Recap of last lecture:

Curvilinear coordinate basis:

$$\partial_\alpha \vec{e}_\beta = \Gamma^\mu{}_{\alpha\beta}\, \vec{e}_\mu$$

$$\Gamma^\mu{}_{\alpha\beta} = g^{\mu\gamma}\, \Gamma_{\gamma\alpha\beta}$$

$$\Gamma_{\gamma\alpha\beta} = \frac{1}{2}\left(\partial_\alpha g_{\beta\gamma} + \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}\right)$$

Γs are symmetric on last two indices in a coordinate basis:

$$\Gamma^\mu{}_{\alpha\beta} = \Gamma^\mu{}_{\beta\alpha} = \Gamma^\mu{}_{(\alpha\beta)}$$

$$\Gamma^\mu{}_{(\alpha\beta)} = \frac{1}{2}\left(\Gamma^\mu{}_{\alpha\beta} + \Gamma^\mu{}_{\beta\alpha}\right)$$

$$\Gamma^\mu{}_{[\alpha\beta]} = \frac{1}{2}\left(\Gamma^\mu{}_{\alpha\beta} - \Gamma^\mu{}_{\beta\alpha}\right) = 0.$$

Covariant derivative:

$$\nabla_\alpha T^\beta{}_\gamma = \partial_\alpha T^\beta{}_\gamma + \Gamma^\beta{}_{\alpha\mu} T^\mu{}_\gamma - \Gamma^\mu{}_{\alpha\gamma} T^\beta{}_\mu$$

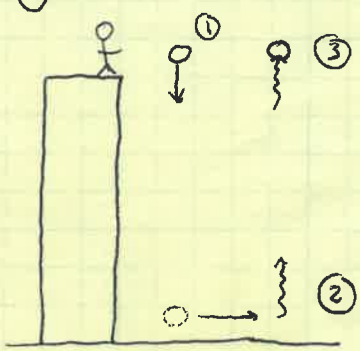
Components of a tensor

Not.

So far, it's all S.R. A good way of describing S.R.: It is the theory which allows us to cover the <u>entire</u> spacetime manifold using <u>inertial</u> <u>reference</u> <u>frames</u>.

<u>Gravity breaks this</u>. As soon as we introduce gravity, <u>global</u> inertial frames cannot work. Two part physical demonstration:

1. Gravitational redshift:



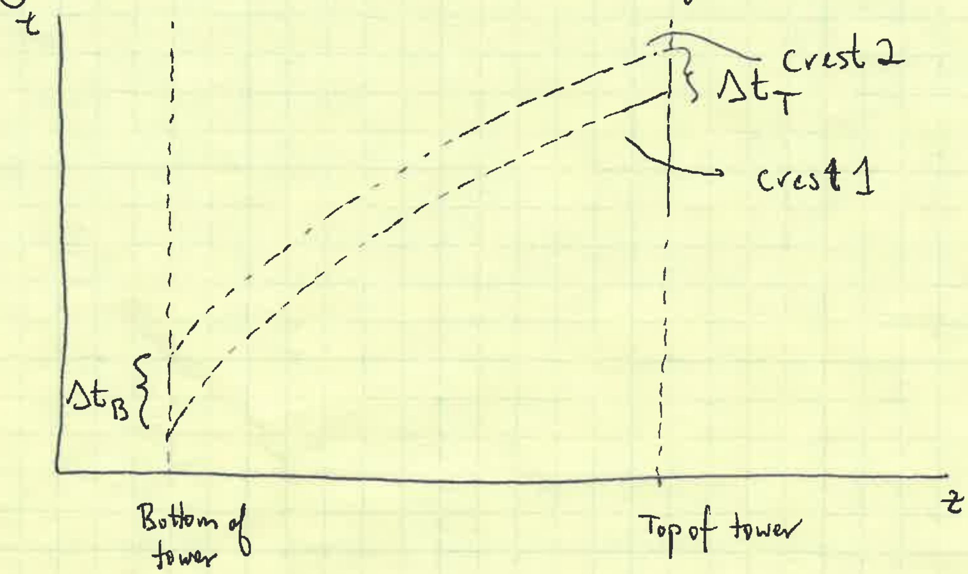
1. Drop rock of rest mass m off tower.
2. At bottom, photonulator converts ball into a single photon: $E_B = m(1+gh) = \hbar\omega_B$
3. At top, re-rockulator converts photon $\hbar\omega_T$ into rock with energy $E_T = \hbar\omega_T$.

What is E_T? We must have $E_T = m$ – otherwise, we could build a perpetual motion machine!

$$\frac{E_T^*}{E_B} = \frac{m}{m(1+gh)} = \frac{\omega_T^*}{\omega_B} \longrightarrow \boxed{\omega_T = \left(1 - \frac{gh}{c^2}\right)\omega_B}$$

Experimentally confirmed: Pound-Rebka; GPS.

2. Now, suppose the region near the earth's surface can be covered by a Lorentz frame. Consider worldlines followed by two successive crests of a light wave. We don't "know" what gravity will do yet, but we can imagine it introduces some kind of a bend:



If there is a global Lorentz frame, then no position or moment is special - manifold is translation invariant (time & space). These trajectories must be congruent.

This means $\Delta t_B = \Delta t_T$.

However, $\Delta t = \frac{2\pi}{\omega}$. Since $\omega_T < \omega_B$, we have a contradiction!

Assumption of a global Lorentz frame dies when we have gravity!

(argument: Alfred Schild)

There could be LOCALLY Lorentz frames!

Next lecture: Will show that freely falling frames are as close as we can get to an inertial frame.

Why? Inertial means nothing accelerates: No forces act. In a freely falling frame, all objects feel the same "acceleration". (Relative to what??) Hence, within the frame, there are no RELATIVE accelerations: in the absense of non-grav. forces, objects maintain their relative velocity.

[This is such a convenience that we will find it useful to regard the F.F.F. as fundamentally NOT accelerated. Ie, define "acceleration" always relative to free fall!!]

Tides break down this notion. Consider an elevator freely falling in earth's gravity. Acceleration at top & bottom are not quite the same.

Tells us that initially "parallel" trajectories (in a spacetime sense) do not remain parallel. Tides cause breakdown of parallelism. Violation of Euclid's parallelism axiom means we have curvature.

Tides also thus set limit on the size of the "Local" Lorentz frame.

Introduced here the <u>equivalence principle</u>:

Gravity cannot be distinguished from a uniform acceleration.

Result of the equivalence of inertial & gravitational mass.

More precisely, this is the <u>WEAK</u> E.P.:

The motion of freely falling particles in gravity cannot be distinguished from uniform acceleration.

A stronger version of the E.P.:

In small enough regions of spacetime, the laws of physics reduce to those of special relativity.

Carroll: Einstein E.P.

Schutz: Strong E.P. ← Deprecated - S.E.P. has a different meaning now.