Massachusetts Institute of Technology
Department of Physics
8.033 Fall 2024

Lecture 2
Electromagnetic radiation and Galilean relativity

2.1 An aside ... why not accelerated reference frames?

Our discussion of Galilean relativity described in some detail how to relate quantities measured in one IRF to quantities measured in another IRF. However, one could imagine frames whose relative motion is accelerated. Could we not broaden our discussion to include such *non-inertial* relative motions?

Certainly we could include accelerations between reference frames. Doing so requires that we introduce *non-inertial forces* in order for Newton's laws, in particular $\mathbf{F} = m\mathbf{a}$, to work. Our main reason for not doing this is simplicity — including accelerations makes the analysis more cumbersome, and is a diversion from the main thrust of our discussion. It is, however, a well-developed topic, and interested students can certainly find discussions of this in many excellent textbooks.

What if the different frames are accelerated, but the frames experience *the same* acceleration? In such a case, one could imagine defining a transformation that takes us from one frame accelerating with $\mathbf{a}$ to another frame that is also accelerating with $\mathbf{a}$. Indeed, in this circumstance it is not hard to see that the Galilean transformations we discussed in the previous lecture work perfectly, translating quantities from one accelerating frame to the other. Given this, one might wonder: if everything experiences the same acceleration, does that acceleration mean anything interesting? Given that all geometric objects in all the frames that we consider experience the same acceleration, perhaps we could just define this as a somewhat peculiar notion of "rest."

This question in fact gets at the heart of the issues and concepts which lie at the core of Einstein's *general* relativity, hinting at the principle of equivalence. We will return to a very similar discussion in several weeks in a more Einsteinian context.

2.2 Galileo meets Maxwell

In our previous discussion, we noted that wave equations have an interesting property: the physics of the wave introduces a special speed, which we labeled w. This describes the speed with which the wave propagates with respect to the medium that supports the wave. An observer moving with respect to the medium will observe the wave propagating with a different speed, in accordance with how velocities add in Newtonian mechanics.

This made perfect sense until roughly the late 1800s. To see what started confusing the situation, consider Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0 \;, \qquad \nabla \cdot \mathbf{B} = 0 \;, \tag{2.1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \;, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \;. \tag{2.2}$$

Let us consider the vacuum limit ($\rho = 0$, $\mathbf{J} = 0$), and let us take the curl of the curl equations. Using some vector calculus identities, this is straightforward. Look at the curl of the left-hand sides of the curl equations first: using vector calculus identities, it is straightforward to show that

$$\begin{aligned}\nabla \times (\nabla \times \mathbf{E}) &= \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \, , \\ \nabla \times (\nabla \times \mathbf{B}) &= \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} \, . \end{aligned} \tag{2.3}$$

Look next at the right-hand sides:

$$\nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \, , \tag{2.4}$$

$$\nabla \times \left(\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \, . \tag{2.5}$$

Putting the right-hand and left-hand sides together, using $\nabla \cdot \mathbf{B} = 0$ and using $\nabla \cdot \mathbf{E} = 0$ when $\rho = 0$, we see that $\mathbf{E}$ and $\mathbf{B}$ each obey wave equations:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \nabla^2 \mathbf{E} = 0 \, , \tag{2.6}$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \nabla^2 \mathbf{B} = 0 \, . \tag{2.7}$$

Further, we see that the parameter w which characterizes the speed of the wave is given by $1/\sqrt{\mu_0 \epsilon_0}$. This speed is given the label c (which comes from the word *celeritas*, meaning swiftness), and takes the value

$$\begin{aligned} c &= 2.99792458 \times 10^8 \, \text{meters/second} \\ &\simeq 3 \times 10^8 \, \text{meters/second} \\ &\simeq 1 \, \text{foot/nanosecond} \, . \end{aligned} \tag{2.8}$$

The equality on the first line is *exact*. A we'll discuss briefly a bit later in the course, we now actually use this value to define the meter. The near equality on the second line is good enough for most of the calculations we do in this class. The final near equality is amusing for those of us educated in parts of the world that still use inches and feet as their common measurement unit, and can be surprisingly useful in a number of practical situations.

If we imagine that $\mathbf{E}$ and $\mathbf{B}$ only depend on t and x, then the wave equations reduce to

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial x^2} = 0 \, , \tag{2.9}$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{B}}{\partial x^2} = 0 \, , \tag{2.10}$$

which have solutions of the form $\mathbf{E}(x \pm ct)$, $\mathbf{B}(x \pm ct)$.

When an analysis of this form was first done in the late 19th century, it was regarded as something of a triumph. In particular, the fact that the equations predicted $c = 1/\sqrt{\mu_0 \epsilon_0}$ was somewhat stunning. Bear in mind that ϵ_0 was an empirically measured parameter that played a role in determining the capacitance of a conductive system; μ_0 was a similar parameter that played a role in determining a system's inductance. The fact that parameters that were determined from static or very slowly varying fields could be so intimately related to the speed of light (whose value had been known to fairly good accuracy for quite some time, and was certainly known to be incredibly fast) was regarded as amazing. This association cemented the connection between light and electromagnetic fields.

Like most wave equation analyses, this calculation picked out a special speed. But, in what frame did we do this analysis?

2.3 The Michelson-Morley experiment

The consensus of the late 19th century was that electromagnetic waves are a disturbance in the so-called "luminiferous ether" (aka the "ether"), and that $c = 1/\sqrt{\mu_0 \epsilon_0}$ is the speed of propagation with respect to that ether. In this framework, the ether defines a preferred rest frame, and the speed of light should only be c if we are in that preferred rest frame.

This line of reasoning tells us that if we measure light propagating across a laboratory that moves relative to the ether, then we should find that it moves with a speed that is not c. Our labs are on the surface of the Earth; the Earth spins on its axis, and orbits the Sun. Even if our lab is at rest with respect to the ether at some moment, it will no longer be at rest later in the day, or later in the year.

Albert Michelson and Edward Morley carried out an ingenious experiment in 1887 to test this hypothesis. Their idea was to use the wave nature of light to build an *interferometer*. The basic experimental setup is sketched in Fig. 1.

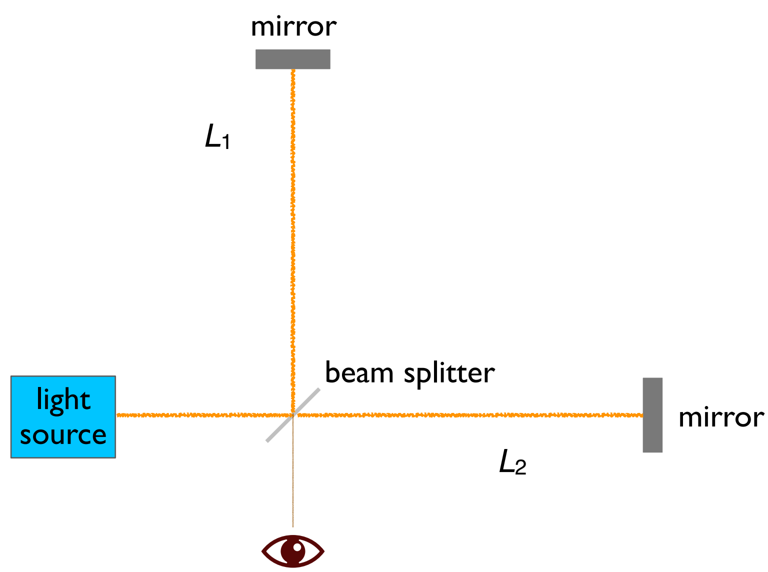


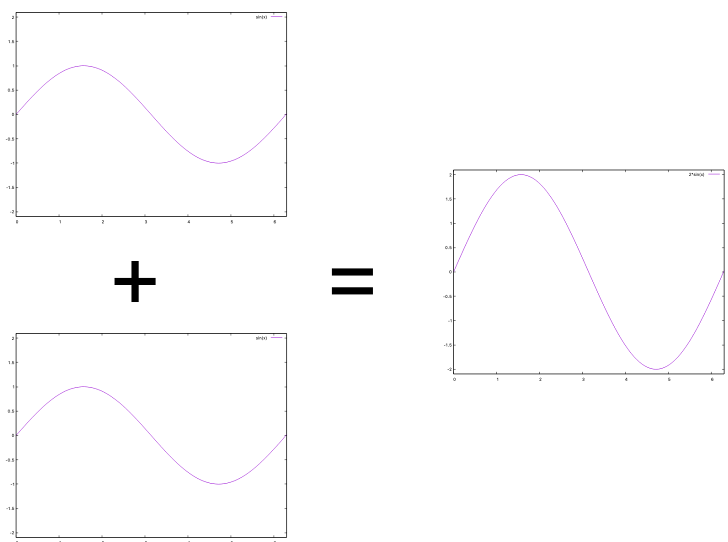
Figure 1: Basic layout of the interferometer used in the Michelson-Morley experiment. A beam of light enters from the source at left and is split by the beam splitter (a piece of partially silvered glass which reflects half of the light up, and allows half to transmit to the right). Both of these beams are reflected at the mirrors at the ends of the arms, return to the beam splitter, and then recombine. Exactly what happens when they recombine depends on the *optical phase difference* they experience along their two travel paths.

What happens when the light returns to the beam splitter? The answer depends on the details of the paths that the light takes in the two arms. To analyze this, let's make some definitions:

- Let t_1 be the travel time for light to go from beam splitter to mirror to beam splitter in arm 1 (of length L_1)
- Let t_2 be the travel time in arm 2
- Define $\Delta t \equiv t_2 - t_1$.

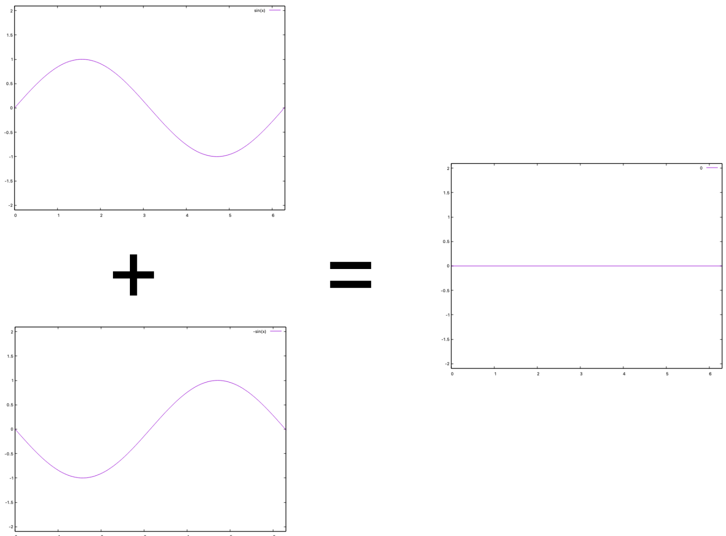
The quantity $c\,\Delta t$ is known as the *optical path difference*; it measures the difference in distance traversed by light as it back and forth through the two arms. Dividing this by λ, the wavelength of the light, yields[1] the *optical phase difference*.

Because light is a wave, the optical phase difference is an extremely important quantity for understanding what happens when light recombines at the beam splitter. If $c\,\Delta t/\lambda = 0, \pm 1, \pm 2, \ldots$, then the light *constructively* interferes: Peaks and troughs in the wave from one arm line up with peaks and troughs in the wave from the other:



[1] Strictly speaking, the optical phase difference is 2π times this quantity.

On the other hand, if $c\,\Delta t/\lambda = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$ then the light *destructively* interferes: Peaks in the wave from one arm line up with troughs in the other:



In general, we expect $c\,\Delta t/\lambda$ to be some value between an integer multiple of 1 and an (odd) integer multiple of $\frac{1}{2}$; the recombined amplitude will be some value between the peak of perfectly constructive interference, and the zero of perfectly destructive interference. In addition, Michelson and Morley used white light as their light source. This means that their measurement using a wide range of wavelengths. As such, we expect the light read out of the beam splitter (where the eye is placed in Fig. 1) to show an *interference fringe pattern*, with constructive interference for some wavelengths, destructive interference for others, and many values in between.

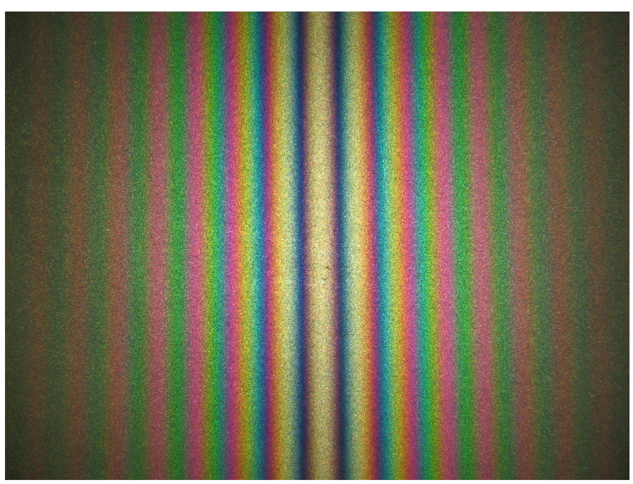


Figure 2: Example of a fringe pattern from readout of a Michelson-type interferometer. Image credit:
`https://commons.wikimedia.org/wiki/File:MichelsonCoinAirLumiereBlanche.JPG`

With this background in mind, let us compute what optical phase difference we expect if c is the speed of light with respect to the ether, and if the Michelson-Morley apparatus moves with speed v with respect to the ether. More specifically, let's imagine that the lab's velocity $\mathbf{v}$ is parallel to arm 1. The time it takes for light to travel up arm 1 and then back to the beamsplitter is

$$t_1 = \frac{L_1}{c-v} + \frac{L_1}{c+v} = \frac{2L_1}{c}\left(\frac{1}{1-v^2/c^2}\right) . \tag{2.11}$$

The asymmetry between the two terms is because of the asymmetry in the light's motion relative to the apparatus along the two legs: in the first term, the mirror is "running away" from the light, so relative to the apparatus the light's speed is $c - v$; in the second term, the light switches direction, and the beam splitter is now "running toward" the light, with a relative speed $c + v$.

Arm 2 is a bit more complicated to analyze, as the light is in this case moving perpendicular to the motion of the apparatus in the rest frame of the ether. Figure 3 lays out the geometry:

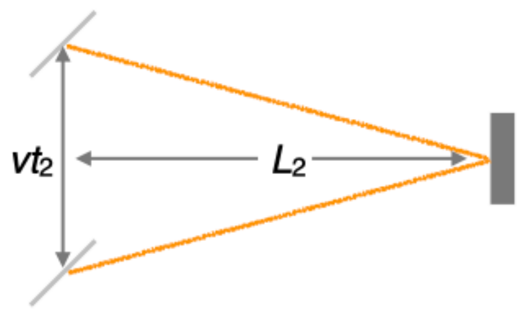


Figure 3: Light travel in arm 2 of the Michelson-Morley apparatus, as viewed in the rest frame of the ether. The light starts at the lower left, travels to the mirror, bounces, and returns to the upper left. During that time, the beam splitter moves from the position in the lower left to the position in the upper left. The light takes a total time t_2 to travel from the beam splitter to the mirror and back to the beam splitter. In that time, it covers a horizontal displacement of L_2 twice, and moves through a vertical displacement vt_2.

We defined t_2 as the time it takes for light to travel from the beam splitter to the mirror and back. As shown in the figure, in the rest frame of the ether the light moves on a diagonal path with horizontal displacement L_2 twice, and with vertical displacement vt_2. The equation governing t_2 is thus given by

$$ct_2 = 2\sqrt{L_2^2 + \left(\frac{vt_2}{2}\right)^2} , \tag{2.12}$$

from which we find

$$t_2 = \frac{2L_2}{c}\frac{1}{\sqrt{1-v^2/c^2}} . \tag{2.13}$$

Combining this with our result for t_1 yields

$$\Delta t = \frac{2}{c}\left[\frac{L_2}{\sqrt{1-v^2/c^2}} - \frac{L_1}{1-v^2/c^2}\right] . \tag{2.14}$$

At this point it is a useful to examine some numbers, in particular what we expect for v/c. The speed of the lab with respect to the ether is roughly bounded by the orbital speed of the Earth about the Sun, so $v \lesssim 2 \times 10^4$ meters/second. The speed of light is 3×10^8 meters/second, so

$$\frac{v}{c} \lesssim 10^{-4} \,. \tag{2.15}$$

This is a small quantity; the expression we've derived for Δt depends on this ratio squared, and even smaller quantity. Examining how this very small quantity enters our analysis, we see it is appropriate to use the binomial expansion, $(1+\alpha x)^n \simeq 1 + n\alpha x$ for $x \ll 1$, to simplify what we have:

$$\Delta t \simeq \frac{2}{c}\left[L_2\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) - L_1\left(1+\frac{v^2}{c^2}\right)\right] \,. \tag{2.16}$$

Michelson and Morley introduced one more very important factor into their experiment: They made it possible to rotate the interferometer's arms, effectively exchanging arms 1 and 2. (They did this by floating their entire optical table, which was built on a very heavy block of sandstone, on a pool of mercury. This both allowed the apparatus to rotate with very little friction, and provided significant isolation from vibrations in the building in which they did the experiment.) Rotating the apparatus, we get new light travel times:

$$t'_1 = \frac{2L_1}{c}\frac{1}{\sqrt{1-v^2/c^2}} \simeq \frac{2L_1}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) \,, \tag{2.17}$$

$$t'_2 = \frac{2L_2}{c}\frac{1}{1-v^2/c^2} \simeq \frac{2L_2}{c}\left(1+\frac{v^2}{c^2}\right) \,, \tag{2.18}$$

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left[L_2\left(1+\frac{v^2}{c^2}\right) - L_1\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\right] \,. \tag{2.19}$$

Let us define δt as the change in the difference of light travel times between the two configurations:

$$\begin{aligned} \delta t &\equiv \Delta t' - \Delta t \\ &= \frac{2}{c}\left[L_2 + \frac{L_2 v^2}{c^2} - L_1 - \frac{L_1 v^2}{2c^2} - L_2 - \frac{L_2 v^2}{2c^2} + L_1 + \frac{L_1 v^2}{c^2}\right] \\ &= \frac{L_1+L_2}{c}\left(\frac{v^2}{c^2}\right) \,. \end{aligned} \tag{2.20}$$

Their ingenious trick of rotating the interferometer means that the quantity they measured only depends on the sum $L_1 + L_2$, rather then depending sensitively on the individual arm lengths L_1 and L_2.

With all this laid out, let's review how this experiment works:

1. We begin with the experiment in a particular configuration. The experimenter monitors light that recombines at the beam splitter (indicated by the eyeball in Fig. 1), seeing a fringe pattern much like that shown in Fig. 2.

2. If there exists an ether and the laboratory is moving with respect to this ether, then light traveling in the two arms experiences the optical path difference $c\,\Delta t$. The initial fringe pattern the experimenter measures corresponds to the optical path difference associated with this initial configuration.

3. The entire interferometer is rotated by 90°. The experimenter (rotating along with it!) monitors the fringe pattern during the rotation. The expectation is that the optical path difference will change by $c\,\delta t$ during this rotation. This will be visible to the experimenter by a shifting of the fringe pattern as the apparatus is rotated.

One of the beautiful features of an interferometry experiment is that a shift of fringe can be measured very precisely. Carefully calibrating the positions of the mirrors, Michelson and Morley were confident that they could measure an optical phase shift $c\,\delta t/\lambda \approx 0.01$. This would have been plenty to detect the effect of motion with respect to the ether, as can be seen by plugging in some numbers for the experiment:

- Size of the apparatus: $L_1 + L_2 \simeq 10$ meters
- Speed with respect to the ether: $(v/c)^2 \simeq 10^{-8}$
- Wavelength of light: $\lambda \simeq 500\,\mathrm{nm} = 5 \times 10^{-7}$ meters

The expected optical phase shift due to motion with respect to the ether is thus

$$\frac{c\,\delta t}{\lambda} \simeq 0.2\ . \tag{2.21}$$

This is a factor of 20 larger than what Michelson and Morley could discern, which is *huge*.

The value they in fact measured was zero. Since their pioneering experiment in 1887, measurements of this kind have been repeated. Measurement technology has improved to the point that we can now measure $c\,\delta t/\lambda \simeq 10^{-10}$. **No motion of an apparatus relative to an ether has ever been detected.**

2.4 Explanations

In 1887, the Michelson-Morley null result was a surprise. Both Michelson and Morley in fact considered it to be "failed experiment," and moved on to other things. However, it became clear over the years that the measurement had been done correctly, and the lack of phase shift was not experimenter error. This result begged explanation. Over time, four possible explanations emerged:

1. The ether is dragged along by the Earth, somehow, so that our labs are always locally at rest with it.

   This hypothesis in fact was the consensus view of how things would work at the time of the Michelson-Morley measurement. Part of what was so confusing about their result was that it contradicted other experiments at the time which preferred the ether to be "partially" dragged by the Earth; Michelson and Morley's result implied that any ether must be completely dragged along, so that the lab is always at rest with respect to the ether. Folding in more modern measurements, the ether drag hypothesis does not hold up when we make measurements on very long baselines (e.g., into space) where the effect of Earth's ether dragging should be greatly reduced or negligible.

2. Maxwell's equations are wrong.

   This hypothesis simply does not work: no wrongness has ever been found which can explain the Michelson-Morley measurements. Electrodynamic effects can be measured with exquisite precision, and Maxwell's equations work tremendously well.

3. The ether squashes moving objects just enough to compensate for the travel time shifts.

   This explanation "works" in the sense that one can design a squashing that fits the data, but raises a new question — how and why do such "length squashings" occur?

4. *There is no ether; there is no special rest frame for Maxwell's equations.* **Light travels at $c = 1/\sqrt{\mu_0 \epsilon_0}$ in ALL inertial reference frames.**

The 4th option is where Einstein chose to begin his analysis. After all, no such frame is called out when Maxwell's equations are written down, so on what grounds should we imagine that this frame exists? This is where we will focus our studies.

2.5 Historical note

It should be noted that the historical record is somewhat unclear regarding the extent to which Einstein was influenced by Michelson and Morley. Some of his statements and writings suggest he was not influenced by their result, though other statements indicate that he was aware of the result and that it had some influence. It is clear, though, that he was aware of similar experiments (particularly those of Fizeau, whose experiment you will explore on problem set #1). It is fair to say that Einstein was aware the ether hypothesis was having trouble finding experimental support.

Einstein's historical motivations aside, with the benefit of over 130 years of hindsight, the importance of Michelson and Morley (and of similar experiments done since then) is clear to *us*: these measurements clearly demonstrate that the simple picture of Maxwell's equations being formulated in the "rest frame of the ether" (whatever that ether might actually be) cannot be correct. Einstein's choice of option #4 on the list above appears to be driven largely by simplicity: there is no ether and no special rest frame referred to anywhere in our formulation of the Maxwell equations, so why would we introduce them? Why not take at face value the fact that c emerges as the speed of light with no reference to a particular rest frame, and see what that implies?

Seeing what this implies will be our focus for the next several weeks.