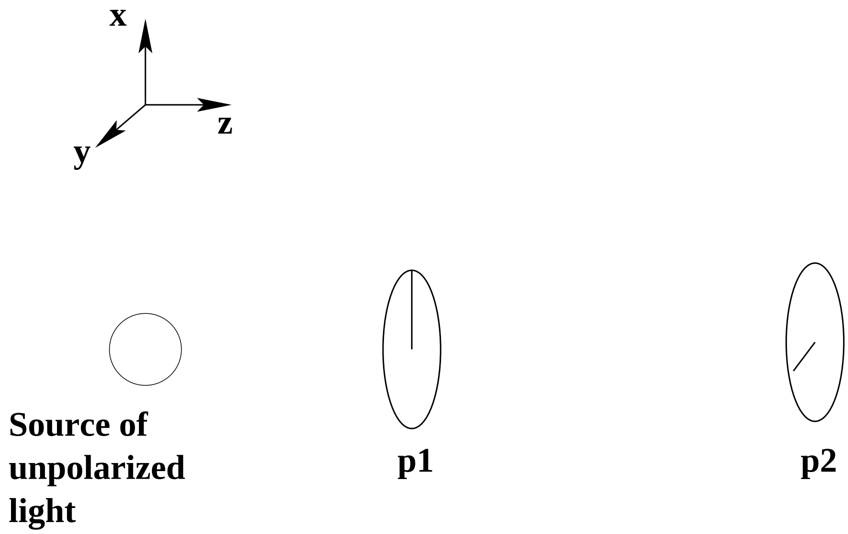
$$\vec{E}_{\text{outgoing}} = E_0 \hat{x} \cos(kz - \omega t) \sum_{j=1}^{\text{huge}} \cos\theta_j \ .$$

Polaroids can thus be used to *produce* linearly polarized light. As we'll discuss in the next lecture, one ends up reducing the intensity (brightness) when this is done; but, the light that comes out has $\vec{E}$ fields all lined up in the same way.

[1] The waves will also in general have an independent phase offset, so that their behavior with respect to time and space should be written as $\cos(kz - \omega t + \varphi_j)$. This complication, though often important, does not play a role in this discussion. An example where it *does* matter, quite a bit, is the laser. Lasers produce light in which all of these "offset phases" line up. This means that the light is *coherent*: the electric and magnetic fields are spatially and temporally in phase with each other.

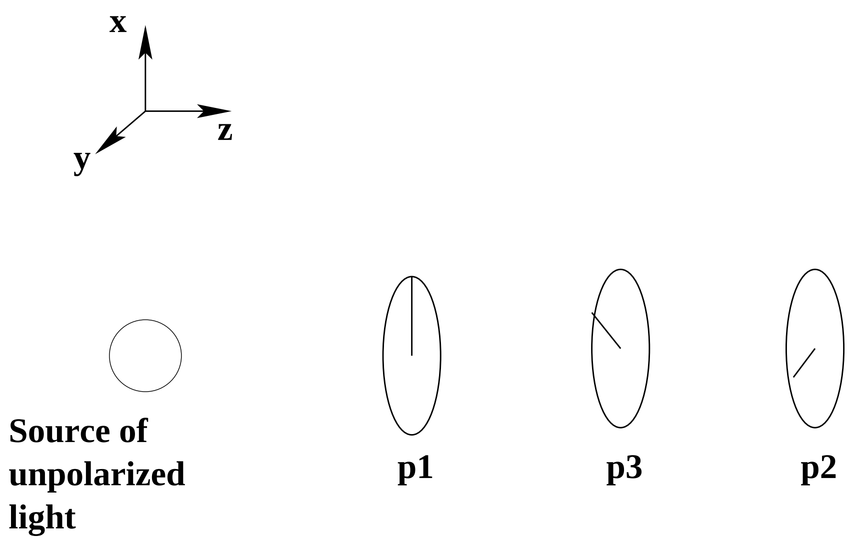
21.3.3 3 polaroids let more light through than 2

From this discussion, we can now easily understand how 3 polaroids let more light through than 2. Consider 2 first: we have one polaroid aligned with the x axis ($\hat{p}_1 = \hat{x}$), and then a second aligned with the y axis ($\hat{p}_2 = \hat{y}$).



After passing through p_1, the light will be polarized along the x axis. It hits p_2, which is aligned with $\hat{y}$. Since $\hat{x}$ and $\hat{y}$ are orthogonal, nothing gets through.

We now place a third polaroid p_3 between p_1 and p_2. We set p_3 to an orientation halfway between p_1 and p_2: $\hat{p}_3 = \cos\theta\hat{x} + \sin\theta\hat{y}$, with $\theta = \pi/4$.

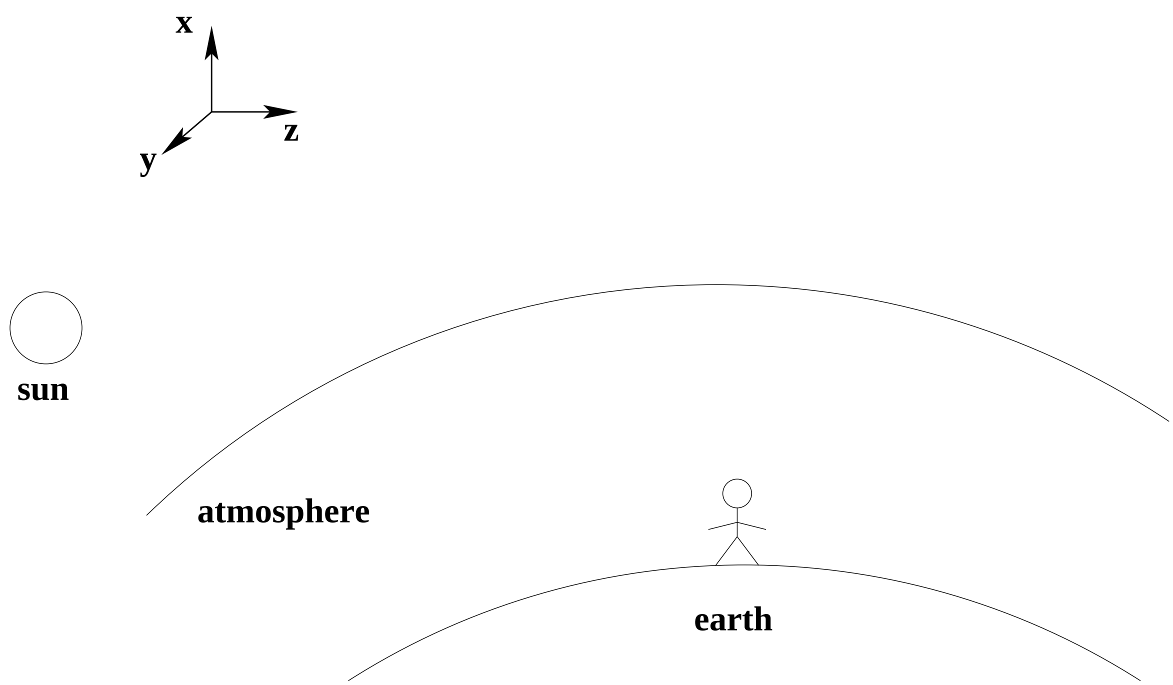


After passing through p_1, the light will be polarized along the x axis. It then hits p_3. The electric fields are reduced by a factor of $\cos\pi/4 = 1/\sqrt{2}$, and the field is then aligned with $\hat{p}_3$. It then hits p_2. The electric fields are *again* reduced by a factor of $\cos\pi/4 = 1/\sqrt{2}$. The fields are *reduced* (by quite a bit!) but they are *not* eliminated.

21.4 Scattering

Many materials *scatter* light: we send light into a medium, and the light is scattered into new directions. This is an extremely rich subject — very dense tomes have been written about the properties and physics of light scattering! Here, we'll content ourselves with a few mostly qualitative observations based on what we've learned so far.

First, consider unpolarized light passing through some medium. The source of unpolarized light could be, for example, the sun; the medium could be the earth's atmosphere.



The (unpolarized) sunlight is initially propagating in the z direction. We look up, in the x direction. What can we say about the light that we see when we look up?

First, we know that, since the light is initially propagating in the z direction, it can have *no* polarization along that axis. Second, since we then "measure" (with our eyes) light that is propagating along the x direction, it can have *no* polarization along that axis. Our conclusion is that **the light that comes down to us from the sky should be linearly polarized along** $\hat{y}$.

In truth, this isn't quite the case. Among other things, the light that we actually measure tends to scatter multiple times. This gives the polarization vectors additional chances to change direction, so we are likely to end up with a mixture rather than a perfectly linear state. But, this argument *does* suggest a general tendency: the light that scatters out of the sky will *tend* to have a preferred polarization axis.

Suppose the light from the sun were somehow made to be linearly polarized. Then, the scattered light will be most intense in the direction *orthogonal* to the polarization direction. If we put a giant polaroid in front of the sun, we could make the sky vary from dark to bright by rotating the polaroid! It would be bright when the polarization axis is orthogonal to "up"; it would be dark when the axis is parallel to "up".

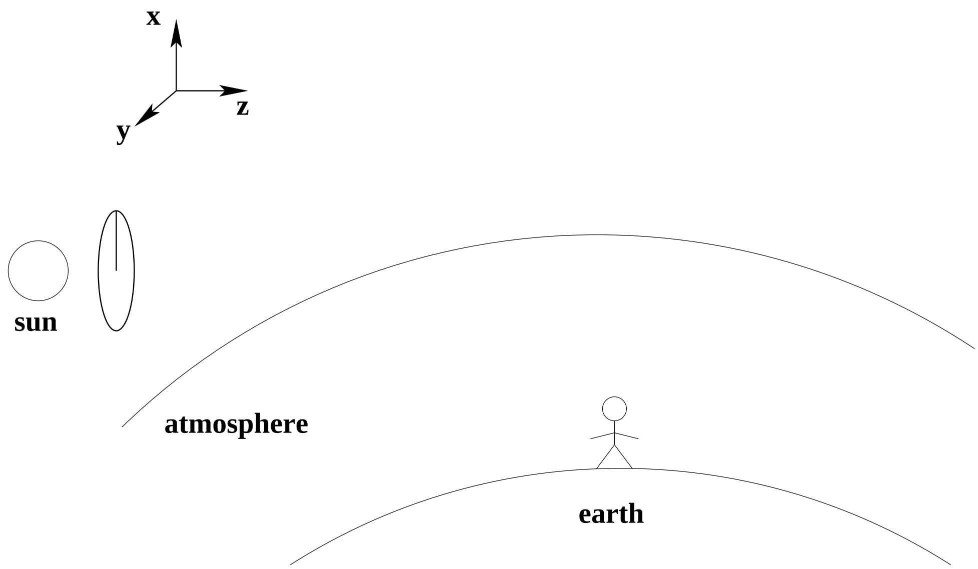


Figure 1: Dark above my head.

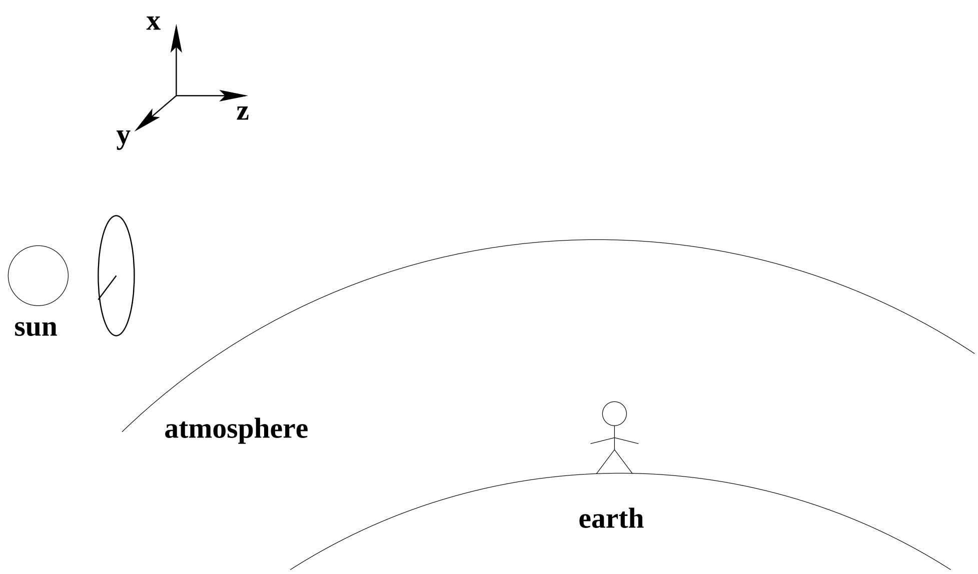


Figure 2: Bright above my head.

Something that has been missing so far is some discussion as to the *likelihood* that light will be scattered. The scattering process works roughly as follows: Light impinges on a molecule. Its electric field shakes that molecule's charges charges at the input frequency ω. The molecule then re-radiates this input light (often changing its direction, subject to the polarization constraints discussed above).

The electric field of the scattered light is determined by the *acceleration* of the charges in the scattering molecule — as we discussed back when we looked at special relativity, radiation comes from accelerating charges. This means that

$$E_{\text{scattered}} \propto \frac{\partial^2 d}{\partial t^2} \propto \omega^2$$

where d is the dipole moment of the shaken molecule. This moment will look something like $\cos\omega t$, which is how we get the ω^2 proportionality for the scattered electric field.

As we will discuss in much greater detail in our next lecture, the *intensity* or *brightness* of electromagnetic radiation is determined by the electric field *squared*:

$$I \propto E^2_{\rm scattered} \propto \omega^4 \propto \lambda^{-4} \ .$$

The higher the radiation's frequency — or the smaller its wavelength — the higher the probability that it will be scattered.

Red light has a wavelength of about 700 nanometers. Blue light has a wavelength of about 350 nanometers. This means that blue light is scattered about $(700/350)^4 = 16$ times more effectively than red light!

This is why the sky is blue during the day: when we look away from the sun, the light we see is light that has scattered out of the initial propagation direction. The input light from the sun includes *all* wavelengths, but only the short wavelength stuff has a very high probability of scattering.

This is also why the sunset is red. At sunset, you are looking right at the sun — along the initial propagation direction — and you are looking through a large thickness of atmosphere. Most of the short wavelength stuff is already scattered out; only the long wavelength stuff remains.

21.5 Circular polarization

There is another polarization state that is encountered a lot in nature. Consider a wave that has the following form:

$$\begin{aligned} \vec{E} &= E_0\hat{x}\sin(kz-\omega t) + E_0\hat{y}\cos(kz-\omega t) \\ \vec{B} &= E_0\hat{y}\sin(kz-\omega t) - E_0\hat{x}\cos(kz-\omega t) \ . \end{aligned}$$

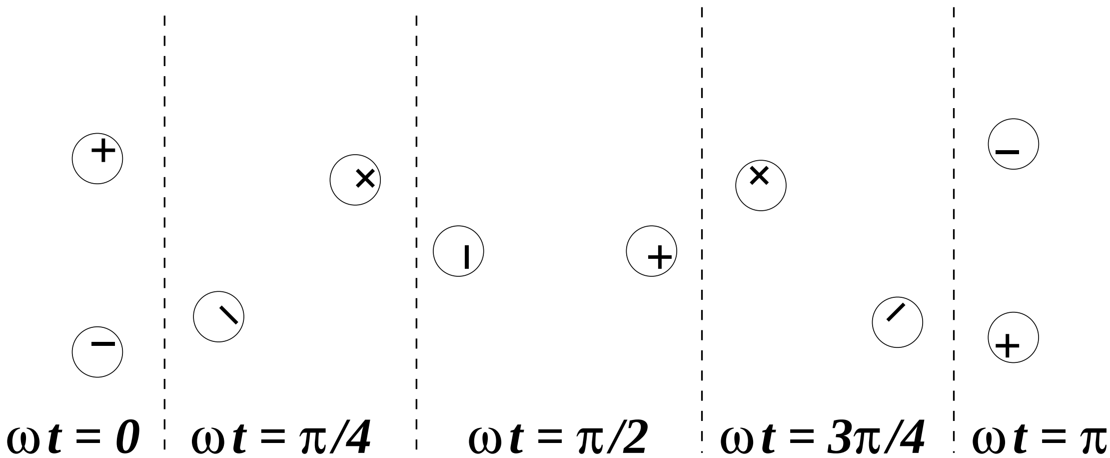
You should be able to show pretty easily that this satisfies all of our requirements for a plane wave. But what exactly is going on with this screwy form???

We can get some intuition for considering what this wave looks like at a fixed position. Suppose we focus on $z = 0$, so

$$\begin{aligned} \vec{E} &= -E_0\hat{x}\sin(\omega t) + E_0\hat{y}\cos(\omega t) \\ \vec{B} &= -E_0\hat{y}\sin(\omega t) - E_0\hat{x}\cos(\omega t) \ . \end{aligned}$$

The electric and magnetic fields *rotate* at the frequency ω! This is a state called *circular polarization*: the tip of the electric (and magnetic) field vector traces out a circle.

Circular polarization can often be particularly important because nature likes to produce it. Recall how we made linearly polarized radiation: we shook a charge configuration back and forth, with the charges oscillating along a simple line. It shouldn't surprise you to know that circular polarization is produced by a rotating charge configuration:



This rotating charge distribution would produce a circularly polarized wave that propagates out of the page.

The general polarization state is more usually somewhere between linear and circular polarization. Not surprisingly, this is called *elliptical* polarization. It is produced from rotating charge configurations that are viewed at some oblique angle. For example, if we looked at the configuration sketched above along its edge, we would get linearly polarized light. If we looked at it face on, we would get circular polarization. Elliptical polarization is what we get in the general case; it is intermediate to circular and linear polarization.

Finally, it's worth knowing that a linearly polarized state can be thought as a superposition of *two* circularly polarized states, but with their vectors propagating in opposite senses. Mathematically, this is so obvious that it may seem stupid. Let $\vec{E}_{\rm lin}$ denote a linearly polarized state; let $\vec{E}_{\rm CW}$ denote a state in which (at fixed location) the electric field is clockwise circular polarized; and let $\vec{E}_{\rm CCW}$ denote a state in which the electric field is counterclockwise circular polarized:

$$\begin{aligned}
\vec{E}_{\rm CW} &= -E_0\hat{x}\sin(kz-\omega t) + E_0\hat{y}\cos(kz-\omega t)\\
\vec{E}_{\rm CCW} &= E_0\hat{x}\sin(kz-\omega t) + E_0\hat{y}\cos(kz-\omega t)\\
\vec{E}_{\rm lin} &= \vec{E}_{\rm CW} + \vec{E}_{\rm CCW} = 2E_0\hat{y}\cos(kz-\omega t)\ .
\end{aligned}$$

This is so obvious you may wonder: why even bother belaboring the point? Well, the reason is that certain media interact with circularly polarized light in *very* interesting ways! For example, many organic molecules have a "chirality", meaning that they twist in a particular way. This turns out to mean that they interact with clockwise circular polarization *differently* than they interact with counterclockwise circular polarization!

If we were to shoot linearly polarized light into a medium containing such molecules, it would interact with the clockwise component of the linear wave differently than it would interact with the counterclockwise component. In other words, the medium would separate the linear polarization into its two circular components. We might expect any number of funky things to happen in this case.